Delegation, Overconfidence, and Welfare in a Differentiated Duopoly

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Abstract

We derive Bertrand and Cournot equilibria in a differentiated duopoly in which each firm hires a manager to undertake research and development (R&D) and production decisions. We show that manager overconfidence and over-investment occur in each market competition. Furthermore, the Cournot game induces a higher level of overconfidence and more R&D investments than Bertrand game. However, if R&D spillovers are strong, the price is lower, and output is more abundant in Bertrand than in Cournot game. Furthermore, Cournot and Bertrand models with overconfident managers induce low prices, massive productions, and are *more* efficient if R&D productivity is low and spillovers are strong. We also show that if firms can only make two types of binding contracts with shareholders, it is a dominant strategy for each firm to choose the delegation contract.

Keywords: Product differentiation, Overconfidence, R&D, Welfare, Delegation

JEL: D43, L13, L21, O32

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1. Introduction

In this paper, we argue that individual characteristics of managers, such as overconfidence or over-optimism, in decentralized corporations improve welfare. In particular, we examine the investment and production decisions of managers who overestimate the intercept of future demands of their companies. We find that firms always hire overconfident managers, and they over-invest in cost-reducing R&D. The efficiency in production allows firms to expand outputs, reduce prices, which then leads to an increase in social welfare—the aggregate sum of consumer surplus and producer profits—in markets.

Recent studies of product-market competition in corporations mostly focus on the reasons firms hire overconfident managers (Goel and Thakor (2008), Englmaier (2010), Yu (2014), Pu et al. (2017)) and the effects of managerial overconfidence on innovation (Einhorn (1980), Eberhart et al. (2004), Malmendier and Tate (2005, 2008), Galasso and Simcoe (2011), Hirshleifer et al. (2012), Chen et al. (2014), Li et al. (2019)), on mergers and acquisitions (Malmendier and Tate (2008)), and leadership (Phua et al. (2018)). From the best of our knowledge, none of these studies explores the effects of managerial overconfidence on social welfare. In this paper, we aim to fill this gap in the literature.

In sections 2 and 3, we first consider a three-stage duopoly model in which two firms compete in the product market. Our model augments the Qiu (1997) model by including the delegation stage with (possible) biased managers. The demand structure is linear and allows products to be substitutes. Before the product competition stage, firms hire managers, who have personal beliefs about the market, to take charge of R&D investments and production decisions. Following the existing literature (Englmaier (2010, 2011), Yu (2014), Li et al. (2019)), we ignore the agency conflict between firms and managers and assume that managers choose the investment level to maximize firms' profits (according to their beliefs) net of R&D expenditures. Following d'Aspremont and Jacquemin (1988), we assume that R&D investments exhibit spillover effects. In this framework, assuming mild assumptions on R&D technology, we determine the equilibrium when either Cournot or Bertrand game characterizes the product-market. Cournot and Bertrand equilibria are unique. We show that overconfidence and over-investment occur as an equilibrium outcome independent of the mode of competition, that is, price or quantities. Cournot game induces a higher level of overconfidence and more R&D investments than Bertrand game. Relative to the models without delegation, differentiated duopoly with overconfident managers produce low prices and large outputs. If R&D spillovers are strong, the price is lower, and production is more abundant in Bertrand than in Cournot mode. Therefore, consumer surplus is higher in Bertrand than in the Cournot game. Furthermore, we show that if R&D productivity is low and spillovers are strong, delegation with overconfident managers is *more* efficient, in the sense that in equilibrium, consumer surplus and producer profits are higher relative to the setting without delegation.

Building on the works of Singh and Vives (1984) and Kyle and Wang (1997), we suppose in section 4, that each firm can make only two types of binding contracts with shareholders: the delegation contract and the non-delegation contract. If a firm chooses the delegation contract, this means that it will have to delegate the R&D investments and production decisions to a manager whatever action the competitor takes. If a firm chooses the non-delegation contract, it commits to making its strategic decisions without hiring a manager independently of the action of the competitor. Consider a four-stage game where firms first simultaneously commit themselves to a type of contract and afterward compete depending upon the chosen models of contracts and the three-stage game under either price or quantity competition. Restricting attention to sub-game perfect equilibria of the dynamic game, we show that it is a dominant strategy for a firm to choose the delegation contract and hire an overconfident manager. This result indicates that delegation and overconfidence may persist and survive in the long run.

2. The Basic Model and Cournot Equilibrium

Consider a sector of an economy with two firms $i = 1, 2$ producing differentiated goods q_1 and q_2 respectively. Following Singh and Vives (1984), we assume that the representative consumer's utility function is $U(q_1, q_2) = a(q_1+q_2) - (q_1^2+2bq_1q_2+q_2^2)/2$, $a > 0$ is the true market size, and $b \in (0, 1)$ is the degree of product differentiation, with differentiation increasing as b is close to zero. The inverse market demands are linear:

$$p_i = a - q_i - bq_j,\ i, j = 1, 2,\ i \neq j. \tag{1}$$

Firms may invest in cost-reducing R&D. The pre-innovation cost for each firm is c, with $a > c$. If firm i engages in R&D, then by spending $V(x_i)$ on R&D, it lowers its marginal cost by $x_i + \beta x_j$: $c_i = c - x_i - \beta x_j$, where the parameter $\beta \in [0, 1]$ captures the extend of output spillovers (d'Aspremont and Jacquemin (1988)). We assume that $V(x_i) = v\frac{x_i^2}{2}$, where v relates to the productivity of the R&D technology (higher v means lower productivity).

Timing is the following.

Stage 1. Each firm hires a manager to take charge of R&D investments and production decisions. Managers have different personalities (types or beliefs) and may over-estimate or under-estimate the size of the market. Following Yu (2014), we assume that there is at least two managers for each type so that the matching between firms and managers is always perfect. Managers' types are observable both to firms and competing managers. If firm i hires manager of type α_i, he or she believes that the inverse market demand of firm i is:

$$p_i = \alpha_i - q_i - bq_j,\ i, j = 1, 2,\ i \neq j. \tag{2}$$

In line with the literature (see, e.g., Malmendier and Tate (2005, 2008), Galasso and Simcoe (2011), Yu (2014), Li et al. (2019)), an *overconfident* manager over-estimate the true (or real) intercept of the demand function, i.e., $\alpha_i > a$.

Stage 2. Managers simultaneously and independently undertake cost-reducing R&D based on their own beliefs about the market. As mentioned in the Introduction, we assume that the objectives of the managers are aligned with shareholders' goals after being hired.

Stage 3. The real market size is realized, and both firms produce and sell their products to consumers.

In this section, we only consider Cournot competition, where firms compete by setting quantities in the market stage. In section 3, we discuss Bertrand competition, a setting in which firms compete in prices. We derive sub-game perfect Nash equilibria using backward induction. For clarity, we relegate the proofs of our results in the appendix.

Assume:

A1. $v > 2a/c$.

Assumption **A1** ensures that every sub-game after the initial stage has a unique Nash equilibrium with both firms (and managers) in the quantity competition.

