Flexible versus simple trade-in strategy for remanufacturing

Zhaojun Yang
Xu Hu
Jun Sun
The University of Texas Rio Grande Valley, jun.sun@utrgv.edu
Yali Zhang

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Flexible versus Simple Trade-in Strategy for Remanufacturing

Abstract: Some enterprises recently start to offer the flexible trade-in option to attract customers from competitors, in contrast to the simple one that only allows them to return used products to the same manufacturers for new. Based on analytical and numerical analyses, this study compares the environmental impacts of two trade-in strategies (simple versus flexible) in combination with different carbon tax policies. From the perspective of consumer switching behavior, a Hotelling model with two market segments is established. Under the flexible trade-in strategy, the carbon emission of enterprises turns out to be significantly higher than that under the simple trade-in strategy. An appropriate carbon tax policy, especially with preferential tax rates on green products, is capable of guiding enterprises to choose a more environment-friendly trade-in strategy included in the model. The findings fill the research gap in comparing the pros and cons of simple and flexible trade-in strategies in terms of sustainable development, and provide managers and policy-makers the insights on how to promote the healthy development of the remanufacturing industry with trade-in strategizing and carbon taxation.

Keywords: remanufacturing; trade-in strategy; Hotelling model; market segment; carbon tax.

Article classification: Research paper
1. Introduction

With the aggravation of global warming and environmental issues, green economy and sustainable development are becoming the consensus of people around the world. In operations and production management, more and more enterprises pay close attention to corporate sustainability and social responsibility (Walker, Klassen, Sarkis et al., 2014). As one effective means, remanufacturing is distinct from traditional manufacturing in that the raw materials come from used products. Rather than dumping them, enterprises disassemble used products and recover the parts that can be processed for new products (Ferrer & Swaminathan, 2006). Through the reuse of raw materials and parts, manufacturers effectively cut greenhouse gas emission (Chen & Chang, 2012; Savaskan, Bhattacharya, & van Wassenhove, 2004).

As a sustainable development endeavor, remanufacturing is supported by governments all over the world. For example, the EU's End-of-life Vehicles (ELVs) Directive and Electronic Equipment Directive came into effect in 2000 and 2003, respectively (Mazzanti & Zoboli, 2006). Maryland and Californian in the United States passed legislations to impose electronic waste recycling fees (Equalization, 2018). Canada and Japan are the two countries that first subsidized the remanufacturing industry (Hicks, Dietmar, & Eugster, 2005). In 2008, China's National Development and Reform Commission (NDRC) initiated 14 pilot projects to support the remanufacturing of auto parts (Wang, Chang, Chen et al., 2014).

One of the major challenges facing the remanufacturing industry today is the acquisition of used products, for which trade-in is indispensable (Gu & Tagaras, 2014; Wang, Chang, Chen et al., 2014). In 2009, the Chinese government introduced a trade-in subsidy to help the remanufacturing industry get more used products from customers. This allows the firms in question to offer consumers incentives in form of rebates or discounts when they return used products for new (especially appliances). Such transactions provide a win-win solution: they save consumers money, give remanufacturers “raw” materials, and promote the sales of newer-generation products (Liu, Zhai, & Chen, 2019).
Smartphones emerge as fast-upgrading consumer electronics, and manufacturers around the world offer all kinds of trade-in programs. Mainly for its iPhones, Apple first launched the first Reuse and Recycling Program in February 2016. When other firms like Samsung and Huawei offered similar options, Apple further allowed consumers to return used smartphones of other brands for its latest products1. Compared with the original practice, this flexible strategy helps Apple attract consumers from its rivals. To get their market shares back, of course, competitors soon implemented similar programs.

The extant research only considers a single manufacturer with the simple trade-in strategy to recycle its own products (Cao, Bo, & He, 2018; Huang, 2018). Whereas the findings provide helpful insights, they are limited due to the fact that multiple manufacturers in the same arena tend to pursue the flexible trade-in strategy instead. It is still unclear which trade-in strategy, simple or flexible, puts the whole remanufacturing industry in a better position for sustainable development from both environmental and financial aspects. In addition to the trade-in subsidy as the positive incentive for remanufacturing, countries like Finland, Norway, Japan and Mexico impose carbon tax as a negative incentive on all enterprises to reduce green-house gas emission (Carl & Fedor, 2016; Haites, 2018). Therefore, this study develops various Hotelling models to compare the two trade-in strategies in the context of carbon taxation. The results of numerical analyses yield a more comprehensive understanding of the phenomenon and useful insights for the healthier development of the remanufacturing industry.

The remainder of this article is organized as below. First, it lays out the research background on remanufacturing and trade-in, carbon tax, and Hotelling model. Then, it describes model development and equilibrium analysis. The modeling enables the comparison between simple and flexible trade-in strategies under different carbon tax settings. The findings are discussed in terms and theoretical and practical implications, followed by the conclusion.

1 https://www.apple.com.cn/shop/trade-in
2. Research Background

2.1 Remanufacturing and trade-in

As a practice of collecting used products and processing them to the condition like new (Ovchinnikov, Blass, & Raz, 2014), remanufacturing is an innovative business strategy that combines elements of marketing (Atasu, Sarvary, & Wassenhove, 2008) and environment protection (Govindan, Parra, Rubio et al., 2019). For enterprises to strengthen competitive advantage, researchers consider the differentiation of utility and price of remanufactured goods from the new (Ferrer, 2010), production planning and product pricing under monopoly and duopoly situations (Ferrer & Swaminathan, 2006), various supply chain leaders (Choi, Li, & Xu, 2013), and multiple cycles (Ferguson & Toktay, 2010). Remanufacturing may create new opportunities for enterprises, but they face various challenges especially the low recycling rate (Ferguson & Toktay, 2010).

To encourage consumers to return used products, the remanufacturing industry implements all kinds of trade-in programs. Initially, the simple trade-in strategy enables manufacturers to recycle their own products, which then ushers in the flexible strategy that allows consumers to return competitors’ products. Remanufacturers make different levels of profits from refurbishing, repairing and reprocessing used products. Table 1 reports the rebate amounts offered by the trade-in programs, which vary across program hosts and product brands, of four major smartphone manufacturers.

[Insert Table 1 here]

Though many enterprises implement flexible trade-in programs for certain products, simple trade-in programs are still common. Except for smartphones, for instance, Apple gives no rebate to consumers when they return other brands of PCs or smartwatches to its Reuse and Recycling Program. Its competitors in the PC industry, Dell and Lenovo, adopt the flexible trade-in strategy in contrast. In the printer industry, HP launched a flexible trade-in program to recycle qualified
non-HP laser and inkjet printers\(^2\), but Canon only implemented the simple trade-in program for its own products\(^3\). Similarly, many enterprises in different industries face a choice between simple and flexible trade-in strategies.

A manufacturer may pursue flexible trade-in to increase its market share. However, is the whole industry better off if all competitors switch to flexible trade-in or hold on to simple trade-in? This study attempts to answer this question with the game theory approach considering all stakeholders, including manufacturers, consumers, and government.

### 2.2 Hotelling model

In the closed-loop supply chain context, researchers consider all kinds of competitions between two entities, such as new and used products (Liu, Zhai, & Chen, 2019), OEM- and re-manufacturers (Dou, Guo, Zhang et al., 2019), platform-run and third-party sellers (Cao, Xu, Bian et al., 2019), regular and trade-in retailers (Huang, 2018), and regular and direct (i.e., manufacturer) retailers (Saha, Sarmah, & Moon, 2016). However, few have examined the competition between two firms that produce both new and remanufactured products.

The Hotelling model (Hotelling, 1990) is capable of modeling the game between two competitors (Chen & Liu, 2014; Chen & Sheu, 2013). It has been used to model the competition between OEM- and re-manufacturers (Pazoki & Zacciyr, 2019; Wu, 2013), retailer and e-tailer (He, Xiong, & Lin, 2016), and regular and direct (i.e., manufacturer) retailers (Ofek, Katona, & Sarvary, 2011). This study uses the Hotelling model to simulate the competition between two firms with remanufacturing capacity.

