Identifying price bubble periods in the energy sector

Shahil Sharma  
*The University of Texas Rio Grande Valley*

Diego Escobari  
*The University of Texas Rio Grande Valley*

Follow this and additional works at: [https://scholarworks.utrgv.edu/ef_fac](https://scholarworks.utrgv.edu/ef_fac)

Part of the [Finance Commons](https://scholarworks.utrgv.edu/finance_commons)

**Recommended Citation**

This Article is brought to you for free and open access by the Robert C. Vackar College of Business & Entrepreneurship at ScholarWorks @ UTRGV. It has been accepted for inclusion in Economics and Finance Faculty Publications and Presentations by an authorized administrator of ScholarWorks @ UTRGV. For more information, please contact justin.white@utrgv.edu, william.flores01@utrgv.edu.
Identifying Price Bubble Periods in the Energy Sector

Shahil Sharma‡  Diego Escobari‡

November 2017

Abstract

In this paper we test for the existence of single and multiple episodes of explosive behavior in three energy sector indices (crude oil, heating oil, and natural gas) and five energy sector spot prices (West Texas Intermediate (WTI), Brent, heating oil, natural gas, and jet fuel). The results from the Supremum Augmented Dickey-Fuller (SADF) and the Generalized SADF tests provide strong statistical evidence of explosive behavior in all of our energy series. A simple theoretical framework of commodity pricing allows us to understand the assumptions to interpret explosive behavior as bubbles. By constructing implied convenience yields using futures prices we test the key assumption and we are able to identify the beginning and the end of bubble periods for the WTI, Brent, heating oil, and natural gas spot prices.

Keywords: Generalized SADF, Energy, Oil, Explosive behavior, Bubbles.

JEL classifications: C22, Q02, Q43

‡ Department of Economics and Finance, The University of Texas Rio Grande Valley, Edinburg, TX 78539, Phone (956) 665-2590, Fax: (956) 665-5020, Email: shahil.sharma@utrgv.edu
‡ Department of Economics and Finance, The University of Texas Rio Grande Valley, Edinburg, TX 78539, Phone (956) 665-3366, Fax: (956) 665-5020, Email: diego.escobari@utrgv.edu, URL: http://faculty.utrgv.edu/diego.escobari
1. Introduction

The rapid growth of the energy sector in recent years, along with its influence in equity markets and the global economy has lured the attention of a growing number of investors. According to the U.S. Energy Information Administration (EIA),\(^1\) the average U.S. household expenditure on gasoline in 2015 was estimated to be about $1,817, the lowest level in more than a decade. Lower energy cost provides the average consumer a wage increase, which boosts the overall economy by improving spending power. The U.S. is rather a commodity buyer than a seller; hence, the low energy prices are a plus, as long as prices do not fall rapidly at an unsustainable pace that they may trigger financial problems. Previous periods of relatively high price volatility of energy commodities (e.g., the 1990 Gulf War, the September 11 attacks, or the 2007-2009 global crisis) led to an increase in the discussion of energy markets from regulators, public, and market participants. Studying energy commodities’ behavior is important due to energy prices direct and indirect impact on consumers, other commodities, equity markets, and the local and global economy.

We have seen repeatedly the connection between asset and commodity pricing bubbles and the economy. Such association was recently observed during the financial crisis of 2007-2009, which most agree erupted from bursting of the U.S. housing bubble. Financial crises are often preceded by a widespread price bubble, and the global financial crisis of 2007-2009 was no exception. It is a complex task to precisely justify an asset bubble rise and bust, however, slowing output growth, widening credit spreads, slumping purchasing-manager indices, declining corporate earnings, falling inflation expectations, rising oil prices and rising inventories, can all signal an upcoming recession. The credit crunch, tightening monetary policy, and the role of

