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Supermajority Politics: Equilibrium Range, Diversity, and Compromise*

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Abstract

Do legislative voting rules affect the diversity of policies observed across structurally similar political economies, and if so, to what extent? To what degree do these rules affect legislative compromise and the stability of the social optimum? Using a spatial model of political competition with single-peaked preferences, we examine these questions in *static* and *dynamic* political economies where changing proposed policies requires supermajority consensus. We develop three findings pertaining to *equilibrium* policies that are immune to change by any supermajority coalition. First, we find the number of equilibrium policies that exist as a function of the supermajority's size. This result implies that under supermajority rules, structurally identical political economies may implement very different policies. Second, we find the optimal level of compromise needed by a leader to ensure that her proposed policy is not defeated and establish that compromise decreases in the supermajority's size. Third, we identify the minimal supermajority rule that ensures the stability of the social optimum. We derive implications for political design, policy diversity, and

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political inclusiveness.

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1. Introduction

Voting bodies in international organizations, countries, and American states rely on various supermajority rules (Alemán and Tsebelis, 2005; Krupnikov and Shipan, 2012; Metcalf, 2000; Posner and Sykes, 2014), but the consequences of these differences remain under-explored. In this paper, we focus on *policy diversity*, the range or number of different equilibrium policies produced in a *political economy*. We also examine how supermajority rules affect political compromise and the stability of the social optimum. More precisely, we answer the following questions:

1. To what extent do supermajority rules affect the diversity of policy outcomes across *structurally similar* legislative bodies?

2. How much should a political party or a leader proposing a policy compromise to ensure that it does not suffer defeat in a pairwise competition under supermajority rule?

3. How do supermajority rules affect the stability of the social optimum?

To investigate these questions, we use a spatial model of political competition between an incumbent policy and an alternative. We consider two different decision-making environments, one being *static* and the other being *dynamic*. For adoption, the alternative must obtain the support of a supermajority of voters who, by assumption, hold single-peaked preferences over a totally ordered policy space, which need not be a uni-dimensional set.[4] In the *static* setup, we focus on the set of equilibrium policies in the *core* (Edgeworth, 1881; Gillies, 1959).[5] *Equilibrium policies*

[4] As discussed in section 2, we do not impose any other structure on the policy space other than it being totally ordered. It therefore belongs to a very large class of sets, including multi-dimensional sets that could be finite or infinite. This precision has important empirical implications, as our findings can be used to understand policy diversity and political compromise in a very wide range of institutional contexts.

[5] The *core* is a popular solution concept for one-shot games (see, e.g., Black (1948), Debreu and Scarf (1963), Boehm (1974), Banks and Duggan (2000), Austen-Smith and Banks (2000, 2005), among others). However, all

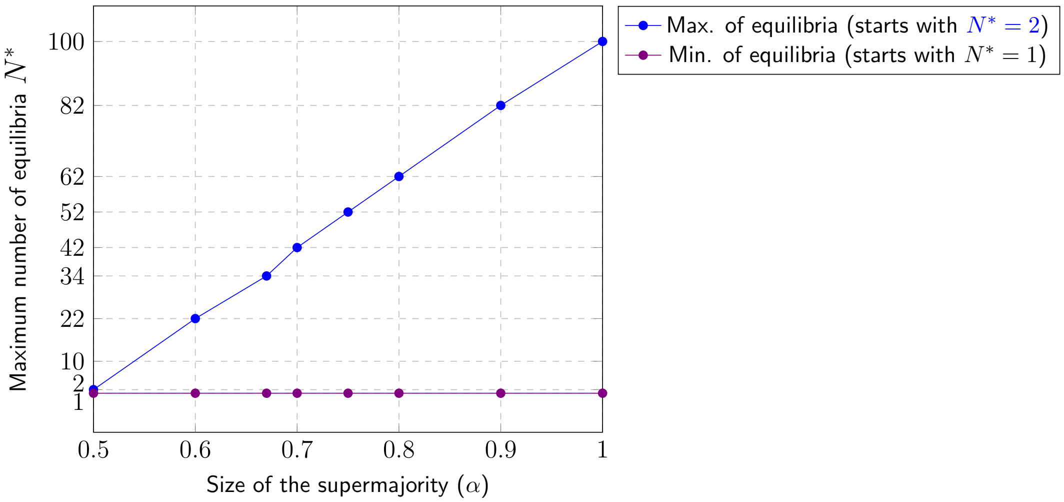


Figure 1: Number of equilibria and the size of supermajority rule in a voting body of 100 agents

are those which, if already the status quo, are never defeated in a pairwise supermajoritarian election against alternatives in the policy space. In the *dynamic* setup, legislators make amendments sequentially and the game can go on indefinitely. In this setting, we focus on the set of equilibrium policies in the *largest consistent set* (Chwe, 1994). This solution concept assumes that legislators are farsighted.

Our main theoretical contributions are three-fold. First, we find the *maximum number* of equilibria as a function of the supermajority needed to pass legislation and as a function of the way the incumbent policy is selected. This number represents the depth of policy diversity across structurally *identical* political economies under supermajority rules. More precisely, if Nature randomly selects a legislator to make a proposal, the maximum number of policies is finite (theorem 1).[6] As shown in figure 1, the maximum number of equilibrium policies is a linear and increasing function of the supermajority needed to replace them. One corollary of this result is the existence of a unique equilibrium under majority rule when there are an odd number of voters and the existence of, at most, two equilibria when there are an even number of voters. The equilibria with an odd and even number of voters are, respectively, the median of the distribution

our results are highly robust to alternative solution concepts (e.g., top cycle, the uncovered set, the Banks set, stationary equilibrium, etc.). Proofs are available upon request.

[6] The idea of selecting legislators and policymakers by a lottery system, also called "sortition", is old; it dates to the fourth century BC and is still practiced today (Manin, 1997; Wantchekon and Neeman, 2002; Procaccia, 2019).

of voters' ideal points and the set of ideal points closest to the median (corollary 1). The familiar Median Voter Theorem (MVT) (Black, 1948; Downs, 1957), then, is a special case of our result, which extends it to a more general setting. If Nature randomly selects the incumbent policy from the policy space, the set of equilibrium policies becomes a continuum—a convex and compact subset of the policy space—and we determine its exact bounds (theorem 2). These findings are robust, in that they continue to hold when legislators display farsighted behavior in the dynamic setting (theorem 3).

Counting the number of equilibria is useful because it can be used to determine the number of "competing" political parties in a given election. Duverger (1963) was among the first scholars to examine the relationship between electoral systems and the number of parties in a political economy. He proposed what are known today as the Duverger' law and the Duverger's hypothesis. The Duverger's law predicts that two major parties will form under the plurality rule (Duverger, 1963, 217), and the Duverger's hypothesis states that "the simple-majority system with second ballot and proportional representation favors multi-partyism" (Duverger, 1963, 239). Our analysis can be seen as a generalization of the Duverger's propositions to supermajority rules in static and dynamic models of political competition. We find, in particular, that simple majority rule favours a two-party system, which extends Duverger's law. However, this law fails under larger majority requirement: the equilibrium number of political parties increases linearly with the supermajority's size.

In deriving the maximum number of equilibrium policies in a supermajoritarian setting, the paper poses and answers a new question hitherto unaddressed in related spatial political competition literature. What is the relationship between a legislative body's voting rule and the diversity of policies that it can sustain? The answer follows from theorems 1, 2, and 3: policy diversity across structurally identical voting bodies increases in a nontrivial manner in the supermajoritarian threshold needed to amend them. When a simple majority can amend proposed policies, outcomes will depend exclusively on the preferences of the median voter. As the threshold to amend policies increases, outcomes will more closely resemble the ideal point of the policy proposer, increasing the number of stable policy outcomes. We precisely quantify this relationship.

By focusing on the *number* of equilibria in a model of spatial political competition, we clearly depart from the extant literature. This literature has primarily studied the question of equilibrium existence (see, e.g., Austen-Smith and Banks (2000, 2005) for a thorough overview of these findings), but has completely overlooked the issue of *number* of equilibria. Our analysis has empirical implications for the extent or the depth of *policy diversity* across structurally similar *political economies*.

It is well-known and quite intuitive that increasing the supermajority size creates a status quo bias and in turn potentially increases the number of equilibria (Hammond and Miller, 1987; Krehbiel, 1998; Tsebelis, 1995; McCarty, 2000). Our main contribution, however, is in exactly quantifying the maximum number and the range of equilibrium policies as a function of the supermajority size and of the way the status-quo policy is selected. To our knowledge, we are the first to do so. When Nature randomly chooses a voter to propose the status-quo policy, we find that the maximum number of equilibria is finite, regardless of whether the policy space is finite or infinite. However, when the status-quo policy itself is randomly chosen by Nature from the policy space, there is a set of equilibria that generally forms a continuum. We precisely determine the bounds of this set.

Second, we extend the model to consider the problem of political compromise. If a political party prefers an incumbent policy to alternatives, how much should it compromise to ensure that it does not suffer defeat in a pairwise competition? Under majority rule, the MVT suggests convergence toward the median voter's ideal point. Parties that are more ideologically distant from the median voter must make greater compromises to avoid defeat. We extend this insight to any supermajority rule and determine the *minimal level* of compromise that a political party should accept to become successful. This level, we find, decreases in the size of the supermajority needed to replace the incumbent policy (proposition 2). In other words, the greater the supermajority needed to adopt a new policy, the less the original policy's proposer or supporters must compromise to ensure that it is reenacted. An implication that follows directly from this result is that political compromise is maximal under the majority rule.