For optimizing trade-in programs in terms of corporate profitability, researchers consider pricing strategy (Liu, Zhai, & Chen, 2019), channel selection (Feng, Li, Xu et al., 2019); (Cao, Wang, Duo et al., 2018), recycler choice (Cao, Bo, & He, 2018; Miao, Fu, Xia et al., 2017), rebate

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\(^3\) Source: http://www.canon.com.cn/support/announce/products/5747.html
mechanism design (Cao, Xu, Bian et al., 2019; Ray, Boyaci, & Aras, 2011), and product upgrading/updating (Liu, Zhai, & Chen, 2019; Yin & Tang, 2014). Customer purchasing behavior is also examined for the economic valuation of remanufacturing efficiency (Zhang & Zhang, 2018). These studies address how different trade-in configurations affect the decision-making and profitability of manufacturers. Yet few have investigated the situation in which two competitors engage in trade-in activities simultaneously with the same or different strategies (i.e., simple vs. flexible).

To capture different types of consumers, the modeling may incorporate market segmentation. Feng, Li, Xu et al. (2019) explored the firms’ optimal trade-in policies in a competitive environment based on two market segments: new consumers and regular consumers. Furthermore, this study compares different trade-in strategies by establishing a Hotelling model considering the market segmentation based on the ownership of different brands of used products.

2.3 Carbon tax

In addition to the market competition, remanufacturers need to consider environment-related government policy in strategizing trade-in. To control and reduce total green-house gas from production, policy-makers impose the cost of emission on manufacturers with carbon tax and cap-and-trade (Barragán-Beaud, Pizarro-Alonso, Xylia et al., 2018). Though both mechanisms are devised to price CO2 emission, carbon tax exerts a more direct intervention over the total amount of emission, whereas cap-and-trade largely relies on market control (Chai, Xiao, Lai et al., 2018; Wittneben, 2009). For illustrative purposes, this study focuses on carbon tax as an exemplary government policy to examine how the regulatory factor impacts corporate decision-making regarding remanufacturing and trade-in.

While trade-in subsidy is only for remanufacturing, carbon tax applies to all production (Cao, He, & Liu, 2019). Thus carbon tax strikes a balance between environment protection and economic development at a larger scale (Yang, Liu, Ji et al., 2016). When the remanufacturing rate is still
relatively low compared with regular production, carbon tax helps reduce the total emission more quickly than other policies (Zhu, Ren, Chu et al., 2019). On the consumer side, levying carbon tax at a reasonable level shifts the demand from brand new products to remanufactured ones due to price differentiation (Pazoki & Zacciyr, 2019).

Like most enterprises, the two manufacturers in the Hotelling model of this study make both brand new and remanufactured products. Widely used in the real world, carbon tax motivates enterprises to engage in remanufacturing for emission reduction (Cao, He, & Liu, 2019). Therefore, carbon tax is incorporated in modeling to see how it affects corporate decision-making regarding trade-in. In particular, simple and flexible trade-in strategies are distinct in their means (high vs. low degrees of remanufacturing) and ends (to upgrade/update products for existing customers vs. to attract new customers). The two strategies have different implications for the fulfillment of environmental responsibility imposed by carbon tax.

As a cleaner production approach, remanufacturing yields less environmental impacts than regular production. Carbon tax, often seen as an effective climate policy, encourages enterprises to pursue remanufacturing for financial benefits. Improving the first-period tax price always reduces the total emission, but improving the second-period tax price may increase the total emission (Dou, Guo, Zhang et al., 2019). A well-designed carbon tax promotes remanufacturing from both economic and environmental aspects. This study examines how carbon tax affects trade-in program implementation, especially the choice between simple and flexible strategies, for optimal industrywide outcomes.

3. Basic Model

3.1 Trade-in and Remanufacturing

There are two enterprises: enterprise $i$ and enterprise $s$ in a competitive relationship, and they make products in the same line but somewhat differentiated (e.g., smartphones of different operating systems). The two enterprises make brand new products as well as remanufactured ones.
Based on the conditions of used products collected from consumers, an enterprise may choose to refurbish, repair or reprocess each. To encourage consumers to recycle the used products, both enterprises implement trade-in programs that allow them to buy new products at discount prices.

In theory, each enterprise has three trade-in options: none, simple and flexible. They lead to the six combinations between two enterprises as shown in Table 2. Take a smartphone manufacturer, for instance, it may choose not to recycle used products at all. Environment-unfriendly, such a strategy is weak at retaining existing customers as well. Once one enterprise allows customers to trade in used products for new, the other will almost certainly follow suit or lost a significant proportion of market share. Thus, Combinations e and f are unstable, and this study focuses on Combinations a-d. Combination a represents the baseline scenario in which the whole industry does not engage in remanufacturing through trade-in. In Combination b, both enterprises only recycle the used products made by themselves. Combination c is mixed in which one enterprise implements a flexible trade-in program while the other sticks to the simple one. In Combination d, both pursue the flexible trade-in strategy to attract customers from competitors.

[Insert Table 2 here]

Enterprises give consumers discounts in various forms (e.g., price reduction, cash, gift card) when they trade in used products for new ones. The discount amount may vary depending on the condition of each used product. Before actual transactions, however, enterprises do not know product conditions, nor are consumers aware of valuation criteria. They can only use the average value for estimation, which can be found on the Internet for well-known products. In this study, the average amount of trade-in discount is denoted as \( v_{jj} \). In addition, consumers may save furthermore by purchasing cheaper remanufactured products, of which the quality is a little bit lower than brand new ones but still acceptable.

**Assumption 1.** New products and remanufactured products are of different qualities and prices. **Assumption 2.** Enterprises give the same total discount to consumers for the same model of used product.
Consuming fewer resources than regular production, remanufacturing always benefits enterprises financially (Zhu, Ren, Chu et al., 2019). Of course, used products are not of the same conditions, leading to different salvage values. In this study, the average salvage value is denoted as $b_{jj}$. Through simple trade-in, an enterprise collects the used products made by itself and directly works on them. Apple, for instance, turns old iPhones into certified refurbished products. Through flexible trade-in, an enterprise collects used products of its competitors but may contract third-party resellers to make uncertified refurbished products. As certified products are pricier than uncertified ones, remanufacturing yields different profit margins for simple and flexible trade-in programs.

3.2 Consumer Behavior and Trade-in

Strategic consumers are considered in this study: each evaluates the utilities of purchase options and chooses the one of the maximum utility. When a consumer decides to buy a new product from enterprise $j$, $j \in \{i, s\}$, the individual has two options: trading in old for new or regular purchase. Figure 1 shows the four buying options and the corresponding utilities to consumers. Each consumer’s demand for a new product in one cycle is a constant, and an individual only uses one product at a time. Once consumers buy new products, their used products have little value to them (i.e., value = 0).

**Assumption 3.** Each consumer has one used product, which is of no value to the person after buying a new product.

Presented the trade-in opportunity offered by enterprise $j$, consumers who decide to purchase its new products are likely to recycle used ones for discounts. As shown in Figure 1, the value of used products to consumers is 0 if not recycled at the purchase of new products. Consumers’ utility derived from buying new products made by enterprise $j$ is denoted as $u_j$, the price of new products made by enterprise $j$ denoted as $P_j$, the total trade-in discount from enterprise $j$ denoted as $v_{jj}$ and mismatch cost denoted as $tx$ or $t(1 - x)$ for the consumer located at $x$.

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[Insert Figure 1 here]
To consumers, purchasing the products without recycling used products is always inferior to the trade-in options offered by enterprises $i$ and $s$. Strategic consumers will exclude the undesirable option first, and choose among the rest to maximize the utility. Depending on whether each enterprise implements a simple or flexible trade-in program, consumers may make different purchase decisions for the pursuit of maximum utility.

3.3 The Basic Model-a

Enterprises $i$ and $s$ manufacture horizontally differentiated products, to which consumers have different preferences. A standard Hotelling setup is used to model the market competition between two enterprises. We assume enterprise $i$ and enterprise $s$ to be respectively situated at locations 0 and 1 on a line of length 1. Consumers are uniformly distributed along the line, each owning the product made by either enterprise.

Table 3 lists the notations in mathematical modeling. The utility a consumer derives, denoted as $U_j$, is affected by product price $P_j$, product utility $u_j$, and disutility from the product-preference mismatch. The closer consumers are to the Hotelling line, the less mismatch between their preferences and the products in question.

