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Modeling Through Model-Eliciting Activities: An Analysis of Models, Elements, And Strategies in High School. The Cases of Students with Different Level of Achievement

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Modeling Through Model-Eliciting Activities: An Analysis of Models, Elements, And Strategies in High School. The Cases of Students with Different Level of Achievement

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***Abstract**: A mathematical client-driven task known as Model-Eliciting Activities was implemented with students of different levels of achievement (i.e., low, average, and high) at the high-school level. The study strived to prove that Model-Eliciting Activities can be solved by students at any achievement level and be used as an assessment tool. Students collaborated in teams of three to develop solutions that met the client's needs. The model-solutions were compared and contrasted among several dimensions and achievement levels, considering the quality of the final product-solution based on the Quality Assessment Guide, the intermedia product composed of the type of models created, the strategies followed, and the elements of the mathematical construct. A Model & Modeling perspective was considered as a framework in which students' teams comprised the cases studied and analyzed. Findings show that students were able to create and elaborate models-solutions regardless of the level of achievement, with a comparable quality but with different strategies. Although student's solutions were similar in sophistication across achievement level, more studies are needed to evaluate high schoolers' solutions to these types of mathematical task.*

***Keywords**:* Modeling, High School, Assessment, Achievement-level

INTRODUCTION

It is not uncommon for students across all levels of achievement to struggle when solving traditional textbook mathematical problems, but below-average students tend to struggle even more, often because their teachers underestimate their abilities (Madon et al., 1997). However, with the right approach, motivation, and engagement, students at any achievement level can solve problem activities beyond their teachers or anybody else's expectations. In particular, the type of problem-solving activities that were used for this project are the ones knows as though-reveling or Model-Eliciting Activities (MEA).

By using MEAs, teachers can help all students without leaving “low-achievers” behind, since these activities enable equally good outcomes across all achievement levels (i.e., low, average, and high). The main difference between MEAs and traditional classroom tasks (Alsup & Sprigler, 2003; Dewey, 1998) is that the design of MEAs leads the problem-solvers to focus on the process toward a solution (Lesh & Doerr, 2003) rather than looking for a single answer using predetermined mathematical algorithms.

That is to say, the solution to a MEA is the model the problem-solver produces—known as problem representation— to provide the best answer to a problem, not the answer itself. MEAs thus enable students to interpret, invent, and find solutions in ways that “jump” the barriers of “achievement stereotype” by focusing on process instead of more rigid strategies. For example, Carmona and Greenstein (2007) compared the solutions brought to the same MEA by a group of 3rd graders and a group of post-graduate math and science students. The activity required the problem-solvers (i.e., the students) to rank 12 teams of players presented on a two-dimensional coordinate system. Working in small groups, the students were required to model a solution to rank the top five teams with the most wins. Each group worked for approximately two hours developing a solution to the problem. By building on their previous knowledge and experiences, the problem-solvers needed to decide which solution-paths led to the best outcome, and which mathematical ideas worked best in developing their constructs.

Each group of students was very different in their respective mathematical skills, age, and grade level, yet Carmona and Greenstein showed that both the elementary and post-graduate students developed adequate solutions. As expected, the post-graduates’ solutions were more mathematically advanced (e.g., they used vector concepts and Pythagorean formulae) and better justified. However, the elementary students’ models were as adequate — in terms of the process and model developed — as the post-graduates’ for providing accurate answers to the proposed problem. If the elementary students are considered “low-achievers” and post-graduates as “high-achievers,” then we can conclude two things: first, that MEAs can work to the “level the playing field” across achievement levels, and I seek to verify and demonstrate that students at these different levels can indeed provide equally adequate solutions to activities like MEAs; second, that the degree of difficulty of any particular problem posed for each achievement level is “better determined by the solution the problem solver produced” (Carmona & Greenstein, p. 253) than by the ability (or lack thereof) teachers presuppose students to have attained. In contrast to Carmona and Greenstein, my research shows that different achievement levels within the same grade level work the same way: although low-achievers often use different processes than their average- or high-achieving peers, they attain equally adequate outcomes, and in some instances even better in terms of the justification of their ideas, the reasoning, and the explanation.

In the context of this research (i.e., high school), more and better strategies for teaching students how to solve real-life problems effectively, creatively, and mathematically are needed. Model-eliciting activities are presented here to illustrate a possible alternative for presenting instructional models in ways that enhance and elicit mathematical literacy for high school students at different achievement levels, and to counter the stereotype that low- and (perhaps) average-achieving students cannot solve open-ended mathematics activities (like the one presented here) as adequately as students stereotyped as high-achieving can, and even to show they can "jump" the boundaries that define these stereotypes. To this end, I am answering the following question: To what extent the quality of the student's final and intermediate product solutions (i.e., mathematical strategies, type of models, and elements) varies both within and across the achievement levels?

