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A Time-Varying True Individual Effects Model with Endogenous Regressors

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Abstract

We propose a fairly general individual effects stochastic frontier model, which allows both heterogeneity and inefficiency to change over time. Moreover, our model handles the endogeneity problems if either at least one of the regressors or one-sided error term is correlated with the two-sided error term. Our Monte Carlo experiments show that our estimator performs well. We employed our methodology to the US banking data and found a negative relationship between return on revenue and cost efficiency. Estimators ignoring time-varying heterogeneity or endogeneity did not perform well and gave very different estimates compared to our estimator.

Keywords: Endogeneity; Panel data; Stochastic frontier; True fixed effects; Time-varying heterogeneity.

JEL Classification: C13, C23, C36.

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1. Introduction

Conventional panel data stochastic frontier models do not disentangle productive unit specific heterogeneity from inefficiency. As a consequence, these models may be picking up the combined effect of heterogeneity and inefficiency. Hence, the heterogeneity is likely to be confused with inefficiency, which may lead to distorted inefficiency estimates. In order to overcome this difficulty, Greene (2005a,b) proposes the true fixed effects (TFE) model, which is the standard fixed effects model augmented by the inefficiency effect. The heterogeneity is captured by productive unit specific dummies and the inefficiency is captured by a one-sided error term. The dummy variable approach can be computationally impractical to implement when the number of productive units is large. Moreover, TFE model is subject to the incidental parameters problem. The simulation results of Greene suggest that, albeit the MLE of the model parameters is consistent, the estimates of error variances are inconsistent unless both the number of time periods and panel units go to infinity.

Wang and Ho (2010) propose first difference and within transformations of the TFE model.¹ After these model transformations, the number of parameters does not depend on the number of productive units. Therefore, the transformed models of Wang and Ho (2010) do not suffer from the incidental parameters problem. Another potential problem with the TFE model is that the heterogeneity is time-invariant but the inefficiency can vary over time. For longer panel data it is likely that both heterogeneity and inefficiency change over time. When the TFE model is used, the time-varying portion of the heterogeneity may distort the inefficiency estimates. We address this issue by proposing a time-varying true individual effects (TVTIE) model where both heterogeneity and inefficiency can change over time. In contrast to Cornwell, Schmidt, and Sickles (1990), who model the inefficiency via second-degree time polynomials, we propose modeling heterogeneity using such polynomials. We deal with the incidental parameters by model transformation (orthogonal projection) so that the number of parameters does not depend on the number of productive units.

The traditional stochastic frontier models assume that the variables included in the cost (or production) equation are exogenous, i.e., they are independent from the two-sided error term. In these models, this assumption is vital for getting consistent parameter and inefficiency estimates. However, in a variety of settings such an assumption may be strong and should be tested. For example, when quality is a part of the production process where quality and quantity decisions are made simultaneously, quality would be an endogenous variable. If the quality variable is included in the cost equation, the parameter estimates would be inconsistent as quality would be correlated with the two-sided error

¹ See also Belotti and Ilardi (2018) and Chen, Schmidt, and Wang (2014).

term.² On the other hand, if the quality variable is cost enhancing and omitted from the cost equation, then the efficiency of a producer with high quality product would be under-estimated. Mutter et al. (2013) discuss and examine this issue in the health economics context.

For standard stochastic frontier models, econometric solutions to this type of endogeneity problem are proposed by Guan et al. (2009), Kutlu (2010), Tran and Tsionas (2013), Griffiths and Hajargasht (2016), and Karakaplan and Kutlu (2017a,b).³ Another assumption that is predominantly used, yet a strong one, is that of the independence of two-sided and one-sided error terms. A potential example where these error terms maybe correlated is when a market power measure, such as Lerner index, or a profitability measure, such as return on revenue⁴ (ROR), is used in the modelling of inefficiency distribution. It is widely accepted that inefficiency and market power are closely related. The quiet life hypothesis of Hicks (1935) states that the firms with market power are likely to be relatively less efficient due to the management's subjective cost of reaching optimal profit levels. By similar reasoning, we would expect the manager of a firm with high ROR to be less careful about reaching optimal profit levels. On the other hand, as suggested by the efficient structure hypothesis (Demsetz, 1973), highly efficient firms can be more successful in achieving higher ROR levels. Hence, the use of market power or profitability measure as inefficiency determinant is justified when modeling the inefficiency component. Note, however, that if this determinant is endogenous in the sense that it is correlated with the two-sided error term, then the independence assumption of two-sided and one-sided error terms would not hold. For instance, the feedback effect between efficiency and market power (similarly, profitability) argued by quiet life hypothesis and efficient structure hypothesis suggests that ROR may be an endogenous variable. Indeed, by definition, ROR is a function of cost, which makes it very likely to be an endogenous variable when estimating a stochastic cost frontier.⁵ Our simulation results indicate that ignoring endogeneity may lead to seriously biased parameter and efficiency estimates. In the cross sectional data setting Karakaplan and Kutlu (2017a) and Amsler, Prokhorov, and Schmidt (2016, 2017) and in the panel data setting Griffiths and Hajargasht (2016), and Karakaplan and Kutlu (2017b) exemplify studies that address this issue.⁶ In order to deal with these endogeneity issues, we extend our TVTIE model further to allow endogenous variables, which we call time-varying true individual

² Jamasb, Orea, and Pollitt (2012) provide, in a cost setting, another source of endogeneity of a quality variable in the electricity distribution industry.

³ See Karakaplan and Kutlu (2018b) for an application of Karakaplan and Kutlu (2017a).

⁴ Return on revenue is equal to the ratio of profit to revenue.

⁵ In general, common determinants of the inefficiency variables (e.g., ROR) and the two-sided error term are also reasons for endogeneity. For example, a negative supply shock may also affect the morale of the managers, which in turn may result in efficiency loss as well.

⁶ See Shee and Stefanou (2014) for some examples and an extension of the Levinsohn and Petrin (2003) approach, which overcomes the problem of endogenous input choice that is due to production shocks that are predictable by the producer yet unknown to the econometrician.

effects with endogenous regressors (TVTIEE) model.

While relatively longer panel data sets became more available, not every time the researcher may have such a long panel data set. Our estimator can also be used when the heterogeneity is time-invariant or it depends on some variables other than functions of time trends. For instance, in a cross-sectional data setting the spatial heterogeneity may be modelled using a distance-varying true individual effects term. Hence, our model can be applied to settings where the heterogeneity occurs through a variety of dimensions such as time and space. For illustrative purposes, we concentrate mostly on time-varying heterogeneity.

In addition to productivity and efficiency estimation, the TVTIEE estimator may also be used for estimating market power. Recently, Orea and Steinbuck (2018) and Karakaplan and Kutlu (2018a) proposed conduct parameter models⁷ in which the firm conducts are modelled by a doubly truncated normally distributed random variable.⁸ These models enable the researchers to estimate firm and time specific conducts without imposing excessive parametric restrictions on the conducts. These models estimate a demand-supply relation system which is, by construction, subject to endogeneity.⁹ Hence, our estimator has an important use in the industrial organization literature. Another application, where endogeneity is present, is joint estimation of firm conduct and marginal cost efficiency without using the total cost data (Kutlu and Wang, 2018). In this setting, rather than estimating a cost function, a demand-supply system is estimated.

As an application to our methodology, we examine the relationship between cost efficiency and profitability, which is measured by ROR, for the big US insured banks for time period between 1976 and 2007. Hence, we concentrate on profitability rather than market power. These two concepts are closely related. However, from the perspective of a manager, ROR is observed relatively easier; and thus ROR may have a more direct effect on the performance of manager. Also, the manager would be interested in market power, mostly, because of its effect on profit. The banking literature generally uses a two-stage approach for examining such a relationship. In the first stage, the efficiencies of banks would be estimated using a standard parametric or non-parametric method, which does not control for endogeneity. In the second stage, the effect of market power on efficiency would be estimated using an instrumental variables method. The problem with this approach is that the efficiency estimates in the first stage may be biased, which would contaminate the second stage estimates. It seems that the lack of proper stochastic frontier methods that can handle endogeneity was the main historical reason

⁷ See Bresnahan (1989) and Perloff, Karp, and Golan (2007) for extensive surveys on conduct parameter approach.

⁸ Almanidis, Qian, Sickles (2014) introduced doubly truncated normal distribution to the stochastic frontier models.

⁹ In this setting, the supply equation has a composed error term where the one-sided component is used for measuring firm conduct.

for using such a two-stage procedure. The TVTIEE estimator enables us to examine the relationship between return on revenue and cost efficiency in a single stage. Although recent developments in the SFA literature enabled solutions to endogeneity problems, TVTIEE is the first estimator that can solve both heterogeneity and endogeneity problems. That is, our proposed TVTIEE is a general model that unifies many models including cross-sectional and panel data SFA models with or without endogeneity. In addition, this estimator allows general heterogeneity patterns in the fixed/individual effects settings, which were not present in the earlier SFA models. If heterogeneity is present and is not controlled, separate identification of inefficiency and heterogeneity in a cross-section setting is a problem, which can be handled by our estimator.

Our empirical findings are striking. It turns out that the Pearson correlation of cost efficiency estimates from the TVTIEE model and Greene's (2005a,b) model is -0.56. Moreover, the Pearson correlation of cost efficiency estimates from the TVTIEE model and Wang and Ho's (2010) model is 0.36, which is a considerably low correlation. We observe a similar pattern when we calculate the corresponding Pearson correlations with TVTIE (instead of TVTIEE). These results indicate that a model that does not consider time-varying heterogeneity may have seriously flawed estimates for efficiency.

The paper is organized as follows. In the next section, we provide a brief summary of the literature. In Section 3, we present our theoretical models. Section 4 gives the Monte Carlo simulations. Our empirical findings are presented in Section 5. Finally, in Section 6, we make our conclusions.

2. A Brief Review of Literature

In this section, we present a short review of the relevant literature and introduce our notation. Consider the following stochastic frontier model:

$$y_{it} = \alpha_{it} + x'_{1it}\beta - su_{it} + v_{it} \quad (1)$$

where hereafter, $s = 1$ for production function and $s = -1$ for cost function; y_{it} is the logarithm of the output or cost of the i^{th} productive unit at time t ; α_{it} is a term that is capturing the heterogeneity; $u_{it} \geq 0$ is a term that is capturing the inefficiency; x_{1it} is an exogenous vector of variables which does not contain the constant; and v_{it} is the usual two-sided error term. Schmidt and Sickles (1984) consider a special case of (1) where $\alpha_{it} = \alpha$ and the inefficiency is time-invariant ($u_{it} = u_i$) so that $a_i = \alpha - u_i$. The slope parameters of this model can be consistently estimated by the within groups

least squares estimator. The inefficiency term u_i is estimated by the relative inefficiency estimator:

$$\hat{u}_i = \begin{cases} \max_i(a_i) - a_i & \text{for a production frontier} \\ a_i - \min_i(a_i) & \text{for a cost frontier} \end{cases} \quad (2)$$

Cornwell, Schmidt, and Sickles (1990) relax the time-invariant inefficiency assumption by replacing a_i with $a_{it} = a_{i0} + a_{i1}t + a_{i2}t^2$.¹⁰ This individual effects approach benchmarks the best productive unit as fully efficient. The inefficiencies of the other productive units are determined relative to this benchmark. This model is easy to estimate and is distribution free. However, this approach attributes all the heterogeneity to inefficiency. Hence, the inefficiency estimates may potentially be seriously distorted.¹¹

It is possible to identify heterogeneity and inefficiency through the distributional assumptions made on the error terms, u_{it} and v_{it} . Surprisingly, many of the stochastic frontier models provide little to no mechanism for disentangling the heterogeneity and inefficiency. The true fixed effects model of Greene (2005a,b) aims to handle this identification issue. Greene's TFE model can be obtained from (1) by setting $\alpha_{it} = \alpha_i$, and by assuming that $u_{it} \sim N^+(0, \sigma_u^2)$ and $v_{it} \sim N(0, \sigma_v^2)$. This model extends the familiar fixed effects linear regression model with normal errors to the stochastic frontier framework. Thus, it is subject to possible inconsistency due to the number of parameters growing with the number of firms (i.e., the "incidental parameters problem"). Indeed, simulation results of Greene (2005a,b) suggest that albeit, the MLE of β is consistent as number of productive firms goes to infinity, the MLE of error variances are inconsistent unless the number of time periods goes to infinity as well.

To alleviate the "incidental parameters problem", Chen, Schmidt, and Wang (2014) take a different approach by first using within transformations to eliminate the fixed effects; and then applying the closed-skew normal (CSN) distribution results to obtain the joint density and the log-likelihood function of the resulting within transformations. Consistent estimation of the model's parameters is obtained by maximizing this log-likelihood function. The main advantage of their approach is that it is no longer subject to the incidental parameters problem. Their simulation results show that the within MLE performs well in finite samples.

¹⁰ See Kutlu (2018) for solutions to endogeneity problems in the context of Schmidt and Sickles (1984) and Cornwell, Schmidt, and Sickles (1990).

¹¹ Another issue with this approach is that the efficiency estimates may be sensitive to outliers. For this reason, some researchers (e.g., Berger, 1993, Berger and Hannan, 1998, and Kutlu, 2012), trim the individual effects estimates from top and bottom. See Kutlu (2017) for an alternative solution that does not require trimming.