[Insert Table 3 here]

As shown in Figure 2, the utility for a consumer located at $x$ from each product can be formulated as

\[
U_i = u_i - tx - P_i, \quad (1)
\]

\[
U_s = u_s - t(1-x) - P_s, \quad (2)
\]

where $u_j$ is the value derived from the new product, and $tx$ represents the mismatch cost. A consumer located at $x$ will compare $U_i$ and $U_s$. If $U_i > U_s$, the consumer will choose enterprise $i$ to maximize the utility. The profit function of the two enterprises can be formulated as

\[
\pi_j = (P_j - C_j)q_j, \quad (3)
\]

where $j \in \{i, s\}$, $C_j$ is the cost of enterprises $j$ manufacturing a new product and $q_j$ the numbers of consumers who purchase products from enterprise $j$. As in the standard setup, by letting $U_i =$
$U_s$, the indifferent consumer’s location can be defined as $x^a = \frac{u_i - u_s - (P_i - P_s) + t}{2t}$.

[Insert Figure 2 here]

Consumers whose mismatch with enterprise $i$ is smaller than that of indifferent consumers choose enterprise $i$’s products and the rest choose enterprise $s’$. The profit function of the two enterprises can be specified as

$$\pi_i = (P_i - C_i)x^a,$$

(4)

$$\pi_s = (P_s - C_s)(1 - x^a).$$

(5)

4. Equilibrium Analysis

In this section, three cases of enterprises’ trade-in strategies are discussed, in which the market is segmented into two by competing enterprises. For each enterprise, the consumers who own its products comprise the loyal market segment, and consumers who own the competitor’s product comprise the new market segment. Interestingly, the loyal market segment of enterprise $i$ is also the new market segment of enterprise $s$. Then the whole market can be divided into two parts, the loyal market segments for enterprises $i$ and $s$, respectively, denoted as market segments 1 and 2. The size of market segment 1 in terms of the number of consumers is $\alpha$, and that of market segment 2 is $1 - \alpha$.

In Case 1, each enterprise adopts the simple trade-in strategy. It is replaced by both enterprises with the flexible strategy in Case 2. Case 3 features a mixture of simple and flexible trade-in options offered by two competitors to consumers.

4.1 Case 1

In Case 1, only consumers who own used products made by enterprise $j$ can trade them in for its new products. Figure 3 integrates the Hotelling lines corresponding to two market segments: each enterprise offers the simple trade-in opportunity to its loyal market segment consumers with a particular price-utility combination for upgrading to new products.

[Insert Figure 3 here]
In market segment 1 where people own used products made by enterprise $i$, consumers can participate in its trade-in program, or purchase enterprise $s$’ products. Similarly, in market segment 2 where people own used products made by enterprise $s$, consumers can upgrade to its new products, or purchase enterprise $i$’s products. The utilities consumers derive from purchasing without trade-in are shown in eq. (1) and (2). When consumers make purchases through trade-in, the utilities change. For consumers in market segment 1, the utility they derive from trade-in with enterprise $i$ can be reformulated as

$$U_i = u_i - tx - P_i + v_{ii}.$$  \hfill (6)

For consumers in market segment 2, the utility they derive from trade-in with enterprise $s$ can be reformulated as

$$U_s = u_s - t(1 - x) - P_s + v_{ss}.$$  \hfill (7)

By letting $U_i = U_s$ in both market segment, indifferent consumer’s locations in two market segments can be defined as $x_1^b = \frac{u_i - u_s - (P_i - P_s) + t + v_{ii}}{2t}$ and $x_2^b = \frac{u_i - u_s - (P_i - P_s) + t - v_{ss}}{2t}$. In market segment 1, consumers whose mismatch with enterprise $i$ is smaller than that of indifferent consumers in segment 1 will choose enterprise $i$, and the rest choose enterprise $s$. In market segment 2, the situation is similar.

In Case 1, two enterprises still pursue the maximal profit. Considering the total discount and the changed sales, the enterprises’ profit function can be reformulated as

$$\pi_i^b = (P_i - C_i - v_{ii} + b_{ii})\alpha x_1^b + (P_i - C_i)(1 - \alpha)x_2^b,$$

$$\pi_s^b = (P_s - C_s)(1 - x_1^b) + (P_s - C_s - v_{ss} + b_{ss})(1 - \alpha)(1 - x_2^b).$$  \hfill (8)

Solving the first-order conditions for the two profit-maximizing enterprises yields the equilibrium prices, total discount, profits, and sales, as summarized by Lemma 1.

**Lemma 1.** When both enterprises take a simple trade-in strategy, the equilibrium prices are

$$p_{i}^{b*} = C_i + \frac{K_2 - b_{ss}}{3},$$

$$p_{s}^{b*} = C_s + \frac{K_3 - b_{ii}}{3}.$$
the equilibrium sales are

\[ q_i^{*} = K_2 + \alpha b_{li} - (1 - \alpha)b_{ss}, \]

\[ q_s^{*} = 1 - q_i^{*}, \]

the equilibrium total trade-in discounts are

\[ v_{li}^{*} = \frac{2b_{li} - b_{ss}}{3}, \]

\[ v_{ss}^{*} = \frac{2b_{ss} - b_{li}}{3}, \]

and the equilibrium profits are

\[ \pi_i^{*} = \frac{\alpha(K_2 + b_{li})^2 + (1 - \alpha)(K_2 - b_{ss})^2}{18t}, \]

\[ \pi_s^{*} = \frac{\alpha(K_3 - b_{li})^2 + (1 - \alpha)(K_3 + b_{ss})^2}{18t}, \]

where \( K_1 = u_i - u_s - C_i + C_s, \; K_2 = 3t + K_1, \) and \( K_3 = 3t - K_1. \)

**Proof:** All proofs are included in Appendix A.

Some observations related to the equilibrium are worth attention. First, when does the salvage of used product \( j \) for enterprise \( j, \; b_{jj}, \) influence the enterprises’ sales and profits? It’s more difficult for an enterprise that has a lower market share in the last cycle to grab the market or maintain the market share. When the market share of enterprise \( i \) is 0.75 in the last cycle, \( \alpha = 0.75, \) enterprise \( s \) needs more advanced remanufacturing technology to keep the salvage of used product \( s \) for enterprise \( s \) three times of that of \( i, \; b_{ss} = 3b_{li}. \) On the contrary, enterprise \( i, \) which had a larger market share in the last cycle, can easily grab the market.

Second, the lower salvage \( b_{jj}, \) the smaller total discount \( v_{jj}^{*}. \) It is rational that enterprises collect used products of lower salvage to reduce the total discount. The total discount is about \( \frac{1}{3} \) of the salvage, and especially, when \( b_{li} = b_{ss}, \) the total discount is exactly \( \frac{1}{3} \) of the salvage, \( v_{jj}^{*} = \frac{1}{3} b_{jj}. \)

Third, consumers get \( \frac{2}{3} \) of salvage value. Compared with model a, consumers get the total
trade-in discount and lower price for the new product in Case 1. The whole difference value is exact \( \frac{2}{3} \) of the salvage value, and the enterprise get the rest \( \frac{1}{3} \). The utility of the used product is almost none to consumers, but valuable to enterprises. Both consumers and enterprises benefit from trade-in transactions.

4.2 Case 2

In Case 2, both enterprises adopt the flexible trade-in strategy. Market segments remain, but consumers can purchase any products through trade-in, as shown in Figure 4. Compared to Case 1, some utility functions change. In segment 1, the utility consumers derived from purchasing products of enterprise \( s \) can be reformulated as

\[
U_s = u_s - t(1-x) - P_s + v_{si}. \tag{10}
\]

In segment 2, the utility consumers derived from purchasing products of enterprise \( i \) can be reformulated as

\[
U_i = u_i - tx - P_i + v_{is}. \tag{11}
\]

By letting \( U_i = U_s \) in both market segments, indifferent consumer’s locations in two market segments can be defined as \( x_1^d = \frac{u_i-u_s-(P_i-P_s)+t+(v_{ii}-v_{si})}{2t} \) and \( x_2^d = \frac{u_i-u_s-(P_i-P_s)+t+(v_{is}-v_{ss})}{2t} \). In market segment 1, consumers whose mismatch with enterprise \( i \) is smaller than that of indifferent consumers in segment 1 will choose enterprise \( i \), and the rest consumers will choose enterprise \( s \). In market segment 2, the situation is similar.