Viewing political compromise as a byproduct of bargaining between a proposer and the pivotal voter, our finding engages a large body of research on the executive veto as a tool to constrain congress (Kiewiet and McCubbins, 1988) and to strengthen the bargaining power of the executive

(McCarty and Poole, 1995; Cameron, 2000). Conceptually, a legislative override can be seen as a supermajority vote to overturn an executive's proposal to veto legislation. So far, the limited theoretical research on overrides and policy stability focuses on a narrow set of supermajoritarian rules (e.g., two-thirds, three-fifths) in the context of regionally specific institutions (Alemán and Schwartz, 2006; Schwartz, 1999; Hammond and Miller, 1987); or concentrates on institutional characteristics other than the supermajoritarian requirements (Tsebelis, 1995; Tsebelis et al., 2002).[7] By developing a model with fewer restrictions, we provide a more general framework for evaluating the role of the veto in legislative bargaining. We also develop a general theory that explains several political phenomena, including empirical relationships between legislative veto override thresholds and executive influence (McGrath et al., 2018) and factors motivating protests, in the United Nations Security Council, to revoke the five permanent members' veto power (United Nations General Assembly, 2018).

Finally, we study the relationship between supermajority rules and the stability of the social optimum—the policy that maximizes aggregate welfare. Unfortunately, the social optimum is generally unstable under majority rule. We determine the minimal supermajority rule that guarantees its stability. An implication of our analysis is that supermajority rules not only lead to policy diversity, but also protect the social optimum from defeat in a pairwise competition.

2. Preliminary Concepts

We consider a voting body $N = \{1, 2, ..., n\}$ composed of a finite number of agents and endowed with a political rule, $\mathcal{L}_\alpha$, and a non-empty *totally ordered* policy space, Z.[8] For a given finite set $X \subset Z$, we denote by $|X|$, the size of X. Possible policies are each represented by a point in Z—the number of points readily corresponds to the number of ideological approaches to a given policy problem—and agents' preferences are defined over Z. We briefly introduce some terms

[7] Tsebelis discusses policy instability resulting from multiple veto players in multi-dimensional policy space.

[8] We impose no structure on the policy space Z other than the fact that it is totally ordered. Consequently, Z may be finite or infinite, uni-dimensional or multi-dimensional or fractal. For example, Z could be the n-Euclidean space $\mathbb{R}^n$ (n greater than or equal to 1) ordered, say, by the lexicographic ordering or by the binary relation induced by any injective real-valued function on $\mathbb{R}^n$. Z may also belong to the class of multi-dimensional sets defined in Brown et al. (2014). For illustrative purposes, we occasionally use a line to represent Z, as this clearly conveys the intuition behind proofs or examples. Nonetheless, this restriction is *unnecessary* for our results to hold.

and notation.

A *political rule* is a distribution of political decision-making power among the various coalitions of agents eligible to vote in the country (for simplicity, we assume that each agent can vote). The political rule is formalized as a function $\mathcal{L}_\alpha$ which maps each subgroup C into 1 or 0. Given a threshold $\alpha \in [1/2, 1]$, $\mathcal{L}_\alpha(C) = 1$ when the coalition C consists of more than αn members. Such *winning coalitions* hold power to modify the status quo. Coalitions for which $\mathcal{L}_\alpha(C) = 0$, on the other hand, do not hold the right to amend policies. As an example of a political rule, $\mathcal{L}_{2/3}$ denotes the two-thirds majority rule, under which a coalition of agents is winning if and only if it comprises more than two-thirds of agents.

We assume that Z is totally ordered by a binary relation denoted $\geq$, and we denote by $>$ the strict part of this relation. We use $\succeq_i$ to denote agent i's preference relation over Z and $(\succeq_i)$ to denote a preference profile over Z. We assume that the profile $(\succeq_i)$ is single-peaked with respect to the strict order $>$ on Z. Following Austen-Smith and Banks (2000, 93–94), it means that each agent has an ideal policy in the policy space Z, and policies that are further from this ideal policy are preferred less. Formally, for each agent $i \in N$, there exists a policy $z_i \in Z$ such that: (1) for any other policy $z \neq z_i$, $z_i \succ_i z$; and (2) for any policy z, z', if $z > z' > z_i$, then $z' \succ_i z$, and, if $z_i > z > z'$, then $z \succ_i z'$.

Throughout the paper, we refer to these characteristics collectively as a *political economy*, or $\mathcal{P}$, defined as the list $(N, Z, (\succeq_i), \mathcal{L}_\alpha)$.

3. Number and Range of Equilibrium Policies

In this section, we examine the existence and the maximum number of equilibrium policies under one-shot political games (section 3.1) and infinite-horizon political games (section 3.2). To perform the analysis in one-shot games, we distinguish two cases: (i) an agent is randomly chosen to make a proposal (section 3.1.1); or (ii) the status quo policy is chosen by Nature (section 3.1.2).[9] Controlling for temporal and geographic factors that affect policymaker preferences and

[9]Neither case accounts for the path-dependency of policies and preferences (Pierson, 2000; Aklin and Urpelainen, 2013). While path-dependency undoubtedly shapes policy outcomes, we disregard it because our findings pertain to the number of stable policies rather than the policies themselves.

status-quo policies, what is the relationship between a legislative body's voting rule and policy stability?

3.1. **One-shot Political Games**

Political contests occur are as follows:

1. At time $t = 0$, a policy (or candidate) $z_0 \in Z$ is in place.
2. At time $t = 1$, a contest is organized between z_0 (the status quo) and an alternative z_1, chosen exogenously from the set $Z \setminus \{z_0\}$.
   a) If z_0 wins, meaning that no winning coalition under $\mathcal{L}_\alpha$ chooses z_1 over z_0, then it remains in place and the contest ends.
   b) If z_0 loses (z_1 wins), then z_1 replaces z_0 and the contest ends.

The rational behavior in the one-shot political game above is straightforward. Each agent chooses between the status quo policy and a political alternative. The incentive driving agents to vote for an opposition policy is the condition that it is preferable to the status quo. Formalizing this behavior, let $\gg_i$ denote the incentive by which agent i decides to support an opposition policy z_1 over the status quo z_0. If $z_1 \succ_i z_0$, agent i will vote for z_1 over z_0, denoted as $z_1 \gg_i z_0$. The policy z_1 wins the pairwise election if there exists a winning coalition C that supports z_1 over z_0 ($z_1 \gg_C z_0$). We can now introduce the equilibrium set, defined as follows.

Definition 1 Let $\mathcal{P}$ be a political economy and C be a winning coalition.

1. z' defeats z thanks to C (i.e., $z' \gg_C z$) if $z' \succ_C z$.
2. z is defeated if there exist a policy z' and a winning coalition C' such that z' defeats z thanks to C'.
3. The *equilibrium set* $\mathcal{E}(\mathcal{P})$ consists of all undefeated policies. □

An equilibrium policy is one that, if chosen as the status quo, could not be defeated or replaced by another policy. In a pairwise contest between two policies, say z and z', the former receives votes from agents whose ideal points are closer than z to z', and vice versa. Each agent's payoff depends on the distance between her ideal policy and the winning policy.

3.1.1. **Nature Randomly Selects a Proposer**

At time $t = 0$, an agent is randomly selected, by Nature or a lottery, to make a proposal. Agents are identical with equal probabilities of being selected. The proposer chooses the status quo. Given single-peakedness and rational behavior, each agent's best choice is to propose the closest equilibrium policy ideal to his ideal point as the status quo.[10] The following theorem demonstrates the existence of a policy that cannot be defeated in a pairwise supermajoritarian competition and provides the maximum number of policies that can be implemented.

Theorem 1 *Let $\mathcal{P}$ be a political economy. Assume that voters are chosen randomly with positive probability to make a proposal. If voters have single-peaked preferences over Z, then there exists at least one equilibrium and the number of equilibria is finite. Formally*[11]*:*

$$1 \leq |\mathcal{E}(\mathcal{P})| \leq (2\beta - 1)n, \ where \ \beta = \begin{cases} 1 & \textit{if } \alpha = 1 \\ \frac{\lfloor \alpha n \rfloor + 1}{n} & \textit{if } \alpha \in [\frac{1}{2}, 1) \end{cases}$$

The maximum number of equilibria in $\mathcal{P}$ is $(2\beta - 1)n$. □

We relegate the proofs of theorem 1 and all the subsequent results to the appendix. A corollary of the theorem 1 is the following result, which derives the size of the equilibrium set under majority rule and thus clarifies the way theorem 1 extends the MVT when the number of voters is even.

Corollary 1 *Let $\mathcal{P}$ be a political economy. Assume that voters are chosen randomly with positive probability to make a proposal. If preferences are single-peaked, and policies are chosen using majority rule, then,*

1. *There is only one equilibrium if the size of voters is odd.*
2. *There exists at least one and at most two equilibria if the size of voters is even.*

[10] One can trace a similar argument from the work of Downs (1957) and the seminal essay of Riker (1982) and the references therein. Even if we assume that the proposer is not rational and he or she proposes his or her ideal point as the status quo, our findings do not change. Throughout the paper, we assume that agents are rational in their decisions.

[11] $\lfloor x \rfloor$ is the largest integer less than or equal to x.

3.1.2. Nature Randomly Chooses a Status Quo

Next, suppose that Nature, rather than choosing the proposer in $t = 0$, instead chooses the status quo $z_0 \in Z$ from the set of all policies. Theorem 2 proves the existence of at least one and possibly an infinite number of equilibrium policies.

