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Abstract

We propose a spatial autoregressive stochastic frontier model, which allows for the endogeneity in both the frontier and environmental variables (i.e., endogeneity due to correlation of inefficiency term and the two-sided error term). The model parameters are estimated using a single-stage control function approach. Monte Carlo simulations show that our proposed model and approach perform well in finite samples. We employed our methodology to the Chinese chemicals firm data and found evidence for both spatial effects and endogeneity.

Keywords: Productivity and competitiveness; Chinese chemical firms; Endogeneity; Spatial autoregressive spillover; Stochastic frontier.

JEL Classification: C13, C23, C36, D24.

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1. Introduction

One of the widely examined issues in the operational research (O.R.) is firm efficiency. In this literature, the data envelopment analysis (DEA) and stochastic frontier analysis (SFA) are two popular choices for estimating efficiency. Zhou et al. (2008) present a survey that covers around 100 O.R. studies for energy and environmental modeling; and Fethi and Pasiouras (2010) present a review of around 200 O.R. studies that use techniques that aim to assess bank performance. Hence, only for banking, energy, and environmental modeling, there are hundreds of performance measurement studies.

The productivity spillover effects at firm-level are well-documented in the literature (e.g., Hu, Jefferson, and Jinchang, 2005; Kuller, 2004; Keller and Yeaple, 2009). If the sources of spillover mechanism are informal conversations, innovative activities, and local competitive pressure, then the firms that are located in closer proximity to each other can experience greater spillover effects. Although efficiency spillovers may be important factors in productivity spillovers, the traditional stochastic frontier models do not consider such spillovers.

One of the contributions of our study is proposing a stochastic frontier model where efficiency and productivity spillovers are present. In particular, we address a variety of endogeneity problems, for the first time, for spatial stochastic frontier models. As stated earlier, traditional stochastic frontier models do not control for spatial lag of the dependent variable, which captures so called spatial autoregressive (SAR) dependence (see Cliff and Ord, 1973, 1981). If such a dependence is present, omitting the SAR term would lead to inconsistent parameter and efficiency estimates. On the other hand, if the SAR term is included, this term would be correlated with the two-sided error term, which means that this term is endogenous. Druska and Horrace (2004), Glass et al. (2013) (GKP), Glass et al. (2014) (GKS), and Kutlu and Nair-Reichert (2019) address this problem via distribution-free approaches.[4] An important advantage of these distribution-free approaches is that we do not assume a specific distribution to the inefficiency term. However, outliers may have serious implications for the magnitudes of the efficiency estimates.[5] Hence, alternatively, in the conventional SFA, it is common to represent inefficiency via a one-sided error term.[6] In the spatial spillover context, Glass et al. (2016) followed this approach and introduced the SAR variable while also making distributional assumptions (i.e., half normal distribution) on the inefficiency component of the error structure.[7] Moreover, Glass and Kenjegalieva (2019) propose a spatial decomposition of total factor productivity growth.

While these stochastic frontier approaches address the endogeneity problem due to the SAR variable being endogenous, they don't address the endogeneity problems resulting from the endogeneity of frontier variables (other than SAR term) and environmental variables (i.e., variables that affect inefficiency), which would lead to inconsistent parameter and efficiency estimates. For example, Mutter et al. (2013) argue that if the quality is a part of the production process where it is cost enhancing and quantity and quality decisions are made simultaneously, then the quality variable (which is a frontier variable) would be endogenous, i.e., correlated with the two-sided error term.[8] Another example for endogeneity is that a determinant of inefficiency (i.e., an environmental variable) and two-sided error term can be correlated. Karakaplan and Kutlu (2019) give such an example from education markets. They assume that cost efficiencies of education districts depend on education market concentration, which is measured by Herfindahl-Hirschman Index (HHI). They argue that if the government simultaneously decides whether to

[4] See Schmidt and Sickles (1984) and Cornwell et al. (1990) for non-spatial distribution-free stochastic frontier models and Duygun et al. (2016) for their Kalman filter counterparts.
[5] See Kutlu (2012, 2017) for a more details about this issue and some potential solutions.
[6] Among others, see, for example, Mester (1997), Bos et al. (2009), Brissimis et al. (2010), Tecles and Tabak (2010, and Galán et al. (2015).
[7] See Han et al. (2016) for an extension of Glass et al. (2014) where the spatial weighting matrix is time-varying.
[8] Note that dropping the quality variable does not address the problem as this would bias efficiency estimates.

consolidate districts (which changes education market concentration) and district expenditure structures, this would result in endogeneity of education market concentration. Karakaplan and Kutlu (2017b) give another example of endogeneity of HHI as an environmental variable from production function of Japanese cotton spinning industry. Both studies find that HHI is endogenous.

In the stochastic frontier context, there is a recent yet growing interest for solutions to these types of endogeneity problems. Guan et al. (2009), Kutlu (2010), Tran and Tsionas (2013, 2015), Amsler et al. (2016, 2017), Griffiths and Hajargasht (2016), Karakaplan and Kutlu (2017a,b) [9], Kutlu (2018a), and Kutlu et al. (2019) exemplify such studies.[10] [11] However, none of studies consider spatial spillovers.

In this paper, we consider a SAR stochastic frontier model where endogeneity of both frontier and environmental variables are allowed. Hence, we address three different endogeneity problems (endogeneity of SAR term, frontier variables, and environmental variables) at the same time. We achieve this by employing a single-stage control function approach, which was first introduced by Kutlu (2010) to the stochastic frontier literature. Our general estimation strategy can easily be modified and applied in both cross-sectional SAR stochastic frontier context as well as conventional SAR models without inefficiency, i.e., full efficiency. Moreover, besides cost and production function estimation, our model can be applied in the industrial organization setting where the one-sided error term captures the market power. For example, Orea and Steinbucks (2018), Karakaplan and Kutlu (2019a), and Kutlu and Wang (2018) propose conduct parameter

[9] See Kutlu and Nair-Reichert (2018) and Karakaplan and Kutlu (2019b) for applications of Karakaplan and Kutlu (2017a, b).
[10] See Kutlu and Tran (2019) for a literature review on endogeneity and heterogeneity in stochastic frontier models.
[11] Kutlu and Sickles (2012) use similar approaches to address endogeneity issues in the Kalman filter estimation context.

models that use stochastic frontier models.[12] Since the conduct parameter models involve estimation of demand and supply equations, they would suffer from endogeneity issues; and thus, these studies utilize techniques that address endogeneity issues. Therefore, the scope of our contribution is beyond the efficiency measurement context.

Our Monte Carlo simulations show that our estimator performs well and ignoring endogeneity or spatial dependence leads to biased parameter and efficiency estimates. Using our estimator, we estimate the efficiencies of Chinese chemicals firms. We find evidence against Hick's (1935) quiet life hypothesis, which argues that efficiency and market power are inversely related. When we allow spatial spillovers, a decrease in market concentration may have a least two effects. First, the managers would perform in a more competitive environment, which would lead to a pressure to work harder to reach more efficient production outcomes (QLH effect). Second, the efficiency spillovers would come from firms that are less concentrated. If being in a proximity of larger firms would help improving inefficiency, then lower concentration may have a negative effect on efficiency. In our case, it seems that the latter effect dominates QLH effect.

The remainder of the paper is organized as follows. Section 2 presents the model and discusses the estimation of efficiency and test for endogeneity. Finite sample behavior of the proposed approach is given in Section 3 using Monte Carlo simulations. Section 4 presents an empirical application to illustrate the usefulness of the proposed model and approach. Section 5 concludes the paper.

2. The Model and Estimation of Efficiency

2.1. The Model

[12] See Bresnahan (1989) and Perloff et al. (2007) for details about conduct parameter approach.

For the sake of fixing the ideas, we present a production function. The same equations can be used for the cost function estimation with minor modifications. We call a variable endogenous if it is correlated with the two-sided error term. A conventional stochastic frontier model is given by:

$$\begin{aligned} y_{it} &= x_{1it}^{'}\beta - u_{it} + v_{it} \\ u_{it} &= h_{it} u_{it}^{*} \\ h_{it} &= f(x_{2it}^{'}\varphi_{u}) > 0, \end{aligned} \tag{1}$$

where y_{it} is the logarithm of the output for productive unit $i = 1,2,...,N$ at time $t = 1,2,...,T$; x_{1it} is a $k_1 \times 1$ vector of exogenous variables; $u_{it} \geq 0$ is a one-sided term that is capturing the inefficiency; $u_{it}^{*} \sim N^{+}(\mu, \sigma_{u}^{2})$; x_{2it} is a $k_2 \times 1$ vector of exogenous variables, which does not contain the constant; v_{it} is the usual two-sided error term for the production function; and β ($k_1 \times 1$) and φ_u ($k_2 \times 1$) are parameters. This model does not incorporate spatial spillovers and/or endogeneity.