Let π_C^i denote firm i's market profit (profit excluding R&D costs) after the real market is revealed in stage 3. Given any R&D outcome (x_i, x_j),

$$\pi_C^i(q_i, q_j; x_i, x_j) = p_i q_i - (c - x_i - \beta x_j) q_i. \tag{3}$$

Firms choose outputs to maximize their respective market profits, and the Cournot-Nash equilibrium is

$$q_i^*(x_i, x_j) = \frac{1}{(4 - b^2)}[(a - c)(2 - b) + (2 - b\beta)x_i + (2\beta - b)x_j], \tag{4}$$

and firm i's overall market profit $\Pi_C^i(x_i, x_j) = \pi_C^i(q_i^*, q_j^*; x_i, x_j) - V(x_i)$ is:

$$\frac{[(2-b)(a-c)+(2-b\beta)x_i+(2\beta-b)x_j]^2}{(4-b^2)^2}-v\frac{x_i^2}{2}. \tag{5}$$

Note from (4) that $\frac{\partial q_i^*}{\partial x_i}=\frac{2-b\beta}{4-b^2}$, and $\frac{\partial q_i^*}{\partial x_j}=\frac{2\beta-b}{4-b^2}$. It follows that $\frac{\partial q_i^*}{\partial x_i}>0$, for any b and β, and $\frac{\partial q_i^*}{\partial x_j}>0$ if $\beta>\frac{b}{2}$. Turning to the second stage, based on their beliefs about the market in (2), each manager undertakes an R&D level to maximize the firm's expected overall profit

$$\Pi_C^i(x_i,x_j)=\frac{[(2-b)(\alpha_i-c)+(2-b\beta)x_i+(2\beta-b)x_j]^2}{(4-b^2)^2}-v\frac{x_i^2}{2}. \tag{6}$$

Using the first-order conditions, manager i's best R&D strategy in response to firm j's choice of R&D level x_j is:

$$x_i(\alpha_i,x_j)=\frac{2(2-b\beta)}{v(4-b^2)^2-2(2-b\beta)^2}[(2-b)(\alpha_i-c)+(2\beta-b)x_j]. \tag{7}$$

Given (7) and symmetry, firm i's R&D investment level is provided by

$$x_i^*(\alpha_i,\alpha_j)=\frac{2(2-b)(2-b\beta)}{\Lambda}\,\overline{x}_i(\alpha_i,\alpha_j), \tag{8}$$

where $\overline{x}_i(\alpha_i,\alpha_j)=[(4-b^2)^2v-2(2-b\beta)^2]\alpha_i+2(2-b\beta)(2\beta-b)\alpha_j+[2(2-b\beta)(2+b)(1-\beta)-(4-b^2)^2v]c$, for $i,j=1,2$ and $i\neq j$, and $\Lambda=[(4-b^2)^2v-2(2-b\beta)^2]^2-4(2\beta-b)^2(2-b\beta)^2$. Given (7), we have $\frac{\partial x_i(\alpha_i,x_j)}{\partial x_j}=\frac{2(2-b\beta)(2\beta-b)}{v(4-b^2)^2-2(2-b\beta)^2}$, and using (8), it follows that, for $i,j=1,2$ and $i\neq j$:

$$\frac{\partial x_i^*}{\partial \alpha_i}=\frac{2(2-b)(2-b\beta)}{\Lambda}[(4-b^2)^2v-2(2-b\beta)^2],\ \text{and,} \tag{9}$$

$$\frac{\partial x_i^*}{\partial \alpha_j}=\frac{4(2-b)(2-b\beta)^2}{\Lambda}(2\beta-b). \tag{10}$$

Under **A1**, $v>\frac{2(2-b\beta)^2}{(4-b^2)^2}$, and $\Lambda>0$. Therefore, it is immediate that $\frac{\partial x_i^*}{\partial \alpha_i}>0$, and $\frac{\partial x_i(\alpha_i,x_j)}{\partial x_j}>0$ if and only if $\beta>\frac{b}{2}$. Similarly, the sign of $\frac{\partial x_i^*}{\partial \alpha_j}$ depends on the sign of

$(2\beta - b)$. Hence, if $0 < \beta < \frac{b}{2}$, then $\frac{\partial x_i^*}{\partial \alpha_j} < 0$, and, if $1 > \beta > \frac{b}{2}$, we have $\frac{\partial x_i^*}{\partial \alpha_j} > 0$.

Now, in the initial stage, firms simultaneously choose a manager of type α_i to maximize their expected profits. Substituting (8) into (5), firm i's profit is

$$\Pi_C^i(x_i^*, x_j^*) = \frac{[(2-b)(a-c) + (2-b\beta)x_i^*(\alpha_i, \alpha_j) + (2\beta - b)x_j^*(\alpha_i, \alpha_j)]^2}{(4-b^2)^2} - v\frac{[x_i^*(\alpha_i, \alpha_j)]^2}{2}.$$

By the first-order conditions and symmetry, firms hire a manager with a confidence level

$$\alpha_C = a + \frac{2(a-c)v}{\Delta_C}(2+b)(2\beta - b)^2 \tag{11}$$

where $\Delta_C = 4(1-\beta)(2-b\beta)(1+\beta)^2 - 2(2+b)[4-b+2(1-b)\beta](2-b\beta)v + (2-b)^2(2+b)^3v^2$, the latter being positive under **A1**.

Note that in case of no delegation in the first stage (i.e., $\alpha_i = \alpha_j = a$), the symmetric equilibrium outcomes are: $x_{nc} = \frac{2(2-b\beta)(a-c)}{\Delta}$, $q_{nc} = \frac{1}{\Delta}v(a-c)(4-b^2)$, and $p_{nc} = a - (1+b)q_{nc}$, where $\Delta = (2+b)(4-b^2)v - 2(1+\beta)(2-b\beta) > 0$ under **A1**.

Substituting α_C into (8) yields the symmetric equilibrium R&D investment level:

$$x_C = \left\{1 + \frac{2(2+b)(2\beta - b)^2 v}{\Delta_C}\right\} x_{nc}. \tag{12}$$

A1 guarantees positive post-innovation costs of production in Cournot competition, i.e., $c - (1+\beta)x_C > 0$. Note that if R&D investments are very productive, the firms will invest more to gain a competitive advantage in the market game which will lead to zero, and even negative post-innovation costs.[2]

[2]Note that Qiu (1997) uses a weaker assumption for Cournot competition, $v > \frac{a}{c}$, and a more robust assumption $v > \frac{2\left(2-b^2-b\beta\right)^2}{(1-b^2)(4-b^2)^2}$ for Bertrand competition. Also, Semenov and Tondji (2019) use the assumption $v > \frac{3\alpha}{c}$ for Cournot-Bertrand game.

Substituting x_C into (4) and (1) yield the (symmetric) equilibrium output and price:

$$q_C = \frac{(a-c)v}{\Delta_C}\left\{(4-b^2)^2 v - 2(2-b\beta)^2\right\}, \text{ and } p_C = a-(1+b)q_C. \tag{13}$$

Finally, equilibrium consumer surplus $[CS = U(q_1, q_2) - p_1 q_1 - p_2 q_2 = (1+b)(q_C)^2]$, producer profits $[\Pi = \Pi_1 + \Pi_2 = 2(q_C)^2 - v(x_C)^2]$, and welfare $[W = CS + \Pi]$ in Cournot competition are given as:

$$CS_C = \frac{(1+b)(a-c)^2 v^2}{\Delta_C^2}\left\{(4-b^2)^2 v - 2(2-b\beta)^2\right\}^2,$$

$$\Pi_C = \frac{2(a-c)^2 v}{\Delta_C^2}\left\{\begin{aligned} & v[(4-b^2)^2 v - 2(2-b\beta)^2]^2 \\ & -2(2-b\beta)^2[2(1-\beta^2)-(4-b^2)v]^2 \end{aligned}\right\},$$

$$W_C = \frac{(a-c)^2 v}{\Delta_C^2}\left\{\begin{aligned} & (1+b)v[(4-b^2)^2 v - 2(2-b\beta)^2]^2 + 2[v((4-b^2)^2 v \\ & -2(2-b\beta)^2)^2 - 2(2-b\beta)^2(2-2\beta^2-(4-b^2)v)^2] \end{aligned}\right\}.$$

Remark 1 An immediate analysis of (11) and (12) suggests that $\alpha_C > a$ and $x_C > x_{nc}$. It follows that in equilibrium, both firms choose overconfident managers, and overinvestment occurs. □