Theorem 2 *Let $\mathcal{P}$ be a political economy and assume that Nature randomly chooses the status quo. Let z_1^* and z_2^* denote, respectively, the minimal and the maximal equilibria when Nature randomly selects a proposer. Then, the equilibrium set is the interval $[z_1^*, z_2^*]$.*

Under majority rule, the equilibrium set described in theorem 2 exhibits an interesting property. When the number of voters is odd and $\alpha = 1/2$, $z_1^* = z_2^*$, and the set $[z_1^*, z_2^*]$ is a singleton, which is the ideal point of the median voter. If the number of voters is even, however, the set of equilibria may be infinite. In this sense, the theorem offers a more complete statement of the MVT compared to Black (1948).

3.2. Infinite-horizon Political Games

In infinite horizon games, contrary to one-shot games, agents (or coalitions) may vote indefinitely. Assume that, a status quo z_0 is randomly chosen from the set of policies. If no winning coalition replaces z_0, then it remains in place on an indefinite basis and the political opposition ends. If a winning coalition S replaces z_0, say with z_1, then z_1 becomes the new status quo, and the process restarts, continuing until a policy has been reached to which no winning coalition is willing to object. Once that policy has been reached, each agent earns and consumes his or her payoff and the political contest ends. We illustrate the predictions of such a game with the *largest consistent set* (Chwe, 1994), one of the prominent equilibrium concepts in infinite-horizon political games. We recall the definition of this equilibrium set below.

Definition 2 Let $\mathcal{P}$ be a political economy, X be a subset of Z, z and z' be two policies.

1. z' is said to *indirectly dominate* z, denoted as $z' \gg z$, if there exists a sequence of policies $z_0,\ z_1,\ \dots,\ z_m \in Z\ (where\ z_0 = z\ and\ z_m = z')$ and a sequence of winning coalitions $S_0,\ S_1,\ \dots, S_{m-1}$ such that $z_i \longrightarrow_{S_i} z_{i+1}$ and $z' \succ_{S_i} z_i$ for $i = 0, 1, \dots, m-1$. The relation $z \longrightarrow_S z'$ means that, if z is a status quo, S can make z' be the new status quo.

2. X is said to be *consistent* if $x \in X$ if and only if $\forall y, S$ (with $|S| > \alpha n$) such that $y \succ_S x$, there exists $z \in X$, where $y = z$ or $z \gg y$, and $not(x \succ_S z)$. The *largest consistent set*, denoted $LCS(\mathcal{P})$, is the union of all the consistent sets of $\mathcal{P}$. □

The largest consistent set formalizes the notion that a coalition that moves from a status quo to an alternative policy anticipates the possibility that another coalition might react. A third coalition might in turn react, and so on, without limit. It is therefore important to act in a way that does not lead a coalition to ultimately regret its action, i.e., coalitions are "*fully farsighted*" (Chwe, 1994, 300). We have the following result.

Theorem 3 *Let $\mathcal{P}$ be a political economy.*

1. *Assume that Nature randomly chooses voters with positive probability to make a proposal, preferences are single-peaked over Z, and $z \longrightarrow_S z'$ if and only if $|S| > \alpha n$. Then, $1 \leq |LCS(\mathcal{P})| \leq (2\beta - 1)n$.*

2. *Assume that Nature randomly selects the status quo, preferences are single-peaked over Z, and $z \longrightarrow_S z'$ if and only if $|S| > \alpha n$. Let z_1^* and z_2^* denote, respectively, the minimal and the maximal equilibria under one-shot games when Nature randomly selects a proposer. Then, the largest consistent set is the interval $[z_1^*, z_2^*]$.*

Remark that theorem 3 generalizes theorems 1 and 2 to infinite-horizon games.

4. Policy Diversity

Having presented theorems 1, 2, and 3, we now use them to explain policy diversity across identical political economies. We describe a simple illustration of these results in the context of ongoing debates over temporary immigration policies. The illustration demonstrates how two countries with identical political, economic, and cultural preferences over immigration resettlement could implement different policies. Throughout the paper, we use the term *policy diversity* to describe the range or number of equilibrium policies invulnerable to amendment under a given voting rule.

> **Example** The government of a country is developing a refugee resettlement program to help asylum seekers. How many refugees should obtain permanent residence? Suppose that

this decision fell to legislators representing the country's citizens and that the country derives utility from the number of refugees it admits. The utility can be in terms of the national and international "warm glow" that it receives (Andreoni, 1990; Facchini et al., 2006), or in terms of the skills or cultural diversity brought by the refugees (Cortes, 2004). Citizens have different perceptions of the utility that they, individually, or the country, overall, will receive from admitting refugees. We assume that these considerations are reflected in the legislators' utility functions. The net utility received by each agent i from z refugees being admitted is $V_i(z) = v_i \ln(z) - z/n$, where $1/n$ is the fraction of the total cost of refugee admission incurred by each constituency (assuming n constituencies), and v_i is legislator i's valuation of the number of refugees.

Suppose nine legislators ($n = 9$) collectively choose the number of refugees to be admitted using majority rule. Observe that V_i is single-peaked, and so voter i's optimum is obtained by solving $V'(z_i) = 0$, leading to the solution $z_i^* = 9v_i$. We assume that $v_i = i$, where $i = 1, 2, ..., 9$. Then, the legislators' optimum points are: $z_1^* = 9$, $z_2^* = 18$, $z_3^* = 27$, $z_4^* = 36$, $z_5^* = 45$, $z_6^* = 54$, $z_7^* = 63$, $z_8^* = 72$, and $z_9^* = 81$ (see also figure 2).

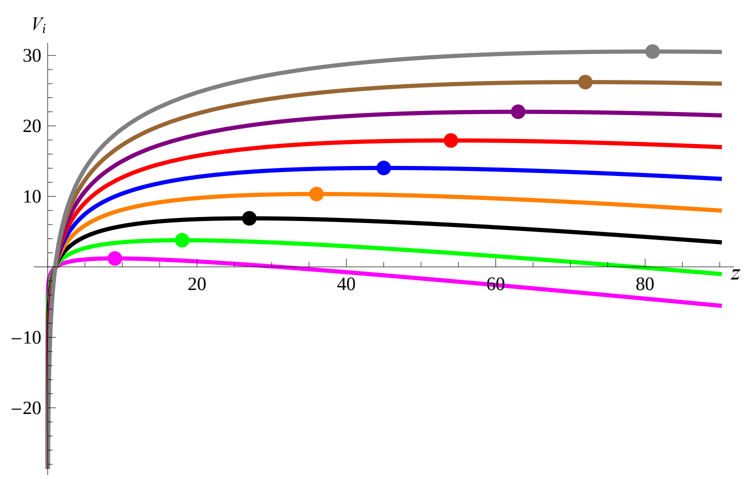


Figure 2: Legislators' utility functions. The bundle on each curve represents legislator i's optimum point.

The legislator with valuation $v_i = 5$ is the median voter. The optimal number $z_5^* = 45$ defeats all other optima in a pairwise competition, and becomes the only option which is not defeated. Therefore, under the majority rule, the country grants permanent residency to 45 refugees.

