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Chapter 2
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ABSTRACT

This chapter examined the trends in reform-based mathematics teaching practices in the United States classrooms. The authors systematically analyzed the journal articles in the Mathematics Teacher: Learning and Teaching PK-12 (MTLT) in order to reveal the current practices that practitioners and experts in mathematics education deem significant and worthy. They found that the most trending reform practices were mathematical discourse, conceptual understanding, task selection, and real-life applications. They discussed each trending practice through sample strategies and provided examples from the reviewed articles. They also identified the least trending reform practices that need attention and discussed associated challenges.

INTRODUCTION

Past and current reform movements in mathematics education in the United States have been largely shaped by the National Council of Teachers of Mathematics (NCTM). State education standards were begun to be established in the early 1990s after the NCTM standard-based education movement in North America to support a systematic improvement in mathematics education (NCTM, 2014). Each state developed and adopted its own learning standards in the early 2000s (Common Core State Standards

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Initiative [CCSSI], 2018). After about a decade, state leaders and commissioners of education emphasized the importance of having consistent goals all over the country through their membership in the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). Then, the NGA and CCSSO (2010) released the Common Core State Standards (CCSS) for English Language Arts (CCSS-ELA) and Mathematics (CCSSM) in 2010 to ensure all students graduate from high school as college and career ready. Since then, 41 States, the District of Columbia, four territories, and the Department of Defense Education Activity (DoDEA) have adopted the CCSS (CCSSI, 2018).

Along with the CCSSM content standards, CCSSI (2010) developed Standards for Mathematical Practices (SMP) in the light of NCTM (2000) process standards (i.e., problem-solving, reasoning and proof, communication, representation, and connections) and based on the strands of mathematical proficiency specified in the National Research Council's (NRC, 2001) report (i.e., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition). The eight SMPs are listed as; (1) make sense of problems and persevere in solving them; (2) reason abstractly and quantitatively; (3) construct viable arguments and critique the reasoning of others; (4) model with mathematics; (5) use appropriate tools strategically; (6) attend to precision; (7) look for and make use of structure; and (8) look for and express regularity in repeated reasoning. The SMPs describe what students are expected to do as they learn mathematics. For example, mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace; consider the available tools when solving a mathematical problem, and communicate precisely to others. In addition, each SMP supports the development of strands of mathematical proficiency. For example, conceptual understanding is developed when students are engaged in one or more SMPs (e.g., make sense of problems and persevere in solving them, construct viable arguments and critique the reasoning of others) (Stein et al., 2017).

BACKGROUND

With the adoption of CCSSM, the focus of mathematics standards moved from developing only skills efficiency to rigorous thinking, conceptual understanding, and reasoning (Stein et al., 2017). NCTM (2014) stated that the progress made, the challenges that have been left, and the necessary actions to ensure mathematics success for all students should be examined to realize the potential of these new standards. The percentage of fourth-graders and eighth-graders scoring "proficient" on the National Assessment of Educational Progress (NAEP) increased by 29 and 22 points, respectively, from 1990 to 2013. Yet, only 44% of United States high school graduates are college-ready according to Scholastic Aptitude Test (SAT) and American College Testing (ACT) scores (NCTM, 2014).

Although NCTM (2014) stated that the implementation of CCSSM is promising to address challenges and expand the progress, the CCSSM document does not indicate how to implement these standards in the classroom. Therefore, NCTM (2014) published the *Principles to Actions* (PtA) book to describe necessary conditions to implement CCSSM and ensure mathematical success for all students. The NCTM's PtA identified the eight research-informed high-leverage effective mathematics teaching practices to support the development of SMPs, based on NCTM's (2000) core principles. These eight effective mathematics teaching practices are: (1) establish mathematics goals to focus learning; (2) implement tasks that promote reasoning and problem solving; (3) use and connect mathematical representations; (4)

facilitate meaningful mathematical discourse; (5) pose purposeful questions; (6) build procedural fluency from conceptual understanding; (7) support productive struggle in learning mathematics; and (8) elicit and use evidence of student thinking. These effective mathematics teaching practices are necessary to promote deep mathematics learning and offer educators and school districts a standard lens to improve instructional practices and ensure mathematics success for all students (NCTM, 2014). They, hence, are one of the components that form reform-based teaching practices in the United States.

Reform-based mathematics teaching is broadly defined as integrating teaching practices that build on students` prior knowledge and involve interdisciplinary and real-life application activities (Joram & Gabriele, 2012). Mathematics teaching and curriculum reform emphasize mathematical thinking and reasoning rather than memorization and computation (Spangler & Wanko, 2017). In this respect, reform-based mathematics teaching practices focus on developing a conceptual understanding of mathematics that connects prior knowledge with new knowledge through problem-solving and inquiry-based learning that allows collaboration and hence student-centered instruction (Jong, 2016). Therefore, setting instructional goals to enhance students` engagement and conceptual and procedural understanding is considered essential for reform-based mathematics teaching (Hiebert et al., 2003). Accordingly, reform-based mathematics teaching emphasizes problem-solving, mathematical discourse, justification of the answers, and using multiple methods or representations to solve the problems (NCTM, 2014).