Let's look at how the degree of product differentiation affects the level of overconfidence in equilibrium. Using (11):

$$\frac{\partial \alpha_C}{\partial b} = \frac{4(2\beta-b)(a-c)v}{\Delta_C^2}A(b,\beta),$$

where $A(b,\beta) = 4(1-\beta)(1+\beta)^2[-4-3b+(2+b+b^2)\beta+2\beta^2] - 2(2+b)^2(1-\beta^2)[(b+4b\beta-2(4+\beta)]v - (2-b)(2+b)^3(4+b^2-4b\beta)v^2 < 0$ under **A1**. Therefore, $\frac{\partial \alpha_C}{\partial b} < 0$ if and only if $\beta > \frac{b}{2}$. The latter implies that when there is closer substitutability between products, firms hire less confident managers when innovation is easily accessible by competitors. Using equation (8):

$$\frac{\partial x_i^*(\alpha_i,\alpha_j)}{\partial b} = \underbrace{\frac{\partial x_i^*(\alpha_i,\alpha_j)}{\partial \alpha_i}}_{+}\underbrace{\frac{\partial \alpha_i}{\partial b}}_{-} + \underbrace{\frac{\partial x_i^*(\alpha_i,\alpha_j)}{\partial \alpha_j}}_{+}\underbrace{\frac{\partial \alpha_j}{\partial b}}_{-} < 0 \text{ if } \beta > \frac{b}{2}, \text{ and} \tag{14}$$

$$\frac{\partial x_i^*(\alpha_i,\alpha_j)}{\partial b} = \underbrace{\frac{\partial x_i^*(\alpha_i,\alpha_j)}{\partial \alpha_i}}_{+}\underbrace{\frac{\partial \alpha_i}{\partial b}}_{+} + \underbrace{\frac{\partial x_i^*(\alpha_i,\alpha_j)}{\partial \alpha_j}}_{-}\underbrace{\frac{\partial \alpha_j}{\partial b}}_{+} \gtrless 0 \text{ if } \beta < \frac{b}{2}. \tag{15}$$

Equation (14) is a direct implication of the argument mentioned above on confidence levels and product substitutability. However, the net effect of product differentiation on the levels of R&D investments is ambiguous when spillover effects are relatively low (Eq. (15)). We also note that:

$$\frac{\partial \alpha_C}{\partial c} = -\frac{2(2+b)(2\beta-b)^2 v}{\Delta_C}, \text{ and } \frac{\partial x_C}{\partial c} = -\frac{2(2-b\beta)\left\{(4-b^2)v-2(1-\beta^2)\right\}}{\Delta_C}.$$

Under **A1**, and $\beta \neq \frac{b}{2}$, it is straightforward to show that the equilibrium level of managers' overconfidence and the size of R&D investment levels decrease with initial production cost, c, i.e., $\frac{\partial \alpha_C}{\partial c} < 0$ and $\frac{\partial x_C}{\partial c} < 0$. This analysis, and Remark 1 both generalize the results of previous findings on delegation and process innovation under Cournot competition (Englmaier (2010), Yu (2014), Li et al. (2019)). In these studies, the authors assume that either the productivity of the R&D technology is constant ($v = 1$) or there is zero R&D spillover effects ($\beta = 0$).

Remark 2 In case of no delegation at the initial stage, the producer surplus is $\Pi_{nc} = \frac{2(a-c)^2 v}{\Delta^2}[(4-b^2)^2 v - 2(2-b\beta)^2]$, the consumer surplus is $CS_{nc} = \frac{(1+b)(4-b^2)^2(a-c)^2v^2}{\Delta^2}$, and welfare is $W_{nc} = \frac{(a-c)^2 v}{\Delta^2}[(3+b)(4-b^2)^2 v - 4(2-b\beta)^2]$. □

Comparisons based on market equilibrium outcomes under quantity competition with possible overconfident managers with those obtained in a game without delegation (Remark 2) yield the following proposition.

Proposition 1 *For any $b \in (0,1)$ and $\beta \in (0,1)$, with $\beta \neq \frac{b}{2}$.*

(a) $q_C > q_{nc}$, and $p_C < p_{nc}$. Consequently, $CS_C > CS_{nc}$.

(b) $\Pi_C > \Pi_{nc}$ if and only if $\beta > \frac{b}{2}$.

(c) There exists a unique $v_1 = v_1(b, \beta)$ such that $W_C - W_{nc} = \begin{cases} > 0 & if \quad v > v_1 \\ < 0 & if \quad v < v_1 \end{cases}$. □

Proposition 1 implies that overconfident managers make decisions that induce higher productions, lower prices, and higher consumer surplus. Also, if the extend of spillover effects is sufficiently strong ($\beta > \frac{b}{2}$), then delegation increases profits, and therefore welfare. In the case of weak spillover effects ($\beta < \frac{b}{2}$), then, according to *(c)*, overconfidence improves welfare if the R&D technology is less productive ($v > v_1$). In fact, for any $b \in (0,1)$ and $\beta \in (0,1)$ so that $\beta > \frac{b}{2}$, the inequality $v > v_1$ always holds under **A1**, and therefore $W_C > W_{nc}$. Also, under **A1**, $v < v_1$ only holds when the R&D spillover effects are relatively low (i.e., β is small), and the product market is almost homogeneous (i.e., b is sufficiently high and close to 1). We illustrate this situation in Figure 1 for parameters: $a = 1$, $c = 0.955$, $\beta = 0.001$, $b = 0.93$, and $v_1 = 2.49068 > \frac{2a}{c} = 2.09424$.

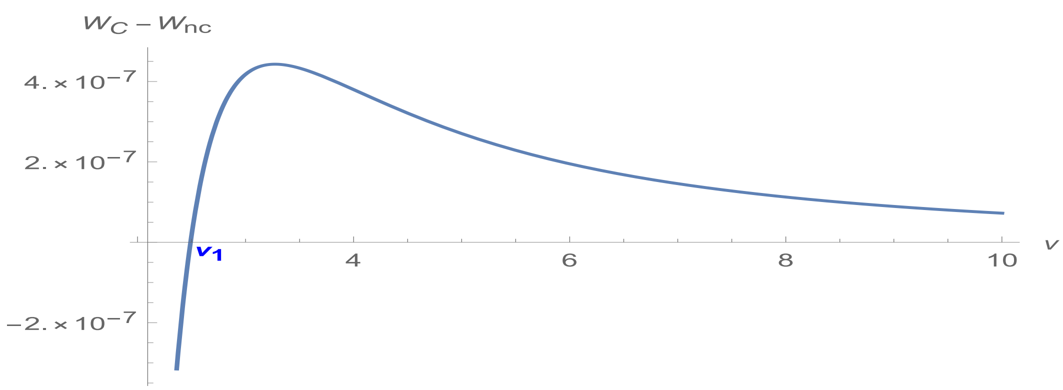


Figure 1: Difference in welfare

It follows that under sufficiently product differentiation, and assuming that the R&D technology is less efficient (i.e., **A1** is satisfied), even if the R&D spillover effects is low, we expect the difference in welfare to be positive. We illustrate this situation in Figure 2 for parameters $a = 1$, $c = 0.955$, $\beta = 0.001$, and $b = 0.9$.