Wang and Ho (2010) extend TFE model to allow for the one-sided term to have scaling property. More specifically, they assume that u_{it} is governed by¹²:

$$\begin{aligned} u_{it} &= h_{it} u_i^* \\ h_{it} &= f(x'_{2it} \phi_u), \end{aligned} \tag{3}$$

where $u_i^* \geq 0$ and x_{2it} is an exogenous vector of variables, which does not contain the constant. It is assumed that $u_i^* \sim N^+(\mu, \sigma_u^2)$, and u_i^* is independent from x_{1it} , x_{2it} , and v_{it} . This model nests some of the earlier stochastic frontier models such as Kumbhakar (1990), Reifschneider and Stevenson (1991), Battese and Coelli (1992), and Caudill and Ford (1993).¹³ To estimate the unknown parameters in the model, Wang and Ho (2010) propose two approaches by using the first difference and within transformations to remove the fixed effects. For each type of transformation, the marginal log-likelihood is derived, and the transformation MLE is obtained by maximizing the marginal log-likelihood function. They show that, albeit the two approaches may seem different, the log-likelihood functions are identical and hence the estimates from the both approaches are numerically identical.

Insofar, all the models described above mainly focus on capturing heterogeneity and assumed that x_{1it} and x_{2it} are exogenous in the sense that they are independent from v_{it} . We now turn our attention to the endogenous stochastic frontier models. In the cross-section data framework, endogeneity in stochastic frontier models have recently been addressed by Tran and Tsionas (2015), Amsler, Prokhorov, and Schmidt (2016, 2017) and Karakaplan and Kutlu (2017a). These approaches can be extended to the case of *pooled* panel data SFM; however, in the context of TFE framework, these approaches are no longer valid and require different strategies to solve the endogeneity problem.

To the best of our knowledge, there exists only a few research on the fixed effects type stochastic frontier panel models with endogenous regressors. Guan et al. (2009) proposed a fixed effect stochastic frontier panel model as in (1) (with $\alpha_{it} = \alpha_i$) along with heteroskedastic errors to measure the excess capital capacity in agricultural production. In their model, they allowed some components of x_{1it} to be correlated with v_{it} but assumed the environmental variables that appeared in the errors variances are exogenous. Guan et al. (2009) proposed a two-stage estimation procedure where in the first stage, frontier parameters associated with time varying regressors are consistently estimated using GMM approach based on the orthogonality conditions of $e_{it} = \alpha_i + v_{it} - u_{it}$. Then, in the second

¹² In order to differentiate the models of Greene (2005a,b) and Wang and Ho (2010), we will refer Greene's TFE model as GTFE.

¹³ For more details about scaling property see Wang and Schmidt (2002).

stage, the remaining parameters are estimated based on the log-likelihood function of an auxiliary stochastic frontier regression where the estimated residuals from the first stage are used as the dependent variable.

Recently, Griffiths and Hajargasht (2016) proposed three different stochastic panel frontier models that allow for endogenous regressors. However, none of these models considered the fixed effects approach. The first model assumed inefficiency is time-invariant (i.e., $u_{it} = u_i$), and the endogeneity entered the model via Chamberlain-Mundlak formulation by allowing for the transformation of u_i to depend on some or all of the firm averages of inputs. The second model extend the first model to also allow for time-varying inefficiency along with time-invariant inefficiency. The final model considered only time-varying inefficiency but endogeneity is extended to also allow for inputs to be correlated with the two-sided error term. Bayesian inferences are used to obtain consistent estimation for all three models.

Another variation of the stochastic panel frontier model which was considered by Colombi et al. (2014), Kumbhakar, Lien, and Hardaker (2014), and Lai and Kumbhakar (2017a,b) is the four-component stochastic frontier model of the form:

$$y_{it} = \alpha_i + x'_{it}\beta - s\nu_i - su_{it} + v_{it}, \quad (4)$$

where α_i is time-invariant heterogeneity, ν_i is time-invariant firm persistent inefficiency, and u_{it} is time-varying transient inefficiency. This is a reasonably general model that nests models of Schmidt and Sickles (1984), Greene (2005a,b), Wang and Ho (2010), and Griffiths and Hajargasht (2016) as special cases.

Kumbhakar, Lien, and Hardaker (2016) use a three-step approach, where in the first step, standard random effects estimation procedure is used to obtain the estimates of β , and the estimates of the remaining parameters are obtained in the second and third-step using standard MLE. Colombi et al. (2014) take a different approach by applying CSN distributions result to obtain the log-likelihood function and then maximized it directly to obtain the parameter estimates in one-step. Lai and Kumbhakar (2017a) extend the model in (4) to allow for the time-invariant and time-varying determinants of inefficiency to enter the variances of ν_i and u_{it} , respectively; whilst Lai and Kumbhakar (2017b) also allow for x_{it} to be correlated with α_i and ν_i . In term of estimation procedures for both models, they use difference and within transformation to first remove the time-invariant components, and then apply CSN distribution results to construct the joint density and the log-likelihood function

of the resulting transformation of the composed-error. Finally, the simulated MLE is used to obtain the consistent estimates of all the parameters in the model. Note that, albeit model (4) is quite general, it does not actually address the time-varying heterogeneity nor the endogeneity issues.

3. Theoretical Model

3.1 Time-Varying Heterogeneity

The TFE model assumes that the heterogeneity is time-invariant but the inefficiency can vary over time. There is no priory reason to believe that the heterogeneity is time-invariant while inefficiency is time-variant. In particular, for longer panel data it is likely that both heterogeneity and inefficiency change over time. If the heterogeneity changes over time, the time-varying portion of the heterogeneity may distort the inefficiency estimates. For example, in the farming context, the soil quality and microclimate change over time due to global warming. Karmalkar and Bradley (2017) argue that the regional warming rates differ substantially (fastest being Northeast) within the US. As a consequence, water availability, plant heat stress, frost damages may vary and reduce yields heterogeneously over timer. In the banking context, new technology adaptation may differ, the effect of regulations may gradually change over time, relative technology improvement costs may change over time, etc. Moreover, the time-varying heterogeneity term serves as an approximation to unobserved time-varying factors that are beyond the control of the firm.

We propose a model that aims to address this issue. First, we introduce some useful notations, and these notations will be used for the rest of the paper unless otherwise noted. Let $y_i = (y_{i1}, y_{i2}, \dots, y_{iT_i})'$ be a $T_i \times 1$ vector, $x_{1i} = (x_{1i1}, x_{1i2}, \dots, x_{1iT_i})'$ be a $T_i \times k_1$ matrix, $x_{3i} = (x_{3i1}, x_{3i2}, \dots, x_{3iT_i})'$ be a $T_i \times k_3$ matrix, where T_i is the number of time periods for panel i , and other variables u_i and v_i are defined similarly. Consider the following time-varying “true” fixed effects model:

$$\begin{aligned} y_i &= x_{3i} \alpha_i + x_{1i} \beta - s u_i + v_i \\ u_i &= h_i u_i^* \\ h_{it} &= f(x_{2it}' \varphi_u) \geq 0 \\ u_i^* &\sim N^+(\mu, \sigma_u^2) \\ v_{it} &\sim N(0, \sigma_v^2), \end{aligned} \tag{5}$$

where x_{3it} is a $k_3 \times 1$ vector of exogenous variables capturing the heterogeneity; x_{1it} , x_{2it} , and x_{3it}

are allowed to be freely correlated with each other; α_i is a productive unit specific coefficient vector, and other variables are defined as earlier. For identification purpose, we assume that x_{3it} and x_{1it} have no elements in common. For example, x_{1it} contains the usual input variables (in case of production) such as capital, labor, materials, etc.; whilst a potential choice for x_{3it} may be $x_{3it} = (1, t, t^2)'$ as in Cornwell, Schmidt, and Sickles (1990). This choice may also (approximately) control for omitted variables and measurement errors. Another interesting choice for x_{3it} would be $x_{3it} = (1, d_{it}, d_{it}^2)'$ where d_{it} stands for spatial distance for panel unit i at time t , which may or may not refer to a physical distance. For example, in a differentiated products setting, the distance may be an index representing the quality differences of a product relative to a benchmark. Alternatively, firm size can be included in the heterogeneity term (e.g., Almanidis, 2013). This would be in line with many productivity studies that model heterogeneity as a function of firm size. Hence, the heterogeneity can be modelled in a variety of ways, and the choice of which variables to include when modelling heterogeneity depends on the particularities of the production process. As we mentioned earlier, for illustrative purposes, we prefer to stay with the time-varying heterogeneity scenario.

For this general setting, the brute force approach is impractical and the incidental parameters problem is more serious. We solve these issues by transforming the model. Equation (5) can be transformed by the matrix $M_{x_{3i}} = I_{T_i} - x_{3i}(x'_{3i}x_{3i})^{-1}x'_{3i}$, which is the orthogonal projection matrix onto the space that is orthogonal to columns of x_{3i} . After applying this transformation to Equation (5), the transformed model becomes:

$$\tilde{y}_i = \tilde{x}_{1i}\beta - s\tilde{u}_i + \tilde{v}_i, \quad (6)$$

where $\tilde{y}_i = M_{x_{3i}}y_i$, $\tilde{x}_{1i} = M_{x_{3i}}x_{1i}$, $\tilde{v}_i = M_{x_{3i}}v_i$, $\tilde{u}_i = M_{x_{3i}}u_i$, and $\tilde{h}_i = M_{x_{3i}}h_i$. Hence, in our notation “ $\sim$ ” stands for the transformed variables. The values of these variables at time t are denoted similarly. Note that the matrix $M_{x_{3i}}$ is a singular idempotent matrix and hence it is not invertible, so the transformed variables would have degenerate distributions, i.e., distribution with rank-deficient correlation matrix.¹⁴ In addition, this distribution is not absolutely continuous, so the density is not well defined. Consequently, the likelihood function has an interpretation only with the reference to

¹⁴ An idempotent matrix A is invertible if and only if $A = I$ where I is an identity matrix. Clearly $M_{x_{3i}}$ is not an identity matrix here.

$T_i - k_3$ linear combination of $\tilde{v}_i$ subspace. To remedy this situation, we make use of the singular multivariate normal distribution of Khatri (1968, Section 3). The density function of $\tilde{v}_i$, which is defined on a $(T_i - k_3)$ subspace, is given by:

$$f_{\tilde{v}_i}(\tilde{v}_i) = 2\pi\sigma_v^2 \left| M_{x_{3i}}^* \right|^{-1/2} \exp \left(-\frac{1}{2\sigma_v^2} \tilde{v}_i' M_{x_{3i}}^- \tilde{v}_i \right), \quad (7)$$

where $\left| \cdot \right|^*$ denotes the pseudo determinant and the superscript “ $-$ ” denotes the generalized inverse of a matrix.¹⁵ We will use Moore-Penrose pseudo inverse, which somewhat simplifies our analysis and decreases computational burden. The likelihood function of this distribution is defined through Radon-Nikodym derivative and is not unique as the dominating measure is not unique. However, each would lead to the same maximum likelihood estimate, which is sufficient for our purposes. Note that an eigenvalue of an idempotent matrix is either one or zero. Hence, the number of non-zero eigenvalues of $M_{x_{3i}}$ equals $\text{tr}(M_{x_{3i}}) = T_i - k_3$. Consequently, (7) can be simplified to:

$$f_{\tilde{v}_i}(\tilde{v}_i) = 2\pi\sigma_v^2^{-(T_i - k_3)/2} \exp \left(-\frac{1}{2\sigma_v^2} \tilde{v}_i' \tilde{v}_i \right). \quad (8)$$

The marginal likelihood of the model is then derived based on the joint distribution of $\tilde{v}_i$ and $\tilde{u}_i$. The marginal log-likelihood contribution of the i^{th} panel is then given by (for constant x_{3i} case see Wang and Ho, 2010):

$$\ln L_i = -\frac{1}{2}(T_i - k_3) \ln(2\pi\sigma_v^2) - \frac{1}{2} \frac{\tilde{e}_i' \tilde{e}_i}{\sigma_v^2} + \frac{1}{2} \left(\frac{\tilde{\mu}_{i*}^2}{\tilde{\sigma}_{i*}^2} - \frac{\mu^2}{\sigma_u^2} \right) + \ln \left(\frac{\tilde{\sigma}_{i*} \Phi(\frac{\tilde{\mu}_{i*}}{\tilde{\sigma}_{i*}})}{\sigma_u \Phi(\frac{\mu}{\sigma_u})} \right), \quad (9)$$

where $\tilde{\mu}_{i*} = \frac{-s\sigma_u^2 \tilde{e}_i' \tilde{h}_i + \mu\sigma_v^2}{\sigma_u^2 \tilde{h}_i' \tilde{h}_i + \sigma_v^2}$, $\tilde{\sigma}_{i*}^2 = \frac{\sigma_v^2 \sigma_u^2}{\sigma_u^2 \tilde{h}_i' \tilde{h}_i + \sigma_v^2}$, $\tilde{e}_i = \tilde{y}_i - \tilde{x}_{1i} \beta$, and Φ denote the standard

normal CDF. By maximizing the total log-likelihood function $\mathcal{L} = \sum_{i=1}^n \ln L_i$, we obtain the ML estimates of all the parameters of the model. Standard results show that the ML estimator is consistent for $n \rightarrow \infty$ and T_i is fixed.

¹⁵ For a positive semi-definite matrix A with at least one non-zero eigenvalue, $|A|^*$ is equal to the product of the non-zero eigenvalues.

Prediction of u_{it} :

Once all the estimates of the parameters are obtained, we can predict u_{it} and efficiency by:¹⁶

$$E[u_{it} | \tilde{e}_i] = h_{it} \left[\tilde{\mu}_{i*} + \frac{\tilde{\sigma}_{i*} \phi\left(\frac{\tilde{\mu}_{i*}}{\tilde{\sigma}_{i*}}\right)}{\Phi\left(\frac{\tilde{\mu}_{i*}}{\tilde{\sigma}_{i*}}\right)} \right], \quad (10a)$$

and

$$EFF_{it} = \exp(-E[u_{it} | \tilde{e}_i]), \quad (10b)$$

which are evaluated at $\hat{\tilde{e}}_i = \tilde{y}_i - \tilde{x}_{1i}'\hat{\beta}$.