[Insert Figure 4 here]

In Case 2, two enterprises still pursue maximal profit. Considering the total discount and the changed sales, the enterprises’ profit function can be reformulated as

\[
\pi_i^d = (P_i - C_i - v_{ii} + b_{ii})\alpha x_1^d + (P_i - C_i - v_{is} + b_{is})(1-\alpha)x_2^d, \tag{12}
\]

\[
\pi_s^d = (P_s - C_s - v_{si} + b_{si})\alpha(1-x_1^d) + (P_s - C_s - v_{ss} + b_{ss})(1-\alpha)(1-x_2^d). \tag{13}
\]

Solving the first order conditions for the two profit-maximizing enterprise yields the equilibrium prices, total discount, profits, and sales, as summarized by Lemma 2.

**Lemma 2.** When both enterprises take a flexible trade-in strategy, the equilibrium prices are
\begin{align*}
p_s^{d*} &= C_s + \frac{3v_{sl}^{d*} + K_3 - 2b_{sl} - b_{ii}}{3}, \\
p_i^{d*} &= C_i + \frac{3v_{is}^{d*} + K_2 - 2b_{is} - b_{ss}}{3},
\end{align*}

the equilibrium sales are
\begin{align*}
q_i^{d*} &= \frac{K_2 + \alpha(b_{ii} - b_{si}) - (1 - \alpha)(b_{ss} - b_{is})}{6t}, \\
q_s^{d*} &= 1 - q_i^{d*},
\end{align*}

the equilibrium total trade-in discounts are
\begin{align*}
3v_{ii}^{d*} - 3v_{is}^{d*} &= 2b_{ii} + b_{sl} - 2b_{is} - b_{ss}, \\
3v_{si}^{d*} - 3v_{ss}^{d*} &= 2b_{si} + b_{li} - 2b_{ss} - b_{ls},
\end{align*}

and the equilibrium profits are
\begin{align*}
\pi_i^{d*} &= \frac{\alpha(K_2 + b_{ii} - b_{si})^2 + (1 - \alpha)(K_2 + b_{is} - b_{ss})^2}{18t}, \\
\pi_s^{d*} &= \frac{\alpha(K_3 + b_{si} - b_{li})^2 + (1 - \alpha)(K_3 + b_{ss} - b_{ls})^2}{18t},
\end{align*}

where \( K_1 = u_i - u_s - C_i + C_s \), \( K_2 = 3t + K_1 \), and \( K_3 = 3t - K_1 \).

Some observations related to the equilibrium are worth attention. First, the greater the gap in remanufacturing technology between the two enterprises, the easier it is for the leading enterprises to seize the market. As \( b_{ii} - b_{si} \) and \( b_{is} - b_{ss} \) increase, the technology gap wider, sales of enterprise \( i \) increase and sales of \( s \) decrease when market share in the last cycle, \( \alpha \), is constant.

For the illustrative purpose, Figure 5 shows an example. When \( b_{ii} - b_{si} \) and \( b_{is} - b_{ss} \) are close to \(-0.3\), the difference between sales of \( i \) and \( s \) is getting lager. Meanwhile, when \( b_{ii} - b_{si} \) and \( b_{is} - b_{ss} \) are close to \(0.3\), the difference disappeared first, then became negative. The remanufacturing technology advantage eliminates the gap in market share.

[Insert Figure 5 here]

4.3 Case 3

In Case 3, enterprise \( i \) switches to the flexible trade-in strategy, while enterprise \( s \) sticks to the simple one. Consumers in market segment 2, where they own the used products of enterprise \( s \), can purchase any products through trade-in. But consumers in the market segment 1 has the
trade-in option only for enterprise i’s products rather than enterprise s’ products. And Figure 6 shows the Hotelling lines in two market segments.

Compared to Case 1, some utility functions change. In segment 2, the utility consumers derived from purchasing products of enterprise i can be reformulated as

$$U_i = u_i - tx - P_i + v_{is}. \tag{14}$$

By letting $U_i = U_s$ in both segments, indifferent consumer’s locations in two segments can be defined as $x_1^c = \frac{u_i - u_s - (P_i - P_s) + t + (v_{ii} - v_{si})}{2t}$ and $x_2^c = \frac{u_i - u_s - (P_i - P_s) + t - v_{ss}}{2t}$. In market segment 1, consumers whose mismatch with enterprise i is smaller than that of indifferent consumers in segment 1 choose enterprise i, and the rest choose enterprise s. In market segment 2, the situation is similar.

![Insert Figure 6 here]

In Case 3, two enterprises still pursue maximal profit. Considering the total discount and the changed sales, the enterprises’ profit function can be reformulated as

$$\pi_i^c = (P_i - C_i - v_{ii} + b_{ii})ax_1^{c1} + (P_i - C_i - v_{is} + b_{is})(1 - \alpha)x_2^c, \tag{15}$$

$$\pi_s^c = (P_s - C_s)\alpha(1 - x_1^c) + (P_s - C_s - v_{ss} + b_{ss})(1 - \alpha)(1 - x_2^c). \tag{16}$$

Solving the first order conditions for the two profit-maximizing enterprise yields the equilibrium prices, total discount, profits, and sales, as summarized by Lemma 3.

**Lemma 3.** When both enterprise i take flexible trade-in strategy, and enterprise s simple, the equilibrium prices are

$$P_s^{c*} = C_s + \frac{K_3 - b_{ii}}{3},$$

$$P_i^{c*} = 2P_s^{c*} + u_i - u_s - t - C_s + v_{ii}^{c*},$$

the equilibrium sales are

$$q_i^{c*} = \frac{K_2 + \alpha b_{ii} - (1 - \alpha)(b_{ss} - b_{is})}{6t},$$

$$q_s^{c*} = 1 - q_i^{c*},$$

the equilibrium total trade-in discounts are
\[ v_{ss}^c = \frac{2b_{ss} - b_{ii} + b_{is}}{3}, \]
\[ v_{li}^c - v_{is}^c = b_{ss} - 2v_{ss}^c, \]

and the equilibrium profits are

\[ \pi_i^c = \frac{\alpha(K_2 + b_{ii})^2 + (1 - \alpha)(K_2 + b_{is} - b_{ss})^2}{18t}, \]
\[ \pi_s^c = \frac{\alpha(K_3 - b_{ii})^2 + (1 - \alpha)(K_3 + b_{ss} - b_{ls})^2}{18t}, \]

where \( K_1 = u_i - u_s - C_i + C_s, K_2 = 3t + K_1, \) and \( K_3 = 3t - K_1. \)

Combined with Lemmas 1, 2 and 3, it can be found that \( q_{s}^{b*} > q_{s}^{c*} < q_{s}^{d*} \) and \( \pi_{s}^{b*} > \pi_{s}^{c*} < \pi_{s}^{d*}. \) Whether considering market share or corporate profits, Case 3 is always the worst-case scenario for enterprise \( s. \) When the enterprise \( i \) adopts the flexible trade-in strategy, the best choice of the enterprise \( s \) is also the flexible trade-in strategy, corresponding to Case 2. In the smartphone industry, as an example, when Apple moved from the simple trade-in strategy to the flexible, the other enterprises had to take the same action, or they would suffer the loss of market share and profit.

\section*{5. Simple versus Flexible}

Which trade-in strategy do enterprises prefer? Is the better performance strategy a greener one? To get the answer, the equilibria in the two cases are compared, and their environmental performance is also assessed.

When one enterprise adopts trade-in strategies, first simple and then flexible, the other is likely to do the same due to marketing and financial pressures (see Appendix B for more information). The more interesting question is: in which situation is an enterprise likely to switch from the simple trade-in strategy to the flexible strategy? The answer to this question is rooted in enterprise profit and sales changes between Case 1 and Case 2. Accordingly, comparing the equilibrium sales of two cases summarized in Lemmas 1 and 2 yields the following proposition.

\textbf{Proposition 1.} If \((1 - \alpha)b_{ls} > ab_{s1},\) the sales of enterprise \( i \) increase more in Case 2 than in Case
1, while those of enterprise $s$ decrease.

The illustration of Proposition 1, as shown in Fig.7 provides additional insights. For sales, enterprise $i$ wants to keep the simple-simple situation (Case 1) when it has a high market share ($\alpha$), but prefers the flexible-flexible situation (Case 2) otherwise. In addition to market share, remanufacturing capability matters as well. When enterprise $i$ has an advantage in remanufacturing (the salvage $b_{ls}$ is bigger than $b_{si}$), it has more incentive to switch to flexible-flexible situation. For an enterprise that has a high market share, even a bigger remanufacturing effort (e.g., technology investment) is needed for flexible trade-in to sustain the same level of sales.