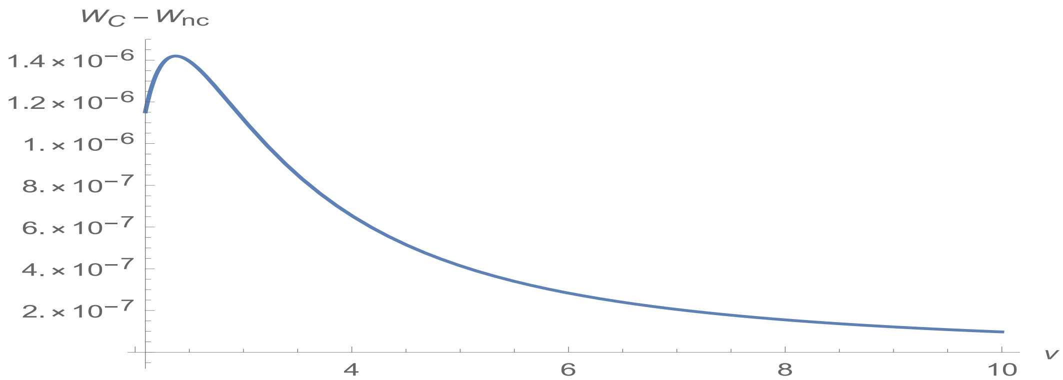


Figure 2: Difference in welfare

In the next section, we discuss Bertrand competition, a framework in which firms compete in prices.

3. Bertrand Equilibrium

Suppose now that in the product market, firms compete in prices. Rewriting market demands in (1) gives us:

$$q_i = \frac{1}{1-b^2}\left\{a(1-b) - p_i + bp_j\right\}, \; i, j = 1, 2, \; i \neq j. \tag{16}$$

Assume:

A2. $\beta > \frac{b}{2-b^2}$.

Assumption **A2** implies that the extend of spillover effects are sufficiently strong. Assumptions **A1** and **A2** are sufficient, but not necessary for the second-order and the stability conditions in the price competition.

Let π_B^i denote firm i's market profit (profit excluding R&D costs) after the real market is revealed. Given any R&D outcome (x_i, x_j) in stage 2, firm i in the market competition stage choose price p_i to maximize its market profits π_B^i. The resulting

Bertrand-Nash equilibrium is

$$p_i^*(x_i,x_j) = \frac{1}{4-b^2}\left\{a(2-b-b^2)+(2+b)c-(2+b\beta)x_i-(b+2\beta)x_j\right\}, \tag{17}$$

and firm i's overall market profit $\Pi_B^i(x_i,x_j) = \pi_B^i(p_i^*,p_j^*;x_i,x_j) - V(x_i)$ is:

$$\frac{[(2-b-b^2)(a-c)+(2-b^2-b\beta)x_i+((2-b^2)\beta-b)x_j]^2}{(1-b^2)(4-b^2)^2} - v\frac{x_i^2}{2}. \tag{18}$$

Using (17), it is straightforward to observe that both $\frac{\partial p_i^*}{\partial x_i} = -\frac{2+b\beta}{4-b^2}$, and $\frac{\partial p_i^*}{\partial x_j} = -\frac{b+2\beta}{4-b^2}$ are negative for any b and β. Similarly, substituting (17) into (16) yield $\frac{\partial q_i^*}{\partial x_i} = \frac{2-b(b+\beta)}{4-5b^2+b^4} > 0$, and $\frac{\partial q_i^*}{\partial x_j} = \frac{-b-(2-b^2)\beta}{4-5b^2+b^4} > 0$ under **A2**. The latter confirms the incentive for firms to invest in cost-reducing R&D. Investments enable firms to extend the production, given the reduction in marginal costs, and this translates to a decline in prices.

Turning to the second stage, on the basis that the market demand is

$$q_i = \frac{1}{1-b^2}\left\{\alpha_i(1-b)-p_i+bp_j\right\},\ i,j=1,2,\ i\neq j, \tag{19}$$

manager of confidence level α_i undertakes an R&D level to maximize the firm overall profit

$$\Pi_B^i(x_i,x_j) = \frac{[(2-b-b^2)(\alpha_i-c)+(2-b^2-b\beta)x_i+((2-b^2)\beta-b)x_j]^2}{(1-b^2)(4-b^2)^2} - v\frac{x_i^2}{2}.$$

Using the first-order conditions, manager i's best R&D strategy in response to firm j's choice of R&D level x_j is:

$$x_i(\alpha_i,x_j) = \frac{2(2-b^2-b\beta)}{v(1-b^2)(4-b^2)^2-2(2-b^2-b\beta)^2}[(2-b-b^2)(\alpha_i-c)+((2-b^2)\beta-b)x_j]. \tag{20}$$

Given (20) and symmetry, firm i's R&D investment level is provided by

$$x_i^*(\alpha_i,\alpha_j) = \frac{2(2-b-b^2)(2-b^2-b\beta)}{\chi}\overline{y}_i(\alpha_i,\alpha_j), \tag{21}$$

and firm j's R&D investment level is given by

$$x_j^*(\alpha_i, \alpha_j) = \frac{2(4-5b^2+b^4)(2-b^2-b\beta)}{(2-b)(1+b)\chi}\overline{y}_j(\alpha_i, \alpha_j), \tag{22}$$

where $\overline{y}_i = [(4-b^2)^2(1-b^2)v - 2(2-b^2-b\beta)(2-b^2-b\beta)]\alpha_i + 2(2-b^2-b\beta)(2\beta - b - b^2\beta)\alpha_j + [2(2-b^2-b\beta)(1+b)(2-b)(1-\beta) - (4-b^2)^2(1-b^2)v]c$, $i, j = 1, 2$, $i \neq j$, and $\chi = [2(2-b^2-b\beta))^2 - (4-b^2)^2(1-b^2)v]^2 - 4(2\beta - b - b^2\beta)^2(2-b^2-b\beta)^2$.

Let's look at the effects of confidence levels on R&D investments. Given (21), we have:

$$\begin{aligned}
\frac{\partial x_i^*(\alpha_i, \alpha_j)}{\partial \alpha_i} &= \frac{2(2-b-b^2)(2-b^2-b\beta)}{\chi}\left\{(4-b^2)^2(1-b^2)v - 2(2-b^2-b\beta)^2\right\}, \text{ and,} \\
\frac{\partial x_i^*(\alpha_i, \alpha_j)}{\partial \alpha_j} &= \frac{4(2-b-b^2)(2-b^2-b\beta)^2}{\chi}\left\{(2\beta - b^2\beta - b)\right\}.
\end{aligned}$$

Similarly, using (22), we have:

$$\begin{aligned}
\frac{\partial x_j^*(\alpha_i, \alpha_j)}{\partial \alpha_i} &= \frac{2(4-5b^2+b^4)(2-b^2-b\beta)^2}{(2-b)(1+b)\chi}\left\{(2\beta - b^2\beta - b)\right\}, \text{ and,} \\
\frac{\partial x_j^*(\alpha_i, \alpha_j)}{\partial \alpha_j} &= \frac{2(4-5b^2+b^4)(2-b^2-b\beta)}{(2-b)(1+b)\chi}\left\{(4-b^2)^2(1-b^2)v - 2(2-b^2-b\beta)^2\right\}.
\end{aligned}$$

Under **A1**, we have $(4-b^2)^2(1-b^2)v - 2(2-b^2-b\beta)^2 > 0$ and $\chi > 0$ so that $\frac{\partial x_i^*(\alpha_i,\alpha_j)}{\partial \alpha_i} > 0$ and $\frac{\partial x_j^*(\alpha_i,\alpha_j)}{\partial \alpha_j} > 0$. Given that $\frac{b}{2-b^2} < \beta < 1$ under **A2**, it follows that $(2\beta - b^2\beta - b) > 0$ so that $\frac{\partial x_j^*(\alpha_i,\alpha_j)}{\partial \alpha_i} > 0$ and $\frac{\partial x_i^*(\alpha_i,\alpha_j)}{\partial \alpha_j} > 0$. It follows that R&D investments are positively correlated with confidence levels.