Prediction of α_i :

It is possible to recover the individual effects term and its parameters by maximizing the log-likelihood of the untransformed model and plugging the parameter estimates into the relevant first order conditions. This gives us the following recursive formula:

$$\hat{\alpha}_i^{rec} = (x'_{3i} x_{3i})^{-1} x'_{3i} (y_i - x_{1i}'\hat{\beta} + sE[u_i | \hat{\tilde{e}}_i]), \quad (11a)$$

where $\hat{\tilde{e}}_i = y_i - x_{3i}'\hat{\alpha}_i^{rec} - x_{1i}'\hat{\beta}$. In practice, we can replace $E[u_i | \hat{\tilde{e}}_i]$ by $E[u_{it} | \hat{\tilde{e}}_i]$ so that the parameters for individual effects term are estimated by:

$$\hat{\alpha}_i = (x'_{3i} x_{3i})^{-1} x'_{3i} (y_i - x_{1i}'\hat{\beta} + sE[u_{it} | \hat{\tilde{e}}_i]). \quad (11b)$$

Basically, this formula is equivalent to regressing “residuals” from untransformed model on x_{3i} and predicting the individual effects parameters by the coefficients obtained from this regression.

3.2 Endogeneity

We are interested in the case where the vectors x_{1it} and x_{2it} may contain endogenous variables. The conventional stochastic frontier models depend on the assumption that these variables are

¹⁶ This predictor of u_{it} is an extension of the conditional expectation introduced by Wang and Ho (2010). Note that the term in the bracket is a predictor for u_i^* .

exogenous. In our framework, a variable is said to be exogenous if it is independent from v_{it} . Hence, the variables that are correlated with v_{it} would be endogenous. For more details on the definition of exogeneity/endogeneity in our context, see Definition 3.2.1 below.¹⁷ If x_{2it} also contains endogenous variables, then u_{it} and v_{it} would not be independent. This contrasts with the conventional independence assumption for u_{it} and v_{it} . Simulations of Kutlu (2010) and Karakaplan and Kutlu (2017a) show that if any of x_{1it} and x_{2it} contains an endogenous variable, the parameter estimates can be substantially biased. We use a single-stage control function approach (i.e., a LIML approach) to deal with the endogeneity issue. In order to make the differences between the assumptions of present model and earlier models clearer, we redefine the variables. Consider the following model:

$$\begin{aligned} y_{it} &= x_{3it}\alpha_i + x_{1it}\beta - su_{it} + v_{it} \\ x_{it} &= z_{it}\delta + \varepsilon_{it} \\ u_{it} &= h_{it}u_i^* \\ h_{it} &= f(x_{2it}'\varphi_u) > 0, \end{aligned} \tag{12}$$

where y_{it} is the logarithm of output (or cost) of the i^{th} productive unit a time t ; x_{3it} is a $T_i \times k_3$ vector of exogenous variables which capture heterogeneity; x_{1it} and x_{2it} are k_1 and k_2 vectors of variables that may contain endogenous variables, and for identification purposes, the dimension of x_{3it} which is $k_3 < T_i$ and neither x_{1it} nor x_{2it} contains constants; x_{1it} , x_{2it} , and x_{3it} are allowed to be freely correlated with each other; x_{it} is a $p \times 1$ (where $p \leq k_1 + k_2$) vector of all the endogenous variables excluding y_{it} , that is, $x_{it} \subseteq (x_{1it} \cup x_{2it})$; z_{it} is an $l \times 1$ vector of exogenous instruments where $l \geq p$;¹⁸ v_{it} is the usual two-sided error term; $u_{it} \geq 0$ is the one-sided error term capturing the inefficiency; and $u_i^* \geq 0$ is a productive unit specific random component independent from v_{it} and ε_{it} . Here δ is an $l \times p$ matrix of coefficients, and ε_{it} is a $p \times 1$ vector of random errors. To be more specific about what we mean by endogenous, it would be useful to provide the following

¹⁷ Note that Definition 3.2.1 restricts the variables to be correlated with only v_{it} . Remark 4 below extends the analysis to the case where the variables are allowed to be correlated with both v_{it} and u_i^* .

¹⁸ When the endogenous variables are functions of each other, $l \geq p$ condition can be relaxed. For example, as explained by Amsler, Prokhorov, and Schmidt (2016), in a translog cost function with 2 endogenous input prices, it is possible to achieve identification by only two control functions, rather than 5.

definition.

Definition 3.2.1: Let $w_{it} = (x'_{3it}, z'_{it})'$ and $w_i = (x_{3i}, z_i)$. In Model (12), (a) x_{it} is endogenous if $E[v_{it} | x_{it}] \neq 0$; (b) w_{it} is exogenous if (i) $E[v_{it} | w_i] = 0$ and; (ii) $E[u_i^* | w_i, x_i] = E[u_i^* | x_i] = E[u_i^*]$.

Let Ω_ε denote a $p \times p$ variance-covariance matrix of ε_{it} , and $\zeta_{it}^* = (\varepsilon_{it}^*, v_{it})' = (\varepsilon_{it}' \Omega_\varepsilon^{-1/2}, v_{it})'$. We make the following assumption:

Assumption 1:

(i) $u_i^* | w_i \sim N^+(\mu, \sigma_u^2)$, and

(ii) $\zeta_{it}^* | w_i \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} I_p & \sigma_v \rho \\ \sigma_v \rho' & \sigma_v^2 \end{bmatrix}\right) = N(0, \Omega)$, where $N(.,.)$ denotes the multivariate normal, σ_v^2

is the variance of v_{it} , and ρ is the vector representing the correlation between ε_{it}^* and v_{it} .

Assumption 1 (i) implies that u_i^* is independent of all the exogenous variables, whilst Assumption 1 (ii) assumes that, conditional on the instruments w_i , the correlation between x_{it} and v_{it} are captured via the correlation between the errors ε_{it} and v_{it} .

LIML Estimation:

To estimate the parameters of the model, we first eliminate the individual effects $x_{3i} \alpha_i$ by multiplying each panel unit in Equation (12) by $M_{x_{3i}}$ as before. The model after the transformation is:¹⁹

$$\begin{aligned} \tilde{y}_i &= \tilde{x}_{1i} \beta - s \tilde{u}_i + \tilde{v}_i \\ \tilde{x}_i &= \tilde{z}_i \delta + \tilde{\varepsilon}_i \\ \tilde{u}_i &= \tilde{h}_i u_i^* \end{aligned} \tag{13}$$

¹⁹ We denote the vectorization operator by $vec(.)$.

and

$$\begin{bmatrix} vec(\varepsilon_{i.}^{*'}) \\ v_{i.} \end{bmatrix} = \begin{bmatrix} (I_{T_i} \otimes \Omega_{\varepsilon}^{-1/2}) vec(\varepsilon_{i.}') \\ v_{i.} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} I_{pT_i} & \sigma_v(I_{T_i} \otimes \rho) \\ \sigma_v(I_{T_i} \otimes \rho') & \sigma_v^2 I_{T_i} \end{bmatrix} \right).$$

By a Cholesky decomposition of the variance-covariance matrix of $(vec(\varepsilon_{i.}^{*'}), v_{i.}')'$ we get:

$$\begin{bmatrix} vec(\varepsilon_{i.}^{*'}) \\ v_{i.} \end{bmatrix} = \begin{bmatrix} I_{pT_i} & 0 \\ \sigma_v(I_{T_i} \otimes \rho') & \sigma_v \sqrt{1 - \rho' \rho} I_{T_i} \end{bmatrix} \begin{bmatrix} vec(\varepsilon_{i.}^{*'}) \\ r_{i.}^* \end{bmatrix}, \quad (14)$$

where $r_{i.}^* \sim N(0, I_{T_i})$, $r_{i.}^*$ and $\varepsilon_{i.}^*$ are independent. Therefore, we have:

$$\begin{aligned} \tilde{v}_{i.} &= M_{x_{3i}} v_{i.} \\ &= \sigma_v M_{x_{3i}} \varepsilon_{i.}^* \rho + \sigma_r M_{x_{3i}} r_{i.}^* \\ &= M_{x_{3i}} \varepsilon_{i.} \eta + \sigma_r M_{x_{3i}} r_{i.}^* \\ &= \tilde{\varepsilon}_{i.} \eta + \tilde{r}_{i.} \end{aligned} \quad (15)$$

where $\sigma_r = \sigma_v \sqrt{1 - \rho' \rho}$, $\eta = \sigma_r \Omega_{\varepsilon}^{-1/2} \rho / \sqrt{1 - \rho' \rho}$, and $\tilde{r}_{i.} = \sigma_r M_{x_{3i}} r_{i.}^*$. Then, the frontier equation can be written as follows:

$$\tilde{y}_{i.} = \tilde{x}_{i.} \beta + (\tilde{x}_{i.} - \tilde{z}_{i.} \delta) \eta + \tilde{e}_{i.}, \quad (16)$$

where $\tilde{e}_{i.} = \tilde{r}_{i.} - s \tilde{u}_{i.}$ and $(\tilde{x}_{i.} - \tilde{z}_{i.} \delta) \eta$ in Equation (16) is a bias correction term. The density function of $\tilde{r}_{i.}$ is given by:

$$\begin{aligned} f_{\tilde{r}_{i.}}(\tilde{r}_{i.}) &= \left| 2\pi \sigma_r^2 M_{x_{3i}}^* \right|^{-1/2} \exp \left(-\frac{1}{2} \tilde{r}_{i.}' (\sigma_r^2 M_{x_{3i}})^{-1} \tilde{r}_{i.} \right) \\ &= (2\pi \sigma_r^2)^{-(T_i - k_3)/2} \exp \left(-\frac{1}{2\sigma_r^2} \tilde{r}_{i.}' \tilde{r}_{i.} \right). \end{aligned} \quad (17)$$

Similarly, the density function of $\tilde{\varepsilon}_{i.}$ is given by:²⁰

²⁰More precisely, this is the density function of $vec(\tilde{\varepsilon}_{i.})$.

$$\begin{aligned}
f_{\tilde{\varepsilon}_i}(\tilde{\varepsilon}_i) &= \left| 2\pi(\Omega_\varepsilon \otimes M_{x_{3i}})^* \right|^{-1/2} \exp \left(-\frac{1}{2} \text{vec}(\tilde{\varepsilon}_i)' (\Omega_\varepsilon \otimes M_{x_{3i}})^- \text{vec}(\tilde{\varepsilon}_i) \right) \\
&= \left| 2\pi\Omega_\varepsilon \right|^{-(T_i - k_3)/2} \exp \left(-\frac{1}{2} \text{tr}(\Omega_\varepsilon^{-1} \tilde{\varepsilon}_i' \tilde{\varepsilon}_i) \right).
\end{aligned} \tag{18}$$

Note that since $\tilde{\varepsilon}_i$ and $\tilde{e}_i$ are independent, the marginal log-likelihood function of panel i is given by:

$$\ln L_i = \ln L_{1i} + \ln L_{2i} \tag{19}$$

where

$$\ln L_{1i} = -\frac{1}{2}(T_i - k_3) \ln(2\pi\sigma_r^2) - \frac{1}{2} \frac{\tilde{e}_i' \tilde{e}_i}{\sigma_r^2} + \frac{1}{2} \left(\frac{\tilde{\mu}_{i*}^2}{\tilde{\sigma}_{i*}^2} - \frac{\mu^2}{\sigma_u^2} \right) + \ln \left(\frac{\tilde{\sigma}_{i*} \Phi(\frac{\tilde{\mu}_{i*}}{\tilde{\sigma}_{i*}})}{\sigma_u \Phi(\frac{\mu}{\sigma_u})} \right)$$

$$\ln L_{2i} = -\frac{1}{2} (T_i - k_3) \ln |2\pi\Omega_\varepsilon| + \text{tr}(\Omega_\varepsilon^{-1} \tilde{\varepsilon}_i' \tilde{\varepsilon}_i),$$

$\tilde{\mu}_i^* = \frac{-s\sigma_u^2 \tilde{e}_i' \tilde{h}_i + \mu\sigma_r^2}{\sigma_u^2 \tilde{h}_i' \tilde{h}_i + \sigma_r^2}$, $\tilde{\sigma}_{i*}^2 = \frac{\sigma_r^2 \sigma_u^2}{\sigma_u^2 \tilde{h}_i' \tilde{h}_i + \sigma_r^2}$, $\tilde{e}_i = \tilde{y}_i - \tilde{x}_{1i} \beta - \tilde{\varepsilon}_i \eta$, and $\tilde{\varepsilon}_i = \tilde{x}_i - \tilde{z}_i \delta$. By maximizing the total log-likelihood of Equation (19), we obtain the LIML estimates for the model's parameters. Under standard conditions, the LIML estimator is consistent as $n \rightarrow \infty$ and T_i fixed.

Prediction of u_{it} :

As earlier, the inefficiency u_{it} and efficiency can be predicted via:

$$E[u_{it} | \tilde{e}_i] = h_{it} \left[\tilde{\mu}_{i*} + \frac{\tilde{\sigma}_{i*} \phi(\frac{\tilde{\mu}_{i*}}{\tilde{\sigma}_{i*}})}{\Phi(\frac{\tilde{\mu}_{i*}}{\tilde{\sigma}_{i*}})} \right] \tag{20a}$$

and

$$EFF_{it} = \exp(-E[u_{it} | \tilde{e}_i]), \tag{20b}$$

respectively. In practice, these equations are evaluated at $\hat{\tilde{e}}_i = \tilde{y}_i - \tilde{x}_{1i} \hat{\beta}$.

