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The Impact of Pictorial Representations in Teaching Math Word Problems to a Child with Autism

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

The study used a single subject A-B-A research design because it used one subject, an 18-year old boy participant who was diagnosed with having higher spectrum Autism (Asperger). The investigator examined the effect of pictorial representations of math word problems on the participant's performance in solving one step, two steps, and three steps math word problems. The investigator found that the use pictorial representations improved the participant's math achievement in solving math word problems. The investigator also found that the participant had difficulty understanding words with mathematical connotations.

Keywords: Autism; asperger; mathematics; special education; learning; divisions; fractions.

1. INTRODUCTION

Understanding how to solve math word problems is important; however, research has shown that many students lack this ability [1]. The literature review of this study has shown there are few studies that have investigated the effect of using visual representations by students with learning disabilities when solving mathematical word problems [2]. Researchers also have found that

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visual representations were positively related to children achievement in math problems [3]. Moreover, studies have shown that typical developing students tend to outperform students with disabilities in learning mathematics. One explanation for this performance difference is that typical developing children used pictorial representation of word problems more often than children with learning disabilities [4]. The purpose of this study is to examine effects of pictorial representations on performance of children with Autism in understanding math word problems.

1.1 Children with Autism

Autism is a spectrum disorder, meaning it includes children with a wide range of deficiencies. A study has discovered a wide range of deficiencies, and every child with autism becomes unique with respect to learning math concepts; therefore, these children have different learning capabilities [5]. At the lower end of the spectrum, there are children with severe impairment, while at the higher end there are children with a lower degree of impairment. The higher end is called children with Asperger, and some researchers call it higher-cognitive functioning (HCF) autism. A study found 28% of children with Autism spectrum have an average IQ, while 3% of these children have an above average IQ [6]. This seems to contradict with previous studies that have found a larger percentage of children with Asperger have an IQ above average intelligent [7]. The major impairment of autistic children is their inability to socialize with other students and to read non-verbal cues, and to have empathy toward other people's actions. Therefore, these impairments could affect these children's learning abilities.

1.2 Brain of Children with Autism

Research has also shown children with autism have less connection activities in the frontal and parietal areas. This could be the cause of impairment of information processing or executive functioning in people. Children with impairment in executive functioning have weaker understandings of applied mathematical problems and comprehending abstract concepts [8]. Moreover, a research study has shown parietal lobe controls visuo-spatial and frontal lope controls self-regulation, planning, and execution [9]. There are a sufficient numbers of fiber connections in the frontal part of brain, but insufficient numbers of these fibers connecting to parietal regions of the brain. Furthermore, a researcher study has found that children with autism should start taking algebra before the adolescence period because after this stage of development the brains of these children start to have synaptic pruning and myelination which affect lower network connections [10].

1.3 Mathematics Learning

The National Council of Teachers of Mathematics (NCTM) [11] has stated all children, including those with disabilities, should be included in an effective learning environment, and teachers should set high expectations for all children. Moreover, the NCTM's standards emphasize the development of mathematical thinking, which includes higher-level thinking and reasoning with emphasis on problem-solving skills relating to the real world. However, a study has found that schools should set higher expectations for these children [12] and include teaching strategies that meet the needs of these students [13]. It is imperative that teachers understand how to teach these children, especially in a subject such as mathematics, which is highly essential in their daily lives. This investigator has mentioned that children with autism have difficulty solving word problems for two reasons: (1) They have lower cognitive ability which slows the process of working on several concepts simultaneously (2) These children have lower reading comprehension ability and a lower attention span which could hinder their short term memory to process analyzing mathematical concepts effectively [12]. Researchers found that when given items such as sports or games to sort; children used concrete-based criteria while those without Autism classified the items based on both concrete and abstract criteria. A concrete criterion is based on colors or shapes while an abstract criterion is based on price or its functionality [14].

A research study has mentioned several teaching strategies in mathematics for these children. The first strategy includes reducing cognitive overload lessons by teaching through conceptual understanding, which is about how's and why's [15]. Cognitive strategy provides deep understanding and helps students to better organize information in solving complicated problems. Therefore, it will compensate for these children's deficits in their executive functioning which weaken students' problem solving or word problem abilities. The second strategy is rehearsal, which is a memory strategy [16]. This study found these children have good memory;

however, they lack strategies to recall facts. The third strategy is the use of teaching through concrete activities rather than abstract ones. These children have impaired visuo-spatial abilities which could affect their mathematical concepts. The fourth strategy is based on Social cognitive theory, which states that children learn through interaction.