In the initial stage (stage 1), firms simultaneously choose a manager of type α_i to maximize their expected profits. Substituting (21) and (22) into (18), firm i's profit is

$$\Pi_B^i(x_i^*, x_j^*) = \frac{[(2-b-b^2)(a-c) + (2-b^2-b\beta)x_i^* + ((2-b^2)\beta - b)x_j^*]^2}{(1-b^2)(4-b^2)^2} - v\frac{(x_i^*)^2}{2}.$$

By the first-order conditions and symmetry, firms hire a manager with a confidence level:

$$\alpha_B = a + \frac{(a-c)v}{\Delta_B}\left\{2(2-b)(2\beta - b^2\beta - b)^2\right\}, \tag{23}$$

where $\Delta_B = 4(1-b)(1-\beta^2)(1+\beta)(2-b^2-b\beta) + 2(2-b)(b+2b\beta-4-2\beta+2b^2+b^2\beta)(2-b^2-b\beta)v + (2-b)(1-b^2)(4-b^2)^2v^2$.

In case of no delegation in stage 1, the symmetric equilibrium outcomes in Bertrand competition are: $x_{nb} = \frac{2(2-b\beta-b^2)(a-c)}{\Phi}$, $p_{nb} = \frac{1}{\Phi}[v(a-ab+c)(1+b)(4-b^2) - 2a(1+\beta)(2-b^2-b\beta)]$, and $q_{nb} = \frac{1}{\Phi}(4-b^2)(a-c)v$, where $\Phi = (2-b)(1+b)(4-b^2)v - 2(1+\beta)(2-b^2-b\beta)$ is positive under **A1** and **A2**.

Substituting α_B into (21) and (22) yield the equilibrium R&D investment level:

$$x_B = \left\{1 + \frac{2(2-b)(2\beta - b^2\beta - b)^2 v}{\Delta_B}\right\} x_{nb}. \tag{24}$$

Both assumptions **A1** and **A2** insure that $c - (1+\beta)x_B > 0$ (positive post-innovation costs). Substituting (24) in (16) and (17) yield the market equilibrium outcome:

$$p_B = \frac{1}{\Delta_B}\left\{\begin{array}{l} 4a(1-b)(1+\beta)(1-\beta^2)(2-b^2-b\beta) + (4-b^2)^2(1-b^2)(a(1-b)+c)v^2 \\ -2(2-b^2-b\beta)\left\{a(1-b)[6+4\beta-b\beta-2b^2-b^2\beta] + (2-b^2-b\beta)c\right\} \end{array}\right\}$$

$$q_B = \frac{(a-c)v}{(1+b)\Delta_B}\left\{v(4-b^2)^2(1-b^2) - 2(2-b^2-b\beta)^2\right\} \tag{25}$$

Finally, we determine the equilibrium consumer surplus, producer profits, and welfare in Bertrand competition:

$$\begin{aligned} CS_B &= (1+b)(q^B)^2 = \frac{(a-c)^2v^2}{(1+b)\Delta_B^2}\left\{v(4-b^2)^2(1-b^2) - 2(2-b^2-b\beta)^2\right\}^2, \\ \Pi_B &= 2(1-b^2)(q_B)^2 - v(x_B)^2, \\ W_B &= CS_B + \Pi_B. \end{aligned}$$

The following remark occurs in equilibrium.

Remark 3 Using (23) and (24), it is evident that $\alpha_B > a$ and $x_B > x_{nb}$. Consequently, in equilibrium, both firms choose overoptimistic managers, and they over-invest in R&D. □

Using a similar approach in Cournot competition, we show that the degree of product differentiation is negatively correlated to the hiring of overconfident managers (i.e., $\frac{\partial \alpha_B}{\partial b} < 0$). Consistent with Cournot, product substitutability between products induce managers to increase R&D investments. In fact:

$$\frac{\partial x_i^*(\alpha_i, \alpha_j)}{\partial b} = \underbrace{\frac{\partial x_i^*(\alpha_i, \alpha_j)}{\partial \alpha_i}}_{+} \underbrace{\frac{\partial \alpha_i}{\partial b}}_{-} + \underbrace{\frac{\partial x_i^*(\alpha_i, \alpha_j)}{\partial \alpha_j}}_{+} \underbrace{\frac{\partial \alpha_j}{\partial b}}_{-} < 0 \text{ if } \beta > \frac{b}{2-b^2}.$$

Furthermore, note that:

$$\frac{\partial \alpha_B}{\partial c} = -\frac{2(2-b)(2\beta - b^2\beta - b)^2 v}{\Delta_B}, \text{ and } \frac{\partial x_B}{\partial c} = -\frac{2(1-b)(2-b^2-b\beta)}{\Delta_B}\left\{(4-b^2)v - 2(1-\beta^2)\right\}.$$

Given **A1** and **A2**, we have $\frac{\partial \alpha_B}{\partial c} < 0$ and $\frac{\partial x_B}{\partial c} < 0$. Therefore, the equilibrium level of managers' overconfidence and R&D investment levels decrease with initial production cost.

Remark 4 In case of no delegation in stage 1, the producer surplus is $\Pi_{nb} = \frac{2(a-c)^2 v}{\Phi^2}[v(4-b^2)^2(1-b^2) - 2(2-b^2-b\beta)^2]$, the consumer surplus is $CS_{nb} = \frac{(a-c)^2 v^2}{\Phi^2}(1+b)(4-b^2)^2$, and welfare is $W_{nb} = \frac{(a-c)^2 v}{\Phi^2}[v(3+b-2b^2)(4-b^2)^2 - 4(2-b^2-b\beta)^2]$. □

By comparing market equilibrium outcomes under Bertrand competition with possible overconfident managers with those obtained in a game without delegation (Remark 4), we get the proposition hereunder.

Proposition 2 *For any $b \in (0,1)$ and $\beta \in (0,1)$, with $\beta \neq \frac{b}{2-b^2}$.*

(a) $p_B < p_{nb}$, and $q_B > q_{nb}$. Consequently, $CS_B > CS_{nb}$.

(b) $\Pi_B > \Pi_{nb}$ *and* $W_B > W_{nb}$ *under* ***A2****.* □

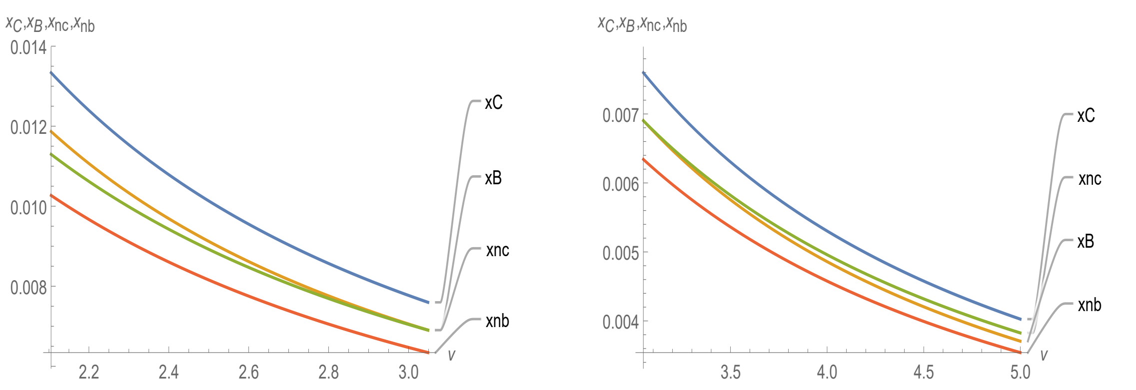
Proposition 2 states that firms hire overconfident managers, and they make decisions that induce higher productions, lower prices, and higher consumer surplus. Moreover, if the extend of spillover effects is sufficiently strong, then, delegation and overconfidence increase profits and welfare. In what follows, we compare the equilibrium outcomes under Cournot and Bertrand.