Now, we present our stochastic frontier model that incorporates spatial spillovers and endogeneity. Consider the following stochastic frontier model:

$$\begin{aligned} y_{it} &= \rho \sum_{j} w_{ij} y_{jt} + x_{1it}^{'}\beta - u_{it} + v_{it} \\ x_{it} &= \delta' z_{it} + \varepsilon_{it} \\ u_{it} &= h_{it} u_{it}^{*} \\ h_{it} &= f(x_{2it}^{'}\varphi_{u}) > 0, \end{aligned} \tag{2}$$

where y_{it} is the logarithm of the output for productive unit $i = 1,2,...,N$ at time $t = 1,2,...,T$; $w_{ij} \geq 0$ is the (spatial) weight for the effect of j^{th} productive unit's output on the output i^{th} productive unit; x_{1it} is a $k_1 \times 1$ vector of variables that may include endogenous variables;

$u_{it} \geq 0$ is a one-sided term that is capturing the inefficiency; $u_{it}^{*} \sim N^{+}(\mu, \sigma_{u}^{2})$;[13] x_{2it} is a $k_2 \times 1$ vector of variables that may include endogenous variables, which does not contain the constant; v_{it} is the usual two-sided error term for the production function; x_{it} is a $p \times 1$ vector of endogenous variables from x_{1it} and/or x_{2it}, i.e., $x_{it} \subseteq (x_{1it} \cup x_{2it})$; z_{it} is an $l \times 1$ vector of instrumental variables; ε_{it} is a $p \times 1$ vector of usual error terms; and β, δ, and φ_u are ($k_1 \times 1$), ($l \times p$), and ($k_2 \times 1$) parameters, respectively.

The main differences between our model and the conventional stochastic frontier model is the $\sum_j w_{ij} y_{jt}$ (SAR) term and endogeneity of frontier and environmental variables. This term captures the total spatial spillovers on the output of i^{th} productive unit from other productive units. The weights, $w_{ij} \geq 0$, capture the relative spillover effect of j^{th} productive unit on the i^{th} productive unit. Glass et al. (2016) incorporate spatial spillovers similarly by including the SAR term in their model but they still assume that all variables are exogenous (except the SAR term). The key difference of our model from Glass et al. (2016) is that we model the stochastic frontier production function simultaneously with the prediction equation for endogenous variables, i.e., $x_{it} = \delta' z_{it} + \varepsilon_{it}$. As we will describe later in this section, this is the key point for addressing the endogeneity issues.

In our benchmark scenario, we assume that the weighting matrix is row-normalized so that sum of each row equals 1, i.e., $\sum_j w_{ij} = 1$. Later, we will also consider weighting matrices with scalar normalizations, i.e., cw_{ij} for some constant $c > 0$. We assume that u_{it}^{*} is independent of

[13] Our method can easily be applied to other conventional distributions for the inefficiency term such as gamma, exponential, and doubly truncated normal distributions.

x_{1it}, x_{2it}, and v_{it}. Let Ω_ε be the variance-covariance matrix of ε_{it}, and $\zeta^*_{it} = (\varepsilon^{*'}_{it}, v_{it})' = (\varepsilon'_{it}\Omega_\varepsilon^{-1/2}, v_{it})'$. Also, assume that:

$$\zeta^*_{it} \sim \boldsymbol{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}I_q & \sigma_v\tau\\ \sigma_v\tau' & \sigma_v^2\end{bmatrix}\right) = \boldsymbol{N}(0,\Omega),$$

where $\boldsymbol{N}(.,.)$ denotes the multivariate normal; σ_v^2 is the variance of v_{it}; and τ is the vector representing the correlation between ε^*_{it} and v_{it}.

A Cholesky decomposition of the variance-covariance matrix of $(\varepsilon^{*'}_{it}, v_{it})'$ gives:[14]

$$\begin{bmatrix}\varepsilon^*_{it}\\ v_{it}\end{bmatrix} = \begin{bmatrix}I_q & 0\\ \sigma_v\tau' & \sigma_v\sqrt{1-\tau'\tau}\end{bmatrix}\begin{bmatrix}\varepsilon^*_{it}\\ r^*_{it}\end{bmatrix}, \tag{3}$$

where $r^*_{it} \sim \mathrm{N}\left(0,1\right)$, r^*_{it}, and ε^*_{it} are independent. Therefore, we have:

$$\begin{aligned} v_{it} &= \varepsilon^{*'}_{it}\sigma_v\tau + \sigma_r r^*_{it} \\ &= \varepsilon'_{it}\eta + r_{it}, \end{aligned} \tag{4}$$

where $\sigma_r = \sigma_v\sqrt{1-\tau'\tau}$, $\eta = \sigma_r\Omega_\varepsilon^{-1/2}\tau / \sqrt{1-\tau'\tau}$, and $r_{it} = \sigma_r r^*_{it}$. Then, the frontier equation can be written as follows:

$$y_{it} = \rho\sum\nolimits_j w_{ij}y_{jt} + x'_{1it}\beta + (x_{it} - \delta' z_{it})'\eta + e_{it}, \tag{5}$$

where $e_{it} = r_{it} - u_{it}$ and $\varepsilon'_{it}\eta = (x_{it} - \delta' z_{it})'\eta$ is a bias correction term. Here, the Cholesky

[14] This is a standard variance-covariance decomposition. In Equation 3, the variance of left-hand-side and right-hand-side are the same.

decomposition enables us to decompose v_{it} into two components: one that is correlated with the regressors and one that is not correlated with the regressors. Hence, by including the correlated component (i.e., ε_{it}) as a regressor in the frontier equation, we can use the existing formulas for spatial stochastic frontier models. However, we do not observe $\varepsilon_{it} = x_{it} - \delta' z_{it}$, which means that we need to estimate this term (by estimating δ) simultaneously along with other parameters. Therefore, the log-likelihood function would have an additional term that controls for the randomness of ε_{it}. Below we outline how the log-likelihood would be calculated.

The density function of r_{it} is given by:

$$f_{r_{it}}(r_{it}) = 2\pi\sigma_r^2{}^{-1/2} \exp\left(-\frac{r_{it}^2}{2\sigma_r^2}\right). \tag{6}$$

Moreover, the density function of ε_{it} is given by:

$$f_{\varepsilon_{it}}(\varepsilon_{it}) = \left|2\pi\Omega_\varepsilon\right|^{-1/2} \exp\left(-\frac{1}{2} tr(\Omega_\varepsilon^{-1}\varepsilon_{it}\varepsilon_{it}^{'})\right). \tag{7}$$

Then, as ε_{it} and e_{it} are independent, the log-likelihood function is given by:

$$\ln L = T \ln L_0 + \sum\nolimits_{i,t} \ln L_{1it} + \ln L_{2it} \ , \tag{8}$$

where $T\ln L_0 = T\ln \left|I_N - \rho W\right|$ is the scaled logged determinant of the Jacobian of the transformation from e to y; W is the $N \times N$ row-normalized matrix for weights with zero diagonals[15] [16]

$$\ln L_{1it} = -\frac{1}{2}\ln(2\pi\sigma_r^2) - \frac{1}{2}\frac{e_{it}^2}{\sigma_r^2} + \frac{1}{2}\left(\frac{\mu_{it*}^2}{\sigma_{it*}^2} - \frac{\mu^2}{\sigma_u^2}\right) + \ln\left(\frac{\sigma_{it*}\Phi(\frac{\mu_{it*}}{\sigma_{i*}})}{\sigma_u\Phi(\frac{\mu}{\sigma_u})}\right);$$

$$\ln L_{2it} = -\frac{1}{2}\ \ln\ \left|2\pi\Omega_\varepsilon\right|\ + tr(\Omega_\varepsilon^{-1}\varepsilon_{it}\varepsilon_{it}^{'})\ ;$$

where $\mu_{it}^* = \dfrac{-\sigma_u^2 e_{it} h_{it} + \mu\sigma_r^2}{\sigma_u^2 h_{it}^2 + \sigma_r^2}$; $\sigma_{it*}^2 = \dfrac{\sigma_r^2\sigma_u^2}{\sigma_u^2 h_{it}^2 + \sigma_r^2}$; $e_{it} = y_{it} - \rho\sum_j w_{ij}y_{jt} - x_{1it}^{'}\beta - \varepsilon_{it}^{'}\eta$; and $\varepsilon_{it} = x_{it} - \delta^{'} z_{it}$. By maximizing the total log-likelihood $\ln L$, we obtain the estimates for the model's parameters. Under standard conditions of maximum likelihood theory, our estimator is consistent as $NT \to \infty$.