MAIN FOCUS OF THE CHAPTER

This chapter examines the trends of reform-based mathematics teaching practices that are focused on in the *Mathematics Teacher: Learning and Teaching PK-12* (MTLT) articles. The MTLT trends that experts in the field deem significant and worthy of writing and publishing would provide information about meaningful and effective reform-based mathematics teaching practices in the United States.

We reviewed the articles in the practitioner journal, MTLT, published by NCTM. We chose MTLT because NCTM is the world's most extensive mathematics education organization and a leading professional organization for mathematics teachers in the United States. MTLT's scope reflects the current practices of mathematics education and sustains an ongoing process of knowledge-based practice and policy regarding the future of the field. Therefore, MTLT articles reveal information about the most recent reform-based mathematics teaching practices emphasized by practitioners and experts in mathematics education. This chapter sought to answer the following question: What are the most trending reform-based mathematics teaching practices in the United States? In an attempt to describe the recent trends in the United States, we examined the themes and focus of the articles in MTLT.

NCTM combined its three practitioner journals—*Teaching Children Mathematics*, *Mathematics Teaching in the Middle School,* and *The Mathematics Teacher*—in a new journal that spans PK-12, MTLT, in January 2020. MTLT is a digital-first journal that allows authors to include screenshots, videos, or applets which allow the readers to understand the messages and implement the ideas in their classrooms (Barlow, 2020). The journal involves peer-reviewed and invited articles from preschool (PK) to 12th grade, which is the only practitioner journal that focuses on PK to 12 mathematics education published by NCTM since January 2020. Twelve issues were published in 2020 and nine issues in 2021 at the time of this writing. Each issue involves a front-and-center article that is designed to span the full PK–12 range. In addition, there is at least one feature and one focus article specified to grade bands.

The journal also involves tasks or problems specific to grade levels, asked and answered sessions, and other mathematics sections.

A systematic analysis of the trends in MTLT revealed how teacher educators and practitioners approach reform-based mathematics teaching. The trends analysis also revealed which reform-based practices received less attention and associated challenges and obstacles in implementing reform-based mathematics teaching practices. Thus, this chapter described the reform-oriented mathematics practices that have been emphasized in MTLT articles in recent years.

METHODOLOGY

This qualitative study relies on document analysis as a stand-alone methodology based on the research questions proposed. As being aware of the limitations of this methodology as "biased selectivity" (Bowen, 2009), we had a specific purpose of selecting NCTM articles since reforms in mathematics education for K-12 in the United States heavily focus on NCTM standards and publications.

A total of 21 issues of MTLT have been published at the time of this writing. The front-and-center article, feature articles, and focus articles were systematically reviewed to explore reform-based mathematics teaching trends by using open coding strategy (Corbin & Strauss, 2014). As we reviewed the front and center, feature, and focus articles of 21 issues of MTLT published between January 2020 and September 2021, we recognized that some of the reform-based teaching practices were emphasized more than others. Four authors independently created codes for 173 articles in 21 issues for the primary focus, grade level, and main practices. The authors coded several articles together in the beginning, resolved any inter-coder disagreement through discussion, and reached a consensus for coding. Two hundred ninety-five codes were created from the articles based on their focus and main ideas. Some of the articles were classified into more than one code. For instance, the article titled "Ready, Set, Launch! —The Engineering Cycle for Productive Struggle" from issue 14-feature article 3 was coded as both "productive struggle" and "problem-solving" practices.

The authors then merged codes into main themes and established a consensus on the coding using QSR International's NVivo qualitative data analysis software (Version 12) (2018). Relying on open coding methodology (Corbin & Strauss, 2014), 59 themes emerged from 173 articles. Then, these 59 themes were merged into bigger categories. For example, a cluster of themes including concept, sense-making, meaningful mathematics learning, making sense of mathematics, deeper level understanding was merged into "conceptual understanding." As displayed in Table 1, the most frequent themes were mathematical discourse, conceptual understanding, task selection, and real-life application, whereas equity, mathematical modeling, and using technology to solve problems were less frequent themes. Other themes that appeared less than or equal to 1% were anticipating student thinking, attending to precision, estimation, feedback, fluency, generalizing, graphing, lesson study, mathematical habits of mind, mathematical intuition, metacognition, objectives, professional noticing, professionalism, proficiency, project-based-instruction, proof teaching, quantitative reasoning, structural reasoning, student voice, and student-centered instruction. We discussed the four most frequent themes in the following sections.

Table 1. Frequency of common themes in NCTM P-12 articles

Themes	Frequency	Themes	Frequency
Mathematical Discourse	12%	Hands-On/Manipulatives	2%
Conceptual Understanding	10%	Culturally Relevant Pedagogy	2%
Task Selection	8%	Justification	2%
Real Life Applications	8%	Productive Struggle	2%
Technology	4%	Student Engagement	2%
Equity	4%	Tool Use	2%
Mathematical Modeling	4%	Problem Solving Strategies	2%
Interdisciplinary Teaching	3%	Productive Disposition	2%
Games	3%	Others	≤1%
Mathematical Language	3%	Total	100%

Mathematical Discourse

The results of the analysis showed that mathematical discourse was the most frequent focus across the MTLT articles. These articles often proposed various strategies that can foster mathematical discourse in the classroom. While some of the strategies were mainly about mathematical discourse, some of them discussed discourse as a secondary focus, with the main focus on another strategy. In our analysis, the mathematical discourse theme included the following codes: discourse action, meaningful discussion, facilitating discussion, productive discussion, purposeful questioning, assessment and advancement questioning, mathematics talk, representation talk, and chalk talk. The proposed strategies, in the reviewed articles, in support of mathematical discourse are discussed in the following sections.