Proposition 3 *Suppose* ***A1*** *and* ***A2*** *hold.*

(a) $\alpha_C > \alpha_B$.

(b) There exists a unique $v_2 = v_2(b, \beta)$ *such that* $\begin{cases} x_C > x_B > x_{nc} > x_{nb} & if \quad v < v_2 \\ x_C > x_{nc} > x_B > x_{nb} & if \quad v > v_2 \end{cases}$. □

In Proposition 3, we show that the degree of overconfidence and the size of over-investment are higher in Cournot competition. Figure 3 illustrates R&D investment levels comparison for parameters: $a = 1$, $c = 0.95$, $b = 0.5$, $\beta = 0.86$, and $v_2 = 3.04891 > 2.10526 = \frac{2a}{c}$, with $\beta > 0.285714 = \frac{b}{2-b^2}$.



(a) $x_C > x_B > x_{nc} > x_{nb}$ and $v < v_2$ (b) $x_C > x_{nc} > x_B > x_{nb}$ and $v > v_2$

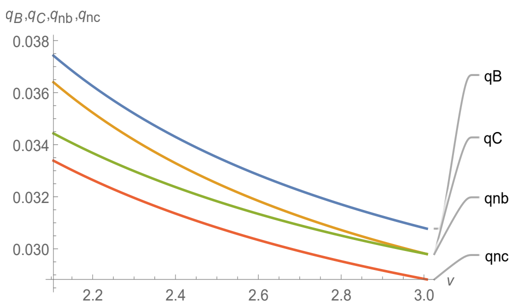
Figure 3: Comparison of R&D investment levels

Note that Qiu (1997) shows that without delegation, $x_{nc} > x_{nb}$. We find that our model with delegation leads to intermediate comparison between x_B and x_{nc}. The next result compares quantity, price, and consumer welfare.

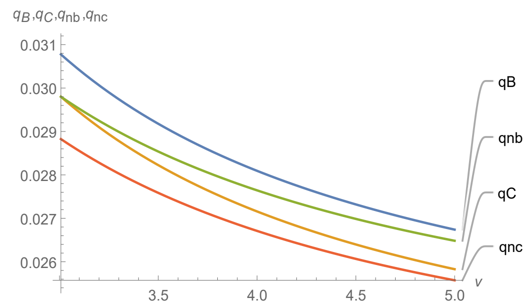
Proposition 4 *Suppose* ***A1*** *and* ***A2*** *hold. There exists a unique* $v_3 = v_3(b, \beta)$*:*

$$\begin{cases} q_B > q_C > q_{nb} > q_{nc} \text{ and } p_B < p_C < p_{nb} < p_{nc} & if \quad v < v_3 \\ q_B > q_{nb} > q_C > q_{nc} \text{ and } p_B < p_{nb} < p_C < p_{nc} & if \quad v > v_3 \end{cases} \square$$

From Proposition 4, it is immediate that $q_B > q_C$ and $p_B < p_C$ whatever the productivity of the R&D technology. Therefore, consumer welfare is higher under Bertrand competition, i.e., $CS_B > CS_C$. Figures 4 and 5 illustrate equilibrium prices and quantities for parameters: $a = 1$, $c = 0.95$, $b = 0.288$, $\beta = 0.86$, and $v_3 = 3.00703 > 2.10526 = \frac{2a}{c}$, with $\beta > 0.15023 = \frac{b}{2-b^2}$. Proposition 4 shows that delegation makes competition stronger and more aggressive under Bertrand and less aggressive under Cournot competition.

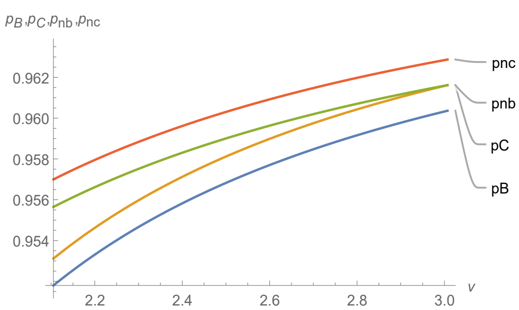


(a) $q_B > q_C > q_{nb} > q_{nc}$ and $v < v_3$

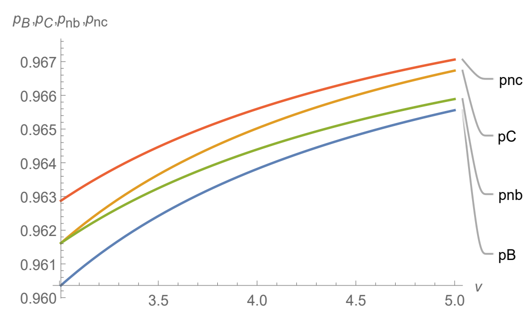


(b) $p_B < p_C < p_{nb} < p_{nc}$ and $v < v_3$

Figure 4: Comparison of equilibrium quantities



(a) $q_B > q_{nb} > q_C > q_{nc}$ and $v > v_3$



(b) $p_B < p_{nb} < p_C < p_{nc}$ and $v > v_3$

Figure 5: Comparison of equilibrium prices

4. The Four-stage game

In this section, we suppose that each firm can make only two types of binding contracts with shareholders: the delegation contract (D) and the non-delegation contract (ND). If firm i chooses the delegation contract, this means that it will have to delegate the R&D investments and production decisions to a manager whatever action the competitor, firm j, $j \neq i$, takes. If firm i chooses the non-delegation contract, it commits to making its strategic decisions without hiring a manager independently of the action of the competitor. Consider a four-stage game where firms first simultaneously commit themselves to a type of contract and afterward compete depending upon the chosen types of arrangements and the three-stage game under quantity competition described in section 2. Restricting attention to sub-game perfect equilibria of the four-stage game, we show that it is a dominant strategy for firm i to choose the delegation contract and hire an overconfident manager.

Let consider firm i, $i = 1, 2$. In the first stage, if both firms choose the delegation contract, D, then both firms enjoy the Cournot profits Π_C. If they decide to choose the non-delegation contract, then each firm receives its profit Π_{nc}. Denote Π^i_{C-ND} firm i's profit when i chooses non-delegation contract, and the competition firm j chooses the delegation contract, and Π^i_{C-D} firm i's profit when i chooses the delegation contract and the competition firm j chooses the non-delegation contract. In the first stage, the firm i, $i = 1, 2$, faces the payoff matrix described in Figure 6.

		Firm j	
		D	ND
Firm i	D	Π_C	Π^i_{C-D}
	ND	Π^i_{C-ND}	Π_{nc}

Figure 6: Payoff Matrix (Four-stage game)

When firms choose different strategies in the first stage, the firm that commits to delegate its investment and production decisions chooses the equilibrium level of confidence:

$$\alpha_D = a + \frac{2(2+b)(b-2\beta)^2(a-c)v}{\Delta_D}\left\{(2-b)^2(2+b)v - 2(1-\beta)(2-b\beta)\right\}$$

with $\Delta_D = -8(2-b\beta)^2(1-\beta^2)^2 + 4(2-b\beta)^2(12-4b\beta-8\beta^2-b^2(2-3\beta^2))v - 6(4-b^2)^2(2-b\beta)^2v^2 + (4-b^2)^4v^3$. Note that under **A1**, $\Delta_D > 0$, and $\alpha_D > a$. After computation, we report that:

$$\Pi^i_{C-D} = \frac{(a-c)^2v}{\Delta_D}\left\{(2-b)^2(2+b)v - 2(1-\beta)(2-b\beta)\right\}^2, \text{ and}$$

$$\Pi^i_{C-ND} = \frac{v(a-c)^2(v(4-b^2)^2 - 2(2-b\beta)^2)}{\Delta_D^2}\left\{\begin{array}{l} 4(1-\beta)^2(1+\beta)(2-b\beta) \\ -2(2-b)(2-b\beta)(4+b-2(1+b)\beta)v \\ +(2-b)^3(2+b)^2v^2 \end{array}\right\}^2.$$

In appendix, we show that, under **A1**, $\Pi_C > \Pi^i_{C-ND}$, and $\Pi^i_{C-D} > \Pi_{nc}$, for $i = 1, 2$. It follows that it is a dominant strategy for firms to choose delegation, and therefore, hire overconfident managers. Proposition 5 states the result.