Unlike the standard stochastic frontier models with endogeneity, equation (8) contains an additional term $T\ln L_0$, and one of the outstanding difficulties we face is that in a large sample, $\left|I_N - \rho W\right|$ term is the determinant of a large matrix, which needs to be re-calculated at each iteration of the optimization procedure. This can be computationally expensive and time consuming. To reduce the computational time, one potential solution, suggested by Pace an Perry (1997), is evaluating $\left|I_N - \rho W\right|$ term using a vector of values for ρ in the interval $[\rho_{\min}, \rho_{\max}]$. These values need to be calculated before optimization and thus would only require calculation of

[15] It is standard to assume in the literature that diagonal elements of W are zero. This rules out self-influence possibility.
[16] For unbalanced panel data the weighting matrix W would be time-varying.

the corresponding vector of determinants once. If we have a sufficiently fine grid of ρ values, we can use interpolated values of $\left|I_N - \rho W\right|$ to obtain intervening points.[17] In what follows, we assume that $\rho \in [0,1)$, the elements of W are non-negative, and all the diagonal elements of W are zero (so as to avoid self-influence). An implication of this this assumption is that $\left|I_N - \rho W\right| \neq 0$ and thus $I_N - \rho W$ is non-singular. As mentioned by LeSage and Pace (1999), $\rho \in [0,1)$ assumption is widely employed in the literature.[18] Moreover, as described by Kutlu (2018b), we will argue later in the paper that this assumption is useful when interpreting the efficiency estimates. Following Glass et al. (2014), we also assume that the rows and columns of W and $(I_N - \rho W)^{-1}$ are uniformly bounded in absolute value before row-normalizing W. This assumption implies that the spatial process for the dependent variable has a fading memory (Kelejian and Pruchas, 1998, 1999). The computational burden can be reduced further by applying variations of concentrated log-likelihood approaches in the literature (e.g., Elhorst, 2009; Glass et al., 2014). Finally, note that when we have cross-sectional data, we can simply assume that $T = 1$, and the rest of the analysis remains the same.

Once we obtain the parameter estimates, the inefficiency term u_{it} can be predicted via:

$$\hat{u}_{it} = E\left[u_{it} \mid e_{it}\right] = h_{it}\left[\mu_{it*} + \frac{\sigma_{it*}\phi(\frac{\mu_{it*}}{\tilde{\sigma}_{it*}})}{\Phi(\frac{\mu_{it*}}{\sigma_{it*}})}\right] \tag{9}$$

In practice, this equation is evaluated at $\hat{e}_{it} = y_{it} - \hat{\rho}\sum_j w_{ij} y_{jt} - x_{1it}^{'}\hat{\beta}$.

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[17] There are several approaches to obtain this determinant (computationally) efficiently. See LeSage and Pace (1999) for details of these approaches as well as numerical approaches used in the maximum likelihood estimation.

[18] Glass et al. (2014) assume that $\rho \in (1/\lambda_{\min}, 1)$ where $\lambda_{\min}$ is the smallest real characteristic root of W.

2.2. Direct, Indirect, and Total Efficiency Estimates

As argued by LeSage (2009), the marginal effect of explanatory variables would be a function of the SAR term; and therefore the β parameter estimates cannot be interpreted as marginal effects. To obtain the interpretable form of the marginal effects, we represent the frontier equation in (2) in matrix form, which is given by:

$$y_{.t} = \rho W y_{.t} + X_{1.t}\beta - u_{.t} + v_{.t}, \tag{10}$$

or equivalently:

$$y_{.t} = (I_N - \rho W)^{-1} X_{1.t}\beta - (I_N - \rho W)^{-1} u_{.t} + (I_N - \rho W)^{-1} v_{.t}, \tag{11}$$

where $y_{.t} = \; y_{1t}, y_{2t}, \ldots, y_{Nt} \; '$, $u_{.t} = \; u_{1t}, u_{2t}, \ldots, u_{Nt} \; '$, $v_{.t} = \; v_{1t}, v_{2t}, \ldots, v_{Nt} \; '$, $X_{1.t} = \; x_{1t}, x_{2t}, \ldots, x_{Nt} \; '$, and $t = 1, 2, \ldots, T$. In what follows, other vectors and matrices are defined similarly.

After renaming the variables, we have:

$$y_{.t} = \tilde{X}_{1.t}\beta - \tilde{u}_{.t} + \tilde{v}_{.t}, \tag{12}$$

where $\tilde{X}_{1.t} = (I_N - \rho W)^{-1} X_{1.t}$, $\tilde{u}_{.t} = (I_N - \rho W)^{-1} u_{.t}$, and $\tilde{v}_{.t} = (I_N - \rho W)^{-1} v_{.t}$. Therefore, the marginal effects are given by:

$$\frac{\partial y_{it}}{\partial x_{1kjt}} = \beta_k [(I_N - \rho W)^{-1}]_{ij}, \tag{13}$$

where x_{1kjt} is the k^{th} frontier variable for productive unit j at time t; β_k is the k^{th} component of

β; and $[(I_N - \rho W)^{-1}]_{ij}$ is the ij^{th} element of $(I_N - \rho W)^{-1}$. The total marginal effect of k^{th} frontier variable at time t is defined as the marginal change in y_{it} as a response to changes in x_{1kjt} for all j:

$$\sum_j \frac{\partial y_{it}}{\partial x_{1kjt}} = \beta_k \sum_j [(I_N - \rho W)^{-1}]_{ij} . \tag{14}$$

As pointed out by Kutlu (2018b), the total inefficiency is captured by the $\tilde{u}_{it}$ term, not by u_{it}. Kutlu (2018b) shows that when W is a row-normalized weighting matrix with diagonal elements being zero and $\rho \in [0,1)$, we have $(I_N - \rho W)^{-1} \geq 0$, i.e., all elements are non-negative. Therefore, $\tilde{u}_{it} \geq 0$, $\tilde{u}_{it}$ is a non-decreasing function of components of $u_{.t}$, and if $u_{.t} = 0$, then $\tilde{u}_{it} = 0$. These imply that we can use $\tilde{u}_{it} = 0$ to represent the full efficiency benchmark. Note that not all weighting matrices satisfy $(I_N - \rho W)^{-1} \geq 0$. For example, a scalar-normalized weighting matrix may or may not satisfy this property. The following proposition shows that this property is satisfied for a certain family of scalar-normalizations.

Proposition: *Let W be a scalar-normalized weighting matrix, i.e., $W = c\tilde{W}$, where $\tilde{W}$ is the weighting matrix (with non-negative elements) before normalization with diagonal elements being equal to zero and c is the normalization constant. Assume that $\rho \in [0,1)$* and $0 < c \leq \dfrac{1}{\max_i \tilde{\lambda}_i}$, where $\{\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_N\}$ are the eigenvalues of $\tilde{W}$. *Then, all elements of $(I_N - \rho W)^{-1}$ are non-zero, i.e., $(I_N - \rho W)^{-1} \geq 0$.*

Proof: Note that inverse of an M-matrix has non-negative elements where an M-matrix is defined as a matrix with non-positive off-diagonal elements with eigenvalues whose real parts are positive. Trivially, all off-diagonal elements of $I_N - \rho W$ are non-positive. Moreover, the eigenvalues of $I_N - \rho W$ are $\{1-\rho\lambda_1, 1-\rho\lambda_2, ..., 1-\rho\lambda_N\}$ where λ_i's are the eigenvalues of W. If $\lambda_i \leq 0$, then we have $1-\rho\lambda_i > 0$. If $\lambda_i > 0$, then $1 \geq \frac{1}{\max_i \ \tilde{\lambda}_i}\tilde{\lambda}_i \geq c\tilde{\lambda}_i = \lambda_i$, which implies that $1-\rho\lambda_i > 0$. Hence, we conclude that $(I_N - \rho W)^{-1} \geq 0$. $\square$

In our spatial model, the usual formula for calculating (total) efficiency is not valid as it ignores the spatial spillovers. The corrected efficiency can be calculated by:

$$E_{it} = \exp(-\tilde{u}_{it}). \tag{15}$$

This is a generalization of the usual formula as when $\rho = 0$, we have $\tilde{u}_{it} = u_{it}$.