Discourse Actions

One of the strategies proposed by the articles is applying discourse actions. Discourse actions are defined as the actions taken to promote mathematical discourse (Boston et al., 2019). The specific discourse actions which are highlighted in some articles are teacher's press (the press for reasoning), the student's providing, teacher's linking, student's linking. The articles identified the actions, addressed their benefits and challenges, and provided tools to improve the use of these actions.

Candela et al. (2020) identified the teacher's press as an act of following up on a student's contributions by asking students to say more about their strategy. Students' providing as an action of students to explain their thinking justifications and validate mathematical accuracy. The author described teacher's linking as an action in which a teacher asks students to restate, extend, compare and contrast or relate another student's contribution. Besides, student's linking was defined as an action in which students pick up and use ideas from their peers, such as explaining, relating to, or building on an idea offered by another student. Some tools were provided in the article to improve the implication of these actions, such as checklists and rubrics for measuring, analyzing, reflecting, and enhancing teachers' (and students') use of discourse actions in the classroom.

While the four actions are highlighted altogether by Candela et al. (2020), the press for reasoning is recognized explicitly by Anderson (2021). The author identified the press for reasoning as an instructional routine of responding to a student's explanation with additional questions to elicit student reasoning. One of the benefits of the strategy is to keep classroom talk focused on students' ideas instead of teacher-led explanations. One challenge of this strategy described in the article was asking questions that shape the students' ideas for them, canalize their thinking toward predetermined pathways, or reduce the cognitive demands of the task. To overcome these challenges and use the strategy for benefit, the author offered three routines: (1) anticipate student responses before the lesson begins, (2) target follow-up questions on specific parts of a more extensive explanation, and (3) use sequences of questions that unfurl the student's reasoning in its entirety.

Purposeful Questioning/Assessment and Advancement Questions

Another strategy that was pointed out by articles is purposeful questioning. This aligns with 'NCTM's (2014) vision that effective mathematics teaching relies on questions that encourage students to explain and reflect on their thinking as an essential component of meaningful mathematical discourse (p.35). Stohlmann (2020a) emphasized that assessment and advancement questions encouraged a rich mathematical discourse by helping students deepen their understanding and prepared teachers to provide informed answers to students' thinking. Besides, Hallman-Thrasher and Spangler (2020) developed strategies and planning tools to help teachers pose questions with specific purposes to facilitate mathematical discussion.

More specifically, Hallman-Thrasher and Spangler described how teachers could develop and pose purposeful questions to engage students in high-cognitive tasks that reveal their thinking, encourage students to communicate and justify their ideas, and maintain a high cognitive demand. They emphasized the use of cognitively demanding tasks, which provide more opportunities for higher-order questioning. In their article, some types of purposeful questions were illustrated by using examples of preservice teachers' work: questioning to elicit students' thinking, questioning to generate student ideas, questioning to clarify student explanation, and questioning to justify.

Stohlmann (2020a) defined the purposeful questions as assessment and advancement questions more specifically. He described how assessment and advancement questions could be planned and implemented based on anticipated solutions of students for a task that was focused on creating an equation to solve a real-world problem. He emphasized thinking about possible student strategies and the questions that could be asked to the students before implementing the task. Students watched a video about a world record for the longest distance traveled rolling across exercise balls and engaged in a real-world problem. He asked the questions that helped students assess their solutions and strategy and helped teachers figure out whether students understood the context. He anticipated possible students' strategies and created assessment and advancement questions for each possible strategy.

Facilitating Discussion

Teachers must determine how to build on and honor student thinking while ensuring that the mathematical ideas at the heart of the lesson remain prominent in class discussions (Engle & Conant, 2002, as cited in NCTM, 2014, p.29). NCTM (2014) emphasized that the teacher must decide how to facilitate the discussion by considering what approaches to share and their order. The teacher must also consider the questions that will help students connect the strategies and critical disciplinary ideas driving lessons to orchestrate a class discussion of student approaches to solving a task. We observed that some articles emphasized the ways of facilitating discussion to develop meaningful mathematical discourse by proposing some strategies. While Ahrendt et al. (2021) pointed out that productive discussion calls for rich tasks, Ghousseini et al. (2021) emphasized using classroom resources and representation to facilitate meaningful discussion. Bennett and Morgan (2020) defined the Chalk Talk as a nonverbal, student-centered, and interactive procedure of facilitating discussion. The article in which Brand et al. (2021) described a teacher's discussion strategies that promote mathematical discourse is another study that emphasized ways of facilitating discussion.