Theoretical Framework

Solving real-life problems in the modern world is neither about following a certain set of rules nor working in isolation to find an answer. People today frequently need to adapt problem-solving strategies to unique and complex situations and to develop collaborative solutions to difficult problems (Lombardi, 2007). However, when students in school are taught to solve problems, they are often presented with unrealistic situations (e.g., textbook problems) that require them to follow particular rules or strategies (already practiced and mastered) to solve a problem (usually individually) that is seldom similar to any real-life situation. Furthermore, the concepts they are tasked to learn are limited to grasping and/or memorizing a set of rules and passing standardized tests about topics that are usually quickly forgotten. In mathematics classrooms, in particular, it is common to find this type of traditional problem-solving perspective — learning to follow specific steps in order to solve formulaic problems. However, this rarely helps students in solving real-life problems that require a level of abstraction. By contrast, in this research project, I took an approach in which students can work collaboratively to interpret situations in multiple ways, evaluate possible solution paths, and enter into a cycle of description, explanation, and prediction as they solve a problem; and in which the development of a useful and powerful solution is beneficial not only to students who excel in memorizing mathematical solution-steps or have attained a high level of achievement in school, but to everyone. This approach is called the Model and Modeling perspective (MM) (Lesh & Doerr, 2003).

Within the MM perspective, models are conceptual tools used to mathematize a real situation and modeling is the process in which a model is adapted or constructed to provide a solution to a problem (Lesh & Doerr, 2003; Zandieh & Rasmussen, 2010; Lesh & Lehrer, R., 2003). Following MM, the model-construction process is an interactive leaning cycle in which students collaboratively (1) examine a situation, (2) identify the problem to be solved and the variables involved, (3) formulate a model or a problem representation, (4) test the model, (5) interpret the results, (6) validate their model solution's applicability to the original situation, and (7) apply the model to other similar situations to test its usefulness (Kang & Noh, 2012). Moreover, since MM emphasizes teamwork when solving a problem, students not only reflect on their own thoughts individually, but also communicate their ideas in ways that other team members can evaluate, reject, or accept. In general, each member learns from the different perspectives that emerge through collaboration (Lesh et al., 2003).

The MM perspective supports and advocates *unstructured*, collaborative learning, since learning occurs in the social context (Vygotsky, 1962) of students' interactions, when team members mutually "negotiate goals, define problems, develop procedures, and produce socially constructed knowledge in small groups" (Springer et al., 1999, p. 24). Many theories and theorists have addressed the benefits of learning in small groups (Springer et al., 1999): Piaget (1926) and Vygotsky (1978) in cognitive psychology, Deutsch (1949) and Lewin (1935) in social psychology, Dewey (1943) in experiential education, and Belenky et al. (1986) in humanist and feminist theory. However, the collaborative learning in the MM perspective is primarily rooted in the developmental theories of Piaget (1926) and Vygotsky (1978) in which "face-to-face work on open-ended tasks—projects with several possible paths leading to multiple acceptable solutions—facilitate cognitive growth" (Springer et al., 1999, p. 25). Based on this principle, MM recognizes that (1) it is crucial for students not simply to argue and discuss their opinions, but to share each other's ideas and perspectives when working collaboratively — i.e., an idea similar to the multiple-perspective principle of Lesh et al. (2003) and (2) that students can uncover their inadequate reasonings as disagreements arise during their discussions, and that working out these disagreements enhances the understanding of all (Springer et al., 1999).

MM thus proposes types of problem-solving activities that furnishes to the learners a space to collaborate with others, to try out, reflect on, and re-enact theirs own and others' ideas — and MEAs are such an activity. In MEAs, the social and communal construction of knowledge takes place (Vygotsky, 1978; Tangney, FitzGibbon, Savage, Mehan, & Holmes, 2001) and "the learners not only construct their own knowledge while interacting with their environment [and other people] but are also actively engaged in the process of constructing knowledge for their learning community" (e.g., while working in teams) (Tangney et al., 2001, p. 3114). In collaborative groups, individuals merge their knowledge to strengthen and broaden their skills while achieving a common goal. This escalates their motivation toward an interest in problem-solving (Zawojewski, et al., 2003). It also increases the possibility for students to create and invent more sophisticated and powerful solutions using mathematical representations (e.g., artifacts and constructs) and inscriptions — e.g., graphs, tables, diagrams — to mediate their thought processes and reasoning.

Model, Modeling and Model-Eliciting Activities

In a classroom context, models are a "student-generated way of organizing their mathematical activity with physical and mental tools" (Zandieh & Rasmussen, 2010, p.68). Moreover, Lesh and Doerr (2003) defined models as: "Conceptual systems (consisting of elements, relations, operations, and rules governing interactions) …used to construct, describe, or explain the behaviors of other system(s)" (p.10). Modeling, then, is the process through which students construct or adapt and structure conceptual systems — i.e., models — in order to solve real-life problems (Zandieh & Rasmussen, 2010; Ekmekci, 2013).