Proposition 5 *In the four-stage game, it is a dominant strategy for form i to choose the delegation contract.* □

Note that Proposition 5 also holds when firms compete under price competition after the first stage. Our result is similar to the finding by Kyle and Wang (1997). In a standard Cournot duopoly model of informed speculation with two traders, Kyle and Wang (1997) show that overconfidence can persist and survive in the long run since it may strictly dominate rationality. We provide a similar conclusion in product-market competition when firms commit to a specific corporate governance before the market competition. In a competitive framework, information about demand is a crucial advantage for profit-maximizing firms. Given that delegation may provide more information about markets to firms or reduce uncertainty, then, delegating strategic decisions to managers could be an optimal managerial decision for firms (e.g., Bhardwaj (2001), Acemoglu et al. (2007), Mackey (2008), Bloom et al. (2012), Liozu and Hinterhuber (2013), Kala (2019)). In our setting, we assume that managers and firms' shareholders have the same interests. However, in other frameworks, managers might be at odds with the principal (or firm's owner). For instance, the manager can use his or her superior informational advantage to make decisions against the objectives of the principal. In that case, delegation in a competitive market might not be an optimal strategy for firms (e.g., Acemoglu et al. (2007), Alonso et al. (2008), Ruzzier (2018)).

5. Conclusion

In this paper, we provide a framework to examine the relationship between personal characteristics of managers in firms and welfare in product markets. For strategic reasons, firms decide to delegate innovation and production decisions to managers who have over-estimate demands in a differentiated duopoly with R&D competition. We show that overconfident managers over-invest in cost-reducing R&D investments and produce more output in markets. The size of over-investment and over-production is large when firms compete in quantities than in prices. If R&D productivity is low, and spillovers are strong, our equilibrium outcomes induce higher welfare relative to competition modes with no delegation.

Appendix A. Proof of Results

PROOF (PROPOSITION 1) (a) Comparing quantities give

$$q_C = \left\{ 1 + \frac{4(2\beta - b)^2(1+\beta)(2-b\beta)}{(4-b^2)\Delta_C} \right\} q_{nc}.$$

Given that $\beta \neq \frac{b}{2}$, it is immediate that $q_C > q_{nc}$. Given that $p_C - p_{nc} = -(1+b)(q_C - q_{nc})$ and $CS_C - CS_{nc} = (1+b)(q_C^2 - q_{nc}^2)$, it follows that $p_C < p_{nc}$ and $CS_C > CS_{nc}$.

(b) With the help of Wolfram Research Inc. (2019), we can write

$$\Pi_C - \Pi_{nc} = \frac{16(a-c)^2(2-b\beta)(2\beta-b)^3v^2}{\Delta_C^2\ \Delta^2} f(b,\beta,v),$$

where $f(b,\beta,v) = f_0(b,\beta) + f_1(b,\beta)v + f_2(b,\beta)v^2$, with $f_0(b,\beta) = 2(1+\beta)^2(2-b\beta)(4+b-2(1+b)\beta)$, $f_1(b,\beta) = -(2+b)^2(2-b\beta)(8+6\beta-b(3+4\beta))$, and $f_2(b,\beta) = (2-b)^2(2+b)^4 > 0$.

The function $f(b,\beta,.)$ is convex in v, and there are two real solutions v_1^f and v_2^f (with $v_1^f < v_2^f$) to equation $f(b,\beta,v) = 0$ for any b and β. By definition, $f(b,\beta,v) > 0$ for $v > v_2^f$. Given that any v that satisfies **A1** is such that $v > v_2^f$, it follows that for any b and β, $f > 0$, and sign $(\Pi_C - \Pi_{nc}) =$ sign $(2\beta - b)$. It is immediate that $\Pi_C > \Pi_{nc}$ if and only if $\beta > \frac{b}{2}$.

(c) In the same manner, as in (b), we can write

$$W_C - W_{nc} = \frac{8(2-b\beta)(2\beta-b)^2(a-c)^2v^2}{\Delta_C^2\ \Delta^2} g(b,\beta,v),$$

where $g(b,\beta,v) = g_0(b,\beta) + g_1(b,\beta)v + g_2(b,\beta)v^2$, with $g_0(b,\beta) = 2(1+\beta)^2(2-b\beta)(8-b^2(3+b)+16\beta+4b\beta-2(1+b)(6-b^2)\beta^2) < 0$, $g_1(b,\beta) = 2(2+b)^2(-2+b\beta)(8+b(-6+(-2+b)b)+28\beta+(-3+b)b(4+3b)\beta+2(8+b(-5+(-2+b)b))\beta^2) > 0$, and $g_2(b,\beta) = (2-b)^2(2+b)^5(1-b+(3-b)\beta) < 0$.

Note that sign $(W_C - W_{nc}) =$ sign $(g(b,\beta,v))$. The function $g(b,\beta,.)$ is convex in v, and there are two real solutions v_1^g and v_2^g (with $v_1^g < v_2^g$) to equation $g(b,\beta,v) = 0$ for any b and β. Any v that satisfies **A1** is such that $v > v_1^g$. Since $g(b,\beta,.)$ is convex, $g(b,\beta,v) < 0$ if $v_1^g < v < v_2^g$ and $g(b,\beta,v) > 0$ if $v > v_2^g$. Therefore, $W_C < W_{nc}$ if $v < v_2^g$ and $W_C > W_{nc}$ if $v > v_2^g$. Consider $v_1 = v_2^g(b,\beta)$. ■

PROOF (PROPOSITION 2) (a) Comparing prices gives:

$$\frac{p_B}{p_{nb}} - 1 = -\frac{(a-c)v}{\Delta_B} \frac{4(1+\beta)(2\beta-b^2\beta-b)^2(2-b^2-b\beta)}{v(4-b^2)(1+b)(a(1-b)+c)-a(1+\beta)(2-b^2-b\beta)}.$$

Under **A1**, $v(4-b^2)(1+b)(a(1-b)+c) - a(1+\beta)(2-b^2-b\beta) > 0$ and $\Phi > 0$. For any b and β, it is evident that $2-b^2-b\beta > 0$. For any b and β, with $\beta \neq \frac{b}{2-b^2}$, we have $2-b^2-b\beta > 0$ and $(2\beta - b^2\beta - b)^2 > 0$ so that $\frac{p_B}{p_{nb}} - 1 < 0$. With $p_{nb} > 0$, we can conclude that $p_B < p_{nb}$. From market demands, $q_B - q_{nb} = -\frac{1}{1+b}(p_B - p_{nb})$, and $CS_B - CS_{nb} = (1+b)(q_B^2 - q_{nb}^2)$. Consequently, $q_B > q_{nb}$, and $CS_B > CS_{nb}$.