We define direct inefficiency of i^{th} productive unit as the part of the inefficiency that is resulting from reasons other than spillovers; and indirect inefficiency as the part of inefficiency that is resulting purely from spillovers of other productive units. The shares of direct and indirect inefficiencies (see Kutlu, 2018b) are given by:

$$\begin{aligned} SIE_{it}^{dir} &= \frac{[(I_N - \rho W)^{-1}]_{ii} u_{it}}{\tilde{u}_{it}} \\ SIE_{it}^{ind} &= \frac{\sum_{i \neq j}[(I_N - \rho W)^{-1}]_{ij} u_{jt}}{\tilde{u}_{it}}. \end{aligned} \tag{16}$$

These shares can be used to decompose inefficiency into direct and indirect efficiency components.

2.3. Testing Endogeneity

Amsler et al. (2016) and Karakaplan and Kutlu (2017a,b) describe a simple test, using similar ideas with the Durbin-Wu-Hausman test, for endogeneity for the non-spatial stochastic frontier models. These tests are applicable in our setting as well. We can test the endogeneity using the F-test for $\eta = 0$. If all components of η are jointly significant, we conclude that the bias correction term is needed and thus we have endogeneity. We can also test the endogeneity of individual variables by testing the significance of the corresponding component of η.

3. Monte Carlo Simulations

To evaluate the performance of our proposed estimator in finite samples, we conduct a Monte Carlo experiment. For simplicity, we consider the cross-sectional setting, i.e., $T = 1$. We consider the following data generating process (DGP):

$$
\begin{aligned}
y_i &= \rho \sum\nolimits_j w_{ij} y_j + z_{1i}\beta_1 + q_{fi}\beta_2 + v_i - u_i \\
u_i &= h_i u_i^* \\
h_i &= [\exp(z_{2i}\varphi_1 + q_{ui}\varphi_2)]^{1/2} \\
u_i^* &\sim N^+(0, \exp(c_u)),
\end{aligned}
$$

where z_{1i} and z_{2i} are exogenous variables; and v_i and u_i^* are independent random variables. As it stands, it is not easy to generate y_i variable directly from these equations. Hence, after generating W and z_{1i}, z_{2i}, q_{fi}, q_{ui}, v_i, and u_i for $i = 1, 2, \ldots, N$, we calculate y_i from the following equality:

$$
y = (I_N - \rho W)^{-1} (z_1\beta_1 + q_f\beta_2 + v - u),
$$

where z_1, q_f, v, and u are represented in the matrix notation. Below, we explain how each of the other variables are generated.

In our primary scenario, the spatial weights w_{ij} are generated using row-normalized exponential distances as follows:

$$w_{ij} = \begin{cases} \exp(-d_{ij}) \,/\, \sum_{i \neq j} \exp(-d_{ij}), & i \neq j \\ 0 \qquad\qquad\qquad\qquad\qquad\quad, & i = j, \end{cases}$$

where d_{ij} are the centroid distances between each pair of spatial units i and j. We also consider the scenario where the weighting matrix is a scalar-normalized matrix. Let $\tilde{W}$ be the weighting matrix before scalar-normalization:

$$\tilde{w}_{ij} = \begin{cases} \exp(-d_{ij}), & i \neq j \\ \quad 0 \qquad\;\; , & i = j, \end{cases}$$

where d_{ij} are the centroid distances between each pair of spatial units i and j. The scalar normalized weighting matrix is given by:

$$w_{ij} = \begin{cases} \dfrac{\tilde{w}_{ij}}{\tilde{\lambda}_{\max}}, & i \neq j \\ \quad 0, & i = j, \end{cases}$$

where $\tilde{\lambda}_{\max} > 0$ is the largest eigenvalue of $\tilde{W}$. Finally, we consider the scenario where spatial weights w_{ij} are generated using row-normalized double-power distance weights as follows:

$$w_{ij} = \begin{cases} \left(1 - \ d_{ij} \,/\, d_{\max} \ ^2\right)^2 \,/\, \sum_{i \neq j} \left(1 - \ d_{ij} \,/\, d_{\max} \ ^2\right)^2, & i \neq j \\ \qquad\qquad 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad , & i = j, \end{cases}$$

where d_{ij} are the centroid distances between each pair of spatial units i and j and

$d_{\max} = \max_{i,j}\{d_{ij}\}$.

For each simulation run, we generated the distance between productive unit i and j as follows: $d_{ij} = \left|d_i - d_j\right|$ where d_i and d_j are drawn independently from a uniform distribution.

The endogenous variables q_{fi} and q_{ui} are generated as follows:

$$\begin{aligned} q_{fi} &= z_{3i}\delta_1 + \varepsilon_{1i} \\ q_{ui} &= z_{4i}\delta_2 + \varepsilon_{2i}, \end{aligned}$$

where $z_i = (z_{1i}, z_{2i}, z_{3i}, z_{4i})' \sim N(0, \Sigma)$; and the correlation among q_{fi}, q_{ui}, and v_i are generated via:

$$\begin{pmatrix} \varepsilon_i \\ v_i \end{pmatrix} \sim \mathbf{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{\varepsilon_1}^2 & 0 & \sigma_{\varepsilon_1}\sigma_v\tau_1 \\ 0 & \sigma_{\varepsilon_2}^2 & \sigma_{\varepsilon_2}\sigma_v\tau_2 \\ \sigma_{\varepsilon_1}\sigma_v\tau_1 & \sigma_{\varepsilon_2}\sigma_v\tau_2 & \sigma_v^2 \end{pmatrix}\right),$$

where $\varepsilon_i = (\varepsilon_{1i}, \varepsilon_{2i})'$. Note that when $\tau_1 = 0$, q_{fi} becomes exogenous and likewise, when $\tau_2 = 0$, q_{ui} becomes exogenous. Finally, when $\tau_1 = \tau_2 = 0$, all variables are exogenous, and we call this "exogenous model." When $\tau_1 \neq 0$ and $\tau_2 \neq 0$, q_{fi} and q_{ui} become endogenous and we call this "endogenous model." We consider the following values for $(\tau_1, \tau_2) = \{(0,0), (0.7, 0.7), (0.9, 0), (0, 0.9)\}$, and we fixed the values of $\rho = 0.5$, $\beta_1 = \beta_2 = 0.5$, $\delta_1 = \delta_2 = 1$, $c_u = -3$, $\sigma_v = 0.2$, $\sigma_{\varepsilon_1} = \sigma_{\varepsilon_2} = 0.3$, $\varphi_1 = \varphi_2 = 1$, and

$$\Sigma = \begin{bmatrix} 1 & 0.3 & 0.3 & 0.3 \\ 0.3 & 1 & 0.3 & 0.3 \\ 0.3 & 0.3 & 1 & 0.3 \\ 0.3 & 0.3 & 0.3 & 1 \end{bmatrix}.$$

In our setting, $(\tau_1, \tau_2) = (0.7, 0.7)$ represent the scenario where we have a somewhat high correlation between endogenous variables and two-sided error term; and $(\tau_1, \tau_2) = (0.9, 0)$ and $(0, 0.9)$ represent the scenarios where this correlation is extreme but only one endogenous variable is present. The variance-covariance matrix Σ that we choose indicates that the exogenous variables have some correlation. Note that our model allows such correlations. We did not impose high correlations for the exogenous variables to see the effects of endogeneity clearly by avoiding multicollinearity related issues.

We consider the following sample sizes: $n = \{100, 150, 200, 400\}$, and the Monte Carlo simulations are conducted with 1000 replications. In some cases, in order to save space, we only announce a subset of results. We examine four different estimators: 1) SSFE: Spatial stochastic frontier model with endogenous regressors; 2) SSF: Spatial stochastic frontier model with exogenous regressors; 3) SFE: Non-spatial stochastic frontier model with endogenous regressors; and 4) SF: Non-spatial stochastic frontier model with exogenous regressors.

The simulation results are given in Tables 1-6. We report the biases and mean squared errors for the parameter estimates, bias for efficiency estimates as well as Pearson and Spearman correlations of true and estimated efficiencies. We see that the parameter estimates are biased when we ignore the spatial component, SAR, or endogeneity in the estimations. Moreover, in terms of bias and correlations, efficiency estimates perform best when the SAR term is included and endogeneity is controlled.