Ahrendt et al. (2021) pointed out that productive discussion calls for rich tasks with multiple solution pathways and gives students opportunities to share their reasoning in ways that inspire others by making them gain a deeper understanding. Ahrendt and colleagues presented a well-designed task for generating a meaningful discussion. They implemented Smith and Stein's (2011) five practices, viewed as crucial parts of orchestrating productive discussion, and a task that supported students to engage in meaningful mathematical discourse. The practices were: (0) choosing a mathematical goal and appropriate task, (1) anticipating student thinking, (2) monitoring student work, (3) selecting students to share work, and (4) sequencing students' work.

Ghousseini et al. (2021) suggested that children as early as kindergarten can reason about mathematics and justify their thinking using both oral language and gestures with the help of available representations in their classroom (e.g., number lines, pictures, and number charts). They highlighted teachers' crucial role in supporting students to engage in mathematical discourse by encouraging them to use classroom resources. Five lessons were observed and videotaped in a kindergarten classroom, and the teachers were interviewed after each lesson. By analyzing the videos and transcripts of class discussions and interviews, they shared three essential practices that yielded from the analysis: (1) establishing expectations that support mathematical talk, (2) eliciting student thinking, and (3) narrating student thinking.

Similarly, Bennett and Morgan (2020) defined the Chalk Talk as a nonverbal, student-centered, and interactive procedure to promote discussion by ensuring that all students have an equitable opportunity to share ideas and provide feedback (Roberts, 2013). They called the chalk talk a silent discussion. This discussion was composed of three steps: (1) student response for initial prompts, (2) student response for initial comment, and (3) whole-class discussion using student responses to highlight the intended outcomes and understandings. The goal was to share ideas and respond to each other. Students were required to express the reason for their agreement and disagreement with different responses. The teacher reminded and emphasized that disagreement is positive interaction that helps them understand others' mathematical ideas and perspectives better. During the first and second steps, teachers observed each station and tried to figure out common themes to lead a whole-class discussion effectively in step three. In other words, Chalk Talk was used as a way for students to engage in mathematical discourse in the form of writing communication.

Likewise, Brand et al. (2021) described a teacher's discussion strategies that promote mathematical discourse during a mathematical modeling activity. They named five notable strategies: (a) promoting common language, (b) encouraging students' perspectives and decision-making, (c) prompting a revision of students' thinking, (d) probing students' responses for assumptions, and (e) emphasizing students' reasoning for further discussion. They observed 6-8 grade students' mathematical modeling activities and categorized the outcome of students' responses associated with each strategy.

Purposefully Structured Lesson Techniques

Mathematical discourse includes the purposeful exchange of ideas through classroom discussion and other verbal, visual, and written communication forms (NCTM, 2014). In line with this, some articles presented instructional and assessment techniques which are purposefully structured to allow students to engage in meaningful mathematical discourse. For example, Thompson and Gay (2020) emphasized the impact of meaningful discourse with fact strategy; and Hicks and Bostic (2021) used a formative assessment technique named whole-class think aloud (WCTAs), which allows students to engage in meaningful mathematical discourse. Additionally, Gunter and Raymond (2020) presented a Spend a Buck strategy (Kagan, 2001) which fostered communication and metacognition through an effective mathematical discourse.

Thompson and Gay (2020) explored the impact of meaningful discourse with fact strategy by working with a group of third-grade teachers. They pointed out that using purposeful structure, instead of just showing the various techniques to students and expecting mastery, helped students engage with the reasoning and thinking behind the strategies. The authors argued that intentional use of discourse and fact strategy instruction could create the bridge students need to connect learning fact strategies to using fact strategies. The combination of using facts strategy and discourse will allow students to explore their thinking and to make sense of the reasoning of others for uncovering understanding and misunderstanding critical concepts.

Hicks and Bostic (2021) shared WCTAs, a formative assessment technique, for problem-solving instruction. They emphasized that high-quality formative assessments need to monitor students' progress toward learning targets and foster deep mathematical learning (NCTM, 2014). Students worked with their partners on one mathematical problem by writing their ideas and draft talk. While the students shared their rough draft thinking aloud, the teacher observed them and took notes about how they read and interpreted the problem, which strategies they applied to solve it, and how they reached a solution. In this technique, while teachers could attend to all students' thinking in real-time, they could monitor students' progress toward the learning target and foster their deep mathematical learning (NCTM, 2014). This technique allowed students to talk about the mathematics they learned and engage in meaningful mathematical discourse.

Similarly, Gunter and Raymond (2020) presented a technique that helped students better understand mathematics and developed their communicative and metacognitive skills. In the article, the Spend a Buck strategy (Kagan, 2001) was used to develop students' thinking strategies for solving equations. To apply this technique, the teacher asked students to "invest" a dollar in a set of strategies for solving equations by considering which strategy was most likely to be more effective. Students discussed their solutions strategy, shared their thinking aloud, and listened to the thoughts of their peers. Since they communicated their thinking during the activity, the Spend a Buck strategy also helped students engage in metacognitive behaviors, including *planning* solutions, *debugging* progress, and *evaluating* their results. As a result, all these activities fostered communication and metacognition through an effective mathematical discourse, which is crucial to help students build their ideas about mathematics.