MEAs are open-ended client-driven problem-solving activities in which students working in small teams are encouraged to develop and generate useful solutions (i.e., conceptual systems or models) for a "client". These solutions are required to be reported to the client in a letter-format way, where students provide a detailed explanation of their model-solution. Students generate solutions by repeatedly communicating, testing, refining, and extending their thoughts (Lesh et al., 2000). These thought processes often involve several modeling cycles. Within MEAs, a modeling cycle is the process by which students describe, manipulate, predict, and verify their mathematical constructs, then adapt, modify, and/or refine their own knowledge and ideas (Lesh & Doerr, 2003). Often, they must go through several cycles of modeling to interpret and improve their products (Kaput, 1998) in ways that go beyond "just providing an answer" to offering unique solutions to a problem (Lesh & Doerr, 2003). Multiple cycles of modeling enhance students' learning because students "have multiple opportunities to invent, revise, and then compare the explanatory adequacy of different models" (Lehrer & Schauble, 2006, p. 382).

Setting and Data Collection

The target population for this study was 11th grade high school students at different achievement levels at a private school located in northeastern Mexico. The 11th grade cohort contained approximately 74 students divided into three sections of 22 to 28. To form the teams, the classroom teacher and I (as a co-teacher) asked the students to form teams of two or three members without forcing or imposing any type of categorization, randomization, or selection rule (i.e., students formed these groups in a spontaneous way). These naturally formed teams represent the cases and focus of this study and analysis. Later, the classroom teacher categorized each team as low-, average, and high-achievement based on the students' individual performance in class (as measured by their grades, test scores, and classroom activities). In the end, 24 teams were formed: 14 average-, 5 low-, and 5 high-achievement.

The fieldwork for this study was divided into three phases. In the first week, I implemented *The Team Ranking Problem* (Greenstein et al., 2008), the following week, *The Hybrid vs. Gas Car* problem (Elliott, 2014), and two to three weeks later *The Historic Hotel* problem (Aliprantis & Carmona, 2003). Students spent approximately between 150-180 min. solving each MEA. For all the three MEAs, I had three information sources: I gathered all the students' paperwork and artifacts, observed and took notes during the project time, and video-recorded the students' interactions, presentations. Students' paperwork and artifacts included all the work the students did individually and in teams when developing their models, and the letter to the "client" each team wrote explaining and detailing their solution or mathematical construct. Observations included notes about the students' mathematical ideas taken by me and the classroom teacher. For the purpose of this manuscript, only solutions for the last MEA implemented is reported (i.e., *The Historic Hotel Problem)*.

The Activity

The Historic Hotel Problem is an MEA developed by Aliprantis and Carmona (2003). It is based on an economics problem (Aliprantis, 1999) that asks and encourages students to develop a mathematical model:

> Mr. Frank Graham has just inherited a historic hotel. He would like to keep the hotel, but he has little experience in hotel management. The hotel has 80 rooms, and Mr. Graham was told by the previous owner that all of the rooms are occupied when the daily rate is \$60 per room. He was also told that for every dollar increase in the daily \$60 rate, one less room is rented. So, for example, if he charged \$61 dollars per room, only 79 rooms would be occupied. If he charged \$62, only 78 rooms would be occupied. Each occupied room has a \$4 cost for service and maintenance per day.
>
> Mr. Graham would like to know how much he should charge per room in order to maximize his profit and what his profit would be at that rate. Also, he would like to have a procedure for finding the daily rate that would maximize his profit in the future even if the hotel prices and the maintenance costs change. Write a letter to Mr. Graham telling him what price to charge for the rooms to maximize his profit and include your procedure for him to use in the future.

The activity has been validated and tested in many different educational settings, with different participants. For example, Aliprantis and Carmona (2003) implemented it in middle schools, Dominguez (2010) also applied it in a calculus class in a southwest Texas university, and Ekmekci (2013) implemented it when working with pre-service mathematics and science teachers. Though implemented in many different contexts, this MEA has not been implemented in a high school or with the intention of comparing and contrasting the model-solutions of low, average, and high-achieving students. The mathematical ideas this activity addresses cover the topics of patterns, variables, and parabola (i.e., quadratic function).

Methodology

The study was conceived as a series of case studies, which were then compared and contrasted along various dimensions (Thomas, 2011; Goodrick, 2014; Hayes, 2000). Case studies are especially well-suited to my purposes here because they allowed me to observe both single individuals and participant groups and to study the students' solving processes as they unfolded, with a higher level of detail than if I were examining a larger population (McLeod, 2008). Case study is an empirical research method that "investigates a contemporary phenomenon within its real-life context…in which multiple sources or evidence are used" (Yin, 2003). In this research, I adopted Creswell et al., definition of case studies as "a qualitative approach in which the investigator explores a case or multiple cases over time through detailed, in-depth data collection involving multiple sources of information" (2007, p. 245). Although I collected audio of all teams, video of some selected teams, only the written work students generated for the last MEA implemented (i.e., The Historic Hotel Problem) is considered in this report.

To evaluate the quality of the student's final solutions I analyzed the students' written report to the client (i.e., the letter to the client) following the Quality Assessment Guide (QAG). In the letter to the client, students are required to explain in detail their solutions. The QAG (See Table 1) is an instrument Lesh and Clarke (2000) developed to assess the quality of solutions students had produced in a specific MEA. The QAG rates the quality of a solution considering five levels of performance, i.e., those that: (1) require redirection, (2) require major extensions or refinements, (3) require only minor editing (4) are useful for the specific data given and (5) are sharable or reusable. All these performance level are based on the client's needs.