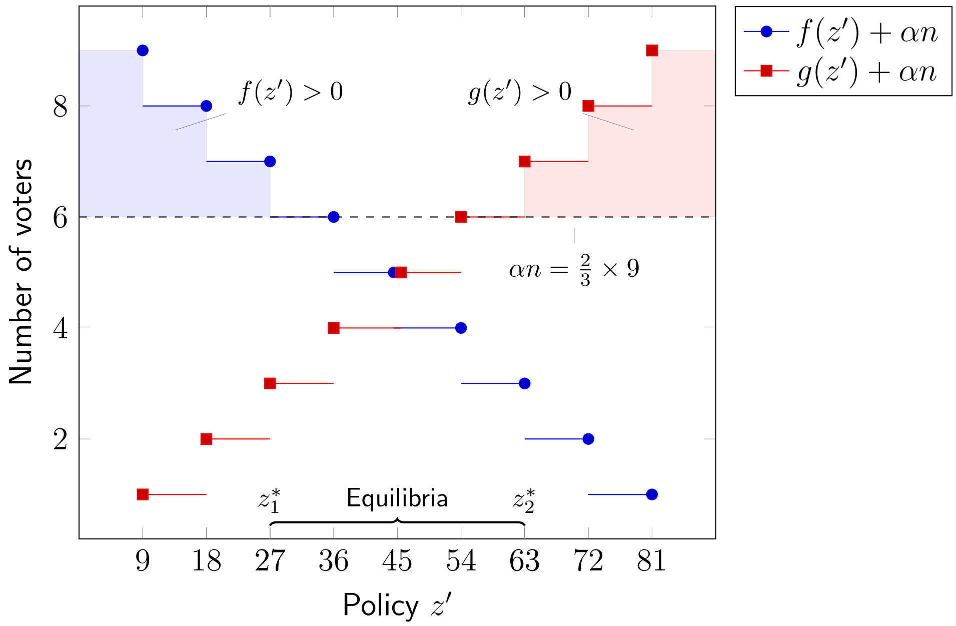


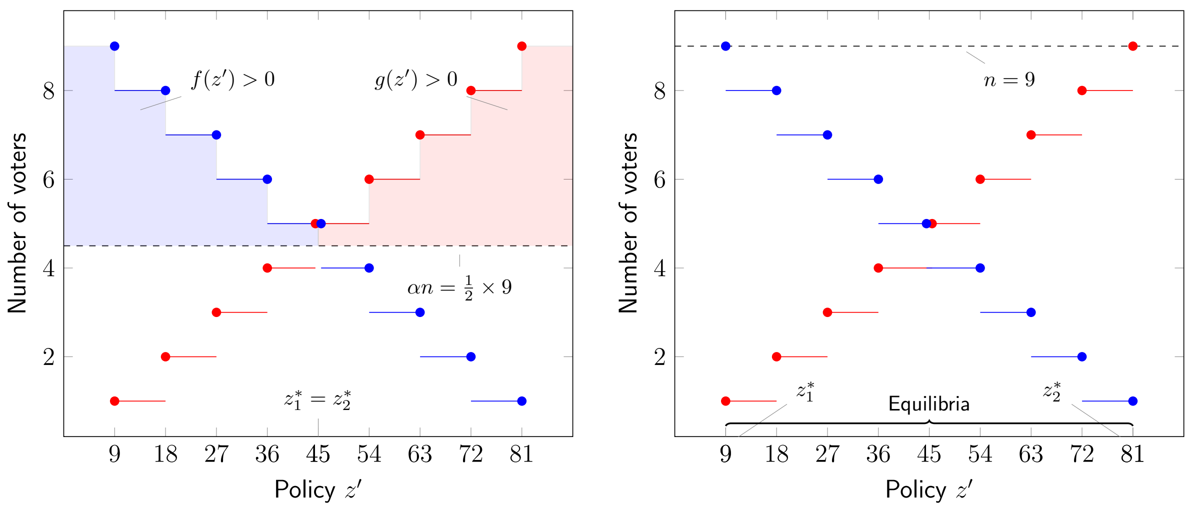
Figure 3: Illustration of equilibria for refugee resettlement program, assuming $\alpha = 2/3$. Using the notation from theorem 2, the number of legislators whose ideal points fall below or above a proposed alternative ideal point z' are given by $f(z') + \alpha n$ and $g(z') + \alpha n$. Equilibrium points are those between $z_1^* = 27$ and $z_2^* = 63$ inclusive.

> Next, suppose that the legislators choose the number of refugees using a two-thirds majority rule. As shown in figure 3, any proposal in the set $\{27, 36, 45, 54, 63\}$ cannot be defeated in a pairwise election, because all alternatives will fail to win support from the necessary supermajoritian coalition. These proposals are shielded from the possibility of amendment on the legislative floor. Moreover, any outcome in the set $\{9, 18, 72, 81\}$ can be defeated by either the option 27 or the option 63.

Any outcome from the set $\{27, 36, 45, 54, 63\}$ may be implemented if chosen as the status quo. In this respect, theorems 1 and 2 distinguish policy diversity from policy volatility, which describes fluctuations in policy over time. Higher supermajority thresholds to amend policy decrease volatility (Diermeier et al., 2015; Breunig and Koski, 2018) but increase diversity, as a wider array of status-quo policies withstand modification through amendment.

Intuitively, both theorems convey a generalized way to identify the number or range of stable policies based on voters' preferences and, more importantly, a legislative body's supermajoritarian rule. Figure 3 provides a helpful illustration. In a pairwise election, status-quo policies $z' < 27$ are defeated by $z_1^* = 27$ and status-quo policies $z' > 63$ by $z_2^* = 63$. The number of legislators who prefer z_1^* to $z' < z_1^*$ or z_2^* to $z' > z_2^*$ exceeds the supermajority threshold. Policies between the shaded region, however, would always defeat alternatives in a pairwise election.

What happens as the supermajoritarian threshold changes? For illustrative purposes, figure 4 compares the set of equilibria under majority rule and unanimity. Under the former, in figure 4a, the single equilibrium policy is the median $z' = 45$, which minimizes $f(z') - 9/2$ and $g(z') - 9/2$. Increasing the threshold to unanimity in figure 4b widens the range of equilibrium policies to encompass *any* policy between 9 and 81.



(a) Illustration of equilibria for refugee resettlement program, assuming $\alpha = 1/2$

(b) Illustration of equilibria for refugee resettlement program, assuming unanimity rule

Figure 4: Illustration of changes in supermajority threshold

A single voter's ability to block policy amendments under unanimity rule decreases the legislative body's ability to modify the status quo, and so any status quo within the range of voters' ideal points constitutes and equilibrium. Higher supermajoritarian thresholds thus increase status-quo proposal power, and under unanimity, this power is absolute. Observed policy will then reflect proposer preferences. To the extent that there is variation across proposers' ideal points, legislative bodies with high supermajority thresholds will also exhibit greater policy diversity than those with majority rule. In policy domains where the status-quo proposers have similar ideal points, there will be little observed policy diversity. When the proposer's ideal point falls within the set of equilibrium policies, the observed policy will be determined by the proposer; when it falls outside the set of equilibrium policies, then it will be amended to that of the pivotal legislator, whose ideal point is the equilibrium policy closest to that of the proposer. In other words, an increase in the supermajority threshold required to amend policy entrenches the status quo, and so — all else held equal — when proposer preferences vary, policy diversity will increase in an institution's supermajority threshold.

Example As an application, consider the question of how the erosion of the Senate filibuster starting in the twentieth century affected policy diversity. Between 1806 and 1917, the Senate lacked rules on when to end debates ("cloture"), and based on parliamentary rules and procedures, coalition-building typically required support from the median or veto-pivot necessary to override an executive veto (Wawro and Schickler, 2004). Under pressure from President Woodrow Wilson, the Senate passed filibuster reform in 1917, voting to invoke cloture with a $2/3$ supermajority. Following a series of filibusters designed to stall civil rights reforms, the Senate reduced the threshold to $3/5$. Finally, votes in 2013 and 2017 eliminated it entirely for executive and judicial nominations.[12] In allowing the senate to amend policies toward the median voter's preferences, these changes also restricted the range of equilibrium policies that could persist unamended. Over the 1940s-60s, liberal Democrats and Republicans sought to tighten the rules of cloture in opposition to southern Democrats and more conservative Republicans, whose preferred status-quo policies would not withstand lower supermajority thresholds (Wawro and Schickler, 2010).

Drawing comparisons across the branches of US government, the findings also predict greater diversity in policies passed by executive order than those passed by the legislature. In contrast to the three-fifths congressional majority necessary to invoke cloture, passing veto-proof legislation to invalidate or overturn an executive order requires two-thirds approval in House and Senate.[13] More broadly, the results also suggest greater constitutional diversity across countries with similar preferences and high supermajority thresholds. They also explain the diversity of procedures used by international organizations, even those that share member states (Posner and Sykes, 2014), and explain why efforts to revise outdated procedures are often stymied by unanimity rules (Lundgren

[12] In 2013, a Senate majority voted to invoke cloture with a simple majority during judicial and executive nominations aside from those to the Supreme Court; and in 2017, it voted to invoke cloture for all nominees with a simple majority vote.

[13] For clarity, we momentarily set aside the possibility of judicial review or presidential compromise.

et al., 2018).

5. Social Optimum and Equilibrium

In this section, we consider two questions:

1. Does there exist a political rule that guarantees the social optimum as an equilibrium in a pairwise opposition?
2. What is the minimal among such rules?

Before proceeding, we illustrate our purpose with a simple example on the provision of public goods (see, Varian (1992)).

Example Suppose that five agents vote to decide the level of public good provision. The current level of the public good is z, and agents vote to increase or decrease it. A voting equilibrium z^* is an amount that no majority prefers to increase or decrease. We assume that the agents all agree that if a majority votes in favor of an increase in the public good, each person will pay a fraction $1/5$ of the additional cost, and assume that all agents have quasi-linear utility functions. If z units of the public good are provided, agent i receives utility $u_i(z) - z/5$. The efficient level of public good (z_e) satisfies the condition $\sum_{i=1}^{n} u_i'(z_e) = 1$. We assume that the net utility of agent i takes the form:

$$U_i(z) = v_i \ln z + x_i - \frac{z}{5},$$

where the factor v_i represents agent i's valuation or taste of the public good, and x_i the amount of agent i's private good. The efficient level of public good is given by $z_e = \sum_{i=1}^{n} v_i$. It is straightforward to show that U is single-peaked. Agent i's optimum is obtained by solving $U'(z_i) = 0$. Agents' optima are therefore the sequence (z_i^*) such that: $z_i^* = 5v_i$ where $i = 1, 2, 3, 4, 5$.

Arbitrarily taking $v_1 = 6$, $v_2 = 2$, $v_3 = 3$, $v_4 = 4$ and $v_5 = 8$, agents' optima are $z_1^* = 30$, $z_2^* = 10$, $z_3^* = 15$, $z_4^* = 20$ and $z_5^* = 40$ (see figure 5) and the social optimum z_e is 23.

The voting equilibrium level of the public good depends on the size of majority required

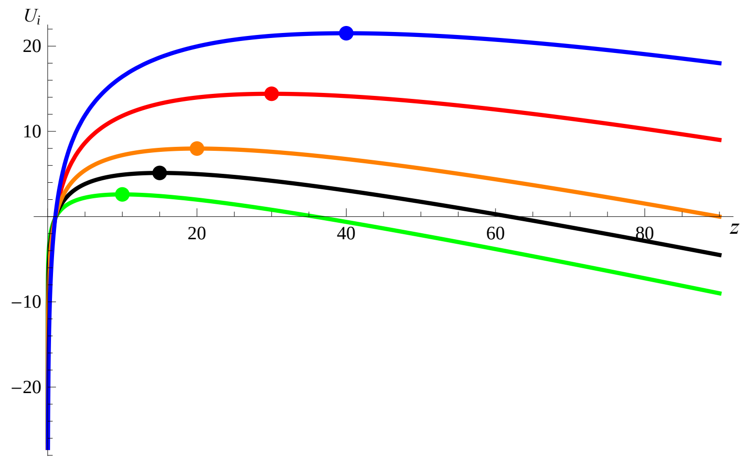


Figure 5: Agents' utility functions. The bundle on each curve represents agent i's optimum point.

to pass the decision. We observe that z_4^*, the optimum of the median policy, will defeat any other policy's peak in a pairwise majoritarian election under majority rule and so the only equilibrium outcome is 20. The social optimum z_e is not an equilibrium. However, if the decision requires a two-thirds majority, then the equilibrium range, given by the interval $[15, 30]$, includes the social outcome z_e.