1.4 Deficiencies with These Children

Children with Autism have difficulty with their coordination which could affect their writing homework and drawing pictures [17]. This problem could also impair children's drawing geometric shapes anddividing shapes for doing fractions. Research has shown about half the children in this category have mathematics learning difficulties [7]. Moreover, these researchers found children with Autism have difficulty with abstract number concepts involving comparison quantities and solving problems with pattern. Furthermore, research has also shown that these children have difficulty working with problems that require multiple steps and knowing how to apply concepts in a given problem [18]. A study which compared these children with typically developing children found that these children underperformed their counterparts in solving mathematical problems even though they have about the same intelligence level [7]. Furthermore, they discovered that lower performance in problem solving for these children could be due to attention deficit or difficulty with reading comprehension. Another study has also found these children have no deficiencies with problems involving procedural memory, but they have difficulty with problems requiring conceptual understanding [19] or real world problem, which involve higher level thinking [17].

1.5 Use of Pictorial Representations in Math Word Problems

Children with learning disability have difficulties working with math problems because they require abstract reasoning. So one strategy in helping children to understand these problems is by using pictorial representations of abstract mathematical concepts [20]. Children with learning difficulties tend to use pictorial representation less frequently in solving math word problems, and they also have difficulty translating word problems into pictorial representations or there is lack of quality in representations [21]. Pictorial representations of problems can enhance student understanding and performance in solving mathematical problems because it makes abstract concepts, which could be perplexing to children, to concrete and visible ones [22]. However, some researchers have found visualizations of problems might not help students' problem solving performances [23]. Moreover, the use of graphing representations helps children to establish meaning, and cognitive construction in problem solving approaches [24].

Studies have shown that children with average or higher average abilities used visual representations more often than other children when encountering math word math word problem. However, children with learning disabilities very infrequently use pictorial representations when working on math problems [25].

On important element of effective comprehension of math problem is that children's ability in using diagrams to represent math word problems [26]. When students are able to use pictorial representations in solving math word problems, they are better able to understand these problems [27]. Representation of math word problem help children toward correct solution and those who have difficulty using diagrams in problem solving will have problem comprehending and solving them [28].

2. METHODS

The focus of this study was to examine the effect of visual representations of word problems on performance of a child with Autism in solving mathematical word problems.

The study used single subject research design, which examined one or few subjects at several time periods. This research design is highly appropriate for children with learning disabilities [29]. In the study, the researcher used A-B-A research design, meaning that there is an initial baseline (A) followed by the treatment (B) and then withdraw of the treatment (A). Single subject design research is a common in area of children with disabilities special education. One significant disadvantage of the single subject design is the generalization of findings to other people; therefore, it has low external validity [30].

The researcher used four steps of Polya when solving math word problem. These steps include first understanding the problem, second, design

plan to do the problem, third to execute the plan and fourth reflection [31].

2.1 Participants

The participant is an 18-year old boy in 12th grade attending a public high school. Since kindergarten, he has been is in special education program for math and English courses. At age 10, he was diagnosed with Autism, specifically Asperger, by a psychologist. Since kindergarten, he has shown a delay in verbal English, mathematics, and coordination motor skills. His math knowledge is limited to basic additions, subtractions, some multiplications, and divisions. His assessment at the end of his high school freshman year was at 1.5 (between first and second grade). Now, he is taking Algebra 1 course, however, he is given, math worksheet at third grade level through eight grade level. His performance in English is at a 6th grade level, and his handwriting is difficult to read. Rez has attention problems when working with a tutor because he is diagnosed having ADHD (Attention Deficit Hyperactivity Disorder). While he is working on mathematics problems, Rez has a tendency to talk to himself about events happened in the school or on characters in Play Station games. Rez's tutor, who is also the investigator for this study has 15 years of high school mathematics teaching experience and holds a doctorate in mathematics education.

2.2 Setting

The location of the interview was in Rez's house. During a 60-minute math session, Rez was allowed to go the bathroom or get a snack two times during the session when he requested. There were two sessions each week, and the treatment lasted for two weeks. Since Rez has been diagnosed with ADHD, he was taken to a remote section of the house because it is very quiet and without any noises that could affect his concentration.