Table 1

Quality Assessment Guide

Performance level	How useful is the product?	What might the client say?
Requires Redirection	The product is on the wrong track. Working longer or harder won't work. The students may require some additional feedback from the teacher.	"Start over. This won't work. Think about it differently. Use different ideas or procedures."
Requires Major Extensions or Refinements	The product is a good start toward meeting the client's needs, but a lot more work is needed to respond to all of the issues.	"You're on the right track, but this still needs a lot more work before it'll be in a form that's useful."
Requires Only Minor Editing	The product is nearly ready to be used. It still needs a few small modifications, additions, or refinements.	"This is close to what I need. You just need to add or change a few small things."

Useful for this Specific Data Given	No changes will be needed to meet the immediate needs of the client.	“This will work well as it is. I won’t even need to do any editing.”
Sharable or Reusable	The tool not only works for the immediate situation, but it also would be easy for others to modify and use it in similar situations.	“Excellent, this tool will be easy for me to modify or use in other similar situations – when the data is slightly different.”

To analyze students’ intermediate-products, I considered several characteristics of the student’s work (Dominguez, 2010; English, 2010; Aliprantis & Carmona, 2003; Ekmekci, 2013; Greenstein & Carmona, 2013):

a. The type of model-solution (Lesh & Zawojewski, 2007) created,

b. The strategies (Kent et al., 2015) followed to obtain their model, and

c. The components (Lohse, Biolsi, Walker, & Rueter, 1994) — also called elements of the visual representation — used to represent the mathematical construct and data.

To deeply analyze and study the intermediate-products, and for the purpose of this report, I considered only the last MEA implemented, *The Historic Hotel*. All 24 teams’ model-intermediate-products solutions were coded based on the characteristics of their model-solutions.

Findings

The quality of the student’s final solution, which is based on the student’s report or letter to the client, was rated using the QAG as framework. The 24 teams’ final products for *the Historic Hotel* MEAs were analyzed by two reviewers with experience in mathematics education at different grade levels. The reviewers scored the students’ final model-products based on the levels of performance of the QAG. The inter-rater agreement score, which represents the percentage of agreement between raters (Tinsley & Weiss, 2000; Gwet, 2014), was approximately 88%. After discussing and resolving the differences, the inter-rater agreement score ended in 92%. The QAG scores for each team are shown in the following table.

Table 2

Teams' Quality Assessment Scores and Means

Team	Level of Achievement	Historic Hotel MEA's Score	MEAN
Guardianes de la Galaxia	Low	4	3.21
Dinamita	Low	2	
VACAOS	Low	3	
LGH	Low	3	
LNA	Low	3	
Mateatletas	High	2	3.40
Carrojan	High	2	
Kam Girls	High	3	
SEJUSA	High	5	
MA^2	High	5	
Princesas	Average	3	2.64
Minions	Average	3	
Los Compadres	Average	2	
APS	Average	1	
CEZAMO	Average	3	
Chetos	Average	2	
FC Valle	Average	4	
Aguilas Doradas	Average	3	
CFN	Average	2	
OP	Average	3	
Fleurs	Average	3	
ICA	Average	3	
Enchiladas	Average	3	
GYF	Average	2	

The QAG is a general instructional approach to scoring teams' work (Hjalmarson et al., 2011; Lesh & Clarke, 2000). It only considers the written reports and/or presentations the students created and generated as results of their model-construction process (Clement, 2008). As a rubric that scores the quality of the students' work, it provides an overall idea on how the students' model solves the client's problem and, at some point, a way to compare the final products based on the client's needs. For example, Table 2 shows that most students, regardless of their performance level, had a score of 2 or 3, meaning that they might need either more time to work on the problem or more practice solving MEAs. In addition, note that low-achieving teams had a higher score mean, when compared to their average peers, meaning that they developed better solutions for the client based on the QAG, and a very similar mean in contrast to their high-performing peers. The QAG is a fairly simple way to understand the teams' final solutions in a very typical manner, by scoring and providing a grade. But it fails to consider a deeper analysis of the type of model the students constructed, the mathematical components/elements they considered for their models, and the types of strategies they used in developing their models. In the end, the QAG does not provide full evidence of the richness of the students' solutions. It is not possible to properly define how good a solution is just by looking at a number from one to five. Certainly, a deeper way to characterize, compare, and contrast the teams' work is required, rather than simply considering the QAG as a unique method of evaluation and analysis of the students' models-solutions. Therefore, in the next section I analyze the intermediate product-solutions that students generated aiming to have a deeper understanding of their models.

Intermedia Product-solutions

The intermedia product solution refers to the models and strategies developed by teams as they solved the activity, and the components and elements they used to create their models. After analyzing the student's data, their models were generalized in different types. Likewise, the elements or components of the mathematical construct were also categorized.

Model-Solution types

I have categorized all 24 teams' solutions for *The Historic Hotel* into five general forms of models the students created. In a general sense, most solutions considered the profit — "Ganancia" in Spanish (G) —, maintenance cost ($MC) —"Mantenimiento" —, the booked rooms (#Rooms), and the room's price ($Price).