Returning to our original questions—whether any rule guarantees a social optimum in equilibrium and, if so, which such rule requires smallest supermajority—we find the answers in proposition 1.

Proposition 1 *Let $\mathcal{P}$ be a political economy. Assume that preferences are single-peaked. Then, when Nature chooses the status quo, there always exists a supermajority voting rule with minimum size under which the social optimum z_e is an equilibrium. When Nature randomly selects a proposer, such a supermajority rule exists if and only if the social optimum coincides with the ideal point of a voter.*

Proposition 1 demonstrates that, because a supermajority rule can be chosen to ensure the stability of any range of policies, it can also ensure that the social optimum is stable. The proposition also shows that this positive result may not hold because individual rational behavior might conflict with collective efficiency. We should note that the proposition differs from various extensions of the Condorcet Jury Theorem, which describe the optimal supermajority threshold when there is uncertainty over which of two outcomes is better than the other.[14] To see this distinction, consider a case where the consequence of retaining the wrong status-quo policy or the probability of finding a better alternative are high. A threshold that ensures the stability of the current social optimum may be suboptimal (too high) because it limits the possibility of changing policies.

Instead of characterizing the optimal threshold under uncertainty, proposition 1 guarantees the existence of a threshold that ensures the stability of the social optimum. In this sense, it validates applied models proposing supermajoritarian systems of governance to preserve socially optimal outcomes. Weitzman (2015), for instance, develops a supermajoritarian voting regime to manage type-I and type-II risks of modifying the environment in response to climate change (solar geoengineering). He designs a voting architecture that chooses an appropriate supermajoritary threshold based on normative judgments about the risks of over-cooling the planet or failing to cool it enough. Proposition 1 proves that any optimal temperature can be accommodated by a set of supermajority rules.

6. Political Compromise

Successful reforms in polarizing policy domains — gun control, abortion, healthcare and immigration among others — requires legislators to make mutual sacrifices and willfully compromise their core values and principles, or interests. In the US, the passage of the 1986 Tax Reform Act (TRA), the 2010 Patient Protection and Affordable Care Act (ACA), and the 2017 Tax Cuts and Jobs Act entailed challenging legislative negotiations and discussions. In their seminal analysis of the TRA and ACA, Gutmann and Thompson (2010) attribute the success of the former to successful bipartisan compromise and that of the latter to mutual sacrifice and mutual opposition by Democratic and Republican leadership.

Today, growing cleavages between parties in many developed and democratic countries have hampered political compromise, as those at the opposite ends of the ideological spectrum find

[14] The question of choosing the optimal system considering uncertainty was first formulated by Condorcet (1793). See Nitzan and Paroush (2017) for further details and extensions.

policy near the median voter's ideal point increasingly unappealing.[15] According to Gutmann and Thompson (2010, 5):

> The chief reason to be concerned is that the greater the resistance to compromise, the greater the bias in favor of the status quo. Little change can happen in democratic politics without some compromise, and almost no major change can happen without major compromises. Without compromise on health care and taxation or other major issues, the status quo prevails even if it preserves a policy that serves everyone's interests less well.

In democratic politics, legislative bodies never share a single political ideology and so, without relaxing their normative visions of society, they cannot pass legislation.[16] In this section, we do not seek to provide the sources of political compromise in democratic settings, but rather to rationalize political compromise that is observed in democratic legislatures.

Below, we introduce a formal definition of the level of political compromise in our framework. Before that, we add an additional structure on the spatial space Z. We assume that Z is endowed with the order topology induced by the total order $\geq$, and we denote by d, a *distance* defined on Z.

Definition 3 Let $\mathcal{P} = (N, Z, (\succeq_i), \mathcal{L}_\alpha)$ be a political economy, and z_i the ideal policy of a proposer i. The *level of political compromise* of proposer i is the distance pc_i defined as:

$$pc_i(\alpha) = \min_{z \in \mathcal{E}(\mathcal{P})} \{d(z, z_i)\}. \qquad \square$$

For principled politicians whose ideological platforms are located either to the left of equilibrium z_1^* or to the right of equilibrium z_2^*, figure 6 illustrates the necessary compromise they must make, as the incumbents, to avoid defeat in all pairwise elections. A political party with ideal policy (or fundamental ideological identity) "z" must compromise by moving toward the closest equilibrium

[15] A 2019 survey by the Pew Research Center finds that most US adults support more political compromise and respect toward opposing political views from *opposing* political parties (Pew Research Center, 2019).

[16] Wantchekon and Neeman (2002) develop a theory that provides mechanisms through which democracy emerges and endures in conflict-ridden societies.

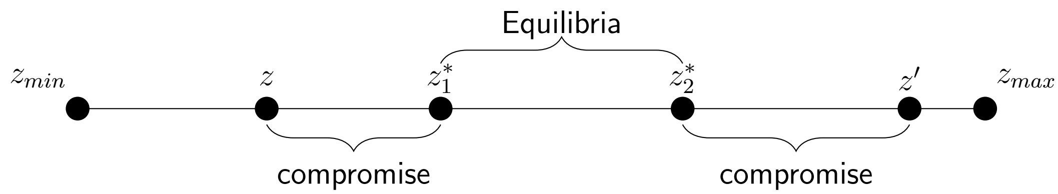


Figure 6: Political compromise.

to its political platform to avoid defeat in a pairwise election against z_2^*. It is rational, then, for this party to run on platform z_1^* if it enters in the race. The same strategy applies to the politician with ideal point z', who must compromise by running on z_2^*. Compromising, then, is rational, as it increases the likelihood of challenging the status quo in a pairwise majoritarian election. The following proposition describes the optimal level of political comprise as a function of the political rule and the voters' ideal points.

Proposition 2 *Let $\mathcal{P} = (N, Z, (\succeq_i), \mathcal{L}_\alpha)$ be a political economy, and Nature randomly chooses a proposer i to make a proposal. Then*

1. *Proposer i's optimal level of compromise is given by the distance between proposer i's ideal point and the closest equilibrium policy to that point.*
2. *The optimal level of political compromise is a decreasing function of the size of the supermajority threshold α.*

It was, in fact, the impracticality of the status quo and the inability to compromise that spurred the adoption of supermajority rules in favor of unanimity. One early adopter was the Catholic church. Prior to the late 11th century, the church administered Papal elections with the aim to achieve broad consensus (*unanimitas*), predicated on the belief that divine ordinance manifested in support from a "flock" of the governed (Burns and Goldie, 1994, 449). The rise of reformist movements and growing political and religious factionalism led to a series of disputed elections in the 11th and 12th centuries. To make their cases, opposing parties in the 12th century cited the doctrine of *sanior et maior pars*—which asserted that the choice must be made by "the sounder and greater" share of the electorate. The doctrine offered an alternative to unanimity but provided no guidance over how to determine an elector's soundness or translate this superiority into votes. Nonetheless, it reflected growing sentiments in favor of equal participation. During the 1159 election of

Alexander III, three cardinals refused to accept the candidate of the rest of the college, leading the Third Lateran Council to adopt a strictly quantitative method between a simple majority and unanimity, i.e, the two-thirds majority rule (Schwartzberg, 2013, 58-60). Supermajority rule, then, was a reduction in the voting threshold to increase political compromise and to prevent "a few fallible cardinals in the minority"—who supported the status quo of no outcome to the elected candidate—"from derailing an otherwise unanimous verdict" (Schwartzberg, 2013, 51).[17]

The *de jure* voting rules for many international institutions continue to require consensus or high supermajority thresholds for amending or implementing policy (Downs et al., 1998; Stone, 2009a; Hovi and Sprinz, 2006). When status-quo policies entail low public investment or countries' beliefs about payoffs diverge, international institutions may under-invest in public goods or fail to insure against long-term global risks (Stone, 2009b). The 1997 Kyoto Protocol to mitigate climate change offers a sadly illustrative example of limited compromise. Its signatories held divergent policy preferences and beliefs about climate change and faced different mitigation costs (Barrett, 2003). Requiring ratification by a double-majority of signatories[18], the protocol's super-majority threshold stymied compromise. Countries with preferences close to the status quo — that is, failing to ratify the treaty—made practically no concessions. India and China refused to bear mitigation costs due to their rapid industrialization and the US, whose population anticipated fewer damages from climate change than its European counterparts, also committed to lower targets (Stone, 2009b). Domestically, too, US foreign policy was the product of a two-level game in which Republicans, who were generally opposed to the agreement, could filibuster any legislation to ratify the treaty (Hovi et al., 2012; Lisowski, 2002). Unsurprisingly, the senate's overwhelming passage of the Byrd-Hagel Resolution repudiating ratification, and the election of George W. Bush, who publicly denounced the agreement, foreclosed US participation. President Barack Obama's decision to enter the Paris Accord spurred discussion about how to "US-proof the Paris Climate Agreement", with proposals including further executive agreements by the US president, the use of majority voting in the United Nations Framework Convention on Climate Change, and Layered Majority Voting with lower thresholds for adjudicating procedural matters

[17]This example is summarized in Schwartzberg (2013, Ch. 1).