During the first 30 minutes, the researcher explained the lesson, which included two or three examples, and then researcher administered two or three problems to assess his understanding. While tutoring Rez, he was instructed to wait a few seconds before answering the investigator's questions or think before he talks (TBT). If Rez could not answer the question correctly, the investigator would ask other questions that activate Rez's prior knowledge that are needed for the new concepts taught. If he had difficulty in working on a problem, the researcher would provide him with cues, which could activate his prior knowledge. However, when he was able to do a problem by himself, the researcher would move to more challenging problems.

2.3 Assessment

2.3.1 One-step problems

An example of the problem without decision making:

1. Rez plans to visit an electronic store. He has $100, if he spends $80 on a camera. Does he have more money left in his pocket?

An example of a problem with decision making:

2. Rez has $160, he spends $120 to buy a printer. He also wants to buy a laptop bag which is for $40. Does he have money to buy both, printer and bag?

Assessment of problem with one step with no decision making: Each problem had a maximum of 100 points, with the following deductions: 100- point deduction for use of wrong operation, 20- point deduction for incorrect answers from computation errors, 40-point deduction for subtracting the larger number from the smaller for problem with decision making, 40-point deduction for selecting incorrect decision.

2.3.2 Two- step problems

You have $120 and would like to buy following items from a store: A Play Station game for $30 and printer for $80. How much money do you have left in your pocket?

An example of decision making problem:

> You have $120 and would like to buy the following items from the store: a Play Station game for $30 and a printer for $80. You are also interested in buying a laptop case which in on sale for $50? Do you have money to buy it?

Assessment with two step problems: First incorrect operation 100-points deduction.

Error in computations will be 20-point deductions and the second incorrect use of operation will be 50-point deduction. In problem with decision making, there will be 40-points deduction for incorrect answer.

2.3.3 Three-step problems

Examples of three-step problems:

You have $150, you buy 3 pairs of pants, each for $20, a coat for $75. How much money do you have after leaving the store?

You have $200. You buy a chair for $50, a camera for $90, and 3 school bags. What was the price of each bag?

Assessment: 100-points deduction for incorrect operation, 40- point deduction for errors in multiplication or division operations and 20-points deduction for minor error in multiplication or division errors.

2.4 Procedure

2.4.1 Material

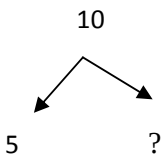
The participants used pencil, paper, and an eraser to complete the math word tests. The use of calculator was not permitted. All data were collected by the investigator in Rez's house.

2.4.2 Baseline

During the baseline, there were four meetings in a two-week period, and the investigator Administered a test in each session with a maximum time of 30 minutes in solving four math word problems. The test contained one question from one-step problems, two questions from two-step problems and one question from three-step problems. Each problem had 100 points, and the investigator computed the average of four problems for the test administered in a given session.

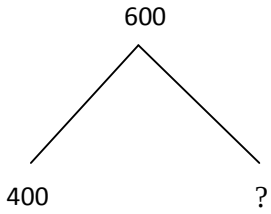
2.4.3 Training

During the first week which includes two 60-minute sessions, the investigator taught Rez how to use pictorial representations to translate math word problems into visible concrete information. The investigator introduced the lesson with simple problems such as: if you have $10, and you buy a t-shirt for $5, how much money do you have left?



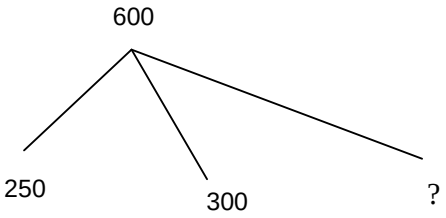
The investigator stated to Rez, what number added to 5 will give you 10, and Rez stated the answer is 5. The same process was used for two-step problems. The interviewer also used fake money to help Reza better understands the concepts. The interviewer introduced problems involving more steps and requiring more thinking. Rez also learned from given representation figures to write math word problems, an example of this as follow:

Write a problem to describe the following figures:



Rez stated that he has $600, and spent $400 on a netbook, how much did he spend on a camera?

Reza was asked to make another story about the same problem. He stated he had $600 before going to an electronic store, and spent $400, how much money does he have left for him
Another example is as follows:



Rez stated that he has $600; he spent $250 on a netbook, $300 on an Xbox. How much did he spend on a laptop bag?

2.4.4 During the treatment

During the first session, the investigator worked with Rez on three sample problems involving one step math word problems. Rez also worked on three similar math word problems that required correct numerical answers. During the second session, Rez worked on two -step problems, and during the third and fourth sessions, Rez worked on three-step math word problems. The investigator got the average score for the problems in each session.