In the first model, the profit is the result of subtracting the maintenance cost of the rooms from the cost of all booking rooms. Two subcategories emerged from this model-type solution: One that considers providing maintenance only to booked rooms (1A), and another that considers providing maintenance service to all rooms, regardless of whether maintenance is needed (1B). In this second subcategory, the maintenance cost is the result of multiplying the daily cost of maintenance ($DCM) — MX$40 — by the total number of rooms (80):

1A. G=(# Rooms • $Price) – (# Rooms • $DCM)
1B. G=(# Rooms • $Price) – $MC $MC=80 • 40 = MX$3200

The second model-type is a factored version (2A) of the original profit equation 1A in which the variable #Rooms is a common factor in all terms. In addition, another subcategory-model emerged that considers a set of alternate models that model the number of rooms to be booked and the maintenance cost (2B). In the former model the initial booking price ($IBP) — MX$600 — and the increased desired price ($IDP) are considered main variables to determine the number of booked rooms (#Rooms). Furthermore, a variable "x" represents any increment in the maintenance cost:

2A. G=($ Price – $DCM) • (# Rooms)
2B. G=($ Price – $DCM) • (# Rooms), where
#Rooms = 80 – [($IDP - $IBP) / 10]
$DCM = 40 + x

The third model-type that emerged in the students' work considers both the number of booked (#Rooms) and unbooked rooms (#UBRooms) as a strategy to determine the increase in the initial booking price ($IBP):

3. G = (# Rooms) • [($IBP + (# UBRooms • 10)] – (# Rooms • $DCM)

The fourth model-solution, which is the least sophisticated model in comparison with the other three models-type showed above, does not take into account the cost of maintenance. The rationale of the teams that decided this solution-path was that the maintenance cost would be included in the booking price, so it was not necessary to include it in the model. The model is only composed of the number of booked rooms and the booking price:

4. G = (# Rooms • $Price)

Finally, the fifth category clusters all models of category 1 of the QAG. These require major reconsiderations, which may reveal a lack understanding of the problem statement of the MEA. Few teams fall into this category, however, and none of the "low-achieving" teams fail to propose a model-solution showing a profit. In fact, only three teams fall into this category: two "average," and one "high-performing."

In Table 3, I show the type of model that each team in the different achievement level created. Teams created different model types, and in many cases the type of model is the same regardless of the level of achieving. From Table 3, I made a rough count to obtain percentages of the different types of models the teams created with the intention of knowing how many model-types fell into each category. For example, I can demonstrate that 46% of the teams constructed the same type of solution (1A) to model the hotel's profit (G) = (#Rooms • $Price) – (#Rooms • $DCM). From this percentage, 55% belong to the average-achieving teams, and 27% and 18% to the low- and high-achieving teams, respectively. In addition, 13% of the teams used a reduced factored version (2A) to model the profit, and only 8% considered providing service to all the rooms — model 1B.

Comparing solutions among teams, 60% of low-achieving teams created a model that fell into the 1A category, which is the most common model created among all teams. Most (80%)

considered following strategy A to model their solution, and all or almost all of them used lists, charts, and tables as components to represent their mathematical construct. In contrast, only 40% of high-achieving teams used the model-type A, and another 40% used the 2B model, which is the more mathematically sophisticated model as it considers an extra set of models to determine the number of rooms to be booked and the maintenance cost. For the average-achieving teams, less than half created a model-type A, 20% used the factored model-version 2A, and a small percentage fell under the model-category type 5, in which the model needs to be restructured and reconsidered.

Model-Strategies

To obtain their models, the teams needed to evaluate the strategy they would follow considering all the characteristics and information provided in *The Historic Hotel* problem statement. In this section, I describe the three types of strategies the teams followed when developing their model-solution.

The first strategy-type (A) implies systematically increasing the initial price by 10, and decreasing the number of booked rooms by 1. For example, if the initial price of MX$600 for a booking room is increased by MX$10, then only 79 rooms get booked, rather than the 80 originally booked. The second strategy (B) considers increasing the initial price by MX$5–instead of by the MX$10 stated in the problem statement and decreasing the number of booked rooms by one *only* when the initial price had been increased by a multiple of 10. This strategy requires increasing the price more times than other strategies and booking fewer rooms, but it generates a higher profit in the long term. For example, increasing the initial price from MX$600 to MX$605, would still generate the profit of 80 booked rooms. However, increasing the price to MX$610 would reduce the number of booked rooms to 79. In the third strategy-type (C), the students considered systematically increasing the initial price by MX$100, and decreasing the number of booked rooms by 10. One more general strategy (D) includes the combination of any of the above strategies, mixing either the first and second, the first and third, or the second and third. In Table 3, I show the strategies used by each team, which fell into one of the three different strategies, and in some cases a combination of any of the strategy's types.