[Insert Figure 7 here]

To better understand Proposition 1, the sales of two enterprises in two market segments are analyzed across two cases. In the two cases, the market is segmented as shown in Figure 8. When owners of enterprise $j$’s products buy its competitor's new products, they are no longer loyal customers. In the last cycle, all consumers in segment 1, $\alpha$, buy the products of enterprise $i$, but in this cycle, some of them “betray” in both Case 1 and 2. While such betrayals exist all the time in either segment, their numbers depend on indifferent consumers’ locations. For example, the percentage of betrayals in segment 1 of Case 1 is equal to 1 minus $x^b_1$, the indifferent consumer’s location. The number of betrayals in segment 1 of Case 1 is equal to this percentage multiplied by the size of segment 1, $\alpha$. The indifferent consumer’s location is the key to the number of betrayals. In segment 1, the more it positions to the left, the bigger number of betrayals, and the opposite holds for segment 2. Four indifferent consumer’s locations are marked in figure 8. It can be observed that $x^d_1$ is to the left of $x^b_1$ and $x^b_2$ is to the left of $x^d_2$, which means more betrayals in Case 2 than in Case 1 in both segments.

[Insert Figure 8 here]

Table 4 lists the indifferent consumer’s locations in two market segments, and the difference reaches the same conclusion numerically. We know the difference of ratio of betrayal between Case 1 and 2, and the key factors making the difference are $b_{si}$ and $b_{ls}$. The higher the $b_{sl}$ or
$b_{ls}$, the higher the ratio of betrayal in segment 1 or 2. It is interesting that improving the salvage value of used products made by competitors in remanufacturing can induce the betrayal of regular consumers of competitors. This provides an impetus for enterprises to improve remanufacturing technology. The salvages, $b_{sl}$ and $b_{ls}$, and the sizes of market segments influence the sales of enterprises in the way of proposition 1.

[Insert Table 4 here]

**Proposition 2.** If $\alpha\left[b_{sl}^2 - 2(K_2 + b_{ll})b_{sl}\right] > (\alpha - 1)\left[b_{ls}^2 + 2(K_2 - b_{ss})b_{ls}\right]$, where $K_2 = K_1 + 3t$, the profits of enterprise $i$ increase more in Case 2 than Case 1, while those of enterprise $s$ decrease.

Fig. 9 illustrates Proposition 2 from the perspective of profitability. The insights obtained were similar to those based on the illustration of Proposition 1 from the perspective of sales. Enterprise $i$ prefers the simple-simple situation (Case 1) when market share ($\alpha$) is high, but the flexible-flexible situation (Case 2) when $\alpha$ is low. An advantage in remanufacturing (the salvage ratio increase) will make the flexible-flexible situation more attractive. An enterprise enjoying a high market share, however, is reluctant to make the switch due to the additional efforts required by flexible trade-in that may hurt its profitability.

[Insert Figure 9 here]

From the government perspective in terms of environment protection, Case 1 and Case 2 are further compared through the assessment of carbon emission for manufacturing new or remanufactured products. The carbon emission of making each new product is denoted as $e_{ln}$ or $e_{sn}$, and each remanufactured product $e_{lir}$, $e_{lsr}$, $e_{ssr}$ or $e_{sir}$. For instance, the carbon emission of enterprise $i$ making a remanufactured product with a used product made by enterprise $s$ is denoted as $e_{lsr}$.

To assess the environmental performance for Case 1, this study obtains the total emission of two enterprises with the following function:

$$CE^b = e_{ln}(1 - \alpha)x_2^b + e_{sn}\alpha(1 - x_1^b) + e_{lir}\alpha x_1^b + e_{ssr}(1 - \alpha)(1 - x_2^b).$$

(17)
For Case 2:

\[ CE^d = e_{lsr}(1 - \alpha) x_2^d + e_{sir} \alpha (1 - x_1^d) + e_{lir} \alpha x_1^d + e_{ssr} (1 - \alpha)(1 - x_2^d). \]  

(18)

**Proposition 3.** If \( a[(e_{sir} - e_{lir})b_{si} + (e_{sn} - e_{sir})(K_2 - b_{li})] + (1 - \alpha)[(e_{lsr} - e_{ssr})b_{is} - (e_{ln} - e_{lsr})(K_2 - b_{ss})] > 0 \), where \( K_2 = K_1 + 3t \), the total carbon emission is more in Case 2 than in Case 1, \( CE^d > CE^b \).

As shown in Proposition 3, the flexible-flexible situation yields more carbon emission than the simple-simple one. For sales and profits, enterprises may switch from the simple trade-in strategy to the flexible one at the cost of the environment. In order to reinforce the fulfillment of corporate social responsibility, the government may resort to the carbon tax mechanism, as discussed in the next section.

In Case 2, all consumers choose to participate in trade-in. With more used products recycled, how can there more carbon emission than in Case 1? Not just the carbon emission of the manufacturing unit new product is more than that of remanufacturing, \( e_{jn} > e_{jkr} \), where \( j \in \{i, s\} \) and \( k \in \{i, s\} \), but also the carbon emission of enterprise \( j \) remanufacturing with used products made by \( k \) is more than that of enterprise \( k \), \( e_{jkr} > e_{kkr} \). In addition, the sales of each enterprise in two market segments differ in Case 1 and Case 2. Two examples are given below to illustrate the paradox.

As shown in Figure 10, when \( b_{is} \) and \( b_{si} \) are small enough, carbon emission in Case 1 is more than that in Case 2; as they increase, the carbon emission in Case 2 gradually exceeds that in Case 1. The parameters set as \( e_{lir} = 0.2 \), \( e_{ssr} = 0.15 \), \( e_{lsr} = 0.34 \) and \( e_{sir} = 0.25 \), which mean the carbon emission of enterprise \( j \) remanufacturing with its own used products is less than that of it remanufacturing with its competitor’s, or \( e_{jkr} < e_{jkr} \). In addition, \( e_{sir} = 0.25 \) and \( e_{ln} = 0.4 \), which mean carbon emission of \( j \) manufacturing a new product is more than that of \( k \) remanufacturing with a used product made by \( j \), or \( e_{kjr} < e_{jn} \). When \( b_{jk} \) increases, the ratio of betrayal increases in both segments, as more consumers trade in the used products made by \( j \) to its competitor for new ones.
Similarly, Figure 11 shows that when $b_{li}$ and $b_{ss}$ is small enough, carbon emission in Case 1 is more than that in Case 2. As $b_{li}$ and $b_{ss}$ increase, the carbon emission in Case 2 gradually exceeds that in Case 1.

As shown in Figure 12, the carbon emission caused by consumers located between $x^d_1$ and $x^d_1$ in segment 1 increases, and that caused by consumers located between $x^d_1$ and 1 decreases. This makes it possible for the carbon emission in Case 2 to be larger than that in Case 1. When $b_{jk}$ is big enough, the carbon emission of Case 2 is equal to that in Case 1.

The above analysis confirms that Case 2 may produce more emission in total. The next section will address how to mitigate the negative impact by employing the carbon tax policy.

6. Carbon Tax Policies

For the purpose of emission reduction, it is a common practice for the government to levy carbon tax. When the taxation policy is introduced, will flexible trade-in strategy still be attractive to enterprises? In this section, basic carbon tax is denoted as $c$ and preferential carbon tax denoted as $\rho c$. The preferential carbon tax is designed for remanufactured products, which are more environment-friendly due to lower unit emission.