---

\(^1\) http://www.eia.gov/todayinenergy/detail.php?id=20752
high-energy prices in the collapsing world economy – overlooked by the regulatory bodies – are widely considered as the root cause for the 2007 crash. However, it is difficult to assign weights to specific factors when analyzing the causes for recessions. Historically, economists have struggled to disentangle the influence of higher oil prices, tightening monetary policy, and credit markets in triggering U.S. recessions (Barsky and Kilian, 2002). In the past, spiking oil prices in 1990, 2001, and 2007 contributed to some degree to the global economic recessions of 1991-1992, 2001-2003, and 2007-2009 (see, e.g., Barsky and Kilian, 2004; and Hamilton, 2009). Kilian and Vigfusson (2017) study oil price shocks as one of the leading factors in explaining U.S. recessions and show that the explanatory power of oil price shocks holds even after accounting for a measure of credit supply, monetary policy stance, and consumer confidence.

Energy commodity prices rose steadily from late the 1990s until the financial crisis in 2007, in part due to strong demand from China and other emerging markets. In recent years, China has announced a transition from an industrial to a service oriented economy, i.e., less demand for energy. According to the World Bank statistics, China’s economic growth rate has been declining in recent years. It appears that there is something more than just the supply side driving energy prices, as the declining industrial demand for energy commodities might also be playing a role. This signals that the global economy may be slowing down more than anticipated. That is mainly troubling because many countries and their respective governments have so much debt, that they cannot do much to fight a recession. In addition, most of the central banks around the world are keeping interest rates close to zero; hence, there might not be much they can do to balance a toppling economy.

---

The expansion over the past decade of unregulated international derivatives trading in the energy sector, especially oil futures, might have contributed to the origination of price bubbles in energy sector before the 2007 financial crisis. In June of 2006, the U.S. Senate Permanent Subcommittee on Investigations report on “The Role of Market Speculation in Oil and Gas Prices…,” noted that “there is considerable evidence supporting the conclusion that the increases in energy prices are a significant result of the large amount of speculation in the current market”. Taking inference from this U.S senate report, we conjecture that the strong outperformance might have been driven by a price bubble. Bohl et al. (2013) shows existence of explosive price behavior in German renewal energy stocks by implementing the Supremum Augmented Dickey-Fuller (SADF) test proposed in Phillips, Wu and Yu (2011, PWY henceforth) and a Markov regime-switching ADF test. More recently, Gronwald (2016) uses the SADF to find evidence of explosive behavior in the oil price series. Caspi et al. (2015) implements the more recent Generalized SADF (GSADF) proposed by Phillips, Shi and Yu (2015, PSY henceforth) to find explosivity in oil prices. The methods in PSY improves the discriminatory power of PWY when more than one period of explosive behavior is present.

In this paper we use time series data from three energy indices (crude oil, heating oil, and natural gas) as well as five energy spot prices (West Texas Intermediate, Brent, heating oil, natural gas, and jet fuel). We initially follow the same approach as in Gronwald (2016) and employ the methods in PWY to find evidence of explosive behavior in our series. Our initial empirical approach is also similar to Caspi et al. (2015) as we employ PSY to identify multiple
episodes of explosive behavior. After identifying episodes of explosive behavior, we further our analysis by presenting a simple theoretical framework of energy commodity pricing. This framework allows us to understand the assumptions to interpret explosive behavior in our energy spot prices as bubbles. In particular, the key assumption is that the corresponding convenience yields is not explosive. We use data on futures and Pindyck’s (1993) implied convenience yield to test this assumption. Because the methods in PSY also help us to date-stamp the periods of explosive behavior, testing for explosiveness in implied convenience yields is helpful to assess which dates of explosive behavior in an energy spot prices series can be classified as bubbles.