Just as I did with the model-types, I made a rough count of the type of strategies that teams used in order to know how many strategies fell into each category. For example, just as with the low-achieving teams, 80% of high-achieving teams considered following strategy A. Similarly, almost 80% percent of the average-achieving teams considered also following strategy A. Only 13% of teams considered this strategy or a combination with any other strategy. Finally, strategy C was only considered by 4% of the teams. Most of the teams, regardless of achievement level, considered using the same type of strategy, which once again shows that MEAs might serve to level the playing field.

Components/Elements of the mathematical constructs

In the definition of models I adopted in this research project, conceptual tools (models) are composed of elements that allow the mathematization of a real-life situation with the ultimate purpose of predicting how that real-life situation would behave (Lesh & Doerr, 2003; Blumm & Ferri, 2009; Mousoulides, 2011; Blomhøj, 2004). For such predictions, the components or elements of the model and the strategies become essential in helping to make connections from real life to the mathematics world and vice versa, to visually represent the collected data, and ultimately predict a situation. While working on their model-product-solution, students created and used different types of components that helped them to reach a better understanding of the situation they were modeling–in this case the maximization of a hotel's profit. The components of the mathematical constructs that emerged in the students' work were: charts (C), tables (T), lists (L), and graphs (G). Examples of how the students used these types of components are given below, but to provide a basic idea here, I present some examples in Figure 1:

Figure 1

Mathematical components of the written solution

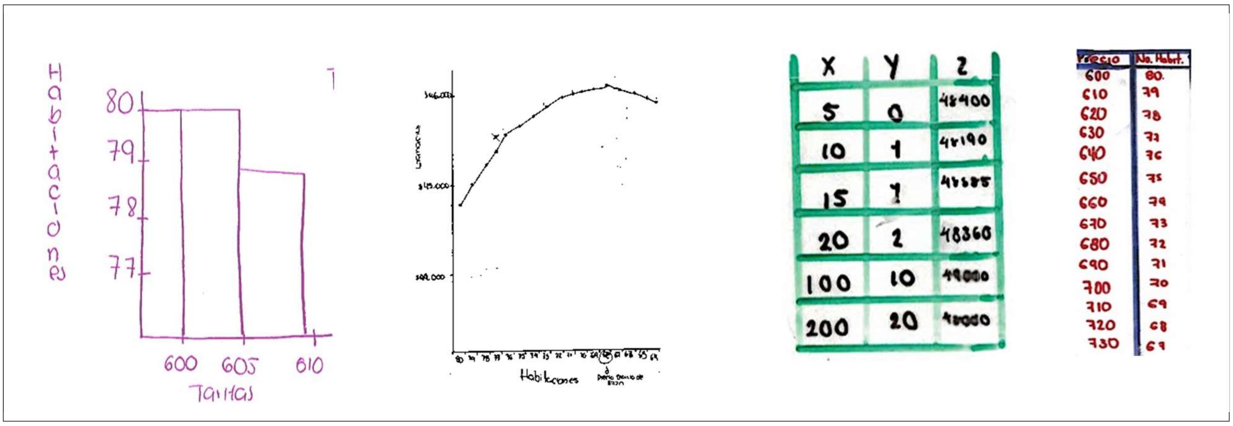

Figure 1. The figures above are examples of the types of components created and used during the process of model-development.

Similar to what I did before with the models and strategy types, I also made a rough count of the elements teams decided to create and use when developing their solution. As can be seen in the summary Table 3, 80% of the low-achieving teams used graphs (G) as a type of element to represent their data and mathematical constructs, and most of them used lists (L) and tables (T). Similarly, all the high-achieving teams used graphs (G) and most of them considered using lists (L) and tables (T). In contrast, few average-achieving teams considered creating graphs or charts, instead more than half used tables or lists as a way to represent their constructs and data.

I have detailed and explained above the three characteristics of the students' intermediate products — i.e., the model, the strategies, and the components/elements — based on what they showed in their written reports, worksheets, artifacts, and final presentations. In Table 3, I show a summary of the models, strategies, and components each team created and used during their model-solving process.

Table 3

Summary of the model-solution characteristics by team

Team	Level of achievement	Type of Model	Type of Strategy	Mathematical Elements used/created
Guardianes de la Galaxia	Low	3	A	G,Ch,L
Dinamita	Low	1A	A	T,G,L
VACAOS	Low	1A	A,C	L
LGH	Low	1A	A	L,T,Ch
LNA	Low	1B	A	Ch,L
MateAtletas	High	5	A	G
Carrojan	High	1A	B	G,L
Kam Girls	High	1A	A	T,L
SEJUSA	High	2B	A	Ch,T,G
MA^2	High	2B	A	G,T,L
Princesas	Average	3	A	T
Minions	Average	4	A	T,Ch,L
Los Compadres	Average	5	A	L
APS	Average	5	C	T
CEZAMO	Average	1A	A	L
Chetos	Average	1A	A	ND
FC Valle	Average	1A	A,B	T,L
Aguilas Doradas	Average	1A	A	T
CFN	Average	1A	B	T
OP	Average	1A	B	G,T
Fleurs	Average	1B	A	T
ICA	Average	2A	A	T,L
Enchiladas	Average	2A	A	G,L
GYF	Average	2A	A	L

Teams at different achievement levels considered similar models, strategies, and components to represent their models. The summary of the intermediate products (Table 3) is useful to get a better idea, in comparison to only considering the QAG, of how students construct their model-solutions and as a measure of comparison among all 24 teams. Only high-achieving teams created, used, and developed solutions that required using more than a single model (e.g., model 2B), but in some aspects (e.g., strategies and components/elements), high-achieving teams' final and intermediate solutions were similar across performance levels. Analyzing the teams' products provides a general perspective on how comparable these solutions are.