To sum up, the review of the MTLT articles that focused on mathematical discourse showed that mathematical discourse is an essential element of any kind of mathematics class, regardless of the focus, content, and grade level. Some critical common strategies of facilitating mathematical discourse discussed in most articles were choosing high-cognitive tasks, assessing and advancing students' thinking by using purposeful questions, encouraging students to share their perspectives and respond to others' perspectives, and anticipating students' possible needs responses to lead a meaningful discussion.

Conceptual Understanding (Mathematical Thinking and Reasoning)

The analysis revealed that conceptual understanding was also one of the most common focuses across the MTLT articles. We grouped the following codes and developed the conceptual understanding category: leveraging students` thinking, meaningful mathematics learning, making sense of mathematics, strategic sense-making, eliciting, and using evidence of students` thinking. A variety of activities, strategies, tools, and teaching methods that promote conceptual understanding and leverage students` thinking was provided in the articles. Some of these tools and strategies that enable conceptual understanding could be listed as mathematical action technologies, real-life applications, exploration, scaffolding, gameplay, and choral counting.

Davis (2021) used TI-Nspire, and Harrow and Merchant (2020) used "Desmos" in order to explore how mathematical action technologies make the static more dynamic. Davis designed a lesson with the TI-Nspire calculators to teach the system of linear inequalities rather than using the test-point procedure that many textbooks have used. The graphing calculator let the students move the point in each region and even on the solid and dashed lines to see if the point was a solution for both inequalities. In this way, students could make conjectures and generalizations about when to use solid lines or dashed or what region was supposed to be shaded. Harrow and Merchant (2020) also found that the Desmos enhanced visualizations improved conceptual understanding across cartesian and polar coordinate systems.

On the other hand, Stephan et al. (2020) explored that real-life context plays a crucial role in learning mathematics conceptually by creating connections between abstract and concrete thinking and engaging students in conceptual mathematical discourse. Roy et al. (2020) also used a real-life application task in which students were asked how to order chicken nuggets from their favorite restaurant for the eighth-grade holiday party. Given that the restaurant sells the nuggets in packages of 6, 9, or 20, their goal was to determine any number of chicken nuggets from 1 to 100 that they could order. The teacher positioned the students as capable contributors and engaged students by using mathematical discourse during the instruction. By the end of the lesson, students were able to leverage their conceptual understanding of algebraic reasoning. Naanou and Rhodes (2020) also used real-life context, focusing on how culturally relevant experiences may promote cultural and conceptual understanding. They designed a lesson about folding (asafiri or crescent) qatayef (a special cultural dessert made on Saint Barbara's Day) to maximize the amount of filling in each qatayef.

Besides, Frank (2021) claimed that lessons that promote conceptual understanding let students learn mathematical structure, not just computations. In Frank's lesson implementation, exploring the quadratic formula allowed students to understand the structure of algebraic representations. Students derived the quadratic formula by quantifying the symmetry of a parabola. At the end of the activity, students were able to use the quadratic formula to solve quadratic equations and reason about the structure of the quadratic formula. Rakes et al. (2020) also showed that a procedural concept such as simplifying radical expressions could be taught in a way that helps students understand conceptually and create a stronger foundation for building on the knowledge rather than memorization. In their lesson, students explored radical simplification through geometry connections, in other words, through semi-concrete geometric representations. Besides, Lischka and Stephens (2020) showed how overarching concepts and models could build connections and sense-making. They provided examples of using the area model in different grade levels and enhancing students' conceptual understanding. They discussed that while the area model is used to multiply whole numbers in the third grade, it could also be used to multiply binomials or factors in Algebra. If students can use the area model across the grade levels and concepts, they could use it to make sense of mathematics (SMP1) and understand the structure of mathematical concepts (SMP7).

In addition to using technology, real-life applications, or geometric representations of procedures, scaffolding was also suggested as one of the strategies to promote conceptual understanding emphasized by Coleman (2020). Coleman (2020) designed a lesson for 6th-grade students who have significant gaps in the content knowledge and showed how scaffolding and building a bridge from a student's basic understanding of an intentionally selected grade-level topic to a strong foundation in the grade-level topic improve their conceptual understanding and procedural fluency.

Game-playing (MacDonald et al., 2020) and choral counting (McMillan & Sagun, 2020) were pointed out as other tools that promote conceptual understanding. MacDonald et al. (2020) showed how gameplays with dominoes could leverage students` quantitative and additive reasoning and how using hands-on tools and game-playing enhanced students` conceptual understanding of subitizing and unit coordination. McMillan and Sagun (2020) used choral counting to engage students in instruction. In their lesson implementation, students counted out loud; and the teacher charted each number in an organized way and asked students what they noticed about the numbers. Students recognized patterns and explored additive or multiplicative ideas using choral counting, which leveraged students' thinking and built conceptual understanding.