Table 1. Simulation Results for $(\tau_1, \tau_2) = (0.7, 0.7)$

Row-Normalized Weighting Matrix with Exponential Distances

		N=100				N=150			
	TRUE	SSFE	SSF	SFE	SF	SSFE	SSF	SFE	SF
$E[\hat{\beta}_1]$	0.5000	0.4985	0.4842	0.5257	0.4977	0.5007	0.4843	0.5242	0.4946
$E[\hat{\beta}_2]$	0.5000	0.4978	0.5389	0.4508	0.5511	0.4981	0.5395	0.4538	0.5480
$E[\hat{c}_u]$	-3.0000	-3.0103	-3.4106	-2.7296	-1.3672	-2.9978	-3.2456	-2.6765	-1.3475
$E[\hat{\varphi}_1]$	1.0000	0.9717	1.2605	0.9171	0.5533	0.9974	1.2123	0.9044	0.5532
$E[\hat{\varphi}_2]$	1.0000	1.0000	0.8704	1.0003	0.3370	0.9899	0.8373	0.9551	0.3315
$E[\hat{\eta}_1]$	0.4667	0.4706	-	0.5593	-	0.4704	-	0.5563	-
$E[\hat{\eta}_2]$	0.4667	0.4685	-	0.5278	-	0.4683	-	0.5250	-
$E[\hat{\rho}]$	0.5000	0.4732	0.5226	-	-	0.4840	0.5212	-	-
$MSE[\hat{\beta}_1]$		0.0036	0.0010	0.0026	0.0017	0.0004	0.0007	0.0019	0.0010
$MSE[\hat{\beta}_2]$		0.0016	0.0022	0.0063	0.0043	0.0010	0.0020	0.0041	0.0032
$MSE[\hat{c}_u]$		0.1144	2.1043	1.0585	2.8480	0.1689	0.8042	0.1705	2.8522
$MSE[\hat{\varphi}_1]$		0.1055	0.4193	0.1140	0.2298	0.0502	0.1839	0.0391	0.2189
$MSE[\hat{\varphi}_2]$		0.1833	0.3063	0.6426	0.4624	0.0663	0.1553	0.0255	0.4615
$MSE[\hat{\eta}_1]$		0.0021	-	0.0293	-	0.0012	-	0.0119	-
$MSE[\hat{\eta}_2]$		0.0015	-	0.0472	-	0.0007	-	0.0067	-
$MSE[\hat{\rho}]$		0.0167	0.0299	-	-	0.0084	0.0183	-	-
Bias[Eff]		0.0051	-0.0277	0.1437	0.0180	0.0021	-0.0182	0.1431	0.0173
Pea. Corr.		0.9868	0.8663	0.9692	0.7721	0.9889	0.8744	0.9767	0.7771
Spe. Corr.		0.9517	0.7225	0.9220	0.6559	0.9592	0.7311	0.9364	0.6546

		N=200				N=400			
	TRUE	SSFE	SSF	SFE	SF	SSFE	SSF	SFE	SF
$E[\hat{\beta}_1]$	0.5000	0.5010	0.4854	0.5224	0.4948	0.5000	0.4850	0.5200	0.4931
$E[\hat{\beta}_2]$	0.5000	0.4987	0.5399	0.4548	0.5477	0.4994	0.5394	0.4581	0.5471
$E[\hat{c}_u]$	-3.0000	-3.0116	-3.1640	-2.6260	-1.3081	-3.0071	-3.1135	-2.6062	-1.2957
$E[\hat{\varphi}_1]$	1.0000	1.0019	1.1968	0.8833	0.5421	0.9974	1.1683	0.8751	0.5346
$E[\hat{\varphi}_2]$	1.0000	0.9953	0.8365	0.9359	0.3371	0.9989	0.8268	0.9329	0.3344
$E[\hat{\eta}_1]$	0.4667	0.4694	-	0.5572	-	0.4677	-	0.5525	-
$E[\hat{\eta}_2]$	0.4667	0.4670	-	0.5248	-	0.4666	-	0.5226	-
$E[\hat{\rho}]$	0.5000	0.4959	0.5210	-	-	0.4986	0.5189	-	-
$MSE[\hat{\beta}_1]$		0.0003	0.0006	0.0014	0.0007	0.0001	0.0004	0.0008	0.0004
$MSE[\hat{\beta}_2]$		0.0005	0.0019	0.0033	0.0029	0.0001	0.0017	0.0024	0.0025
$MSE[\hat{c}_u]$		0.0304	0.3939	0.1820	2.9423	0.0099	0.1736	0.1760	2.9438
$MSE[\hat{\varphi}_1]$		0.0227	0.1207	0.0358	0.2234	0.0082	0.0657	0.0253	0.2228
$MSE[\hat{\varphi}_2]$		0.0207	0.1023	0.0229	0.4500	0.0083	0.0654	0.0137	0.4480
$MSE[\hat{\eta}_1]$		0.0009	-	0.0109	-	0.0003	-	0.0087	-
$MSE[\hat{\eta}_2]$		0.0004	-	0.0055	-	0.0002	-	0.0040	-
$MSE[\hat{\rho}]$		0.0042	0.0097	-	-	0.0007	0.0041	-	-
Bias[Eff]		-0.0015	-0.0102	0.1414	0.0151	0.0001	-0.0050	0.1388	0.0145
Pea. Corr.		0.9909	0.8797	0.9795	0.7803	0.9925	0.8850	0.9839	0.7838
Spe. Corr.		0.9642	0.7393	0.9420	0.6594	0.9690	0.7473	0.9517	0.6611

Table 2. Simulation Results for $(\tau_1, \tau_2) = (0,0)$

Row-Normalized Weighting Matrix with Exponential Distances

		N=100				N=150			
	TRUE	SSFE	SSF	SFE	SF	SSFE	SSF	SFE	SF
$E[\hat{\beta}_1]$	0.5000	0.4975	0.4977	0.5100	0.5090	0.4993	0.4994	0.5101	0.5102
$E[\hat{\beta}_2]$	0.5000	0.4997	0.4999	0.5130	0.5096	0.4976	0.4978	0.5080	0.5060
$E[\hat{c}_u]$	-3.0000	-3.1983	-3.2083	-1.3630	-1.3498	-3.1568	-3.1608	-1.3405	-1.3371
$E[\hat{\varphi}_1]$	1.0000	1.0405	1.0432	0.4794	0.4738	1.0433	1.0440	0.4745	0.4751
$E[\hat{\varphi}_2]$	1.0000	1.0492	1.0567	0.4871	0.4726	1.0277	1.0309	0.4721	0.4640
$E[\hat{\eta}_1]$	0.0000	0.0021	-	-0.0161	-	0.0015	-	-0.0131	-
$E[\hat{\eta}_2]$	0.0000	-0.0089	-	0.0249	-	-0.0034	-	0.0188	-
$E[\hat{\rho}]$	0.5000	0.4857	0.4884	-	-	0.4990	0.5002	-	-
$MSE[\hat{\beta}_1]$		0.0008	0.0008	0.0020	0.0018	0.0005	0.0005	0.0011	0.0010
$MSE[\hat{\beta}_2]$		0.0008	0.0007	0.0021	0.0016	0.0005	0.0005	0.0012	0.0010
$MSE[\hat{c}_u]$		0.8426	0.8007	2.8353	2.8852	0.5554	0.5316	2.8475	2.8620
$MSE[\hat{\varphi}_1]$		0.1591	0.1565	0.2945	0.2993	0.1198	0.1164	0.2931	0.2921
$MSE[\hat{\varphi}_2]$		0.1796	0.1673	0.2912	0.3023	0.1024	0.0996	0.2935	0.3014
$MSE[\hat{\eta}_1]$		0.0088	-	0.0270	-	0.0056	-	0.0131	-
$MSE[\hat{\eta}_2]$		0.0086	-	0.0292	-	0.0051	-	0.0132	-
$MSE[\hat{\rho}]$		0.0224	0.0208	-	-	0.0126	0.0119	-	-
Bias[Eff]		-0.0008	-0.0008	0.0204	0.0166	-0.0055	-0.0055	0.0175	0.0162
Pea. Corr.		0.8664	0.8703	0.7656	0.7790	0.8757	0.8778	0.7781	0.7855
Spe. Corr.		0.7244	0.7286	0.6562	0.6669	0.7367	0.7389	0.6652	0.6708
		N=200				N=400			
	TRUE	SSFE	SSF	SFE	SF	SSFE	SSF	SFE	SF
$E[\hat{\beta}_1]$	0.5000	0.4988	0.4988	0.5077	0.5081	0.4995	0.4994	0.5072	0.5073
$E[\hat{\beta}_2]$	0.5000	0.4992	0.4989	0.5083	0.5060	0.4996	0.4997	0.5083	0.5073
$E[\hat{c}_u]$	-3.0000	-3.1282	-3.1291	-1.3342	-1.3299	-3.0489	-3.0488	-1.3173	-1.3148
$E[\hat{\varphi}_1]$	1.0000	1.0307	1.0301	0.4688	0.4691	1.0053	1.0051	0.4635	0.4645
$E[\hat{\varphi}_2]$	1.0000	1.0316	1.0332	0.4757	0.4666	1.0127	1.0133	0.4738	0.4675
$E[\hat{\eta}_1]$	0.0000	-0.0047	-	-0.0189	-	0.0005	-	-0.0097	-
$E[\hat{\eta}_2]$	0.0000	-0.0015	-	0.0250	-	-0.0010	-	0.0171	-
$E[\hat{\rho}]$	0.5000	0.5031	0.5030	-	-	0.5003	0.5003	-	-
$MSE[\hat{\beta}_1]$		0.0003	0.0003	0.0008	0.0007	0.0002	0.0002	0.0004	0.0004
$MSE[\hat{\beta}_2]$		0.0003	0.0003	0.0008	0.0006	0.0002	0.0002	0.0004	0.0003
$MSE[\hat{c}_u]$		0.3556	0.3521	2.8566	2.8724	0.1248	0.1236	2.8688	2.8776
$MSE[\hat{\varphi}_1]$		0.0760	0.0752	0.2943	0.2940	0.0298	0.0297	0.2936	0.2926
$MSE[\hat{\varphi}_2]$		0.0697	0.0691	0.2874	0.2963	0.0274	0.0272	0.2824	0.2889
$MSE[\hat{\eta}_1]$		0.0039	-	0.0087	-	0.0020	-	0.0039	-
$MSE[\hat{\eta}_2]$		0.0040	-	0.0096	-	0.0019	-	0.0038	-
$MSE[\hat{\rho}]$		0.0093	0.0093	-	-	0.0033	0.0032	-	-
Bias[Eff]		-0.0053	-0.0051	0.0182	0.0173	-0.0036	-0.0036	0.0165	0.0161
Pea. Corr.		0.8802	0.8812	0.7844	0.7887	0.8861	0.8866	0.7909	0.7925
Spe. Corr.		0.7399	0.7412	0.6675	0.6706	0.7509	0.7516	0.6744	0.6754