Conclusion

High school students worked in teams of two or three members to solve open-ended mathematics activities known as Model-Eliciting Activities (Lesh & Doerr, 2003; Lesh et al., 1999). As they engaged in these activities, they "stretched and developed their conceptual understanding" (Capraro, et al., 2007, p. 124), and viewed the emerging path-solutions from many different perspectives (Dominguez, 2010; Greenstein & Carmona, 2007).

The Achievement level as considered and established in this research project does not effectively determine the ability of students to generate solutions to open-ended mathematics activities like MEAs, which require complex thinking (Iversen & Larson, 2006; Lesh & Sriraman, 2005; Carmona & Greenstein, 2010). However, it was noticeable that some but not all high-achieving teams developed solutions that were more sophisticated, mathematically speaking, than those of the low-and average-achieving teams, based upon the Quality Assessment Guide, and the analysis of the intermedia products. Therefore, although the open-ended nature of the MEAs allows students to consider different solution paths for the same problem, much more evidence would be needed to show that MEAs could indeed serve to level the playing field among students of different achievement levels.

Many aspects of modeling research still need to be investigated further, particularly in the high school level, including student's perceptions and belief of using open-ended task like the ones used here. Although—from the perspective of the teachers and myself as a co-teacher and researcher—students seemed to enjoy the collaborative work and the opportunities to externalize their thoughts, these was not considered in the current study.

The research I have presented here is a first step in empowering low-achiever students in mathematics, too often sent to remedial classes, and in breaking the stereotype of them as being unable to develop solutions to mathematical activities like MEAs as creatively, adequately, and powerfully as average- or high-achievement students. In fact, this study contributes to breaking the stereotype of high-achiever students always performing outstandingly well.

References

Baxter, P., & Jack, S. (2008). Qualitative case study methodology: Study design and implementation for novice researchers. *The qualitative report*, *13*(4), 544-559.

Belenky, M., Clinchy, B., Goldberger, N., & Tarule, J. (1986). *Women's ways of knowing*. New York: Basic Books.

Blomhøj, M. (2004). Mathematical modelling-A theory for practice. *International perspectives on learning and teaching mathematics*, 145-160.

Blum, W., & Ferri, R. B. (2009). Mathematical modelling: Can it be taught and learnt?. *Journal of mathematical modelling and application*, *1*(1), 45-58.

Capraro, M. M., Capraro, R. M., & Cifarelli, V. V. (2007). What are students thinking as they solve open-ended mathematics problems. In Proceedings of the ninth international conference of Mathematics Education in a Global Community. Charlotte, NC.

Carmona, G., & Greenstein, S. (2010). Investigating the relationship between the problem and the solver: Who decides what math gets used? *In Modeling Students' Mathematical Modeling Competencies: ICTMA 13*. (pp. 245-254). Springer US

Chamberlin, S. A., & Moon, S. M. (2005). Model-eliciting activities as a tool to develop and identify creatively gifted mathematicians. *Prufrock Journal*, *17*(1), 37-47.

Clement, J. (2008). Creative model construction in scientists and students: The role of imagery, analogy, and mental simulation. Springer Science & Business Media.

Cresswell, J.W. (2008). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*. 3rd edition. Jersey: Merrill Prentice Hall.

DelMas, R., Garfield, J., & Zieffler, A. (2009). The tyranny of context. *International Collaboration for Research on Statistical Reasoning, Thinking, and Literacy, Brisbane, Australia*.

Deutsch, M. (1949). *A theory of cooperation and competition. Human Relations,* 2, 129-152.

Dewey, J. (1943). *The school and society* (Rev. ed.). Chicago: University of Chicago Press.

Dominguez, A. (2010). Single Solution, Multiple Perspectives. In *Modeling Students' Mathematical Modeling Competencies* (pp. 223-233). Springer US.

Ekmekci, A. (2013). Models and Modeling in Mathematics Education: Making Learners' Mathematical Thinking, Knowledge and Skills Visible. *Mediterranean Journal for Research in Mathematics Education*, *12*(1-2), 57-76.

Elliott, T. (2014). Hybrid-Electric Vehicles vs. Gasoline-Powered Vehicles. Retrieved online at: http://www.cpalms.org/Public/PreviewResourceLesson/Preview/69715.

English, L. D. (2010). Modeling with complex data in the primary school. In Modeling students' mathematical modeling competencies (pp. 287-299). Springer US.

Goodrick, D. (2014). Comparative case studies. *Methodological briefs: Impact evaluation*, *9*.

Gwet, K. L. (2014). Handbook of inter-rater reliability: The definitive guide to measuring the extent of agreement among raters. Advanced Analytics, LLC.