To sum up, the conceptual understanding was the second frequent theme emphasized in the MTLT articles over the last two years. Classroom stories presented various strategies and tools that can enhance conceptual understanding and leverage students` thinking. The main idea across the presented strategies and tools was helping students visualize mathematics by using technology or real-life or pictorial application; and assisting students in developing reasoning skills through games, interactions, and discussions.

Task Selection

Task selection has been highly emphasized in the reviewed MTLT articles. The authors presented diverse perspectives on task selection. Thus, four main codes emerged in the reviewed articles focusing on task selection: perseverance/curiosity/engagement/productive struggle; features of the quality tasks; and instructional approaches for implementing quality tasks. The following sections provide specific examples from some articles to illustrate how task selection was discussed in the reviewed articles.

Perseverance/Curiosity/Engagement/Productive Struggle

Task selection was often discussed and studied in relation to SMP1, making sense of problems and persevering in solving them. For example, Lischka et al. (2020) suggested implementing extended tasks to promote perseverance in students. These extended tasks must have multiple entry points and solution approaches within students' current understanding offering perspectives on future math topics. The swimmer task example was given as an extended task, and how it let two students persevere in solving the problem was reported in the article:

Two swimmers start at opposite ends of a 90 feet rectangular lap pool, each in their lane. One swims 30 feet per minute, and the other swim 20 feet per minute. If they swim for 30 minutes, how many times will they meet each other, and at what times will they meet? (p. 49).

This problem can be solved with both simple and complex strategies to promote perseverance in students. Similar to implementing extended tasks, Gilbertson (2021) presented "no-solution problems" to engage students in sense-making and perseverance in problem-solving. Gilbertson argued that "no-solution problems" provide students with multiple opportunities:

- Motivating meaningful discussion and positive engagement
- Establishing mathematical truth
- Sparking curiosity through open problems
- Motivating new mathematics
- No solution in the real world
- Engaging in mathematical practices (p.209-210)

Yee et al. (2020), Manousaridis (2021), and Taylor and Lee (2021) articulated the productive struggle, which is "a necessary facet of effective mathematics teaching" (p.124) via the example of STEM Rocket Launch Task. NCTM (2014) indicated that "Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and support to engage in productive struggles as they grapple with mathematical ideas and relationships" (p.10). STEM Rocket Launch Task required high school students to design a prototype rocket, fly it, and make revisions based on its flight. This task not only emphasized that "mistakes are valued" but also "persistence is valued" in classrooms engaging in productive struggle.

On the other hand, Manousaridis (2021) suggested using tasks promoting curiosity in students. The authors asked the following subtraction questions to the students:

$54 - 45 = ?$
$43 - 43 = ?$
$98 - 89 = ?$

Once students recognized the result of all problems equal to nine, they became curious about why it was always nine, which led to a great discussion in the classroom. Manousaridis (2021) confirmed 'Warshauer's (2015) claim that such tasks genuinely engage students in productive struggle.

Features of the Tasks

Features of the tasks that could lead to desired learnings and understandings in students were described in a variety of ways in MTLT articles. For example, Bostic et al. (2021) claimed that implementing mathematics tasks with high cognitive demand and multiple entry points (i.e., low-floor and high-ceiling tasks) is essential in fostering rich mathematics for each student, while O'Connor et al. (2021) described such tasks as simply high level. Similarly, Ahrendt et al. (2021) pointed to the significance of choosing high-level tasks that support students in meeting the goal of the lesson before implementing Smith and Stein's (2011) five practices for orchestrating productive discussions in the classroom.

Russo et al. (2020) provided a more comprehensive description of features of tasks: meaningful context, visualization, offering opportunities to make predictions, and comparison to other concepts. On the other hand, Wasserman et al. (2020) highlighted the desired task features as encouraging problem-solving, offering conceptual insights, and preparing students for future mathematics studies.

While Jung and Brand (2021) offered social-justice-oriented mathematical tasks to view mathematics as a way of understanding social justice issues, Barlow et al. (2020) relied on Substitution, Augmentation, Modification, and Redefinition. (SAMR) model (Puendetura, 2013) that intended to encourage educators to significantly enhance the quality of education provided via technology for selecting tasks from MTLT front-and-center articles to use in a virtual setting.

Instructional Approach

MTLT articles relating to task selection with desired features also proposed some instructional approaches for successful classroom implementation. For example, for word problems that are challenging to understand and solve, Melissa et al. (2021) offered reading comprehension strategies that could help students make sense of the context while solving: visualizing, retelling, making connections, and asking questions. By interviewing ten Algebra 2 teachers, Snider (2021) found that teachers employed a variety of considerations as general and specific examples, misconceptions, splitting a concept or procedure into cases or essential ideas, boundary cases, non-examples, using examples to cause cognitive dissonance, sequencing different types of examples, how students will engage with the examples, and coherence across lessons and units.

While Ivy et al. (2020) emphasized the importance of asking "why" questions while students are solving reversibility tasks which are planned to build flexibility in students' thinking, Lim (2020) offered Superficially Similar but Structurally Different (SS-SD) problems as a way of addressing hammer-and-nail phenomenon which is a tendency of students' using a proportion (hammer) to solve a missing-value problem even when it is non-proportional (mistaken as a nail) (p.488).