First, Case CT (Carbon Tax) is established in which basic carbon tax is levied on all products. Case CT is solved in a way similar to Case 2 in Lemma 2. As the two enterprises need to pay carbon tax for their products, the profits function can be reformulated as:

$$
\pi^C_T = (P_i - C_i - v_{li} + b_{li} - ce_{iir}) \alpha x^{CT}_i
+ (P_i - C_i - v_{ls} + b_{ls} - ce_{lsr})(1 - \alpha)x^{CT}_2,
$$  \hfill (19)

$$
\pi^C_S = (P_s - C_s - v_{sl} + b_{sl} - ce_{slr}) \alpha(1 - x^{CT}_1)
+ (P_s - C_s - v_{ss} + b_{ss} - ce_{ssr})(1 - \alpha)(1 - x^{CT}_2).
$$  \hfill (20)
Second, Case DCT (Differentiated Carbon Tax) is established to accommodate preferential carbon tax for environment-friendly products. In this case, preferential carbon tax applies when enterprise \( j \) engages in remanufacturing with used products made by itself. The profits function can be reformulated as:

\[
\begin{align*}
\pi_i^{DCT} &= (P_i - C_i - v_i + b_{ii} - \rho c e_{iir}) \alpha x_1^{DCT} \\
&\quad + (P_i - C_i - v_{is} + b_{is} - c e_{isr})(1 - \alpha) x_2^{DCT},
\end{align*}
\]

\[
\begin{align*}
\pi_s^{DCT} &= (P_s - C_s - v_{si} + b_{si} - ce_{sir}) \alpha (1 - x_1^{DCT}) \\
&\quad + (P_s - C_s - v_{ss} + b_{ss} - \rho c e_{ssr})(1 - \alpha)(1 - x_2^{DCT}).
\end{align*}
\]

**Proposition 4.** Carbon emission in Case CT is lower than that in Case 2 all time; if \( e_{iir} < e_{sir} \) and \( e_{ssr} < e_{isr} \), carbon emission in Case DCT is lower than that in Case CT. Carbon tax policy is effective in controlling greenhouse gas emission.

The carbon tax policy urges enterprises to produce more environment-friendly products, lowering the total carbon emission. An example is given in Figure 13: when carbon tax rate increases, total carbon emission in both Case CT and Case DCT decreases, while the latter outperforms the former. Therefore, the preferential carbon tax policy favoring environment-friendly products is more effective than the basic one.

[[Insert Figure 13 here]]

**Proposition 5.** Carbon tax policy does not always mean a loss of profits. If \( \alpha[c(e_{sir} - e_{iir})^2 + 2(e_{sir} - e_{iir})(K_2 + b_{ii} - b_{si})] + (1 - \alpha)[c(e_{ssr} - e_{isr})^2 + 2(e_{ssr} - e_{isr})(K_2 + b_{is} - b_{ss})] > 0 \), where \( K_2 = K_1 + 3t \), the profit of enterprise \( i \) is higher in Case CT than in Case 2; if \( \alpha[c(e_{sir} - \rho e_{iir})^2 + 2(e_{sir} - \rho e_{iir})(K_2 + b_{ii} - b_{si})] + (1 - \alpha)[c(e_{ssr} - \rho e_{sir})^2 + 2(\rho e_{ssr} - e_{isr})(K_2 + b_{is} - b_{ss})] > 0 \), where \( K_2 = K_1 + 3t \), the profit of enterprise \( i \) is higher in Case DCT than in Case 2.

When carbon tax is levied, it may have a positive impact on an enterprise’s profitability. As shown in Figure 14, when carbon tax rate \( c \) increases within the low range (below 0.4-0.5), the profit of enterprise \( i \) decreases. When it continues rising in the high range, however, the profit of
enterprise $i$ increases in both Case Tax and Case DCT. Beyond the carbon tax rate of around 0.8, the profit of enterprise $i$ is even higher in Case DCT than in Case 2. Thus, a strong preferential carbon tax policy is conducive to the healthy development of the remanufacturing industry.

[Insert Figure 14 here]

7. Conclusion

The purpose of the trade-in is to protect the environment by recycling used products for remanufacturing. In order to meet the needs of consumers, many enterprises have adopted the flexible trade-in strategy to replace the simple one, but such a practice may compromise the environment protection purpose. This study focuses on enterprise choice between simple and flexible trade-in strategies in terms of their business and environment impacts considering carbon tax mechanism. The findings suggest that remanufacturing based on simple trade-in is optimal for both environment and business. Flexible trade-in is more of a marketing strategy that attracts consumers from competitors but increases total emission.

The findings yield some helpful implications for managers and policy-makers:

a. Being the first enterprise to implement a trade-in program (simple or flexible) helps capture more market share. The other enterprises are better off to follow suit, or lose their edge. In reality, it is recommended that an enterprise adopt the same trade-in strategy as its competitor’s.

b. Whether an enterprise shall be the first one to take a simple or flexible trade-in strategy depends on certain conditions. An enterprise having a relatively large market share wants to be the first to offer consumers the simple trade-in opportunity, and an enterprise having relatively small market share wants to make the flexible trade-in option available. An advantage in remanufacturing technology also motivates an enterprise to adopt the flexible trade-in strategy.

c. The flexible trade-in strategy is less effective in cutting down carbon emission than the
simple trade-in strategy. The carbon tax mechanism helps enterprises fulfill their corporate social responsibility by making an environment-sensible choice between simple and flexible trade-in strategies.

The trade-in activity is designed for environment protection by reducing pollution in the manufacturing process. The flexible trade-in strategy provides consumers with more flexibility, but it may not be more environmentally friendly than the simple one. It is necessary for the government to implement environmental policies, such as carbon tax, to guide enterprises to choose greener strategies.

Enterprises must use the flexible trade-in strategy with caution. It is true that one enterprise may get more profit by taking the flexible trade-in strategy when competitors are taking the simple one. But profit will diminish when competitors switch to the flexible one as well. Actually, the profit may be lower than the previous phase when enterprises all take the simple trade-in strategy. It is up to the government to avoid the damage to both the environment and industry by discouraging enterprises from using flexible trade-in strategy.

This study has limitations that point to future research. The single-cycle models established are simpler but not very realistic, and they can be extended to multiple-cycle ones in the future. Also, a marketplace for used product exchange is likely to improve the utilization rate of used products. Further analyses may consider the possibility that enterprises trade used products with each other. Finally, cap-and-trade policy can be included to compare with carbon tax in terms of similar or different roles that they play in corporate choice of trade-in strategies.
References


Appendix A

Proof of Lemma 1. As \( q_i^b = \frac{u_i - u_s - (P_i - P_s) + t + \alpha v_{il} - (1 - \alpha) v_{ss}}{2t} \), we can rewrite the two profit functions as

\[
\pi_i^b = (P_i - C_i - v_{il} + b_{il}) \frac{u_i - u_s - (P_i - P_s) + t + v_{il} - v_{sl}}{2t} \alpha + (P_i - C_i) \frac{u_i - u_s - (P_i - P_s) + t + v_{il} - v_{ss}}{2t} (1 - \alpha),
\]

(A.1)

\[
\pi_s^b = (P_s - C_s) \left[ 1 - \frac{u_i - u_s - (P_i - P_s) + t + v_{il} - v_{sl}}{2t} \right] \alpha + (P_s - C_s - v_{ss} + b_{ss}) \left[ 1 - \frac{u_i - u_s - (P_i - P_s) + t + v_{il} - v_{ss}}{2t} \right] (1 - \alpha).
\]

(A.2)

Using the first-order conditions, \( \frac{\partial \pi_i^b}{\partial v_{il}} = 0, \frac{\partial \pi_i^b}{\partial P_i} = 0, \frac{\partial \pi_i^b}{\partial v_{ss}} = 0 \) and \( \frac{\partial \pi_s^b}{\partial P_s} = 0 \), we have

\[
p_i^b = C_i + \frac{K_2 - b_{ss}}{3},
\]

(A.3)

\[
p_s^b = C_s + \frac{K_3 - b_{il}}{3},
\]

(A.4)

\[
v_{il}^b = \frac{2b_{il} - b_{ss}}{3},
\]

(A.5)

\[
v_{ss}^b = \frac{2b_{ss} - b_{il}}{3}.
\]

(A.6)

The second-order conditions are negative in both cases. The four equations above yield the equilibrium prices of \( p_i^b, p_s^b, v_{il}^b \) and \( v_{ss}^b \) in Lemma 1. We can then drive the equilibrium \( q_i^b, q_s^b, \pi_i^b \) and \( \pi_s^b \) using the equilibrium prices.