(b) We can write

$$\Pi_B - \Pi_{nb} = \frac{8(a-c)^2 v^2((2-b^2)\beta - b)^3(2-b^2-b\beta)}{(1+b)\Phi^2\ \Delta_B^2} h(b,\beta,v),$$

where $h(b,\beta,v) = h_0(b,\beta) + h_1(b,\beta)v + h_2(b,\beta)v^2$, with $h_0(b,\beta) = 2(1-b)(1+\beta)^2(4+b-b^2(2-\beta)-2\beta-2b\beta)(2-b^2-b\beta) > 0$, $h_1(b,\beta) = -(2-b)^2(1+b)(2-b^2-b\beta)(8+6\beta-b(3+4\beta+b(4+3\beta))) < 0$, and $h_2(b,\beta) = (2-b)^4(1-b)(1+b)^2(2+b)^2 > 0$.

The function $h(b,\beta,.)$ is convex in v, and there are two real solutions v_1^h and v_2^h to equation $h(b,\beta,v) = 0$ for any b and β. The comparison between v_1^h and v_2^h depends on the difference between β and $\frac{b}{2-b^2}$. Also, sign $(\Pi_B - \Pi_{nb}) =$ sign $[(2-b^2)\beta - b)h(b,\beta,v)]$. Under **A2**, we have $\beta > \frac{b}{2-b^2}$. It follows that $(2-b^2)\beta - b > 0$, and $v_1^h > v_2^h$. Under **A1**, any v is such that $v > v_1^h$. By definition, $h(b,\beta,v) > 0$ for any $v > v_1^h$ so that $\Pi_B - \Pi_{nb}$ is positive and $\Pi_B > \Pi_{nb}$. Thus $W_B > W_{nb}$ when $\beta > \frac{b}{2-b^2}$ under **A2**. ■

PROOF (PROPOSITION 3) (a) We have

$$\alpha_B - \alpha_C = \frac{1}{\Delta_C \Delta_B}\left\{I(b,\beta,v)\Delta_C - J(b,\beta,v)\Delta_B\right\},$$

where $I(b,\beta,v) = a(1-b)[(4-b^2)v - 2(1-\beta^2)][(2-b)^2(1+b)(2+b)v - 2(1+\beta)(2+-b^2-b\beta)] - 2(2-b)(2\beta - b^2\beta - b)^2 cv$, and $J(b,\beta,v) = a[v(4-b^2) - 2(1-\beta^2)][(2-b)(2+b)^2 v - 2(1+\beta)(2-b\beta)] - 2(2+b)(2\beta - b)^2 cv$. Under **A1** and **A2**, the expression $I(b,\beta,v)\Delta_C - J(b,\beta,v)\Delta_B < 0$. Therefore $\alpha_C > \alpha_B$.

(b)

$$\frac{x_C}{x_B}-1=\frac{b^3v}{(1-b)(2-b^2-b\beta)\Delta_C}[v(1-b)(4-b^2)^2(1+\beta)-2(1-\beta)(2-b\beta)(2-b^2-b\beta)].$$

Given **A1** and **A2**, it is immediate that $v(1-b)(4-b^2)^2(1+\beta)-2(1-\beta)(2-b\beta)(2-b^2-b\beta)>0$. Then, $x_C>x_B$. It is also straightforward to verify that $x_{nc}>x_{nb}$ under the same conditions.

We also write

$$\frac{x_B}{x_{nc}}-1=\frac{v(2-b)}{(2-b\beta)\Delta_B}k(b,\beta,v),$$

where $k(b,\beta,v)=k_0(b,\beta)+vk_1(b,\beta)$, with $k_0(b,\beta)=4b^3-2b^5+10b^3\beta-2b^5\beta+16\beta^2+2b^5\beta^2+4b^3(-1+\beta)\beta^2-2b^4(1+\beta)+2b^4\beta^2(1+\beta)+b^2(4-8\beta^2)-8b\beta(2+\beta^2)>0$, and $k_1(b,\beta)=-8b^3+6b^5-8b^3\beta+6b^5\beta+4b^4(1+\beta)-b^6(1+\beta)-(b^7)(1+\beta)<0$.

Note that sign $(x_B-x_{nc})=$ sign $k(b,\beta,v)$. Let denote $v^k=-\frac{k_0}{k_1}$. Given any b and β, $k(b,\beta,v)>0$ if and only if $v<v^k$. We conclude that $x_B>x_{nc}$ if $v<v^k$, and the result follows. Take $v_2=v^k$. ■

Proof (Proposition 4) (a) We write

$$q_B-q_C=\frac{(a-c)v}{(1+b)\Delta_C\Delta_B}L(b,\beta,v),$$

where $L(b,\beta,v)=[v(4-b^2)^2(1-b^2)-2(2-b^2-b\beta)^2]\Delta_C-(1+b)[v(4-b^2)^2-2(2-b\beta)^2]\Delta_B$. Under both **A1** and **A2**, $L(b,\beta,v)$ is positive for any v, b and β so that $q_B-q_C>0$, i.e., $q_B>q_C$. Therefore, given that $p_B-p_C=-(1+b)(q_B-q_C)$, it follows that $p_B<p_C$, and $CS_B>CS_C$.

$$q_C-q_{nb}=\frac{(a-c)v}{\Phi\Delta_C}M(b,\beta,v),$$ ■

where $L(b,\beta,v)=m_0(b,\beta)+m_1(b,\beta)v+m_2(b,\beta)v^2$, with $m_0(b,\beta)=-8b^2+12b^3\beta+12b^3\beta^2-4b^4\beta^2-4b^4\beta^3+32\beta^2(1+\beta)-16b\beta(1+\beta)(2+\beta^2)+8b^2\beta(-1+2\beta(1+\beta))$,

$m_1(b,\beta) = 64b^2 - 24b^4 + 2b^6 + 32b^2\beta - 32b^3\beta - 16b^4\beta + 8b^5\beta + 2b^6\beta + 8b^4\beta^2 - 2b^6\beta^2$, and $m_2(b,\beta) = -64b^2 + 48b^4 - 12b^6 + b^8 < 0$. The function $M(b,\beta,.)$ is a concave function in v, and there are two real solutions v_1^m and v_2^m (with $v_2^m > v_1^m$) to equation $M(b,\beta,v) = 0$ for any b and β. Under **A1**, any v is such that $v > v_1^m$. By definition $M(b,\beta,v)$ is positive if $v < v_2^m$, and $M(b,\beta,v)$ is negative if $v > v_2^m$. Consider $v_3 = v_2^m$, and the inequalities follow.

(c) $\Pi_C - \Pi_B = \frac{(a-c)^2 v}{\Delta_C^2 \Delta_B^2} \{N_1(b,\beta)\Delta_B^2 - N_2(b,\beta)\Delta_C^2\} > 0$, and $W_B - W_C > 0$.

Proof (Proposition 5)

$$\Pi_C - \Pi_{C-ND}^i = \frac{v(a-c)^2}{\Delta_C^2 \Delta_D^2} O(b,\beta,v),$$

where $O(b,\beta,v) = [-2(2-b\beta)^2(2-2\beta^2-(4-b^2)v)^2 + v(-2(2-b\beta)^2 + (4-b^2)^2 v)^2]\Delta_D^2 - [(-2(2-b\beta)^2 + (4-b^2)^2 v)(4(1-\beta^2)(1-\beta)(-2+b\beta) + 2(2-b)(2-b\beta)(4+b-2(1+b)\beta)v - (2-b)^3(2+b)^2 v^2)^2]\Delta_C^2$. Under **A1**, $O(b,\beta,v) > 0$, for all b and β. Therefore, $\Pi_C > \Pi_{C-ND}^i$.

Similarly,

$$\Pi_{C-D}^i - \Pi_{nc} = \frac{v(a-c)^2}{\Delta^2 \Delta_D^2} 8(b-2\beta)^4(2-b\beta)^2 v. \qquad \blacksquare$$

Assuming $\beta \neq \frac{b}{2}$, it is evident that $\Pi_{C-D}^i > \Pi_{nc}$.

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