Brakoniecki et al. (2021) presented a common proportional reasoning problem, the orange juice task, which offers multiple solution strategies to solve the problem. They implemented the presentation of multiple strategies in response to the problem as an instructional strategy. The strategies were unitizing, partitioning, building up, norming, reasoning up and down, using multiplicative comparison, and percentage and decimal comparison.

Real-Life Applications

Real-life applications are also highly emphasized in the reviewed articles. The real-life application can be categorized as real-life contexts and multiple representations based on our coding criteria. The real-life context includes the following codes: Situation-oriented contexts, real-world contexts, real-world problems, real-life experiences, real-life applications, real-life problem solving, and real-life situations. The multiple representations include the following codes: Mathematical representations, visual representations, and using visual representations.

Real-Life Context

Effective mathematics teaching practices allow students to learn new mathematics by solving contextual problems (NCTM, 2014). Most mathematics teachers receive the question of "when/where will we use mathematics in real life?" from their students. Therefore, there is a need to implement activities that will allow students to make connections between school mathematics and real life. Cunningham (2021) claimed that mathematics needs to permeate the school day to assist students in making connections between mathematics at school and the mathematics they encounter in everyday life. Bartell (2020) stated that reimagining homework and assigning work that released information about students` backgrounds and lived experiences empowered the way of mathematical teaching and learning.

Stephan et al. (2020) used the work of several middle school teachers who used realistic contexts as both hooks and anchors to illuminate three features of real-world contexts: (1) support the transition from concrete to abstract reasoning; (2) engage students conceptually mathematical discourse; and (3) capitalize on real-world and experiential real-world situations. They concluded that real-life contexts increase students' interest and motivation and demonstrate how mathematics is used to make sense of their world. Also, Brown (2021) found that real-world tasks provide students with learning opportunities that allow mathematics to be used to understand the world, strengthen mathematical knowledge, and provide students with opportunities to use mathematics to analyze the world they live in.

Multiple Representations

Stohlmann (2020b) stated that teachers could support students' persistence in problem-solving by explaining how to be successful in an escape room, emphasizing and supporting productive teamwork, and combining multiple representations. The author chose the escape room challenges with these ideas in mind, so students solve the challenges by using multiple representations. The escape room challenge task had seven questions, and students were encouraged to use multiple representations to solve problems.

Students increase their problem-solving skills by choosing among many representations that they believe will most likely lead to the solution of a problem (NCTM, 2000). Monson et al. (2020) discussed that it is essential to represent increasingly complex ideas about fractional multiplication and division concepts, such as selecting problem-solving strategies and using mathematical representations to solve problems. Using multiple representations to teach key fraction ideas in learning mathematics expanded students' experience to develop a solid understanding of units, divisions, and fractions (Monson et al., 2020). The authors found the most effective teaching models for building fraction understanding, including fraction circles, paper-folding strips, and number lines.

CONCLUSION

The front and center, focus, and feature articles of 21 issues of MTLT were systematically reviewed by four researchers to examine the trends of reform-based instruction focused in MTLT articles. MTLT is the only practitioner journal that NCTM, one of the leading organizations in mathematics education, has been publishing since 2020. The analysis of the reviewed articles revealed information about the insight into what the authors of these articles and editors of MTLT journal feel is vital regarding reform-based instruction. The results showed that the most frequently published reform-based teaching practices were mathematical discourse, conceptual understanding, task selection, and real-life applications. Mathematical modeling, technology integration into teaching mathematics, questioning, gaming, interdisciplinary mathematics were the practices that were also paid attention to in the reviewed articles. Some of the infrequent practices were anticipating students` thinking, attending to precision, estimation, feedback, fluency, generalizing, and more.

MTLT articles provided many examples about reform-based mathematics teaching practices and how these practices promote deep learning of mathematics. The mathematical discourse was the most frequent practice across the reviewed articles over the last two years. Mathematical discourse is a set of tools and practices which give students opportunities to share their ideas and make their understandings clear, construct convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives by including purposeful exchange of ideas through classroom discussion and other forms of verbal, visual, and written communication (NCTM 1991, 2000, as cited in NCTM 2014 p.29). While some of the articles focused on discourse actions as *teacher's press student's providing*, *teacher's linking,* and *student's linking* (Anderson, 2021; Candela et al., 2020), some of them provided practices as choosing mathematical goals, anticipating students` thinking that will provide opportunities to have productive mathematical discussions (Ahrendt et al., 2021).

The second most frequent practice was conceptual understanding and leveraging students` thinking. The articles coded in this category provided strategies, tools, and practices that promote conceptual understanding. These strategies and tools could be listed as integrating mathematical action technologies, real-life application, activities that allow students to explore, scaffolding between grade levels, game-playing, and choral counting. Conceptual understanding is one of the five strands of mathematical proficiency and is defined as comprehension of mathematical concepts, relations, and operations (NRC, 2001). Conceptual understanding establishes the foundation needed for procedural fluency as it is emphasized in building procedural fluency from conceptual understanding effective mathematics teaching practice by NCTM (2014).