Proof of Lemma 2. As \( q_i^d = \frac{u_i - u_s - (P_i - P_s) + t + \alpha (v_{il} - v_{sl}) + (1 - \alpha) (v_{il} - v_{ss})}{2t} \), we can rewrite the two profit functions as

\[
\pi_i^d = (P_i - C_i - v_{il} + b_{il}) \frac{u_i - u_s - (P_i - P_s) + t + v_{il} - v_{sl}}{2t} \alpha + (P_i - C_i - v_{il} + b_{il}) \frac{u_i - u_s - (P_i - P_s) + t + v_{il} - v_{ss}}{2t} (1 - \alpha),
\]

(A.7)

\[
\pi_s^d = (P_s - C_s - v_{ss} + b_{ss}) \left[ 1 - \frac{u_i - u_s - (P_i - P_s) + t + v_{il} - v_{sl}}{2t} \right] \alpha + (P_s - C_s - v_{ss} + b_{ss}) \left[ 1 - \frac{u_i - u_s - (P_i - P_s) + t + v_{il} - v_{ss}}{2t} \right] (1 - \alpha).
\]

(A.8)

Using the first-order conditions, \( \frac{\partial \pi_i^d}{\partial v_{il}} = 0, \frac{\partial \pi_i^d}{\partial P_i} = 0, \frac{\partial \pi_i^d}{\partial v_{ss}} = 0 \) and \( \frac{\partial \pi_s^d}{\partial P_s} = 0 \), we have

\[
p_s^d = C_s + \frac{3v_{sl} + K_3 - 2b_{sl} - b_{il}}{3},
\]

(A.9)
\[ p_i^{d*} = C_i + \frac{3v_i^{d*} + K_2 - 2b_{ls} - b_{ss}}{3}, \quad (A.10) \]

\[ 3v_{il}^{d*} - 3v_{ls}^{d*} = 2b_{il} + b_{sl} - 2b_{ls} - b_{ss}, \quad (A.11) \]

\[ 3v_{sl}^{d*} - 3v_{ss}^{d*} = 2b_{sl} + b_{li} - 2b_{ss} - b_{ls}, \quad (A.12) \]

And the second-order conditions are negative in both cases. Then the four equations above yield the equilibrium prices of \( p_i^{d*}, p_s^{d*}, v_{il}^{d*} \) and \( v_{ss}^{d*} \) in Lemma 2. We can then drive the equilibrium \( q_i^{d*}, q_s^{d*}, \pi_i^{d*} \) and \( \pi_s^{d*} \) using the equilibrium prices.

Proof of Lemma 3. As \( q_i^c = \frac{u_i - u_s - (P_i - P_s) + t + \alpha v_{il}}{2t} \), we can rewrite the two profit functions as

\[ \pi_i^c = (P_i - C_i - v_{il} + b_{il}) \frac{u_i - u_s - (P_i - P_s) + t + v_{il}}{2t} \alpha \]

\[ + (P_i - C_i - v_{is} + b_{is}) \frac{u_i - u_s - (P_i - P_s) + t + (v_{is} - v_{ss})}{2t} \alpha, \quad (A.13) \]

\[ \pi_s^c = (P_s - C_s - v_{is} + b_{ss}) \frac{1 - (u_i - u_s - (P_i - P_s) + t + v_{il})}{2t} \alpha \]

\[ + (P_s - C_s - v_{ss} + b_{ss}) \frac{1 - (u_i - u_s - (P_i - P_s) + t + v_{is})}{2t} \alpha. \quad (A.14) \]

Using the first-order conditions, \( \frac{\partial \pi_i^c}{\partial v_{il}} = 0, \frac{\partial \pi_i^c}{\partial P_i} = 0, \frac{\partial \pi_s^c}{\partial v_{ss}} = 0 \) and \( \frac{\partial \pi_s^c}{\partial P_s} = 0 \), we have

\[ p_s^{c*} = C_s + \frac{K_3 - b_{il}}{3}, \quad (A.15) \]

\[ p_i^{c*} = 2p_s^{c*} + u_i - u_s - t - C_s + v_{il}^{c*}, \quad (A.16) \]

\[ v_{ss}^{c*} = \frac{2b_{ss} - b_{il} + b_{is}}{3}, \quad (A.17) \]

\[ v_{il}^{c*} - v_{is}^{c*} = b_{ss} - 2v_{ss}^{c*}. \quad (A.18) \]

And the second-order conditions are negative in both cases. Then the four equations above yield the equilibrium prices of \( p_i^{c*}, p_s^{c*}, v_{il}^{c*} \) and \( v_{ss}^{c*} \) Lemma 3. We can then drive the equilibrium \( q_i^{c*}, q_s^{c*}, \pi_i^{c*} \) and \( \pi_s^{c*} \) using the equilibrium prices.

Proof of Proposition 1. Because \( q_i^{d*} = \frac{K_2 - a(b_{il} - b_{si}) + (1 - \alpha)(b_{ss} - b_{ls})}{6t} \) and \( q_i^{b*} = \frac{K_2 - \alpha b_{il} + (1 - \alpha)b_{ss}}{6t} \), \( q_i^{d*} > q_i^{b*} \) if and only if \( -\alpha b_{sl} + (1 - \alpha)b_{ls} > 0 \). The same reason applies to manufacture s.

Proof of Proposition 2. Because \( \pi_i^{b*} = \frac{\alpha(K_2 + b_{il})^2 + (1 - \alpha)(K_2 - b_{ss})^2}{18t} \) and \( \pi_i^{d*} = \frac{3v_i^{d*} + K_2 - 2b_{ls} - b_{ss}}{3} \).
\[ \frac{\alpha(K_2 + b_{li} - b_{sl})^2 + (1-\alpha)(K_2 + b_{li} - b_{ss})^2}{18t} \], \, \pi_i^{d*} > \pi_i^{b*} \quad \text{if and only if} \quad \alpha[b_{sl}^2 - 2(K_2 + b_{li})b_{sl}] + (1 - \alpha)[b_{ls}^2 + 2(K_2 - b_{ss})b_{ls}] > 0, \quad \text{where} \ K_2 = K_1 + 3t. \ \text{The same reason applies to manufacture s.} \]

**Proof of Proposition 3.** Because \( CE^{b*} = e_{in} \frac{K_2-b_{ss}}{6t} (1-\alpha) + e_{sm} \frac{K_2-b_{li}}{6t} \alpha + e_{ssr} \frac{K_3+b_{ss}}{6t} (1-\alpha) \) and \( CE^{d*} = e_{isr} \frac{K_2+b_{li}-b_{ss}}{6t} (1-\alpha) + e_{sr} \frac{K_3+b_{li}-b_{ss}}{6t} \alpha + e_{ssr} \frac{K_3-b_{li}+b_{ss}}{6t} (1-\alpha) \), \( CE^{d*} > CE^{b*} \) \quad \text{if and only if} \quad \alpha[(e_{sr} - e_{in})b_{si} + (e_{sr} - e_{sm})(K_3 - b_{li})] + (1 - \alpha)[(e_{isr} - e_{ssr})b_{ls} + (e_{isr} - e_{in})(K_2 - b_{ss})] > 0, \quad \text{where} \ K_1 = u_i - u_s - C_i + C_s, \ K_2 = K_1 + 3t \quad \text{and} \ K_3 = 3t - K_1.

**Proof of Proposition 4.** Because \( CE^{d*} = e_{isr} \frac{K_2+b_{li}-b_{ss}}{6t} (1-\alpha) + e_{sr} \frac{K_3+b_{li}-b_{ss}}{6t} \alpha + e_{ssr} \frac{K_3-b_{li}+b_{ss}}{6t} (1-\alpha) \), \( CE^{CT*} = e_{isr}(K_2-ce_{ir}+ce_{sr}+b_{li}-b_{sl}) \alpha + e_{ssr}(K_2-ce_{ir}+ce_{sr}+b_{li}-b_{sl}) \alpha \), \( CE^{DCT*} = e_{isr}(K_2-\rho ce_{ir}+ce_{sr}+b_{li}-b_{sl}) \alpha + e_{ssr}(K_2-\rho ce_{sr}+ce_{ir}+b_{li}-b_{sl}) \alpha \). \( CE^{CT*} - CE^{d*} \) is equivalent to \( -\alpha(e_{sr} - e_{in})^2 < (1 - \alpha)(e_{isr} - e_{ssr})^2 \), and \( CE^{CT*} < CE^{d*} \), where \( K_1 = u_i - u_s - C_i + C_s \) and \( K_2 = K_1 + 3t \).