Prediction of α_i :

It is possible to estimate the individual effects term parameters by:

$$\hat{\alpha}_i = (x'_{3i} x_{3i})^{-1} x'_{3i} (y_i - x_{1i} \hat{\beta} + sE[u_i | \hat{e}_i]). \quad (21)$$

Before concluding this section, we make the following remarks:

Remark 1: The model that we presented in Equation (13) assumes heterogeneity and solves the incidental parameters problem by model transformation. Even when there is no heterogeneity so that:

$$y_i = \alpha + x_{1i} \beta - s u_i + v_i, \quad (22)$$

our general model applies. In this case, since there is no need for model transformation, it would be enough to replace $M_{x_{3i}}$ by I_{T_i} and maximize the log-likelihood function given in Equation (19). Note that when the transformation is the identity, we would have $k_3 = 0$ as $\text{tr}(I_{T_i}) = T_i$. Hence, our model generalizes the models of Karakaplan and Kutlu (2017a,b).

Remark 2: Although simultaneous estimation of the model may be desirable, sometimes the researcher may want to estimate the model in two steps. One of the prominent advantages of two-stage estimation is that the second stage would be estimated by using the exogenous counterpart of the model. In the first stage, the transformed endogenous variables would be regressed on the transformed exogenous variables by OLS. That is, we obtain the prediction of $\tilde{e}_{it}$, i.e., $\hat{\tilde{e}}_{it} = \tilde{x}_{it} - \tilde{z}_{it} \hat{\delta}$. In the second stage, $\ln L_{1i}$ is maximized taking the second set of parameters as given. Hence, the model in the second stage is:

$$\tilde{y}_i = \tilde{x}_{1i} \beta + \hat{\tilde{e}}_i \eta + \tilde{e}_i. \quad (23)$$

For the (time-invariant) true fixed effects model, the second stage can be estimated by the within estimator of Wang and Ho (2010). The more general case where heterogeneity is time-varying can be estimated by our time-varying true individual effects estimator. The two-step estimation problem suffers from the generated regressors problem. Consequently, the conventionally-calculated standard errors from the second stage are not correct. Kutlu (2010) suggests using bootstrapping in order to correct the standard errors. Kutlu and Karakaplan (2017a,b) and Amsler, Prokhorov, and Schmidt (2016) propose analytical solutions. One such solution is based on Murphy and Topel (1985). Greene (2008) gives a concise presentation of this two-stage maximum likelihood estimation method.

Remark 3: Solution to the endogeneity problem is not complete without a test for endogeneity. Following Amsler, Prokhorov, and Schmidt (2016) and Karakaplan and Kutlu (2017a,b), we provide a simple test using similar ideas with the standard Durbin-Wu-Hausman test. When $\eta = 0$ the two-stage estimator is asymptotically as efficient as the one-stage estimator. Moreover, the standard errors do not need correction for the second stage. An implication of these is that the standard F-test of the hypothesis that $\eta = 0$ can be used to test endogeneity. Hence, if all components of η are jointly significant, then we would conclude that there is endogeneity in our model. It is also possible to test the endogeneity of a specific variable by testing the significance of the corresponding component of η .

Remark 4: The endogenous variables x_{it} as defined by Definition 3.2.1 do not cover the general correlation case, since it does not allow these variables to be correlated with u_i^* . Extension of our model to allow for the general correlation can be done similarly to Amsler, Prokhorov, and Schmidt (2017), which is based on the copula approach. To do this, we first redefine the endogeneity in the form of definition below.

Definition 3.2.2: In Model (11), (a) x_{it} is endogenous if $E[v_{it} | x_{it}] \neq 0$ and $E[u_i^* | x_{it}] \neq E[u_i^*]$; (b) w_{it} is exogenous if (i) $E[v_{it} | w_{it}] = 0$ and; (ii) $E[u_i^* | w_{it}] = E[u_i^*]$.

Thus, Definition 3.2.2 allows for x_{it} to be correlated with both v_{it} and u_i^* . Let $\zeta_{it} = (\varepsilon_{it}', v_{it}')'$ and $\zeta_i = (\varepsilon_i', v_i')'$. To derive the log-likelihood function, we make the following assumption:

Assumption 2:

- (i) $u_i^* | w_{it} \sim N^+(0, \sigma_u^2)$,
- (ii) $\zeta_{it,j} | w_{it} \sim N(0, \Omega_{jj})$ where $\zeta_{it,j}$ is the j^{th} element of the vector ζ_{it} and Ω_{jj} is the j^{th} diagonal element of Ω , for $j = 1, \dots, p+1$;
- (iii) The joint distribution of $(\zeta_{it}', u_i^*)'$ conditional on w_{it} is characterized by the marginal distribution in (i) and (ii) and the Gaussian copula²¹.

To obtain the log-likelihood function, we first need to calculate the joint density of $(\zeta_{it}', u_i^*)'$

²¹ For a formal definition of copula and its properties, see for example, Nelson (2006).

implied by Assumption 2. Appendix 1 provides details for calculation of the log-likelihood as well as how to estimate the parameters of the model via a simulated ML procedure.

Remark 5: In production function estimation literature, Olley and Pakes (1996), Levinsohn and Petrin (2003) (LP), and Akerberg, Caves, and Frazer (2015) present control function methods to overcome this endogeneity issue by modelling choice of inputs to find a control function that aims fix the estimation bias in parameters. However, these alternative models make strong assumptions about the competitive environment, the level of investment, the monotonicity of intermediate input demand function, etc. In the context of stochastic production frontiers, Shee and Stefanou (2014) propose a modified version of LP for obtaining consistent estimates of production parameters and technical efficiencies. They construct a control function from a third order polynomial approximation of an unknown function of capital and energy input variables. Recall that intrinsically our TVTIEE method is also a control function approach. Hence, in practice the control function of Shee and Stefanou (2014) can be boosted by adding our control functions that are obtained by using existing instruments. The method of Shee and Stefanou (2014) only solves LP type of endogeneity problems. For example, if the inefficiency term contains endogenous variables, this problem would not be handled. However, with addition of our control function, one can solve both the LP type and other types (e.g., endogenous variable in one-sided error term) of endogeneity problems that can arise in production function estimation.

4. Monte Carlo Simulations

To examine the finite sample performance of our proposed estimator with and without endogeneity, we conduct the following Monte Carlo experiments. First, we concentrate on correlation structures that assume that u_i^* is independent of ε_{it} and v_{it} . Then, we introduce general correlation structures to the experiments. To this end, we consider the following data generating process:

$$\begin{aligned} y_{it} &= x'_{3it} \alpha_i + \beta_1 x_{1it}^{(1)} + \beta_2 x_{1it}^{(2)} + u_{it} + v_{it} \\ u_{it} &= h_{it} u_i^* \\ h_{it}^2 &= \exp(\varphi_1 x_{2it}^{(1)} + \varphi_2 x_{2it}^{(2)}) \\ \sigma_u^2 &= \exp(c_u) \\ u_i^* &\sim \mathbf{N}^+(0, \sigma_u^2) \end{aligned} \tag{24}$$

where $x_{3it} = 1, t/T, (t/T)^2$ and $\alpha_i = (\alpha_{0i}, \alpha_{1i}, \alpha_{2i})'$ is a 3×1 productive unit specific coefficient vector for the heterogeneity term; $x_{1it}^{(1)}$ and $x_{2it}^{(1)}$ are exogenous variables; and u_i^* and v_{it} are

independent random variables. To generate the time-varying heterogeneity term, we draw α_{ji} 's, $j = 0, 1, 2$, from a uniform distribution in $\gamma[-0.5, 0.5]$ where γ is a parameter representing the extend of heterogeneity. After generating $\alpha_{it} = x'_{3it}\alpha_i$, $x_{1it}^{(1)}$ is generated so that the correlation of α_{it} and $x_{1it}^{(1)}$ is equal to $\rho_{1\alpha} = 0.7$. Similarly, $x_{2it}^{(1)}$ is generated so that the correlation of α_{it} and $x_{2it}^{(1)}$ is equal to $\rho_{2\alpha} = 0.7$. Specifically, we generate $x_{1it}^{(1)}$ and $x_{2it}^{(1)}$ as follows: $x_{jit}^{(1)} = \alpha_{it} + \omega_{jit}$ where $\omega_{jit} \sim \mathbf{N}(0, \sigma_\alpha^2(1 - \rho_{j\alpha}^2) / \rho_{j\alpha}^2)$, $j = 1, 2$ is a random variable that is independent from α_{it} and σ_α^2 is the variance of α_{it} . Remaining variables are generated as follows: $(z_{1it}, z_{2it})' \sim \mathbf{N}((\mu_{z_1}, \mu_{z_2})', \Sigma)$ and

$$\begin{aligned} x_{1it}^{(2)} &= z_{1it}\delta_1 + \varepsilon_{1it} \\ x_{2it}^{(2)} &= z_{2it}\delta_2 + \varepsilon_{2it} \end{aligned}$$

$$\begin{pmatrix} \varepsilon_{it} \\ v_{it} \end{pmatrix} \sim \mathbf{N} \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{\varepsilon_1}^2 & 0 & \sigma_{\varepsilon_1}\sigma_v\rho_1 \\ 0 & \sigma_{\varepsilon_2}^2 & \sigma_{\varepsilon_2}\sigma_v\rho_2 \\ \sigma_{\varepsilon_1}\sigma_v\rho_1 & \sigma_{\varepsilon_2}\sigma_v\rho_2 & \sigma_v^2 \end{pmatrix} \right),$$

where $\varepsilon_{it} = (\varepsilon_{1it}, \varepsilon_{2it})'$. Note that when $\rho_1 = 0$, $x_{1it}^{(2)}$ becomes exogenous and likewise, when $\rho_2 = 0$, $x_{2it}^{(2)}$ becomes exogenous. Finally, when $\rho_1 = \rho_2 = 0$, all variables are exogenous and we call this “exogenous model.”²² In each experiment, we consider the following values for $(\rho_1, \rho_2) = \{(0, 0), (0.7, 0.7)\}$, and we fixed the values of $\beta_1 = \beta_2 = 0.5$, $\delta_1 = \delta_2 = 1$, $c_u = -0.5$, $\sigma_v = \sqrt{0.1}$, $\sigma_{\varepsilon_1} = 0.5$, $\sigma_{\varepsilon_2} = 0.75$, $\varphi_1 = 0.4$, $\varphi_2 = 0.9$, $(\mu_{z_1}, \mu_{z_2})' = (1, 1)'$, $\gamma = 2$, and

$$\Sigma = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}.$$

While this data generating process allows direct correlations between ε_{it} and v_{it} , one-sided error term, u_{it} , is only indirectly correlated with these error terms through the explanatory variables.

²² Note that indirect correlation of x_{1it} and $u_{it} = h_{it}u_i^*$ through x_{2it} would not lead to inconsistent parameter estimates (given that x_{1it} and x_{2it} are independent of u_i^*) as x_{1it} and x_{2it} are allowed to be correlated. Therefore, it is reasonable to label this model “exogenous model.” In an earlier version of our paper, we implemented simulations that specifically impose correlation between x_{1it} and x_{2it} . These simulation results show that our estimator performs well.

In particular, u_i^* is independent of ε_{it} and v_{it} . In the second case, we examine the behavior of our estimator under a general correlation structure for ε_{it} , v_{it} , and u_i^* . To introduce a general correlation structure, we keep everything the same as earlier but replace ε_{it} and v_{it} with $\tilde{\varepsilon}_{1it} = \varepsilon_{1it} + 3.5(u_i^* - E[u_i^*])$, $\tilde{\varepsilon}_{2it} = \varepsilon_{2it} + 3.5(u_i^* - E[u_i^*])$, and $\tilde{v}_{it} = v_{it} + 1.5(u_i^* - E[u_i^*])$. Hence, u_i^* is correlated with ε_{1it} , ε_{2it} , and v_{it} ; and correlations are approximately, 0.8, 0.8, and 0.9, respectively. As earlier, we set $(\rho_1, \rho_2) = (0.7, 0.7)$.

Finally, we consider the following sample sizes: $(n, T) = \{(100, 10), (100, 20), (200, 10), (200, 20)\}$, and the Monte Carlo experiments are conducted with 1,000 replications.

4.1 Simulation Results for Time-Varying Heterogeneity

The purpose of this section is examining the performance of our TVTIEE estimator and consequences of ignoring time-varying heterogeneity. For the sake of concentrating only on heterogeneity, we assume a setting without endogenous variables. In particular, we assume that $(\rho_1, \rho_2) = (0, 0)$. The estimators considered in our Monte Carlo experiments are: Time-varying true individual effects estimator which allows endogeneity and time-varying heterogeneity (TVTIEE); true fixed effects estimator which allows endogeneity and time-invariant heterogeneity (TFEE); time-varying true individual effects estimator which doesn't allow for endogeneity but allows for time-varying heterogeneity (TVTIE); true fixed effects estimator which doesn't allow for endogeneity but allows time-invariant heterogeneity (TFE); and untransformed version of time-varying true individual effects estimator which doesn't allow for endogeneity but allows for time-varying heterogeneity (TVTIEU) by including $\alpha_{it} = (1, t, t^2)\alpha_i$ term in the estimations. We include TVTIEU estimator to examine the relative performances of estimators corresponding to transformed and untransformed models. For the TVTIEU estimator the number of parameters for heterogeneity term would be $3TN$, which can be very large even for a moderate-sized data set. In our terminology, the estimator of Wang and Ho (2010) corresponds to the TFE estimator. When estimating the parameters using the models that consider endogeneity (i.e., TVTIEE and TFEE), we assume that both $x_{1it}^{(2)}$ and $x_{2it}^{(2)}$ are endogenous.