Table 3. Simulation Results for $(\tau_1, \tau_2) = (0.9, 0)$

Scalar-Normalized Weighting Matrix with Exponential Distances

		N=100				N=200			
	TRUE	SSFE	SSF	SFE	SF	SSFE	SSF	SFE	SF
$E[\hat{\beta}_1]$	0.5000	0.4994	0.4850	0.5257	0.4972	0.5004	0.4854	0.5224	0.4956
$E[\hat{\beta}_2]$	0.5000	0.4991	0.5547	0.4789	0.5670	0.4990	0.5551	0.4845	0.5632
$E[\hat{c}_u]$	-3.0000	-3.0359	-3.2191	-2.0375	-1.4146	-3.0443	-3.0340	-1.9401	-1.3686
$E[\hat{\varphi}_1]$	1.0000	0.9844	1.0844	0.6994	0.5190	1.0142	1.0467	0.6751	0.5154
$E[\hat{\varphi}_2]$	1.0000	0.9835	1.0689	0.7042	0.5110	1.0041	1.0345	0.6840	0.5030
$E[\hat{\eta}_1]$	0.6000	0.6044	-	0.7458	-	0.6026	-	0.7283	-
$E[\hat{\eta}_2]$	0.0000	-0.0030	-	0.0362	-	-0.0006	-	0.0379	-
$E[\hat{\rho}]$	0.5000	0.4737	0.4856	-	-	0.5000	0.4812	-	-
$MSE[\hat{\beta}_1]$		0.0006	0.0009	0.0025	0.0016	0.0003	0.0006	0.0013	0.0007
$MSE[\hat{\beta}_2]$		0.0006	0.0036	0.0027	0.0060	0.0003	0.0033	0.0011	0.0046
$MSE[\hat{c}_u]$		0.2184	1.5982	1.0414	2.7045	0.0702	0.1841	1.1855	2.7526
$MSE[\hat{\varphi}_1]$		0.0885	0.2533	0.1372	0.2603	0.0324	0.0517	0.1239	0.2483
$MSE[\hat{\varphi}_2]$		0.0793	0.1655	0.1267	0.2646	0.0307	0.0506	0.1182	0.2590
$MSE[\hat{\eta}_1]$		0.0032	-	0.0347	-	0.0013	-	0.0226	-
$MSE[\hat{\eta}_2]$		0.3665	-	0.3308	-	0.3619	-	0.3217	-
$MSE[\hat{\rho}]$		0.0169	0.0255	-	-	0.0030	0.0068	-	-
Bias[Eff]		0.0029	-0.0176	0.0868	0.0196	-0.0012	-0.0025	0.0770	0.0183
Pea. Corr.		0.9452	0.8772	0.8982	0.7972	0.9546	0.8879	0.9149	0.8067
Spe. Corr.		0.8482	0.7407	0.8145	0.6925	0.8641	0.7541	0.8357	0.6985

Table 4. Simulation Results for $(\tau_1, \tau_2) = (0, 0.9)$

Scalar-Normalized Weighting Matrix with Exponential Distances

		N=100				N=200			
	TRUE	SSFE	SSF	SFE	SF	SSFE	SSF	SFE	SF
$E[\hat{\beta}_1]$	0.5000	0.4992	0.4931	0.5157	0.5075	0.4999	0.4950	0.5134	0.5044
$E[\hat{\beta}_2]$	0.5000	0.4990	0.4940	0.5161	0.5068	0.5006	0.4959	0.5160	0.5040
$E[\hat{c}_u]$	-3.0000	-3.0115	-3.3313	-2.0080	-1.3013	-3.0467	-3.3143	-1.9483	-1.2763
$E[\hat{\varphi}_1]$	1.0000	0.9758	1.2319	0.6697	0.5279	1.0122	1.2530	0.6587	0.5243
$E[\hat{\varphi}_2]$	1.0000	0.9950	0.7707	0.8028	0.2735	1.0097	0.7872	0.7629	0.2715
$E[\hat{\eta}_1]$	0.0000	0.0019	-	-0.0225	-	-0.0010	-	-0.0188	-
$E[\hat{\eta}_2]$	0.6000	0.6027	-	0.7513	-	0.6016	-	0.7358	-
$E[\hat{\rho}]$	0.5000	0.4715	0.5145	-	-	0.5037	0.5491	-	-
$MSE[\hat{\beta}_1]$		0.0006	0.0007	0.0018	0.0019	0.0003	0.0004	0.0009	0.0007
$MSE[\hat{\beta}_2]$		0.0006	0.0007	0.0022	0.0017	0.0003	0.0004	0.0010	0.0007
$MSE[\hat{c}_u]$		0.2005	1.2668	1.0820	3.0300	0.0706	0.8518	1.1567	3.0427
$MSE[\hat{\varphi}_1]$		0.0797	0.2988	0.1532	0.2512	0.0317	0.2061	0.1345	0.2391
$MSE[\hat{\varphi}_2]$		0.0798	0.2965	0.0775	0.5506	0.0312	0.1687	0.0722	0.5411
$MSE[\hat{\eta}_1]$		0.3612	-	0.4012	-	0.3626	-	0.3886	-
$MSE[\hat{\eta}_2]$		0.0027	-	0.0325	-	0.0011	-	0.0234	-
$MSE[\hat{\rho}]$		0.0135	0.0255	-	-	0.0033	0.0142	-	-
Bias[Eff]		0.0050	-0.0036	0.0805	0.0131	-0.0034	-0.0165	0.0747	0.0134
Pea. Corr.		0.9454	0.8533	0.8999	0.7495	0.9547	0.8717	0.9173	0.7601
Spe. Corr.		0.8489	0.7033	0.8217	0.6274	0.8642	0.7266	0.8420	0.6308