[18]55 countries that collectively account for 55 percent of 1990 carbon dioxide emissions of the 38 Annex I (industrialized) countries.

and higher ones for more substantive decision-making (Kemp, 2016, 2017).

Beyond explaining policy developments in international negotiations, theorem 2 and proposition 2 are also validated by McGrath et al. (2018), who conduct a comparative study across US states. Leveraging cross-country variation in state legislative override requirements, they find that legislatures with higher override requirements demonstrate less ability to override an executive veto. Mapping the legislative process to our model, state governors first propose budgets and then legislatures pass their own. The budget is then sent to the governor for approval and, if vetoed, can only be enacted if a legislative supermajority overrides the veto. The supermajority thresholds used in the study—which, in this case, are the proportions of the legislature needed to override an executive veto—vary between $1/2$ and $2/3$.[19] In accordance with proposition 2, budgets passed in states with higher override requirements were substantially closer to those proposed by the governor, with the strongest effects in states where executives' preferences diverged sharply from those of legislative veto players.

7. Conclusion

In this study, we derive the maximum number of equilibria in static and dynamic political games under supermajority rule when agents have single-peaked preferences over a totally ordered policy space. We fully characterize the relationship between this number and a voting body's supermajority rule, showing that the number increases in a nontrivial manner in the size of the supermajority coalition needed to change policy. Our results can explain why highly divergent policies may persist, even across democracies with identical political preferences and voting rules. Moreover, in deriving the maximum number of equilibrium policies in a supermajoritarian setting, our results generalize Duverger's propositions on institutions and political parties. Our study also provides a simple yet sharp rationale for political compromise based on supermajority rules. In only imposing the assumption that voters hold single-peaked preferences over a totally ordered policy space, our model is quite general and applies to a variety of policies beyond those chosen from a uni-dimensional set.

[19] Three states with a $3/5$ or majority veto override were excluded from some models because they also had supermajority budgetary requirements (McGrath et al., 2018, 165).

Our theory generalizes dynamics in other theoretical work (Dixit et al., 2000), and its implications align with voting behavior in institutions ranging from state legislatures (McGrath et al., 2018) to international institutions (Stone, 2009b). Additionally, we contribute to existing social choice literature. Focusing on supermajority voting rules—a topic that has, to date, received limited attention—the article raises and answers novel questions. What, precisely, is the relationship between supermajority thresholds and the number of equilibrium policies? And how does this relationship manifest in the diversity of policies across institutions with one threshold as opposed to another?

Our model also offers avenues for future empirical and theoretical research. To sharpen the model's key insight, we do not restrict which policies can be proposed as the status quo or as alternatives. In reality, of course, legislators may bear informational, monetary, or reputational costs to propose or amend policies. We consider a simple extension to describe these cases in Appendix B, demonstrating that the size of the equilibrium set with amendment costs is no smaller than those without them (lemma 6) and that the size of the equilibrium set with proposal costs is no greater than that without them (lemma 7). Intuitively, sufficiently high amendment costs may prevent legislators from proposing amendments which would have won the requisite supermajority, and sufficiently high proposal costs may induce some legislators to forego proposals that would have withstood amendments.

Further extensions can consider proposal or amendment costs that vary based on legislators' ideal points or on the location of the proposed policy or amendment. The model is also amenable to accommodating "decision-costs" from policy gridlock (Buchanan and Tullock, 1962) and to introducing uncertainty in legislators' policy preferences. The latter extension would draw connections between policy diversity and the extensive literature examining the Condorcet Jury Theorem.

Empirically, the model offers several testable predictions. Do reductions in amendment thresholds—such as revisions to the senate requirements to invoke cloture, discussed in section 4—decrease policy diversity and increase the extent to which proposers compromise? And, comparing legislative bodies whose members have similar preferences, do those requiring high supermajoritarian thresholds to amend proposals generate more diverse policies than those with low thresholds? And how does the distribution of agenda power mediate the relationship between policy diversity and voting rules? Despite the challenges in finding variation in voting rules across otherwise comparable legislative bodies (Cameron, 2009), recent research has employed innovative data to discern such relationships, both globally and domestically (McGrath et al., 2018; Blake and Payton, 2015; Brutger and Li, 2019).

Appendix A. Proof of Main Results

PROOF (THEOREM 1) Let $z \in Z$ be a policy and define $S(z)$ as the number of agents for whom z is the peak. A coalition of voters S has a veto right to propose or to amend a given status quo if $|S| > \alpha n$ or $|S| \geq \beta n$. Consider the functions f and g defined on the policy set Z as follows: for any policy $z' \in Z$,

$$f(z') = \sum_{z \geq z'} S(z) - \alpha n, \text{ and } g(z') = \sum_{z \leq z'} S(z) - \alpha n. \tag{A.1}$$

Let defined the following sets:

$$Z_f = \{z' \in Z : \; f(z') > 0\}, \text{ and } Z_g = \{z' \in Z : \; g(z') > 0\}. \tag{A.2}$$

To prove theorem 1, the following lemmas proved useful.

Lemma 1 *There exist two peaks z_1^* and z_2^* such that: z_1^* minimizes f over Z_f, and z_2^* minimizes g over Z_g.* □

PROOF Notice that neither Z_f, nor Z_g is empty. In fact, given that z_{min} and z_{max} are respectively the smallest and the greatest peaks of Z, then $f(z_{min}) = n - \alpha n = (1 - \alpha)n > 0$ and $g(z_{max}) = n - \alpha n = (1 - \alpha)n > 0$, since $\alpha < 1$, which implies that $z_{min} \in Z_f$ and $z_{max} \in Z_g$, in turn implying that $Z_f \neq \emptyset$ and $Z_g \neq \emptyset$. Given that Z_f is finite and f is a strictly decreasing function, there exists a unique peak z_1^* which minimizes f over Z_f. In addition, for any peak $z' > z_1^*$, $f(z') \leq 0$, which implies that $\sum_{z \geq z'} S(z) \leq \alpha n$. Similarly, given that Z_g is finite and g is a strictly increasing function, there exists a unique peak z_2^* which minimizes g over Z_g. In addition, for any peak $z' < z_2^*$, $g(z') \leq 0$, which implies that $\sum_{z \leq z'} S(z) \leq \alpha n$. ■

Lemma 1 *Assume that* $z_1^* = z_2^* = z^*$. *Then,* $\mathcal{E}(\mathcal{P}) = \{z^*\}$. □

PROOF We claim that z^* (see figure A.7) is the Condorcet winner. Indeed, let $z \in Z$ be a peak.

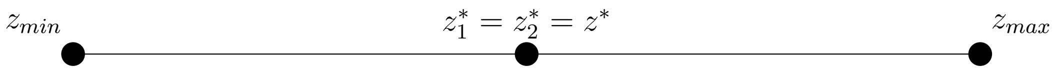


Figure A.7: $z_1^* = z_2^* = z^*$

If $z < z^*$, by definition of z^*, $\sum_{z' \leq z} S(z') \leq \alpha n$ and $\sum_{z' \geq z^*} S(z') > \alpha n$, which implies that z^* beats z in a pairwise opposition. Similarly, if $z > z^*$, we show in the same way that z^* defeats z in a pairwise competition. It follows that z^* defeats any other peak in a pairwise opposition. Since there is no other option which defeats z^*, then $\mathcal{E}(\mathcal{P}) = \{z^*\}$. ■

Lemma 2 *If* $z_1^* \neq z_2^*$, *then* $z_1^* < z_2^*$. □

PROOF Assume by contradiction that $z_1^* > z_2^*$ (see figure A.8). By definition of z_1^* and z_2^*, we have $\sum_{z \geq z_1^*} S(z) > \alpha n$ and $\sum_{z \leq z_2^*} S(z) > \alpha n$, then $\sum_{z \in Z} S(z) > 2\alpha n$. Given that $\sum_{z \in Z} S(z) = n$, it follows that $n > 2\alpha n$, meaning that $\alpha < \frac{1}{2}$, a contradiction, since by assumption $\frac{1}{2} \leq \alpha < 1$. Hence, the only remaining possibility is $z_1^* < z_2^*$ (see figure A.9). ■

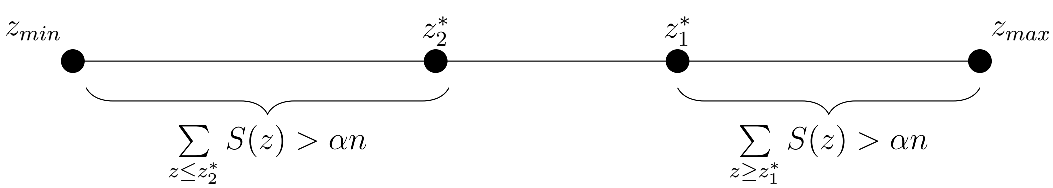


Figure A.8: $z_2^* < z_1^*$.

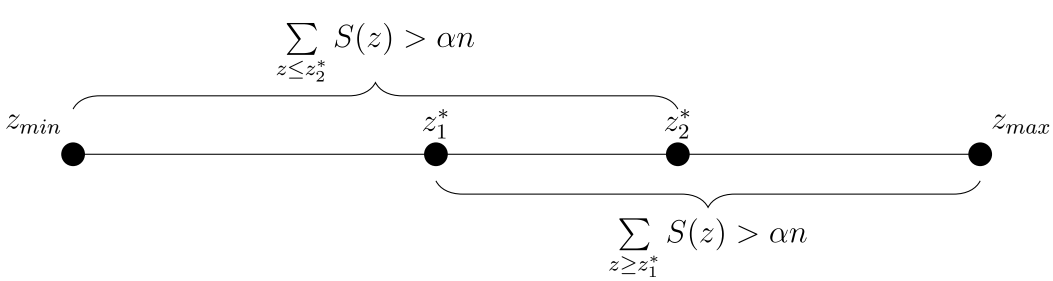


Figure A.9: $z_1^* < z_2^*$.