Hayes, N. (2000). *Doing psychological research*. Abingdon: Taylor & Francis Group.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., & Murray, H. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann, 34, 40.

Hjalmarson, M. A., Moore, T. J., & Delmas, R. (2011). Statistical Analysis When the Data is an Image: Eliciting Student Thinking About Sampling and Variability. Statistics Education Research Journal. Chicago

Iversen, S. M., & Larson, C. J. (2006). Simple thinking using complex math vs complex thinking using simple math—A study using model eliciting activities to compare students' abilities in standardized tests to their modeling abilities. *ZDM*, *38*(3), 281-292.

Kent, L. B., Empson, S. B., & Nielsen, L. (2015). The Richness of Children's Fraction Strategies. Teaching Children Mathematics, 22(2), 84-90.

Larson, C., Harel, G., Oehrtman, M., Zandieh, M., Rasmussen, C., Speiser, R., & Walter, C. (2010). Modeling perspectives in math education research. In *Modeling Students' Mathematical Modeling Competencies* (pp. 61-71). Springer US.

Lehrer, R., & Schauble, L. (2006). *Cultivating model-based reasoning in science education*. Cambridge University Press.

Lesh, R., & Clarke, D. (2000). Formulating Operational Definitions of Desired Outcomes of Instruction in Mathematics and Science Education. In A. Kelly, R. Lesh (Eds.), *Research Design in Mathematics and Science Education.* (pp. 120-150). Lawrence Erlbaum Associates, Mahwah, New Jersey.

Lesh, R., Cramer, K., Doerr, H., Post, T., & Zawojewski, J. (2003). *Model development sequences. In Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching* (May 2003), pp. 35-58.

Lesh, R., & Doerr, H. M. (2003). *Beyond Constructivism, Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Lesh, R., & Lehrer, R. (2003). Models and modeling perspectives on the development of students and teachers. *Mathematical thinking and learning*, *5*(2-3), 109-129.

Lesh, R., Hoover, M., Hole, B., Kelly, A., Post, T., (2000) Principles for Developing Thought-Revealing Activities for Students and Teachers. In A. Kelly, R. Lesh (Eds.), *Research Design* *in Mathematics and Science Education.* (pp. 591-646). Lawrence Erlbaum Associates, Mahwah, New Jersey.

Lesh, R., & Sriraman, B. (2005). John Dewey Revisited-Pragmatism and the models-modeling perspective on mathematical learning. In *Proceedings of the 1st International Symposium on Mathematics and its Connections to the Arts and Sciences* (pp. 32-51).

Lewin, K. (1935). *A dynamic theory of personality.* New York: McGraw-Hill.

Lohse, G. L., Biolsi, K., Walker, N., & Rueter, H. H. (1994). A classification of visual representations. Communications of the ACM, 37(12), 36-50.

Lombardi, M. M. (2007). Authentic learning for the 21st century: An overview. *Educause learning initiative*, *1*(2007), 1-12. Martin, L. C. (1999).

McLeod, S. A. (2008). Case study method. *Simply Psychology*.

Mousoulides, N. G. (2011). GeoGebra as a conceptual tool for modeling real world problems. In Model-Centered Learning (pp. 105-118). SensePublishers.

Piaget, J. (1964). Part I: Cognitive development in children: Piaget development and learning. *Journal of research in science teaching*, *2*(3), 176-186.

Springer, L., Stanne, M. E., & Donovan, S. S. (1999). Effects of small-group learning on undergraduates in science, mathematics, engineering, and technology: A meta-analysis. *Review of educational research*, *69*(1), 21-51.

Tangney, B., FitzGibbon, A., Savage, T., Mehan, S., & Holmes, B. (2001). Communal Constructivism: Students constructing learning for as well as with others. In *Society for Information Technology & Teacher Education International Conference* (Vol. 2001, No. 1, pp. 3114-3119).

Tinsley, H. E., & Weiss, D. J. (2000). Interrater reliability and agreement. Handbook of applied multivariate statistics and mathematical modeling, 95-124. Chicago

Thomas, G. (2011). A typology for the case study in social science following a review of definition, discourse, and structure. *Qualitative inquiry*, *17*(6), 511-521.

Vygotsky, L.S. (1978). Mind in Society. Cambridge, MA: Harvard University Press.

Vygotsky, L.S. (1962). Thought and Language. Cambridge, MA: MIT Press. (Original work published in 1934).

Yin, R. K. (2003). Case study research: Design and methods (3rd ed.). Thousand Oaks, CA: Sage.

Yildirim, T. P., Shuman, L., Besterfield-Sacre, M., & Yildirim, T. P. (2010). Model eliciting activities: assessing engineering student problem solving and skill integration processes. *International Journal of Engineering Education*, *26*(4), 831-845.

Zawojewski, J. S., Lesh, R. A., & English, L. D. (2003). A models and modeling perspective on the role of small group learning activities. *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*, 337-358.

Zandieh, M., & Rasmussen, C. (2010). Defining as a mathematical activity: A framework for characterizing progress from informal to more formal ways of reasoning. *The Journal of Mathematical Behavior*, *29*(2), 57-75