Table 1 is Here

In Table 1, we present simulation means and mean squared errors (MSE) for $\beta = (\beta_1, \beta_2)'$, $\varphi = (c_u, \varphi_1, \varphi_2)'$, and $\eta = (\eta_1, \eta_2)'$.²³ Also, bias, MSE, and Spearman correlation (Spe. Corr.) for efficiency estimates are presented. The TVTIEU estimator is clearly the worst performing estimator in the list in terms of estimating inefficiency parameters. The empirical sizes of inefficiency parameters for 5% significance level are close to 1 for $(n, T) = (200, 20)$.²⁴ The β parameters for this estimator are only slightly biased. This bias decreases as the sample size increases. However, the biases for efficiency estimates are substantial. The performances of TVTIEE and TVTIE estimators are similar and they perform better than TFEE and TFE estimators. In particular, the estimators that do not control for time-varying heterogeneity give biased efficiency and β estimates and these biases are not small. Hence, the TVTIEE estimator avoids potential biases for efficiency estimates when heterogeneity is time-varying and performs well even when there is no endogeneity.

Finally, the bias for efficiency as a function of the number of time periods for the TVTIE estimator goes away relatively fast as the number of time periods increases. For example, for $(n, T) = (100, 50)$ case the bias is 0.0034. However, increasing the number of panel units reduces bias only marginally.²⁵ The bias for efficiency goes away as the heterogeneity, i.e., γ , increases. For example, for $(n, T) = (100, 10)$ case with $\gamma = 4$ and $\gamma = 5$ the biases for efficiency are 0.0066 and 0.0044, respectively.

4.2 Simulation Results for Time-Varying Heterogeneity and Endogenous Regressors

4.2.1 When $x_{1it}^{(2)}$ and $x_{2it}^{(2)}$ are correlated with v_{it}

The purpose of this section is examining the performance of our estimator and consequences of ignoring endogeneity. We start with the case where ε_{it} and v_{it} are not correlated with u_i^* . In Table 2, we present the simulation results for $(\rho_1, \rho_2) = (0.7, 0.7)$. In this scenario, both $x_{1it}^{(2)}$ and $x_{2it}^{(2)}$ are correlated with the two-sided error term. When estimating the models that consider endogeneity, we assume that both $x_{1it}^{(2)}$ and $x_{2it}^{(2)}$ are endogenous. Since TVTIE and TFE estimators do not control for endogeneity, as expected, β_2 estimates from these estimators are biased and the biases do not

²³ The Monte Carlo simulations are conducted using MATLAB software.

²⁴ Due to space limitations we do not announce the empirical sizes. They are available upon request.

²⁵ Even for the $(n, T) = (2000, 10)$ case the bias hasn't reduced much.

seem to go away for larger samples. Moreover, β_1 estimates for TFEE and TFE are biased as these estimators neglect time-varying heterogeneity. Except the TVTIEE estimator all estimators give biased efficiency estimates as expected. Overall, the TVTIEE estimator not only controls for time-varying heterogeneity but also endogeneity, which in turn performs good.

Table 2 is Here

4.2.2 When $x_{1it}^{(2)}$ and $x_{2it}^{(2)}$ are correlated with both v_{it} and u_i^*

Next, we consider the general correlation structure where $x_{1it}^{(2)}$ and $x_{2it}^{(2)}$ are allowed to be correlated with both v_{it} and u_i^* . Table 3 presents the simulation results for this general correlation scenarios. In these experiments u_i^* is correlated with ε_{1it} , ε_{2it} , and v_{it} . Although the correlations of u_i^* with other three error terms are reasonably high (approximately, 0.8, 0.8, and 0.9, respectively), the simulation results indicate that our estimator performs reasonably well. The intuition is that even though u_i^* is correlated with other error terms, all we need is conditional independence. It turns out that our model performs reasonably well even when general correlations are present.

Table 3 is Here

4.2.3 Other Simulation Results

In Table 4, we present 5% empirical size values for TVTIEE. The empirical sizes are reasonably close to 0.05 even for small samples.²⁶

Table 4 is Here

As earlier, the bias from the TVTIEE estimator goes away relatively fast as the number of time periods increases. However, increasing the number of panel units does not seem to help much in terms

²⁶ We also obtained 1% and 10% empirical size values as well. They were reasonably good too. To save the space we do not announce them but they are available upon request.

of reducing the bias. In any case, when there is sufficient heterogeneity (i.e., large enough γ) our efficiency estimates are consistent.

Now, we examine the consequences of using our time-varying heterogeneity models (TVTIEE and TVTIE) when the heterogeneity is time-invariant. For this purpose, we assume that the data generating processes are as earlier but $\alpha_{it} = (1, 0, 0)\alpha_i$ so that the heterogeneity is time-invariant. In order to have some idea about statistical efficiency loss due to the larger number of parameters used in the time-varying heterogeneity model, we announce the MSEs for both time-invariant and time-varying heterogeneity models and the ratios of MSEs. The simulation results for this case are provided in Table 5. As expected, the ratios of MSE values get smaller as the number of time periods decreases. Based on these results, we conclude that the efficiency loss can be negligible compared to potential negative consequences of not using the time-varying heterogeneity models.

Table 5 is Here

Another concern is testing for heterogeneity. We only consider $(n, T) = (100, 10)$ and $(\rho_1, \rho_2) = (0.7, 0.7)$ case. The data generating process is the same as earlier except that we assume there is no heterogeneity, i.e., $\alpha_i = (\alpha_{0i}, \alpha_{1i}, \alpha_{2i})' = (0, 0, 0)$ and $x_{jit}^{(1)} \sim \mathbf{N}(0, 1)$ for $j = 1, 2$.²⁷ The estimations are done by TVTIEE under homogeneity assumption, i.e., transformation matrix is the identity matrix. As earlier, Monte Carlo experiments are run 1,000 times. When calculating our heterogeneity test statistic, we use the sample error term, $\hat{v}$, and regress it on the constant, individual-specific dummies, t , and t^2 terms, which is a $nT \times 3n$ matrix. We use the F-test for significance of the model, i.e., significance of all parameters except the constant term. For 5% significance level, we reject homogeneity in 4.80% of simulation runs. Therefore, we can deduce the presence of homogeneity reasonably well.

5. Empirical Example: Cost Efficiency of the US Banks

Our purpose is investigating the cost efficiencies of the US insured banks between 1976 and 2007. In the banking literature, there have been many studies examining the relationship between market power and efficiency such as Jayaratne and Strahan (1996), Berger and Hannan (1998), Kroszner

²⁷In the benchmark scenario $x_{1it}^{(1)}$ and $x_{2it}^{(1)}$ were correlated with the heterogeneity term.

and Strahan (1999), and Koetter, Kolari, and Spierdijk (2012). The quiet life hypothesis claims that firms with market power are more likely to operate inefficiently. A concept that is related to market power is that of return on revenue²⁸ as higher market power levels would enable firms to extract higher profits. The inefficiency level of a firm is presumably more directly affected by ROR compared with market power; because a main objective of the manager would be profit maximization and ROR is a measure that can be observed more directly by the manager. Hence, when modeling inefficiency, we concentrate on ROR. We first briefly describe our data, and then present our results.

5.1 Data

Our data set is based on the data set collected by Koetter, Kolari, and Spierdijk (2012).²⁹ This data set is gathered from annual year-end data from all US insured banks between 1976 and 2007. The main source of the individual commercial bank data is the Reports of Condition and Income (or Call Reports) of the Federal Reserve System. Missing observations and negative observations for variables that are supposed to be positive are dropped. The monetary values are deflated to 2005 prices using the consumer index obtained from Bureau of Economic Analysis. Moreover, we only consider the banks with at least five observations.

Almanidis, Karakaplan, and Kutlu (2016) argue that for banking industry, which is likely to be characterized by heterogeneous technologies, a common frontier assumption for all banks may potentially lead to inconsistent parameter estimates and distorted efficiency rankings.³⁰ Hence, in order to avoid such problems, we concentrate on the top US banks. In particular, we consider a bank as top if the bank is among the 100 largest banks measured in total assets in the country at some year between 1976 and 2007. The number of top banks in our estimations is 342. Variable definitions and summary statistics are given in Table 6 and Table 7, respectively. We assume that ROR is endogenous and instrument this variable by share of Asian employees (SHA), Herfindahl-Hirschman Index for ethnicity (ETH_{HHI}), disposable personal income (DPI), and unemployment rate (UR). Koetter, Kolari, and Spierdijk (2012) argue that SHA and ETH_{HHI} capture different occupational accomplishments based on race, which may be used to instrument competition levels in the banking industry. Chirinko and Fazzari (2000) show that macroeconomic conditions affect competition. Based on this, Koetter, Kolari, and Spierdijk (2012) include DPI and UR as instruments for competition level as well. Hence, we

²⁸Remember that ROR is the ratio of profit to revenue.

²⁹For more details about the data set we direct the reader to Koetter, Kolari, and Spierdijk (2012). The data is publicly available in archives of the Review of Economics and Statistics website.

³⁰See also Tsionas (2002) and Almanidis (2013) among others.

follow them in our choice of instruments. In the estimations, in order to impose the homogeneity restriction, we normalize the logarithms of TEX , W_1 , and W_2 , by the logarithm of W_3 .

Table 6 is here

Table 7 is here

5.2 Results

In this section, we present our empirical model and estimation results. Given the length of time period that we are considering both the inefficiency and heterogeneity maybe varying over time. As our simulation results suggest, ignoring such time-varying patterns may lead to biased efficiency estimates. For example, when studying the overall effect of a deregulation on the social welfare, biased efficiency estimates may lead to an invalid evaluation of deregulation policies; because the forgone welfare due to inefficiency should also be considered when comparing pre-deregulation and post-deregulation social welfares (Kutlu and Sickles, 2012).

Another important, yet mostly ignored, issue in stochastic frontier models is the endogeneity. The literature on banking industry seems to accept some sort of simultaneity between market power and efficiency (e.g., Berger and Hannan, 1998; and Koetter, Kolari, and Spierdijk, 2012). However, the way in which such an endogeneity is handled may lead to inconsistent parameter and efficiency estimates. A common approach is, first, estimating a stochastic frontier model from which the efficiency is calculated. Then, in a second stage, the efficiency estimates from the first stage are regressed on, potentially endogenous, variables (e.g., market power measures) that determine efficiency. The second stage is done using an instrumental variables approach. Even when there is no endogeneity, as Wang and Schmidt (2002) argue, such a two-stage method may lead to biased estimates. When there is no endogeneity, a solution would be estimating a standard one-stage stochastic frontier model where the distribution of inefficiency is directly modelled by the determinants of inefficiency. If the frontier variables are correlated with the two-sided error term, then the two-stage method proposed by Guan et al. (2009) would give consistent parameter estimates. However, this method does not work when the endogeneity is due to correlation of one-sided error and two-sided error terms, which is likely to be the case for the banking industry. Hence, since our control function approach allows correlation between one-sided and two-sided error terms, it may be a more proper method in the banking context. For the sake of comparison, we estimated five different models. The first model is a standard Battese-Coelli model, which does not consider the heterogeneity (BC); the second model is the transformed

true fixed effects model of Wang and Ho (2010) (TFE); the third model is the untransformed model of Greene (2005a,b) (GTFE); the fourth model is TVTIE model; and the final model is TVTIEE model. Except the GTFE model, as in our theoretical section, we assume that $u_{it} = h_{it} u_i^*$, $h_{it}^2 = \exp(x_{2it} \varphi_u)$, and $u_i^* \sim N^+(0, \sigma_u^2)$, where $x_{2it} = ROR_{it}$. For the GTFE model, we assume that $u_{it} = h_{it} u_{it}^*$, $h_{it}^2 = \exp(x_{2it} \varphi_u)$, and $u_{it}^* \sim N^+(0, \sigma_u^2)$, where $x_{2it} = ROR_{it}$. Hence, in the GTFE model u^* is observation specific. Our estimation results for these models are given in Table 8.^{31 32} One important observation is that the signs for ROR variable are negative for BC and GTFE models, which are untransformed.³³ The estimates from BC estimator suffer from omitted variable bias (along with endogeneity) due to omitted heterogeneity term; and the estimates from GTFE model suffers from incidental parameters problem, which may explain the different sign estimates for ROR variable.

Table 8.1 is here

Table 8.2 is here

Table 9 is here

Figure 1 is here

Figure 2 is here

In Figure 1, we compare the (empirical) distributions of the efficiency estimates from these five models. The distribution for the TVTIEE model looks considerably different from the other distributions. We tested the pairwise equality of these distributions by the Kolmogorov-Smirnoff test. At any conventional significance level, we rejected the null hypothesis that the density functions are the same. The average efficiencies from these distributions are not very close to that of TVTIEE as well. Even when the means and distributions of efficiency estimates are different, the efficiency rankings

³¹The estimations in the empirical section are done by MATLAB software. In particular, we used the global optimization package and an algorithm that switches between different optimization methods.

³² For TVTIEE model, the F-value for testing the joint significance of excluded instruments is 74.52, which is much larger than the rule of thumb F-value (i.e., 10) for testing weakness of instruments.