Lemma 3 *There exists* $z^* \in (z_1^*, z_2^*)$, *with* $S(z^*) \neq 0$. □

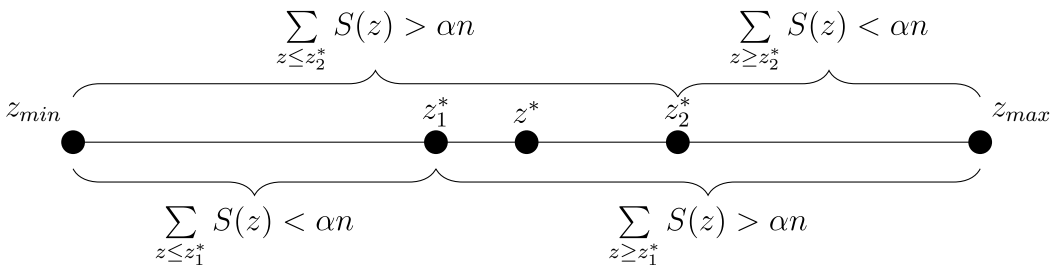


Figure A.10: $z_1^* < z_2^*$, $S(z^*) \neq 0$, $z^* \in (z_1^*, z_2^*)$.

PROOF Assume the contrary. By the definition of policies z_1^* and z_2^*, we have $\sum_{z \geq z_1^*} S(z) > \alpha n$ and $\sum_{z \leq z_2^*} S(z) > \alpha n$. These imply that $\sum_{z \geq z_1^*} S(z) + \sum_{z \leq z_2^*} S(z) > 2\alpha n$ or $\sum_{z \in Z} S(z) > 2\alpha n$ leading to $\alpha < \frac{1}{2}$, which is a contradiction. It follows that there exists a policy $z^* \in (z_1^*, z_2^*)$ such that $S(z^*) \neq 0$. Note that, in this case, z_1^* and z_2^* are such that $\sum_{z \leq z_1^*} S(z) < \alpha n$, and $\sum_{z \geq z_2^*} S(z) < \alpha n$ (see figure A.10). ∎

Lemma 4 *If* $z \in Z \setminus [z_1^*, z_2^*]$*, then* $z \notin \mathcal{E}(\mathcal{P})$. □

PROOF Consider $z \in Z$ distinct to z_1^* and z_2^*. Assume that z is the closest peak to the left of z_1^*. In a pairwise opposition between z and z_1^*, the former receives at most $\sum_{z' \leq z} S(z')$ number of votes, while the latter receives at most $\sum_{z' \geq z_1^*} S(z')$. Since, $\sum_{z' \leq z} S(z') \leq \sum_{z' \leq z_1^*} S(z') < \alpha n$, and $\sum_{z' \geq z_1^*} S(z') > \alpha n$, then, z_1^* wins. We can also show that z_2^* defeats any peak z strictly greater than z_2^*. ∎

Lemma 5 *If* $z \in [z_1^*, z_2^*]$*, then* $z \in \mathcal{E}(\mathcal{P})$. □

PROOF Consider a peak $z \in [z_1^*, z_2^*]$. Assume that there exists $z' \in \,]z_1^*, z_2^*[$ such that z' defeats z. Without loss of generality, assume that z' is the closest peak to z with $z' < z$. z' defeats z implies that $\sum_{x \leq z'} S(x) > \alpha n$, which is a contradiction, because by definition of z_2^*, $z' < z_2^*$ implies that $\sum_{x \leq z'} S(x) \leq \alpha n$. Thus, $\mathcal{E}(\mathcal{P}) = [z_1^*, z_2^*]$. ∎

Now, we can conclude the proof of theorem 1.

First, if $\alpha = 1$, then the only winning coalition is the grand set N. Assuming that individuals make proposals, then each peak is a predicted outcome of the game. In fact, the maximum number of votes that an alternative in a pairwise opposition can receive is $n - 1$. If the rule requires

n votes to win, then no alternative can be defeated, and the maximum number of predicted outcomes is the size of N, i.e., n. Second, if n is odd, and $\alpha = 1/2$, then the median peak is the unique prediction of the one-shot game. Proving that assertion is straightforward since, the median peak is the Condorcet winner, i.e., it defeats any other candidate in a pairwise opposition. Third, from lemmas 1, 4, and 5, we show that any alternative which is not part of the interval bounded by the peaks z_1^* and z_2^* can be directly dominated by either z_1^* or z_2^*, and any peak in this interval cannot be defeated. Therefore, the maximal number of equilibria is equal to the size of individuals who have a peak between z_1^* and z_2^*. Given that the proportion of individuals required to form a winning coalition is at least $\beta = \frac{\lfloor \alpha n \rfloor + 1}{n}$, then the upper bound of $\mathcal{E}(\mathcal{P})$ is $n - 2(1-\beta)n = (2\beta - 1)n$. ∎

Proof (Corollary 1) From theorem 1, if $\alpha = \frac{1}{2}$, then the size of equilibrium set $\mathcal{E}(\mathcal{P})$ depends on the size of n. If n is odd, the number $(2\beta - 1)n$ equals 1, meaning that a unique equilibrium exists. It is, in fact, the ideal policy of the median voter. If n is even, there exist at most two equilibria since $(2\beta - 1)n$ equals 2. ∎

Proof (Theorem 2) Let $z \in Z$, and define $S(z)$ as the number of agents for whom z is the peak. Consider the functions f and g defined on Z as follows: for any policy $z' \in Z$,

$$f(z') = \sum_{z \geq z'} S(z) - \alpha n, \text{ and } g(z') = \sum_{z \leq z'} S(z) - \alpha n,$$

and define the sets

$$Z_f = \{z' \in Z : \quad f(z') > 0\}, \text{ and } Z_g = \{z' \in Z : \; g(z') > 0\}$$

There exists a unique peak z_1^* which minimizes f over Z_f, and a unique peak z_2^* which minimizes g over Z_g (for further details, see proof of theorem 1 above.). The status quo, chosen randomly by Nature, can take any position in spatial space Z (see figure A.11, $z_{min} = \min Z$ and $z_{max} = \max Z$).

In fact, the status quo may not correspond to any individual's ideal policy. From theorem 1, any position between and including z_1^* and z_2^*, is invulnerable to pairwise opposition. In this case, the

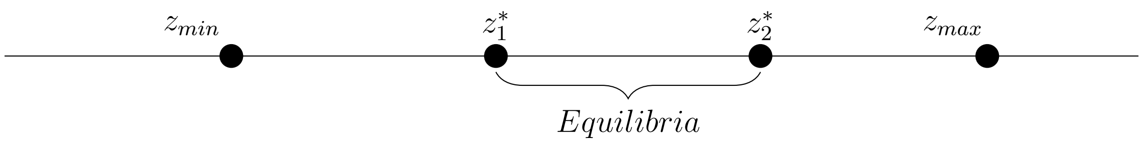


Figure A.11: Nature chooses the status quo.

interval bounded by the peaks z_1^* and z_2^* is the equilibrium set. ■

Proof (Theorem 3) Let $z \in Z$. If $z < z_1^*$, then z_1^* indirectly dominates z; If $z > z_2^*$, then z_2^* indirectly dominates z. The only alternatives that are indirectly dominated belong to the interval $[z_1^*, z_2^*]$. A subset $X \subseteq Z$ is consistent if

$$f(X) = \left\{ \begin{array}{l} x \in Z : \forall y \in Z, \forall S,\ |S| > \alpha n,\ \exists z \in X,\ \text{where} \\ y = z \text{ or } z \gg y \text{ and } not(x \succ_S z) \end{array} \right\} = X.$$

For each agent $i \in N$, we denote by z_i his or her ideal point. By definition of z_1^* and z_2^*, the sets $S = \{i \in N : z_i \geq z_1^*\}$ and $T = \{i \in N : z_i \leq z_2^*\}$ are winning coalitions. Let $z \in Z$ be a proposal: (a) if $z < z_1^*$, then any deviation from z by any winning coalition to z_1^* is not deterred. Similarly; (b) if $z > z_2^*$, then any deviation from z by any winning coalition to z_2^* is not deterred. Hence, in these two cases, $z \notin f(Z)$. However, if $z \in [z_1^*, z_2^*]$, any deviation from z is deterred. Indeed, without loss of generality, assume $x = z_1^*$, and consider $y \in Z$ and a winning coalition S', such that $x \longrightarrow_{S'} y$. (c) If $y < z_1^*$, then there exists $z = z_1^*$, with $z_1^* \gg y$ via T, and $not(z_1^* \succ_S z_1^*)$; (d) If $y \in]z_1^*, z_2^*[$, then, there exists $z = y$, such that $not(z \succ_{S'} z_1^*)$, with $|S'| > \alpha n$, because $y < z_2^*$; (e) If $y > z_2^*$, then, there exists $z = z_2^*$, with $z_2^* \gg y$ via S, and $not(z_2^* \succ_{S'} z_1^*)$, with $|S'| > \alpha n$. It follows that $f(Z) = [z_1^*, z_2^*]$. It is straightforward to check that $f(f(Z)) = f(Z)$; therefore $f(Z)$ is the largest consistent set, and point 2. of the theorem is proved. We conclude the proof of point 1. by using the same argument in the end of the proof of theorem 1. ■