Table 5. Simulation Results for $(\tau_1, \tau_2) = (0.7, 0.7)$

Scalar-Normalized Weighting Matrix with Exponential Distances

		N=100				N=200			
	TRUE	SSFE	SSF	SFE	SF	SSFE	SSF	SFE	SF
$E[\hat{\beta}_1]$	0.5000	0.4996	0.4829	0.5245	0.4958	0.5006	0.4850	0.5208	0.4934
$E[\hat{\beta}_2]$	0.5000	0.4989	0.5387	0.4568	0.5502	0.4990	0.5389	0.4621	0.5462
$E[\hat{c}_u]$	-3.0000	-3.0093	-3.2529	-2.6360	-1.3378	-3.0138	-3.1822	-2.5556	-1.3134
$E[\hat{\varphi}_1]$	1.0000	0.9835	1.1982	0.8892	0.5438	0.9943	1.1811	0.8559	0.5330
$E[\hat{\varphi}_2]$	1.0000	1.0006	0.8547	0.9724	0.3388	0.9985	0.8370	0.9224	0.3345
$E[\hat{\eta}_1]$	0.4667	0.4690	-	0.5624	-	0.4690	-	0.5555	-
$E[\hat{\eta}_2]$	0.4667	0.4707	-	0.5427	-	0.4666	-	0.5317	-
$E[\hat{\rho}]$	0.5000	0.4584	0.4955	-	-	0.4890	0.5197	-	-
$MSE[\hat{\beta}_1]$		0.0006	0.0010	0.0024	0.0018	0.0003	0.0006	0.0012	0.0007
$MSE[\hat{\beta}_2]$		0.0006	0.0022	0.0044	0.0041	0.0003	0.0018	0.0025	0.0028
$MSE[\hat{c}_u]$		0.0550	1.0052	0.2375	2.9151	0.0268	0.4342	0.2523	2.9167
$MSE[\hat{\varphi}_1]$		0.0444	0.2415	0.0655	0.2365	0.0194	0.1190	0.0418	0.2301
$MSE[\hat{\varphi}_2]$		0.0481	0.2209	0.0427	0.4588	0.0204	0.1024	0.0248	0.4529
$MSE[\hat{\eta}_1]$		0.0019	-	0.0160	-	0.0007	-	0.0107	-
$MSE[\hat{\eta}_2]$		0.0013	-	0.0114	-	0.0005	-	0.0068	-
$MSE[\hat{\rho}]$		0.0137	0.0212	-	-	0.0029	0.0097	-	-
Bias[Eff]		0.0136	-0.0002	0.1387	0.0151	0.0021	-0.0080	0.1323	0.0145
Pea. Corr.		0.9855	0.8646	0.9668	0.7700	0.9901	0.8771	0.9769	0.7802
Spe. Corr.		0.9494	0.7206	0.9192	0.6554	0.9901	0.8771	0.9769	0.7802

Table 6. Simulation Results for $(\tau_1, \tau_2) = (0.7, 0.7)$

Row-Normalized Weighting Matrix with Double-Power Distances

		N=100				N=200			
	TRUE	SSFE	SSF	SFE	SF	SSFE	SSF	SFE	SF
$E[\hat{\beta}_1]$	0.5000	0.4988	0.4844	0.5235	0.4984	0.5007	0.4849	0.5220	0.4937
$E[\hat{\beta}_2]$	0.5000	0.4970	0.5380	0.4537	0.5495	0.4990	0.5388	0.4568	0.5464
$E[\hat{c}_u]$	-3.0000	0.4988	0.4844	0.5235	0.4984	-3.0066	-3.1974	-2.5980	-1.3082
$E[\hat{\varphi}_1]$	1.0000	0.4970	0.5380	0.4537	0.5495	0.9906	1.1868	0.8688	0.5318
$E[\hat{\varphi}_2]$	1.0000	0.4988	0.4844	0.5235	0.4984	0.9954	0.8413	0.9322	0.3340
$E[\hat{\eta}_1]$	0.4667	0.4720	-	0.5636	-	0.4690	-	0.5571	-
$E[\hat{\eta}_2]$	0.4667	0.4688	-	0.5396	-	0.4666	-	0.5265	-
$E[\hat{\rho}]$	0.5000	0.4743	0.5018	-	-	0.4969	0.5259	-	-
$MSE[\hat{\beta}_1]$		0.0006	0.0009	0.0024	0.0018	0.0003	0.0006	0.0013	0.0007
$MSE[\hat{\beta}_2]$		0.0006	0.0021	0.0050	0.0040	0.0003	0.0018	0.0032	0.0028
$MSE[\hat{c}_u]$		0.0506	0.9013	0.2275	2.8841	0.0250	0.4427	0.2114	2.9352
$MSE[\hat{\varphi}_1]$		0.0444	0.2267	0.0690	0.2339	0.0186	0.1213	0.0376	0.2311
$MSE[\hat{\varphi}_2]$		0.0453	0.2067	0.0490	0.4615	0.0197	0.1017	0.0228	0.4536
$MSE[\hat{\eta}_1]$		0.0018	-	0.0162	-	0.0006	-	0.0110	-
$MSE[\hat{\eta}_2]$		0.0012	-	0.0111	-	0.0004	-	0.0058	-
$MSE[\hat{\rho}]$		0.0133	0.0203	-	-	0.0025	0.0099	-	-
Bias[Eff]		0.0078	-0.0029	0.1402	0.0153	-0.0011	-0.0095	0.1374	0.0144
Pea. Corr.		0.9869	0.8653	0.9659	0.7666	0.9910	0.8787	0.9776	0.7785
Spe. Corr.		0.9525	0.7228	0.9160	0.6507	0.9647	0.7393	0.9386	0.6578

4. Empirical Example

In this section, we estimate the technical efficiencies of the Chinese firms in the chemical industry in 2006. First, we briefly describe our data, and then present our results.

4.1. Data

Our firm-level dataset is based on the dataset of Baltagi et al. (2016).[19] The dataset contains 12,552 Chinese firms in the chemical industry for 2006, which is compiled by the National Bureau of Statistics of China. The output variable is the sales (Y) for firms; and the input variables are employment (L), capital (K) used in production; and material inputs (M). As control variables we also include the share of high-skilled labor (H), which is defined as the fraction of workers with university (or equivalent) education level. In addition to these variables, in the frontier we include the following variables, constructed by Baltagi et al. (2016): dummy for being a state-owned firm (SOWND), dummy for being exporter (EXPD), dummy for using intangible asset intensely (IASSETD), and a variable measuring the fraction of foreign-owned to total capital ratio (FOWNR).

The firm-level dataset has information about the postcodes of firms, which enables us to identify geographic location of firms in terms of latitude and longitude. Based on these, the great circle distances between all firms are calculated using haversine formula. This enabled us to obtain the weighting matrices for each of our specifications.

In line with Hick's (1935) quiet life hypothesis[20], we assume that the technical efficiencies depend on market power, which is proxied by Herfindahl-Hirschman Index (HHI). When

[19] The dataset of Baltagi et al. (2016) is hosted by the Journal of Applied Econometrics archive. For further details about the dataset see Baltagi et al. (2016).

[20] See also Jayaratne and Strahan (1996), Berger and Hannan (1998), Kroszner and Strahan (1999), Koetter et al. (2012), and Kutlu et al. (2019).

calculating the HHI, rather than using political boundaries defined by province borders, we assume that the markets for a firm is defined as the area within 400 km radius.[21] We also assume that spillover effects are effective within this region. Hence, our definition of HHI would be in line with the definition of spillovers, i.e., their range is the same. We assume that the HHI is endogenous. We use 1-year lagged HHI as an instrumental variable. The descriptive statistics of variables are given in Table 7.

Table 7. Descriptive Statistics

Variable	State-Owned		Non-State-Owned		All	
Variable	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
SALES	286,435	665,088	104,292	409,854	113,898	429,038
L	789.3	1,605.6	172.5	376	205	537.3
K	232,892	693,490	33,690	163,219	44,196	229,229
M	176,893	434,383	67,524	270,372	73,292	282,454
H	0.2260	0.1839	0.1718	0.1874	0.1746	0.1876
FOWNR	0.0660	0.1911	0.1443	0.3266	0.1402	0.3214
EXPD	0.2795	0.4491	0.2263	0.4185	0.2291	0.4203
IASSETD	0.5801	0.4939	0.4248	0.4943	0.4330	0.4955
SOWND	1	0	0	0	0.0527	0.2235
HHI	22.10	130.47	6.76	117.59	7.57	118.35
Number of Obs.	662		11890		12552	

4.2. Empirical Model and Estimation Results

We estimate a spatial stochastic frontier function where HHI is allowed to be an endogenous variable. As in the Monte Carlo simulations section, for a given weighting matrix, we estimate four different models: 1) SSFE: Spatial stochastic frontier model with endogenous regressors; 2) SSF: Spatial stochastic frontier model with exogenous regressors; 3) SFE: Non-spatial stochastic frontier model with endogenous regressors; and 4) SF: Non-spatial stochastic frontier model with exogenous regressors. Our benchmark setting assumes that the weighting matrix is row-normalized with exponential distances.