³³ The correlation of ROR variable with w_1 , w_2 , y_1 , y_2 , and z are 0.41, 0.64, 0.14, 0.27, and 0.35, respectively. Moreover, the correlation of ROR variable with TVTIEE estimates of u^* , v , and heterogeneity term are -0.03, -0.16, -0.21, respectively.

may be similar. Hence, we also announce the Pearson and Spearman correlations in Table 9. The upper triangular part gives the Pearson correlations; and the lower triangular part gives the Spearman correlations. The results are striking. The efficiency estimates from the BC and TFE estimators are only weakly correlated with that of TVTIEE estimator. The corresponding correlation for the GTFE estimator is negative. The correlations of TVTIE estimator are fairly high. In Figure 2, we provide the scatter plots of cost efficiency estimates. In this figure, the estimates from TVTIEE are given on the x-axis and the other estimates are given on the y-axis. This figure is in line with the correlations that we announced. Hence, if the cost efficiency estimates from TVTIEE model are more precise and we use these estimates as a proxy for the true cost efficiencies, then our findings indicate that ignoring time-varying heterogeneity may have serious consequences.

Finally, one may argue that heterogeneity is time-invariant. Although for longer panel data we believe that this may not be the case, it is still a relevant and righteous question, which requires an empirical test. In the banking context, not every bank may have the same pattern for adjusting new developments. Hence, time-varying heterogeneity reflects the heterogeneity in banks' adjustment speed over the years. In our empirical study, we tested time-invariant heterogeneity for each firm and found out that 79.24% of the banks show time-varying heterogeneity based on 5% significance level. Overall, the no heterogeneity is rejected with a p-value = 0.0000. While we need more empirical support for the existence of time-varying heterogeneity, we may argue that it is likely to be a possibility rather than an irrelevant abstract concept.

6. Conclusion

In this paper, we presented a general stochastic frontier model that can address a variety of important yet mostly ignored issues in the literature. In particular, we extended a variation of Greene's true fixed effects model so as to allow time-varying heterogeneity. The conventional stochastic frontier models may confuse heterogeneity with inefficiency. Greene's true fixed effects model solve this issue by separating out the inefficiency and heterogeneity. However, in this model, the inefficiencies of productive units may vary over time but heterogeneity is assumed to be fixed over time. In a variety of interesting cases, this seems to be a rather strong assumption and time-varying portion of heterogeneity may be confused with inefficiency as was illustrated in our simulations. Moreover, since the true fixed effects model captures heterogeneity by including productive unit specific dummy variables, it is subject to the incidental parameters problem. We solved this issue by transforming the relevant model so that the heterogeneity term is eliminated. Although we mostly concentrated on the heterogeneity in time dimension, our estimator can handle heterogeneity that occurs through a variety of dimensions such as space. For example, we may introduce spatial heterogeneity by using a distance-varying true

individual effects term.

We further generalized this model to allow endogeneity. The stochastic frontier models are subject to additional complications compared to the standard production and cost function models. The complication is due to the fact the one-sided error term is generally assumed to be independent of the two-sided error term, which enables us to get a closed form solution for the log-likelihood function and conditional expectation of one-sided error term. If any of the regressors or one-sided error term is correlated with the two-sided error term, the parameter estimates would be inconsistent. This is an overlooked issue in the stochastic frontier literature. Historically, the endogeneity problem is ignored because the econometric methods for handling endogeneity were not well-developed. Hence, in order to address these concerns, our paper provided a solution to the endogeneity issue for a fairly general family of stochastic frontier models. Our Monte Carlo experiments indicate that our estimator (TVTIEE) performs better than other competing estimators.

To demonstrate the usefulness of our proposed approach, we estimated the cost efficiencies of the top US insured banks and examined the relationship between return on revenue and cost efficiency. Our estimates suggest that this relationship is negative. Our empirical analysis provides further evidence that ignoring time-varying heterogeneity may lead to substantially inaccurate efficiency estimates. Hence, overall we conclude that the researcher should be careful about time-varying heterogeneity and endogeneity.

Finally, whilst the approach discussed in this paper confines to the case where the reduced form equations are available, there is wider literature on instrument-based and instrument-free based estimation that perhaps can be applied to the stochastic frontier setting, see for example, Lewbel (2012) and Park and Gupta (2012). In addition, there is recent literature on alternative methods of handling endogeneity in stochastic frontier models, see for example, Tran and Tsionas (2015) and Amsler, Prokhorov, and Schmidt (2016, 2017). However, some of these approaches require different estimation strategies, we will leave them for future research.

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Appendix 1: The Case of General Correlation

In this Appendix, we provide a brief description of how to extend our model to allow for x_{1it} , x_{2it} to be correlated with both v_{it} and u_{it} . Our derivation follows analogously to Amsler, Prokhorov and Schmidt (2017). First, let us redefine the following notions. Let $\tilde{e}_i = \tilde{v}_i - s\tilde{u}_i = \tilde{v}_i - s\tilde{h}_i(\tilde{x}'_{2i}\varphi_u)u_i^*$ which implies $s u_i^* = (\tilde{v}_i - \tilde{e}_i)\tilde{h}_i^{-1}(\tilde{x}'_{2i}\varphi_u)$. Also, let $\sigma_{\tilde{v}_i}^2 = \Omega_{p+1}$, $\sigma_{\tilde{e}_i,j}^2 = \Omega_{jj}$ for $j = 1, \dots, p$.

Now, Assumption 2 implies that we can construct the joint density $f_{\tilde{e}_i, \tilde{v}_i, u_i^*}(\tilde{e}_i, \tilde{v}_i, u_i^*)$ and it is given by:

$$f_{\tilde{e}_i, \tilde{v}_i, u_i^*}(\tilde{e}_i, \tilde{v}_i, u_i^*) = c(F_{\tilde{e}_{i,1}}, \dots, F_{\tilde{e}_{i,p}}, F_{\tilde{v}_i}, F_{\tilde{u}_i})(\prod_{j=1}^p f_{\tilde{e}_{i,j}}) f_{\tilde{v}_i} f_{\tilde{u}_i} \quad (\text{A.1})$$

where $c(\cdot)$ is Gaussian copula density, F represents the cdf of the random variable as indexed by subscript evaluated at the point of standardized variate. That is, if $\Phi(\cdot)$ represents the standard normal cdf, then $F_{\tilde{e}_{i,j}} = \Phi(\tilde{e}_{i,j} / \sigma_{\tilde{e}_{i,j}})$, $F_{\tilde{v}_i} = \Phi(\tilde{v}_i / \sigma_{\tilde{v}_i})$ and $F_{\tilde{u}_i}$ is the CDF of $N^+(0,1)$ evaluated at (u_i^* / σ_u) . Similarly, $f(\cdot)$ represents the corresponding pdf. In addition, the Gaussian Copula density, based on Assumption 2 can be written as:

$$c(\omega_1, \dots, \omega_n) = |\Gamma|^{-1/2} \exp(-\frac{1}{2} a'(\Gamma^{-1} - I)a) \quad (\text{A.2})$$

where $n = p + 2$, $a = [\Phi^{-1}(\omega_1), \dots, \Phi^{-1}(\omega_n)]'$, Γ is the correlation matrix of the copula, and the typical elements of a are given by: $\Phi^{-1}(F_{\tilde{e}_{i,j}}) = \Phi^{-1}(\Phi(\tilde{e}_{i,j} / \sigma_{\tilde{e}_{i,j}})) = \tilde{e}_{i,j} / \sigma_{\tilde{e}_{i,j}} = \tilde{e}_{i,j}^{(l)}$, $\Phi^{-1}(F_{\tilde{v}_i}) = \Phi^{-1}(\Phi(\tilde{v}_i / \sigma_{\tilde{v}_i})) = \tilde{v}_i / \sigma_{\tilde{v}_i} = \tilde{v}_i^{(l)}$ and $\Phi^{-1}(F_{\tilde{u}_i}((u_i^* / \sigma_u)))$. Let $\tilde{e}_i^{(l)}$ be the vector with elements $\tilde{e}_{i,j}^{(l)}$.

To obtain the likelihood function, we need an expression for the joint density of $(\tilde{e}_i, \tilde{v}_i, \tilde{e}_i)$. Given the joint density of $(\tilde{e}_i, \tilde{v}_i, u_i^*)$ in (A.1), we can transform this joint density to the joint density of $(\tilde{e}_i, \tilde{v}_i, \tilde{e}_i)$ as follows.

$$f_{\tilde{e}_i, \tilde{v}_i, \tilde{e}_i}(\tilde{e}_i, \tilde{v}_i, \tilde{e}_i) = \tilde{h}_i^{-1}(\tilde{x}'_{2i}\varphi_u) |\Gamma|^{-1/2} \exp(-\frac{1}{2} a'_{vi}(\Gamma^{-1} - I)a_{vi}) (\prod_{j=1}^p f_{\tilde{e}_{i,j}}) f_{\tilde{v}_i} f_{\tilde{u}_i} ((\tilde{v}_i - \tilde{e}_i)\tilde{h}_i^{-1}(\tilde{x}'_{2i}\varphi_u))$$

where $a_{vi.} = (\tilde{\varepsilon}_i^{(l)'}, \tilde{v}_i^{(l)}, \Phi^{-1}[F_{\tilde{u}}((\tilde{v}_i - \tilde{e}_i)\tilde{h}_i^{-1}(\tilde{x}_{2i.}'\varphi_u))])'$. Next, we integrate out $\tilde{v}_i$,

$$\begin{aligned} f_{\tilde{\varepsilon}, \tilde{e}}(\tilde{\varepsilon}_i, \tilde{e}_i) &= \int f_{\tilde{\varepsilon}, \tilde{v}, \tilde{e}}(\tilde{\varepsilon}_i, \tilde{v}_i, \tilde{e}_i) d\tilde{v}_i \\ &= \tilde{h}_i^{-1}(\tilde{x}_{2i.}'\varphi_u) |\Gamma|^{-1/2} \left(\prod_{j=1}^p f_{\tilde{\varepsilon}_i, j} \right) \times \\ &\quad E_{\tilde{v}} \left[\exp \left(-\frac{1}{2} a_{vi.}' (\Gamma^{-1} - I) a_{vi.} \right) f_{\tilde{u}}((\tilde{v}_i - \tilde{e}_i)\tilde{h}_i^{-1}(\tilde{x}_{2i.}'\varphi_u)) \right] \end{aligned} \quad (\text{A.3})$$

where $E_{\tilde{v}}[\cdot]$ denotes the expectation over the distribution of $\tilde{v}_i$. The integral in (A.3) does not have a closed form solution, and hence it needs to be evaluated numerically. Given the values of all parameters, the expression contained in $E_{\tilde{v}}[\cdot]$ can be computed by taking the average of over many repeated random draws from the distribution of $\tilde{v}_i$.

Finally, by substituting out $\tilde{\varepsilon}_i = \tilde{x}_i - \tilde{z}_i\delta$, and $\tilde{e}_i = \tilde{y}_i - \tilde{x}_{3i}\alpha_i - \tilde{x}_{1i}\beta$, we obtain the joint density of $\tilde{x}_i$ and $\tilde{y}_i$,

$$f_{\tilde{x}, \tilde{y}}(\tilde{x}_i, \tilde{y}_i) = f_{\tilde{\varepsilon}, \tilde{e}}(\tilde{x}_i - \tilde{z}_i\delta, \tilde{y}_i - \tilde{x}_{3i}\alpha_i - \tilde{x}_{1i}\beta)$$

and the log-likelihood function is:

$$\ln L = \sum_{i=1}^n \ln(f_{\tilde{\varepsilon}, \tilde{e}}(\tilde{x}_i - \tilde{z}_i\delta, \tilde{y}_i - \tilde{x}_{3i}\alpha_i - \tilde{x}_{1i}\beta)).$$

Table 1. Simulation Results: Exogenous Model, $(\rho_1, \rho_2) = (0, 0)$

	TRUE	T=10, N=100					T=20, N=100				
		TVTIEE	TFEE	TVTIE	TFE	TVTIEU	TVTIEE	TFEE	TVTIE	TFE	TVTIEU
$E[\hat{\beta}_1]$	0.5000	0.4996	0.5967	0.4996	0.5967	0.4965	0.5002	0.5926	0.5002	0.5926	0.4992
$E[\hat{\beta}_2]$	0.5000	0.5000	0.4959	0.4998	0.4975	0.5024	0.5001	0.4956	0.5002	0.4976	0.5017
$E[\hat{c}_u]$	-0.5000	-0.4854	0.1753	-0.4848	0.1624	-1.6568	-0.4904	0.2005	-0.4913	0.1860	-1.1031
$E[\hat{\phi}_1]$	0.4000	0.3990	0.3181	0.3988	0.3157	0.5781	0.3984	0.3140	0.3982	0.3116	0.4868
$E[\hat{\phi}_2]$	0.9000	0.8970	0.8069	0.8968	0.8121	1.2891	0.8960	0.7912	0.8964	0.7967	1.0882
$E[\hat{\eta}_1]$	0.0000	-0.0013	0.0037	-	-	-	0.0004	0.0051	-	-	-
$E[\hat{\eta}_2]$	0.0000	-0.0006	-0.0109	-	-	-	-0.0007	-0.0116	-	-	-
$MSE[\hat{\beta}_1]$		0.0003	0.0097	0.0003	0.0097	0.0003	0.0001	0.0088	0.0001	0.0088	0.0001
$MSE[\hat{\beta}_2]$		0.0002	0.0002	0.0001	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
$MSE[\hat{c}_u]$		0.1096	0.5581	0.1079	0.5419	1.4816	0.0490	0.5492	0.0489	0.5291	0.4346
$MSE[\hat{\phi}_1]$		0.0020	0.0084	0.0020	0.0088	0.0351	0.0007	0.0083	0.0007	0.0087	0.0085
$MSE[\hat{\phi}_2]$		0.0090	0.0168	0.0087	0.0160	0.1639	0.0031	0.0160	0.0030	0.0149	0.0396
$MSE[\hat{\eta}_1]$		0.0008	0.0009	-	-	-	0.0003	0.0004	-	-	-
$MSE[\hat{\eta}_2]$		0.0004	0.0006	-	-	-	0.0002	0.0003	-	-	-
Bias[Eff]		-0.0182	-0.1124	-0.0182	-0.1096	0.1114	-0.0094	-0.1122	-0.0092	-0.1088	0.0609
MSE[Eff]		0.0174	0.0328	0.0173	0.0321	0.0314	0.0082	0.0258	0.0081	0.0250	0.0125
Spe. Corr.		0.8787	0.8666	0.8789	0.8666	0.8678	0.9415	0.9155	0.9416	0.9154	0.9376