Proof (Proposition 1) We show that the social optimum outcome belongs to the range $[z_{min}, z_{max}]$ (see figure A.12). By contradiction, assume that $z_e < z_{min}$. Then, in a pairwise opposition between z_e and z_{min}, the former receives zero share of the votes, meaning that each agent strictly prefer z_{min} to z_e, or $u_i(z_{min}) > u_i(z_e),\ \forall i \in N$. The latter expression leads to

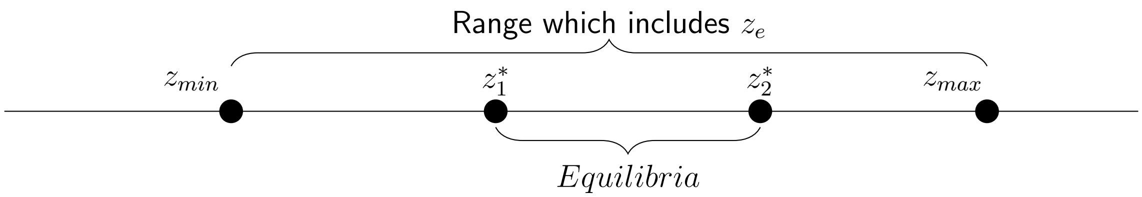


Figure A.12: The social optimum and voting equilibrium.

$\sum_{i \in N} u_i(z_{min}) > \sum_{i \in N} u_i(z_e)$, which is absurd by the definition of z_e. We obtain the same conclusion if we assume that $z_e > z_{max}$.

If the decision is made by using the unanimity rule, then the social optimum is an equilibrium, because there will never be enough agents who can form a winning coalition to defeat it in a pairwise competition. Assume that Nature chooses the status quo, and the minimum size of the majority required to pass a decision is less than the size of all voters (unanimity rule), then the social optimum could be an equilibrium thanks to theorems 1 and 2. Moreover, these results stipulate that the size of the equilibrium set increases as the size of the majority required to pass social decision increases. Then, depending on agents' preferences, there will always exists a minimum threshold $\alpha_{min} \in [1/2, 1]$, such that $z_e \in \mathcal{E}(\mathcal{P})$. However, if Nature chooses randomly a proposer, and the social optimum does not coincide with the ideal point of a legislator, then it will not have a chance to be submitted for a vote whatever the threshold required by the voting rule. For that reason, there will not exist a rule under which the social optimum is undefeated in a pairwise competition. ∎

Proof (Proposition 2) Figure 7 in the main text (replicated below as A.13) clearly elucidates the intuition behind the proposition.

1. For a given proposer i, the optimal amount of compromise is the minimal distance between i's ideal policy to the equilibrium point that maximizes i's preferences. Given that preferences are single-peaked, the minimal distance is obtained from the closest equilibrium policy to i's ideal point.

2. The results (theorems 1, 2, and 3) show that the size of the equilibrium set varies increasingly as the size of majority required by the decision rule increases. This reduces the distances of political ideologies to possible equilibria, and therefore diminishes the level of

political compromise. ∎

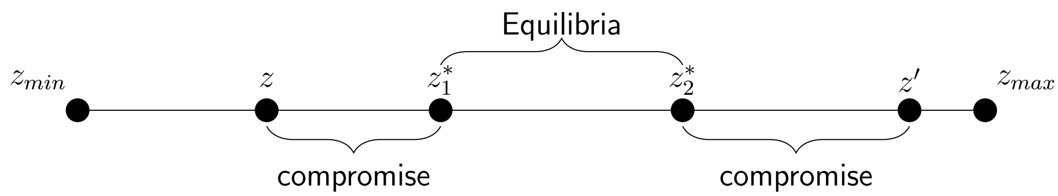


Figure A.13: Political compromise.

Appendix B. Extensions

Appendix B.1. Costs for making alternative proposal

So far, we assume legislators can costlessly propose status quo and alternative policies. What happens, however, if candidates must incur costs for making alternative proposals? Consider a pairwise majority competition $\mathcal{P}' = (N, Z', (\succsim_i), \mathcal{L}_\alpha)$ between a status quo policy q and tuple $(z, k) \in Z'$, where $k_i \in K = \{0, 1\}$ and $k_i = 1$ if the legislator bears the cost of proposing an alternative to the status-quo and zero otherwise; $z \in Z$ is an alternative proposal; and $Z' = Z \times K$. We assume that for all $z \in Z$, $(z, 0) \succeq_i (z, 1)$, for each $i \in N$, meaning that, all else equal, each legislator prefers proposing an alternative without bearing the cost to proposing it while bearing the cost. Additionally, we assume that the status quo policy is that of a legislator chosen by Nature.

Proposing a policy that will be defeated in a pairwise election is strictly dominated by choosing not to propose one at all, and so the set of proposed policies are limited to those that can beat the status quo in a pairwise election.[20] Costs secure equilibrium policies by adding barriers to proposing alternatives, and so policies that are equilibria in a costless environment will remain so in a costly one. As with costless proposals, the number of costly equilibrium policies when $\alpha = 1$ is n, since it includes all costless equilibria but cannot exceed n.

Lemma 6 *(i) If $z \in \mathcal{E}(\mathcal{P})$ then it is also in $\mathcal{E}(\mathcal{P}')$; (ii) When $1/2 \leq \alpha < 1$, however, the converse does not always hold.* □

[20] If the status quo is a costless equilibrium, then it defeats all alternatives in a pairwise election and so alternatives are never proposed.

PROOF

(i) Suppose $z \in \mathcal{E}(\mathcal{P})$. Then, z cannot be defeated in a pairwise election. Because $(z,0) \succeq_i (z,1)$ for each $z \in Z$ and for each legislator, none would choose to propose an alternative. So $z \in \mathcal{E}(\mathcal{P}')$.

(ii) We can construct a political economy in which the policy sets

$$\{z > z_2^* : \forall q \in [z_2^*, z) \, \forall i : z_i < \frac{z_2^* + z}{2}, \, (z,0) \succeq_i (q,1)\}$$
$$\{z < z_1^* : \forall q \in (z, z_1^*] \, \forall i : z_i > \frac{z_2^* + z}{2}, \, (z,0) \succeq_i (q,1)\}$$

are non-empty. Then for any policy z in one of the sets, $z \in \mathcal{E}(\mathcal{P}') \setminus \mathcal{E}(\mathcal{P})$. Intuitively, we can choose a policy $z \notin \mathcal{E}(\mathcal{P})$ and proposal costs such that all voters would prefer z to the closest equilibrium policy. ■

We can illustrate this with an example. Suppose $n = 3$, $\alpha = 1/2$ and $S(z) = 2$ and so $z \notin \mathcal{E}(\mathcal{P})$. If $(z^*, 1) \succeq_i (z, 0)$ for the legislators with ideal points less than z then $z \in \mathcal{E}(\mathcal{P}') \setminus \mathcal{E}(\mathcal{P})$.

Appendix B.2. **Costs for proposing status-quo**

Presidents may face costs from choosing to veto legislation (Groseclose and McCarty, 2001; Rice and Kernell, 2019) and drafting new legislation to become the status-quo may require a politician's time and resources. Suppose, then, that in a political economy $\mathcal{P}'' = (N, Z'', (\succeq_i), \mathcal{L}_\alpha)$, legislators face costs not only to challenge the status quo, but also to propose it in the first place.

Each legislator's decision is denoted by the tuple $(\tilde{z}, m, k) \in Z''$ where $Z'' = \tilde{Z} \times M \times K$. If the legislator chooses a status quo policy then $m_i = 1$ and if the legislator abstains then $m_i = 0$. The set $\tilde{Z} = \{Z, np\}$ where $\tilde{z} = np$ if all legislators abstain and so *no policy* is implemented. To ensure the existence of an equilibrium, there exists at least one legislator who prefers some policy in $\mathcal{E}(\mathcal{P}')$ to no policy at all, i.e., there exists $q^* \in \mathcal{E}(\mathcal{P}')$ such that $(q^*, 1, k) \succeq_i (np, 0, k)$.

Lemma 7 *If $z \in \mathcal{E}(\mathcal{P}'')$, then it is also in $\mathcal{E}(\mathcal{P}')$. Policies in $\mathcal{E}(\mathcal{P}')$, however, may be unattainable given sufficiently high status-quo proposal costs.* □

Proof Suppose $z \in \mathcal{E}(\mathcal{P}'') \setminus \mathcal{E}(\mathcal{P}')$. Because $z \notin \mathcal{E}(\mathcal{P}')$, there exists some policy q such that, $(q, 1) \succeq_i (z, 0)$ for at least one legislator and $(q, 0) \succeq_i (z, 0)$ for at least $\lfloor \alpha n \rfloor$ legislators, contradicting the assertion that $z \in \mathcal{E}(\mathcal{P}'')$. So, if $z \in \mathcal{E}(\mathcal{P}'')$ then it is also in $\mathcal{E}(\mathcal{P}')$. If $|\mathcal{E}(\mathcal{P}')| > 1$ and for some $q \in \mathcal{E}(\mathcal{P}')$, $(np, 0, k) \succeq_i (q, 1, 0)$ for all legislators, then q is unattainable, i.e., $q \in \mathcal{E}(\mathcal{P}') \setminus \mathcal{E}(\mathcal{P}'')$. ∎

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