2.4.5 After treatment

The investigator administered four tests in period of four weeks. Each test contained four questions, one from step-one, two from step-two, and one from step-three.

3. RESULTS AND DISCSSIONS

The Fig. 1 shows the participant's scores in math word problems for the past 12 weeks. The graph shows Rez's having an average score of 30% during the baseline (during the first four week). During the treatment, beginning fifth week through the end of the eight week, Rez improved his cores to about 90%, an increase of 200%. The results also shows that during the post treatment, beginning with the ninth week through the end of the eleventh week, there is an average score of 80% which is much higher than the average score for the baseline. The investigator also found that Reza had difficulty understanding words with mathematical connotations. For example, he thought the word "most" meant a lot, but he didn't know it meant more than half. Other math-related words were as follows: included most, minimum, half, each, discount, together, multiplication, division, at least, at least

3.1 Implications for Teachers

Teachers of students with autism should be cognizant that they are teaching children who have lower IQ than typically developing children [6]; therefore, they are in need of teaching strategies that are suitable for them [13]. Moreover, research studies have shown that these students have difficulty understanding mathematical problems when presented in abstract forms [8]. For example, Rez had been taught these four basic operational concepts (additions, multiplications, divisions, subtractions) in schools, however, he did not learn them well because they were presented abstractly. Therefore, teachers must present the lesson by using more concrete strategies because children will better understand the concepts [16], as the participant's performance showed in this study. The use of pictorial presentation is less abstract and effective in teaching math to students [24]. When four basic operational concepts or problems are presented in visual or pictorial form, these children can better manipulate and understand them [22]. The finding from this study showed Rez was able to improve his performance in solving math problem by using more visual representation. Teachers should also know that pictorial representation is so effective that children with higher IQ employ this method when they want to solve math problems [25].

Teachers should consider several issues that teachers should consider when implementing this intervention. First, teachers should also be aware that children with autism may lack the skills needed in the use of pictorial representations in math problems [21]; therefore, the researcher for this study spent several days training Rez to translate the math word problem into their visual representations. Second, these children also have problems with their coordination [17]; the researcher observed Rez having difficulty drawing straight lines; for example, his lines were drawn either too close or were curved. Third, the literature review has shown [12] that children had difficulty reading comprehension of words; this supports the findings from this study that Rez had difficulty with words, especially the ones with mathematical connotations such as, most, at least, at most, more etc.

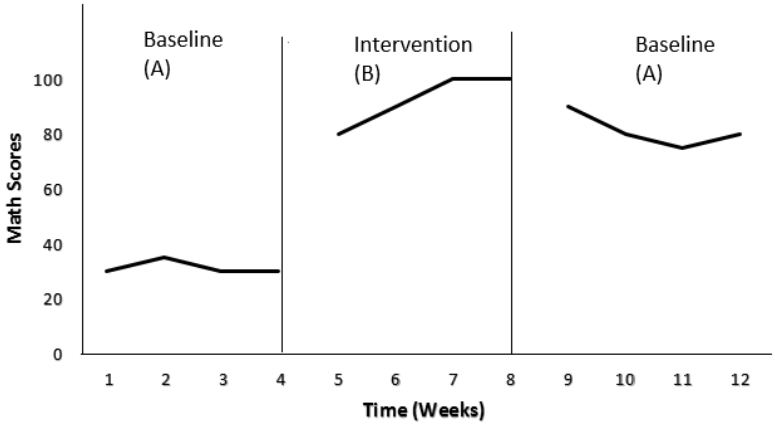


Fig. 1. Shows the performance of math test score in the A-B-A design

4. CONCLUSIONS

This study examined the effectiveness of pictorial representations of math word problems for a child in higher spectrum of autism (Asperger). The finding shows that the participant had a significant improvement in working with math word problems when using pictorial representations of them. The result also showed a month after the treatment, the child could still perform much better than the prior to the treatment. Another important finding was that Rez had difficulty understanding some English words that had mathematical meaning. He had problems Understanding words such as: most, minimum, half, each, discount, together, multiplication, division, at least, at most. Children sometimes exhibit a high level of knowledge in some areas of content, but they may have difficulty to understand some simple words; that is reason that some researchers name these children as “little professor”. Therefore, teachers should aware of this problem so they do not assume they know the meaning of some simple vocabulary words with mathematical connotations. One major limitation of single subject study is its generalization of its results to other students, or even to children with autism because every child may be at different level of autism spectrum. Research has shown no two children with autism are alike; therefore, the researcher should implement a strategy that addresses needs of that child.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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