	TRUE	T=10, N=200					T=20, N=200				
		TVTIEE	TFEE	TVTIE	TFE	TVTIEU	TVTIEE	TFEE	TVTIE	TFE	TVTIEU
$E[\hat{\beta}_1]$	0.5000	0.4997	0.5970	0.4997	0.5971	0.4987	0.5000	0.5899	0.5000	0.5900	0.5002
$E[\hat{\beta}_2]$	0.5000	0.5002	0.4961	0.5002	0.4977	0.5029	0.5000	0.4961	0.5000	0.4978	0.5015
$E[\hat{c}_u]$	-0.5000	-0.4869	0.1727	-0.4872	0.1613	-1.6470	-0.5016	0.1131	-0.5021	0.0980	-1.1104
$E[\hat{\phi}_1]$	0.4000	0.3985	0.3160	0.3985	0.3140	0.5774	0.3995	0.3240	0.3995	0.3219	0.4868
$E[\hat{\phi}_2]$	0.9000	0.8950	0.8139	0.8951	0.8184	1.2804	0.8991	0.8197	0.8993	0.8254	1.0906
$E[\hat{\eta}_1]$	0.0000	-0.0002	0.0040	-	-	-	0.0000	0.0039	-	-	-
$E[\hat{\eta}_2]$	0.0000	-0.0004	-0.0093	-	-	-	-0.0003	-0.0110	-	-	-
$MSE[\hat{\beta}_1]$		0.0001	0.0096	0.0001	0.0096	0.0001	0.0001	0.0082	0.0001	0.0082	0.0001
$MSE[\hat{\beta}_2]$		0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0001	0.0000	0.0000	0.0000
$MSE[\hat{c}_u]$		0.0518	0.5075	0.0514	0.4925	1.3874	0.0252	0.4045	0.0250	0.3860	0.4090
$MSE[\hat{\phi}_1]$		0.0009	0.0080	0.0009	0.0083	0.0331	0.0003	0.0062	0.0003	0.0065	0.0080
$MSE[\hat{\phi}_2]$		0.0041	0.0117	0.0041	0.0110	0.1510	0.0015	0.0084	0.0015	0.0075	0.0383
$MSE[\hat{\eta}_1]$		0.0004	0.0005	-	-	-	0.0002	0.0002	-	-	-
$MSE[\hat{\eta}_2]$		0.0002	0.0003	-	-	-	0.0001	0.0002	-	-	-
Bias[Eff]		-0.0176	-0.1128	-0.0175	-0.1102	0.1118	-0.0079	-0.1007	-0.0078	-0.0973	0.0617
MSE[Eff]		0.0169	0.0328	0.0169	0.0322	0.0313	0.0080	0.0228	0.0080	0.0221	0.0125
Spe. Corr.		0.8746	0.8583	0.8747	0.8582	0.8637	0.9409	0.9145	0.9410	0.9142	0.9371

Table 2. Simulation Results: $x_{1it}^{(2)}$ and $x_{2it}^{(2)}$ are correlated with v_{it} , $(\rho_1, \rho_2) = (0.7, 0.7)$

	TRUE	T=10, N=100				T=20, N=100			
		TVTIEE	TFEE	TVTIE	TFE	TVTIEE	TFEE	TVTIE	TFE
$E[\hat{\beta}_1]$	0.5000	0.4995	0.5976	0.5017	0.5966	0.4993	0.5929	0.5008	0.5913
$E[\hat{\beta}_2]$	0.5000	0.4998	0.4938	0.5681	0.5657	0.5002	0.4938	0.5687	0.5662
$E[\hat{c}_u]$	-0.5000	-0.5072	0.0683	0.0438	0.5343	-0.5086	0.1217	0.0131	0.5397
$E[\hat{\phi}_1]$	0.4000	0.4005	0.3351	0.4134	0.3453	0.4002	0.3286	0.4135	0.3417
$E[\hat{\phi}_2]$	0.9000	0.9012	0.8160	0.7163	0.6851	0.9004	0.7944	0.7268	0.6790
$E[\hat{\eta}_1]$	0.4427	0.4427	0.4503	-	-	0.4425	0.4501	-	-
$E[\hat{\eta}_2]$	0.2951	0.2954	0.2824	-	-	0.2953	0.2807	-	-
$MSE[\hat{\beta}_1]$		0.0002	0.0098	0.0002	0.0096	0.0001	0.0088	0.0001	0.0085
$MSE[\hat{\beta}_2]$		0.0002	0.0002	0.0048	0.0045	0.0001	0.0001	0.0048	0.0044
$MSE[\hat{c}_u]$		0.0242	0.3783	0.3869	1.1632	0.0210	0.4248	0.3036	1.1309
$MSE[\hat{\phi}_1]$		0.0001	0.0051	0.0019	0.0045	0.0000	0.0056	0.0008	0.0042
$MSE[\hat{\phi}_2]$		0.0003	0.0109	0.0403	0.0528	0.0001	0.0131	0.0328	0.0522
$MSE[\hat{\eta}_1]$		0.0002	0.0005	-	-	0.0001	0.0003	-	-
$MSE[\hat{\eta}_2]$		0.0000	0.0004	-	-	0.0000	0.0003	-	-
Bias[Eff]		0.0005	-0.0926	-0.1133	-0.1846	0.0003	-0.0991	-0.1094	-0.1880
MSE[Eff]		0.0008	0.0200	0.0299	0.0551	0.0003	0.0176	0.0209	0.0497
Spe. Corr.		0.9948	0.9272	0.8903	0.8774	0.9982	0.9530	0.9465	0.9250

	TRUE	T=10, N=200				T=20, N=200			
		TVTIEE	TFEE	TVTIE	TFE	TVTIEE	TFEE	TVTIE	TFE
$E[\hat{\beta}_1]$	0.5000	0.5000	0.5980	0.5006	0.5957	0.5001	0.5908	0.4998	0.5881
$E[\hat{\beta}_2]$	0.5000	0.5002	0.4946	0.5692	0.5668	0.5002	0.4944	0.5681	0.5661
$E[\hat{c}_u]$	-0.5000	-0.5064	0.0603	0.0112	0.4961	-0.5062	0.0405	0.0309	0.4821
$E[\hat{\phi}_1]$	0.4000	0.4002	0.3343	0.4166	0.3483	0.4000	0.3385	0.4113	0.3492
$E[\hat{\phi}_2]$	0.9000	0.9004	0.8265	0.7256	0.7004	0.9001	0.8221	0.7209	0.6948
$E[\hat{\eta}_1]$	0.4427	0.4426	0.4489	-	-	0.4425	0.4490	-	-
$E[\hat{\eta}_2]$	0.2951	0.2953	0.2833	-	-	0.2951	0.2813	-	-
$MSE[\hat{\beta}_1]$		0.0001	0.0098	0.0001	0.0093	0.0001	0.0083	0.0000	0.0079
$MSE[\hat{\beta}_2]$		0.0001	0.0001	0.0048	0.0045	0.0000	0.0001	0.0047	0.0044
$MSE[\hat{c}_u]$		0.0125	0.3463	0.3024	1.0375	0.0107	0.3101	0.3025	0.9874
$MSE[\hat{\phi}_1]$		0.0000	0.0048	0.0011	0.0034	0.0000	0.0040	0.0005	0.0030
$MSE[\hat{\phi}_2]$		0.0001	0.0075	0.0336	0.0430	0.0001	0.0069	0.0335	0.0436
$MSE[\hat{\eta}_1]$		0.0001	0.0003	-	-	0.0000	0.0001	-	-
$MSE[\hat{\eta}_2]$		0.0000	0.0002	-	-	0.0000	0.0002	-	-
Bias[Eff]		0.0001	-0.0927	-0.1094	-0.1811	-0.0002	-0.0888	-0.1123	-0.1808
MSE[Eff]		0.0008	0.0201	0.0284	0.0532	0.0003	0.0154	0.0215	0.0466
Spe. Corr.		0.9947	0.9218	0.8862	0.8705	0.9982	0.9516	0.9455	0.9230

Table 3. Simulation Results: $x_{1it}^{(2)}$ and $x_{2it}^{(2)}$ are correlated with v_{it} and u_i^* , $(\rho_1, \rho_2) = (0.7, 0.7)$

	TRUE	T=10, N=100				T=20, N=100			
		TVTIEE	TFEE	TVTIE	TFE	TVTIEE	TFEE	TVTIE	TFE
$E[\hat{\beta}_1]$	0.5000	0.4994	0.5914	0.4991	0.5883	0.5002	0.5909	0.5007	0.5876
$E[\hat{\beta}_2]$	0.5000	0.4993	0.4913	0.5725	0.5682	0.5000	0.4916	0.5728	0.5685
$E[\hat{c}_u]$	-0.5000	-0.5024	-0.2146	-0.4264	0.0079	-0.5043	-0.1727	-0.2823	0.0861
$E[\hat{\varphi}_1]$	0.4000	0.4010	0.3905	0.4526	0.4094	0.4004	0.3814	0.4257	0.3925
$E[\hat{\varphi}_2]$	0.9000	0.9015	0.8929	0.8792	0.8455	0.9005	0.8759	0.8416	0.8201
$E[\hat{\eta}_1]$	0.4427	0.4433	0.4522	-	-	0.4428	0.4524	-	-
$E[\hat{\eta}_2]$	0.2951	0.2939	0.2761	-	-	0.2946	0.2758	-	-
$MSE[\hat{\beta}_1]$		0.0002	0.0087	0.0002	0.0081	0.0001	0.0085	0.0001	0.0079
$MSE[\hat{\beta}_2]$		0.0001	0.0003	0.0054	0.0048	0.0001	0.0002	0.0053	0.0048
$MSE[\hat{c}_u]$		0.0244	0.1229	0.0675	0.3102	0.0220	0.1327	0.0799	0.3733
$MSE[\hat{\varphi}_1]$		0.0000	0.0005	0.0036	0.0008	0.0000	0.0005	0.0009	0.0003
$MSE[\hat{\varphi}_2]$		0.0001	0.0016	0.0034	0.0058	0.0000	0.0011	0.0047	0.0078
$MSE[\hat{\eta}_1]$		0.0002	0.0006	-	-	0.0001	0.0003	-	-
$MSE[\hat{\eta}_2]$		0.0000	0.0005	-	-	0.0000	0.0004	-	-
Bias[Eff]		-0.0031	-0.0593	-0.0589	-0.1153	-0.0012	-0.0614	-0.0724	-0.1247
MSE[Eff]		0.0008	0.0139	0.0171	0.0310	0.0003	0.0109	0.0134	0.0288
Spe. Corr.		0.9947	0.9389	0.9187	0.9051	0.9980	0.9595	0.9544	0.9360

	TRUE	T=10, N=200				T=20, N=200			
		TVTIEE	TFEE	TVTIE	TFE	TVTIEE	TFEE	TVTIE	TFE
$E[\hat{\beta}_1]$	0.5000	0.4999	0.5995	0.4984	0.5953	0.5001	0.5926	0.5007	0.5892
$E[\hat{\beta}_2]$	0.5000	0.4995	0.4913	0.5729	0.5689	0.5000	0.4912	0.5727	0.5681
$E[\hat{c}_u]$	-0.5000	-0.4953	-0.2426	-0.4175	-0.0202	-0.4972	-0.1778	-0.2827	0.0735
$E[\hat{\varphi}_1]$	0.4000	0.4010	0.3950	0.4517	0.4130	0.4002	0.3829	0.4254	0.3941
$E[\hat{\varphi}_2]$	0.9000	0.9012	0.9071	0.8831	0.8614	0.9001	0.8809	0.8444	0.8273
$E[\hat{\eta}_1]$	0.4427	0.4432	0.4525	-	-	0.4428	0.4526	-	-
$E[\hat{\eta}_2]$	0.2951	0.2936	0.2748	-	-	0.2945	0.2756	-	-
$MSE[\hat{\beta}_1]$		0.0001	0.0101	0.0001	0.0093	0.0001	0.0087	0.0000	0.0080
$MSE[\hat{\beta}_2]$		0.0001	0.0002	0.0054	0.0048	0.0000	0.0001	0.0053	0.0047
$MSE[\hat{c}_u]$		0.0118	0.0885	0.0402	0.2568	0.0116	0.1165	0.0633	0.3437
$MSE[\hat{\varphi}_1]$		0.0000	0.0002	0.0031	0.0005	0.0000	0.0004	0.0008	0.0002
$MSE[\hat{\varphi}_2]$		0.0000	0.0008	0.0019	0.0029	0.0000	0.0006	0.0038	0.0061
$MSE[\hat{\eta}_1]$		0.0001	0.0003	-	-	0.0000	0.0002	-	-
$MSE[\hat{\eta}_2]$		0.0000	0.0005	-	-	0.0000	0.0004	-	-
Bias[Eff]		-0.0038	-0.0582	-0.0609	-0.1136	-0.0017	-0.0615	-0.0727	-0.1241
MSE[Eff]		0.0008	0.0138	0.0176	0.0307	0.0003	0.0109	0.0134	0.0286
Spe. Corr.		0.9945	0.9375	0.9158	0.9024	0.9980	0.9584	0.9533	0.9344