\( CE^{DCT*} < CE^{CT*} \) \quad \text{if and only if} \quad e_{sr} > e_{in} < 0 \quad \text{and} \quad e_{ssr} - e_{isr} < 0.

**Proof of proposition5.** Because \( \pi_i^{d*} = \frac{1}{18t} \left[ \alpha(K_2 + b_{li} - b_{sl})^2 + (1-\alpha)(K_2 + b_{li} - b_{ss})^2 \right] \), \( \pi_i^{CT*} = \frac{1}{18t} \left[ \alpha(K_2 - ce_{ir} + ce_{sr} + b_{li} - b_{sl})^2 + (1-\alpha)(K_2 - ce_{ir} + ce_{sr} + b_{li} - b_{ss})^2 \right] \) and \( \pi_i^{DCT*} = \frac{1}{18t} \left[ \alpha(K_2 - \rho ce_{ir} + ce_{sr} + b_{li} - b_{sl})^2 + (1-\alpha)(K_2 - ce_{ir} + ce_{sr} + b_{li} - b_{ss})^2 \right] \). Where \( K_1 = u_i - u_s - C_i + C_s \) \quad \text{and} \quad K_2 = K_1 + 3t.

\( \pi_i^{d*} < \pi_i^{CT*} \) \quad \text{if and only if} \quad \alpha[c(e_{sr} - e_{in})^2 + 2(e_{sr} - e_{in})(K_2 + b_{li} - b_{sl})] + (1 - \alpha)[c(e_{ssr} - e_{isr})^2 + 2(e_{ssr} - e_{isr})(K_2 + b_{li} - b_{ss})] > 0.

\( \pi_i^{d*} < \pi_i^{DCT*} \) \quad \text{if and only if} \quad \alpha[c(e_{sr} - \rho e_{in})^2 + 2(e_{sr} - \rho e_{in})(K_2 + b_{li} - b_{sl})] + (1 - \alpha)[c(e_{ssr} - e_{isr})^2 + 2(e_{ssr} - e_{isr})(K_2 + b_{li} - b_{ss})] > 0.
\[ \alpha [c(\rho e_{ssr} - e_{isr})^2 + 2(\rho e_{ssr} - e_{isr})(K_2 + b_{ls} - b_{ss})] > 0. \]

**Appendix B**

This appendix illustrates why enterprise \( s \) takes the same simple trade-in strategy as enterprise \( i \) had taken, by comparing Combination \( b \) and Combination \( e \) in terms of enterprise profit. Combination \( e \) where enterprise \( i \) takes simple trade-in strategy and enterprise \( s \) takes none trade-in strategy is modeled, with consumer utility function in Eq. (1), Eq. (2) and Eq. (6), and enterprise profit function in Eq. (5) and Eq. (8). Solving the first-order conditions for the two profit-maximizing enterprises yields the equilibrium profit:

\[
\pi^*_s = \frac{\alpha (K_3 - 2b_{ii})(K_3 - \frac{b_{ii}}{2}) + (1 - \alpha)(K_3 - 2b_{ii})(K_3 + b_{ii})}{18t}. \tag{B.1}
\]

Based on the equilibrium profits of enterprise \( s \) in Combination \( b \) (as shown in Lemma 1) and Combination \( e \) (as shown in Eq. B.1), there is:

\[
\pi^*_{bb} - \pi^*_{ee} = \frac{b_{ii}(K_3 + (1 - \alpha)(2b_{ii}^2 + b_{ss}^2 + K_3(2b_{ss} + b_{ii}))}{18t} > 0. \tag{B.2}
\]

Enterprise \( s \) gets more profit in Combination \( b \) than Combination \( e \), and enterprise \( s \) will take the same simple trade-in strategy as enterprise \( i \) has taken. Similarly, it can be verified that enterprise \( s \) will take the same flexible trade-in strategy as enterprise \( i \) has taken.
**Figure Captions**

**Figure 1.** Consumers’ buying options and the responding utilities

**Figure 2.** Hotelling line of model-a

**Figure 3.** Hotelling line in Case 1

**Figure 4.** Hotelling line in Case 2

**Figure 5.** Sales of two enterprises with the different remanufacturing technology gap

**Figure 6.** Hotelling line in Case 3

**Figure 7.** The impact of market segment size and salvage difference on enterprises’ trade-in strategy choice

**Figure 8.** The segment of the market in two cases

**Figure 9.** The impact of market segment size and salvage difference on enterprises’ trade-in strategy choice

**Figure 10.** Carbons emission with different salvage \( b_{jk} \)

**Figure 11.** Carbons emission with different salvage \( b_{jj} \)

**Figure 12.** The carbon emission caused by consumers in Case 2

**Figure 13.** Total carbon emission with different carbon tax rate \( c \)

**Figure 14.** Profit of enterprise \( i \) with different carbon tax rate \( c \)
Figure 1. Consumers’ buying options and the responding utilities

\[ U_i = u_i - tx - P_i \]
\[ U_s = u_s - t(1-x) - P_s \]
\[ U_i = u_i - tx - P_i + v_{i(i)} \]
\[ U_s = u_s - t(1-x) - P_s + v_{ss} \]

Figure 2. Hotelling line of model-a

Figure 3. Hotelling lines in Case 1

Figure 4. Hotelling lines in Case 2
Note: In this example, we set $C_i = 0.4$, $C_s = 0.3$, $u_i = 0.4$, $u_s = 0.3$, $t = 0.1$, $\alpha = 0.7$

**Figure 5.** Sales of two enterprises with the different remanufacturing technology gap

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$x_1^i$</th>
<th>$x_1^s$</th>
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**Market segment 1**

<table>
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<tr>
<th>$1 - \alpha$</th>
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<th>$x_2^s$</th>
</tr>
</thead>
<tbody>
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</table>

**Market segment 2**

**Figure 6.** Hotelling lines in Case 3
Figure 7. The impact of market segment size and salvage difference on enterprises’ trade-in strategy choice (with ratio $\frac{b_{ls}}{b_{si}}$).

Figure 8. The segment of the market in two cases
Figure 9. The impact of market segment size and salvage difference on enterprises’ trade-in strategy choice (with ratio $\frac{b_{ij}^2 + 2(K - b_{ss})b_{ij}}{b_{si}^2 - 2(K + b_{li})b_{si}}$).

Note: This example sets $C_i = 0.4$, $C_s = 0.3$, $u_i = 0.4$, $u_s = 0.3$, $t = 0.1$, $\alpha = 0.7$, $e_{lir} = 0.2$, $e_{s;r} = 0.15$, $e_{i;r} = 0.34$, $e_{s;i} = 0.25$, $e_{sn} = 0.3$, $e_{ln} = 0.4$, $b_{li} = 0.3$, $b_{ss} = 0.2$

Figure 10. Carbons emission with different salvage $b_{jk}$.
Note: This example sets $C_i = 0.4$, $C_s = 0.3$, $u_i = 0.4$, $u_s = 0.3$, $t = 0.1$, $\alpha = 0.7$, $e_{iir} = 0.2$, $e_{ssr} = 0.15$, $e_{sir} = 0.34$, $e_{ssr} = 0.25$, $e_{sn} = 0.3$, $e_{in} = 0.4$, $b_{ls} = 0.1$, $b_{sil} = 0.15$

**Figure 11.** Carbons emission with different salvage $b_{jj}$

**Figure 12.** The carbon emission caused by consumers in Case 2
Note: In this example, we set $C_i = 0.4$, $C_s = 0.3$, $u_i = 0.4$, $u_s = 0.3$, $t = 0.1$, $\alpha = 0.7$, $e_{iir} = 0.2$, $e_{ssr} = 0.15$, $e_{isr} = 0.34$, $e_{sir} = 0.25$, $b_{ls} = 0.1$, $b_{sl} = 0.15$, $\rho = 0.7$

Figure 13. Total carbon emission with different carbon tax rate $c$

Note: In this example, we set $C_i = 0.4$, $C_s = 0.3$, $u_i = 0.4$, $u_s = 0.3$, $t = 0.1$, $\alpha = 0.7$, $e_{iir} = 0.2$, $e_{ssr} = 0.15$, $e_{isr} = 0.34$, $e_{sir} = 0.25$, $b_{ls} = 0.1$, $b_{sl} = 0.15$, $\rho = 0.7$

Figure 14. Profit of enterprise $i$ with different carbon tax rate $c$