Task selection was the third most frequent focus in the reviewed articles. Roy et al. (2020) pointed out that one of the most critical features of effective mathematics teaching is task selection, allowing students to explore and discuss mathematics. While Gilbertson (2021) provided an example of "no solution problems" for sense-making and reasoning, Taylor and Lee (2021) presented the STEM Rocket Launch task to point out that mistakes and persistence are valued. Effective teaching of mathematics and reform-based mathematics teaching practices encourage teachers to engage students in solving and discussing tasks that promote mathematical thinking, reasoning, and problem-solving (NCTM, 2014). These tasks allow multiple-entry points, a variety of ways of representations, tools, and methods.

Last but not least, the fourth most frequent trend in the reviewed articles was real-life applications categorized as real-life context and multiple representations. Real-life applications allow students to connect school mathematics and everyday life (Cunningham, 2021). Mathematical concepts could be represented through multiple representations: symbolic, verbal, graphical, and numeric (NCTM, 2000). NCTM (2014) identified several key aspects of what students should do in the classroom: (a) using multiple representations to make sense and understand mathematics; (b) explaining and verifying their mathematical understanding and reasoning with drawings, diagrams, and other representations; and (c) contextualizing mathematical ideas by connecting them to real-world situations. By linking the mathematical content and impactful practices such as connecting mathematical representations and implementing tasks promote reasoning and problem-solving, powerful mathematics learning experiences, and meaningful classroom discourse (NCTM, 2014).

In summary, these four trending practices are central to reform-based mathematics teaching as mathematical discourse, conceptual understanding, mathematical thinking, integrating real-life application to mathematics teaching, using multiple representations are often pointed out as the prominent aspects of reform-based teaching by mathematics educators (e.g., Hiebert et al., 2003; Joram & Gabriele, 2012; Spangler & Wanko, 2017). On the other hand, other prominent aspects that are central to reform-based teaching practices, such as setting instructional goals (Hiebert et al., 2003) and problem-solving (NCTM, 2014), did not receive as much attention in the reviewed articles. It was surprising that only a few articles have focused on problem-solving per se in MTLT, although problem-solving is identified as one of the eight effective mathematics teaching practices; and making sense of problems and persevering in solving them is one of the eight SMPs in Common Core State Standards (2010). Although problem-solving was not often the main focus and goal of the articles, it was used as a tool for discourse and or conceptual understanding. We also noticed that each of the four most trendy practices is closely related to NCTM's (2014) eight effective mathematics teaching practices. For instance, while the mathematical discourse theme refers to facilitating meaningful mathematical discourse, the conceptual understanding theme directly links to building procedural fluency from conceptual understanding and has indirect links to elicit and use evidence of students` thinking. Besides, one can see explicit connections between the task selection theme and implementing tasks that promote problem-solving and reasoning, and between the real-life application theme and using and connecting mathematical representations. Other effective mathematics teaching practices, such as asking purposeful questions, were a stand-alone focus only in a few articles. Yet, some articles discussed it as a strategy supporting mathematical discourse. Anderson (2021) pointed to the challenges of implementing this strategy: reducing the task's cognitive demand and shaping students' reasoning. This is only one example of the challenges of implementing reform-based teaching practices mentioned in the reviewed articles.

Interestingly, only a few articles focused on productive struggle despite its popularity both in informal and formal education forums and presentations. This could be because of the challenge in observing productive struggle in students, and it's one of the most challenging practices among the eight effective mathematics teaching practices for teachers. When the United States is compared to other countries, students are rarely asked reasoning and thinking questions about mathematical ideas (Banilower et al., 2006; Heibert & Stigler, 2004). United States teachers want to rescue their students because they may perceive students' frustration as an immediate failure (NCTM, 2014). On the other hand, it is surprising that only a single article was found that focused on establishing focused mathematics goals. Although setting instructional goals is emphasized as an aspect of reform-based teaching, and also it is one of the NCTM's effective teaching practices, the reviewed articles paid no attention to this practice. Establishing clear goals helps teachers make decisions about the tasks and activities and provide opportunities for students to check their progress (NCTM, 2014). These eight effective mathematics teaching practices represent a core set of high-leverage practices that promote deep learning of mathematics. Therefore, each practice needs attention and should be observable in the MTLT articles. Problem-solving, supporting productive struggle, and establishing mathematical goals need more attention so that practitioners may better understand how to integrate these practices into teaching.

TAKE-AWAYS, FUTURE DIRECTIONS, AND LIMITATIONS

The trends that were focused on the most frequently by experts in mathematics education provide valuable information about the reform-based mathematics teaching practices and their impacts on students` performance. Classroom teachers may benefit from learning specific techniques in support of each practice that they can implement in their classrooms.

This chapter is limited to reviewing only 21 issues of MTLT. Other three journals of NCTM that are published until 2020 were not reviewed and included in this chapter. This is relatively small sample size when the number of journals is considered. Future researchers could review more articles from different journals and compare the most common trends in each journal. Besides, reviewing more articles may provide more information about experts` perspectives on reform-based mathematics teaching practices.

Future researchers may also group the articles according to the grade level as elementary, middle, high school and compare the most frequent reform-based mathematics teaching practices focused by experts.

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