Table 4. 5% Empirical Size Values for TVTIEE

(ρ_1, ρ_2)	T=10, N=100			T=20, N=100		
	(0,0)	(0.7,0.7)	General	(0,0)	(0.7,0.7)	General
$\hat{\beta}_1$	0.0430	0.0410	0.0580	0.0490	0.0520	0.0540
$\hat{\beta}_2$	0.0660	0.0590	0.0540	0.0420	0.0500	0.0500
$\hat{c}_u$	0.0390	0.0400	0.0640	0.0380	0.0480	0.0620
$\hat{\varphi}_1$	0.0420	0.0460	0.0560	0.0530	0.0480	0.0380
$\hat{\varphi}_2$	0.0320	0.0430	0.0530	0.0580	0.0480	0.0370
$\hat{\eta}_1$	0.0580	0.0540	0.0500	0.0510	0.0530	0.0490
$\hat{\eta}_2$	0.0460	0.0480	0.0610	0.0550	0.0490	0.0450
	T=10, N=200			T=20, N=200		
	(0,0)	(0.7,0.7)	General	(0,0)	(0.7,0.7)	General
$\hat{\beta}_1$	0.0420	0.0430	0.0450	0.0520	0.0500	0.0560
$\hat{\beta}_2$	0.0470	0.0580	0.0460	0.0440	0.0530	0.0510
$\hat{c}_u$	0.0460	0.0450	0.0510	0.0450	0.0440	0.0660
$\hat{\varphi}_1$	0.0510	0.0540	0.0650	0.0390	0.0480	0.0460
$\hat{\varphi}_2$	0.0460	0.0340	0.0560	0.0420	0.0450	0.0500
$\hat{\eta}_1$	0.0470	0.0590	0.0440	0.0450	0.0510	0.0550
$\hat{\eta}_2$	0.0460	0.0510	0.1070	0.0420	0.0510	0.0720

Table 5. Simulation Results for Time-Varying Heterogeneity vs Time-Invariant Heterogeneity

	T=10, N=100						T=20, N=100					
	$(\rho_1, \rho_2) = (0.7, 0.7)$			$(\rho_1, \rho_2) = (0, 0)$			$(\rho_1, \rho_2) = (0.7, 0.7)$			$(\rho_1, \rho_2) = (0, 0)$		
	TVTIEE	TFEE	Ratio	TVTIE	TFE	Ratio	TVTIEE	TFEE	Ratio	TVTIE	TFE	Ratio
	TVTIEE	TFEE	Ratio	TVTIE	TFE	Ratio	TVTIEE	TFEE	Ratio	TVTIE	TFE	Ratio
$MSE[\hat{\beta}_1]$	0.0004	0.0003	1.23	0.0005	0.0004	1.29	0.0002	0.0002	1.10	0.0002	0.0002	1.10
$MSE[\hat{\beta}_2]$	0.0002	0.0001	1.31	0.0001	0.0001	1.31	0.0001	0.0001	1.11	0.0001	0.0000	1.09
$MSE[\hat{c}_u]$	0.0283	0.0263	1.08	0.1829	0.1437	1.27	0.0223	0.0222	1.01	0.0782	0.0718	1.09
$MSE[\hat{\varphi}_1]$	0.0001	0.0001	1.32	0.0037	0.0028	1.32	0.0000	0.0000	1.12	0.0013	0.0012	1.12
$MSE[\hat{\varphi}_2]$	0.0007	0.0005	1.26	0.0172	0.0137	1.25	0.0002	0.0002	1.08	0.0066	0.006	1.09
$MSE[\hat{\eta}_1]$	0.0002	0.0001	1.33	-	-	-	0.0001	0.0001	1.11	-	-	-
$MSE[\hat{\eta}_2]$	0.0000	0.0000	1.17	-	-	-	0.0000	0.0000	1.07	-	-	-
$MSE[Eff]$	0.0011	0.0009	1.33	0.0223	0.0182	1.22	0.0004	0.0004	1.12	0.0106	0.0097	1.10
	T=10, N=200						T=20, N=200					
	$(\rho_1, \rho_2) = (0.7, 0.7)$			$(\rho_1, \rho_2) = (0, 0)$			$(\rho_1, \rho_2) = (0.7, 0.7)$			$(\rho_1, \rho_2) = (0, 0)$		
	TVTIEE	TFEE	Ratio	TVTIE	TFE	Ratio	TVTIEE	TFEE	Ratio	TVTIE	TFE	Ratio
	TVTIEE	TFEE	Ratio	TVTIE	TFE	Ratio	TVTIEE	TFEE	Ratio	TVTIE	TFE	Ratio
$MSE[\hat{\beta}_1]$	0.0143	0.0132	1.24	0.0002	0.0002	1.28	0.0001	0.0001	1.11	0.0001	0.0001	1.20
$MSE[\hat{\beta}_2]$	0.0003	0.0002	1.25	0.0001	0.0001	1.23	0.0000	0.0000	1.12	0.0000	0.0000	1.12
$MSE[\hat{c}_u]$	0.0143	0.0132	1.08	0.0885	0.0681	1.30	0.0112	0.0110	1.02	0.0384	0.0351	1.10
$MSE[\hat{\varphi}_1]$	0.0001	0.0000	1.34	0.0017	0.0013	1.30	0.0000	0.0000	1.12	0.0006	0.0006	1.11
$MSE[\hat{\varphi}_2]$	0.0003	0.0002	1.35	0.0086	0.0065	1.31	0.0001	0.0001	1.13	0.0032	0.0029	1.13
$MSE[\hat{\eta}_1]$	0.0001	0.0001	1.24	-	-	-	0.0000	0.0000	1.12	-	-	-
$MSE[\hat{\eta}_2]$	0.0000	0.0000	1.26	-	-	-	0.0000	0.0000	1.11	-	-	-
$MSE[Eff]$	0.0010	0.0008	1.34	0.0211	0.0176	1.20	0.0004	0.0003	1.13	0.0103	0.0093	1.10

Table 6. Variable Definitions

Variable	Description
TEX	Total operating expenses
Y_1	Total securities
Y_2	Total loans
W_1	Price of fixed assets
W_2	Price of labor
W_3	Price of borrowed funds
Z	Gross total equity
t	Time trend
SHA	Share of Asian employees
ETH _{HHI}	HHI for Ethnicity
DPI	Disposable personal income in millions
UR	Unemployment rate
ROR	Return on revenue
tex	$\ln(\text{TEX}) - \ln(W_3)$
w ₁	$\ln(W_1) - \ln(W_3)$
w ₂	$\ln(W_2) - \ln(W_3)$
y ₁	$\ln(Y_1)$
y ₂	$\ln(Y_2)$
z	$\ln(Z)$

Table 7. Descriptive Statistics

Variable	Mean	Std. Dev.	Min	Max
tex	11.1342	1.6827	4.6827	16.8996
y ₁	13.4201	1.6829	4.1248	19.2182
y ₂	14.6590	1.6839	4.6347	20.2894
w ₁	2.0621	0.6637	-0.2516	5.0769
w ₂	1.9430	0.7824	0.3914	4.4312
z	12.5337	1.6153	5.9628	18.4405
t	14.4975	8.4051	1	32
SHA	3.4206	3.5403	0.4987	20.3049
ETH _{HHI}	0.6194	0.1193	0.4675	0.9254
DPI	0.1793	0.1756	0.0057	1.1630
UR	6.2697	1.9472	2.1877	17.3133
ROR	0.3125	0.1414	-0.6641	0.8805
# of Obs.	7060			

Table 8.1. Stochastic Frontier Model Estimates: Frontier Parameters

tex	BC		TFE		GTFE		TVTIE		TVTIEE	
w ₁	-0.0700		-0.0378		-0.0835		0.1139	**	0.0978	**
	(0.0594)		(0.0585)		(0.0582)		(0.0420)		(0.0439)	
w ₂	0.6295	***	0.6087	***	0.7052	***	0.1572	**	0.1628	**
	(0.0669)		(0.0668)		(0.0646)		(0.0527)		(0.0556)	
y ₁	0.5141	***	0.4244	***	0.5137	***	0.3024	***	0.2846	***
	(0.0326)		(0.0333)		(0.0320)		(0.0243)		(0.0254)	
y ₂	0.3195	***	0.3087	***	0.3100	***	0.1544	***	0.1566	***
	(0.0533)		(0.0526)		(0.0551)		(0.0427)		(0.0443)	
z	0.1398	**	0.2350	***	0.1791	**	0.5242	***	0.4921	***
	(0.0660)		(0.0667)		(0.0660)		(0.0514)		(0.0540)	
0.5w ₁ ²	0.0477	**	0.0742	***	0.0373	**	-0.0002		0.0184	
	(0.0157)		(0.0155)		(0.0152)		(0.0102)		(0.0107)	
w ₁ w ₂	-0.0767	***	-0.0885	***	-0.0657	***	0.0243	**	0.0155	
	(0.0136)		(0.0138)		(0.0133)		(0.0102)		(0.0106)	
0.5w ₂ ²	0.1023	***	0.1316	***	0.0974	***	0.0884	***	0.0874	***
	(0.0192)		(0.0193)		(0.0185)		(0.0136)		(0.0142)	
0.5y ₁ ²	0.0351	***	0.0330	***	0.0329	***	0.0343	***	0.0376	***
	(0.0032)		(0.0033)		(0.0031)		(0.0021)		(0.0022)	
y ₁ y ₂	-0.0440	***	-0.0399	***	-0.0432	***	-0.0434	***	-0.0432	***
	(0.0048)		(0.0049)		(0.0048)		(0.0038)		(0.0040)	
0.5y ₂ ²	0.0547	***	0.0700	***	0.0492	***	0.0855	***	0.0907	***
	(0.0069)		(0.0068)		(0.0068)		(0.0055)		(0.0057)	
y ₁ w ₁	-0.0279	***	-0.0232	***	-0.0256	***	-0.0254	***	-0.0272	***
	(0.0064)		(0.0064)		(0.0062)		(0.0045)		(0.0047)	
y ₁ w ₂	0.0294	***	0.0101		0.0306	***	0.0220	***	0.0124	**
	(0.0069)		(0.0069)		(0.0066)		(0.0049)		(0.0052)	
y ₂ w ₁	-0.0548	***	-0.0488	***	-0.0398	**	-0.0291	**	-0.0343	***
	(0.0132)		(0.0130)		(0.0134)		(0.0093)		(0.0098)	
y ₂ w ₂	-0.0342	**	-0.0418	***	-0.0372	**	0.0349	***	0.0273	**
	(0.0129)		(0.0126)		(0.0128)		(0.0096)		(0.0101)	
w ₁ z	0.1127	***	0.0945	***	0.0931	***	0.0546	***	0.0625	***
	(0.0158)		(0.0156)		(0.0158)		(0.0110)		(0.0116)	
w ₂ z	-0.0144		0.0086		-0.0180		-0.0671	***	-0.0509	***
	(0.0156)		(0.0153)		(0.0153)		(0.0112)		(0.0119)	
y ₁ z	-0.0152	**	-0.0072		-0.0141	**	0.0027		0.0029	
	(0.0057)		(0.0059)		(0.0056)		(0.0040)		(0.0042)	
y ₂ z	0.0088		-0.0097	**	0.0106	**	-0.0233	***	-0.0263	***
	(0.0049)		(0.0049)		(0.0047)		(0.0037)		(0.0038)	
t	-0.0077	***	-0.0036	**	-0.0102	***				
	(0.0018)		(0.0017)		(0.0017)					
0.5t ²	-0.0001		-0.0002	***	0.0000					
	(0.0001)		(0.0001)		(0.0001)					
constant	-3.3309	***								
	(0.1903)									

Table 8.2. Stochastic Frontier Model Estimates: Composed Error Term Parameters

σ_w	BC		TFE		GTFE		TVTIE		TVTIEE	
constant	-2.9532	***	-3.0864	***	-3.1153	***	-4.5190	***	-4.6000	***
	(0.0174)		(0.0178)		(0.0193)		(0.0185)		(0.0187)	
σ_u										
ROR	-0.5432	***	3.7570	***	-10.1382	***	14.9333	***	7.0519	
	(0.0813)		(0.6737)		(0.6807)		(1.2560)		(0.5642)	***
constant	-0.1596		-2.7837	***	-2.3360	***	-12.6084	***	-5.7098	
	(0.0922)		(0.6264)		(0.1606)		(0.8607)		(0.4468)	***
Ave. Efficiency	47.28		77.53		93.97		97.66		86.31	
Log-likelihood	-324.19		611.14		697.01		5006.26		13361.17	

Note: Standard errors are given in parenthesis. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 9. Pearson and Spearman Correlations for Efficiency Estimates

SPE/PEAR	BC	TFE	GTFE	TVTIE	TVTIEE
BC		-0.30	0.02	-0.02	0.09
TFE	-0.12		-0.29	0.32	0.33
GTFE	0.16	-0.39		-0.34	-0.41
TVTIE	-0.08	0.42	-0.87		0.77
TVTIEE	0.09	0.36	-0.56	0.82	

Figure 1. Densities of Efficiency Estimates

Figure 2. Relationships Between Efficiency Estimates