Our estimation results are presented in Table 8 and Figure 1. Based on our endogeneity

[21] For the school district markets, Karakaplan and Kutlu (2019b) calculate HHI using a similar approach.

test, we find evidence for endogeneity of HHI as η is statistically significant at any conventional significance level. Similarly, we find evidence for spatial interaction between firms (i.e., spatial spillovers) based on the significance of SAR term parameter.

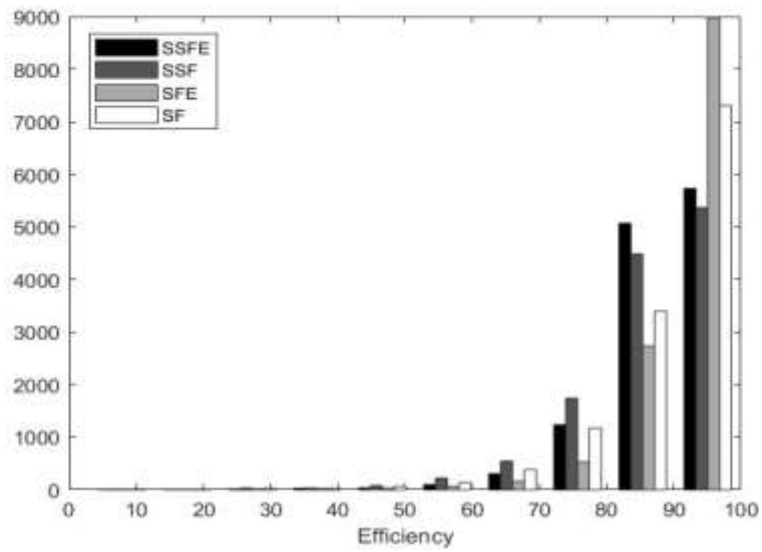
In what follows, the results that we state are for the SSFE model, which we select based on statistical tests (i.e., endogeneity and spillover tests) that we mentioned. Estimated median of efficiency is 89.62%, which is a reasonable number. From Figure 1, we can see that compared to spatial models, the non-spatial models predicted many more firms with efficiency greater than 90%. Hence, the distributions of efficiencies are distorted if spatial effects are not considered. Indeed, our Kolmogorov-Smirnov test rejected pairwise equality of distribution for efficiency estimates obtained from SSFE and other models at any conventional significance level. Unlike our simulations, the pairwise Spearman correlations of efficiencies were still high (more than 0.98). So, in our empirical example, while the magnitudes of efficiencies are distorted, the rankings are not distorted much.

Table 8. Estimates for Stochastic Frontier Models

	SSFE			SSF			SFE			SF		
ln(SALES)	**COEF.**	**SE**		**COEF.**	**SE**		**COEF.**	**SE**		**COEF.**	**SE**	
ln(L)	0.4755	(0.0251)	***	0.4472	(0.0252)	***	0.4462	(0.0251)	***	0.4358	(0.0252)	***
ln(K)	0.2035	(0.0177)	***	0.2271	(0.0177)	***	0.2377	(0.0176)	***	0.2492	(0.0177)	***
ln(M)	-0.0350	(0.0239)		-0.1323	(0.0239)	***	-0.0207	(0.0240)		-0.1237	(0.0240)	***
$0.5 \times \ln(L)^2$	0.0556	(0.0065)	***	0.0526	(0.0064)	***	0.0504	(0.0065)	***	0.0489	(0.0064)	***
$0.5 \times \ln(K)^2$	0.0317	(0.0024)	***	0.0314	(0.0024)	***	0.0305	(0.0024)	***	0.0304	(0.0024)	***
$0.5 \times \ln(M)^2$	0.1534	(0.0032)	***	0.1631	(0.0033)	***	0.1557	(0.0032)	***	0.1651	(0.0033)	***
ln(L) × ln(K)	0.0145	(0.0030)	***	0.0163	(0.0029)	***	0.0168	(0.0030)	***	0.0182	(0.0030)	***
ln(L) × ln(M)	-0.0719	(0.0035)	***	-0.0698	(0.0035)	***	-0.0695	(0.0035)	***	-0.0692	(0.0035)	***
ln(K) × ln(M)	-0.0494	(0.0022)	***	-0.0521	(0.0022)	***	-0.0528	(0.0022)	***	-0.0543	(0.0022)	***
H	0.2935	(0.0152)	***	0.2841	(0.0151)	***	0.2669	(0.0151)	***	0.2652	(0.0151)	***
FOWNR	0.0914	(0.0126)	***	0.0881	(0.0124)	***	0.0887	(0.0127)	***	0.0889	(0.0126)	***
EXPD	0.0082	(0.0092)		0.0115	(0.0091)		-0.0080	(0.0094)		0.0046	(0.0092)	
IASSETD	0.0304	(0.0078)	***	0.0313	(0.0078)	***	0.0304	(0.0078)	***	0.0257	(0.0079)	***
SOWND	-0.0012	(0.0161)		-0.0214	(0.0161)		-0.0125	(0.0161)		-0.0091	(0.0164)	
CONSTANT	1.4366	(0.1933)	***	2.9446	(0.1948)	***	4.1579	(0.1190)	***	4.7720	(0.1190)	***
ρ	0.2800	(0.0151)	***	0.1892	(0.0156)	***	-	-	-	-	-	-
σ_w												
CONSTANT	-2.6806	(0.0146)	***	-2.6186	(0.0150)	***	-2.6392	(0.0144)	***	-2.5980	(0.0148)	***
σ_u												
FOWNR	0.5957	(0.1762)	***	0.3377	(0.1489)	**	0.4750	(0.2049)	**	0.2901	(0.1599)	*
EXPD	0.1613	(0.1644)		0.2841	(0.1319)	**	-0.2495	(0.2044)		0.1621	(0.1421)	
IASSETD	0.9412	(0.1139)	***	0.8444	(0.0947)	***	1.2067	(0.1274)	***	0.8587	(0.0988)	***
SOWND	1.3138	(0.1655)	***	0.8780	(0.1600)	***	1.3832	(0.1720)	***	1.2181	(0.1577)	***
ln(HHI)	-0.6857	(0.0291)	***	-0.7420	(0.0245)	***	-0.6925	(0.0325)	***	-0.7245	(0.0253)	***
CONSTANT	-7.7464	(0.2393)	***	-7.4298	(0.1919)	***	-8.0310	(0.2749)	***	-7.4272	(0.1999)	***
η	0.1249	(0.0036)	***	-	-	-	0.1152	(0.0036)	***	-	-	-
Median Efficiency	89.62			88.89			94.06			91.97		
Number of Obs.	12552			12552			12552			12552		

Notes: Standard errors are in parentheses. Asterisks indicate significance at the 1% (***), 5% (**) and 10% (*) levels.

Figure 1. Histogram for Efficiency Estimates



5. Concluding Remarks

The conventional stochastic frontier models neither allow spatial spillovers nor endogeneity. If any of the frontier or environmental variables are correlated with the two-sided error term; or the SAR component is omitted while being a relevant term, then parameter and efficiency estimates would be inconsistent. We presented the first model that can address both issues simultaneously by employing a control function approach. Our Monte Carlo simulations show that ignoring either of endogeneity or spatial dependence may have serious negative implications on the parameter and efficiency estimates. In particular, we would have biased parameter and efficiency estimates, which distorts efficiency rankings.

Given that chemicals industry affects many other industries and well-being of this industry may have substantial direct or indirect effect on overall economy, understanding the factors that affect efficiency is essential for both policymakers and relevant firms themselves. This objective, however, requires using proper econometric methods that are robust to potential econometric issues. We employed our estimation method to the Chinese firms in the chemicals industry. It turns out that, spillover effects are statistically significant and have economic impact on the sales of these firms. We also found evidence for endogeneity of HHI. Hence, as illustrated in the simulations and in our empirical example, ignoring efficiency or spatial effects may have negative implications, e.g., the distribution of inefficiency estimates may differ depending on whether we have a SAR term or not. Therefore, using conventional stochastic frontier estimation results risk being irrelevant if either a necessary SAR term or endogeneity is ignored.

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