Our results show strong evidence of explosive behavior episodes in each of our eight energy series. Moreover, we are able to date-stamp the beginning and end periods of each episode of price explosiveness using the 95% critical values obtained from Monte Carlo simulations. For the real crude oil index, the episodes of explosive behavior match the years of the Gulf War, and the years leading to the Asian crisis and the 2007-2009 global recession. Overall, most episodes identified by the test statistics are short-lived (i.e., lasting fewer than 12 weeks) with several of the episodes being consistent across crude oil and its derivatives, including an episode of price implosion around 2015. We also analyze an alternative source of energy (i.e., natural gas), which showed a significantly different pattern of explosive behavior dates compared to crude oil and its derivatives. After constructing the implied convenience yields for four of our energy spot prices, we find strong evidence supporting the assumption of non-explosiveness for the West Texas Intermediate (WTI), Brent and natural gas throughout our period of study. For the heating oil, we

---

3 Previous work that uses the SADF and GSADF to study explosive behavior and bubbles in agricultural commodities as well as in other prices and indices include Gilbert (2010), Phillips and Yu (2011), Gutierrez (2013), and Escobari and Jafarinejad (2015).

4 Natural gas supply typically depends on expectations of sufficiently high long-term prices. Moreover, the demand for natural gas is far more consistent compared to volatile crude oil. On the one hand, natural gas suffers price stagnancy but on the other hand, its price does not fluctuate much during economic downturns.
are able to identify the periods in which the assumption holds. This allows us to interpret price explosiveness as evidence of price bubbles.

Our results are additionally important given the link between energy markets and macroeconomic factors. Hamilton (1983) has documented a strong correlation between crude oil price changes and the U.S. GNP growth (see, e.g., Mork, 1989; Lee et al., 1995; and Gronwald, 2008). Energy price shocks can affect corporate cash flows since energy is an input in production and because energy price changes can influence the demand for output at industry and national levels. Energy price shocks can affect the discount rate for cash flows by influencing the expected rate of inflation and the expected real interest rate. Recent trends in energy prices have been widely discussed in the regulatory and public arena, and have been linked with consumer spending, prices of other commodities, and performance in financial markets. On the effect of oil price shocks on stock market returns, Jones and Kaul (1996) and Sadorsky (1999) report a significant negative connection (see also, e.g., Chen et al., 1986; and Huang et al., 1996).  

Nandha and Faff (2008) find that oil price rises have a detrimental effect on stock returns in all sectors except mining and the oil and gas industries. O'Neill et al. (2008) find that oil price increases lead to reduced stock returns in the U.S., the United Kingdom and France, while Park and Ratti (2008) report that oil price shocks have a statistically significant negative impact on real stock returns in the U.S. and 12 European oil-importing countries. In new strands in the literature, Kilian and Park (2009) report that only oil price increases driven by precautionary demand for oil negatively affect stock prices. Gogineni (2007) finds that industry stock price returns depend on demand and cost side reliance on oil and on the size of oil price changes.

\[5\] Sadorsky (2012) employs several multivariate GARCH models to study volatility dynamics of alternative energy stocks.
Our work is additionally related to previous studies that have proposed and implemented different time series methods to capture bubbles in asset and commodity prices. This includes integration and cointegration tests (Diba and Grossman, 1988a, 1988b), variance bound tests (LeRoy and Poter, 1981; Shiller, 1981), specification tests (West, 1988) as well as Chow and CUSUM-type tests (Homm and Breitung, 2012).

The remainder of the paper proceeds as follows. Section 2 describes the data, while Section 3 describes a theoretical framework of commodity prices to understand the conditions under which explosive behavior can be interpreted as a bubble. Section 4 presents the empirical approach, while Section 5 describes and discusses the results. Section 6 concludes.

2. Data

Our time series weekly data contains three value-weighted indices, five spot prices, and four futures prices. Table 1 presents the descriptive statistics for all twelve series. We have series with different starting dates due to data availability. For the crude oil index, WTI spot, heating oil spot, and for three of our futures series (i.e., WTI, heating oil, and natural gas) the sample starts on May 22, 1987, while the sample start date for other series varies as reported in Table 1. We use the Consumer Price Index (CPI), as obtained from the Federal Reserve Bank of St. Louis, to adjust for inflation.

[Table 1, about here]

The source of all price series is the U.S. EIA, further retrieved from Thomson Reuters. The jet fuel spot price is a type of United States Gulf Coast spot price free on board measured in US$ per gallon. The heating oil number 2 New York harbor spot price is free on board, also reported in US$ per gallon. The crude oil WTI Cushing and the crude oil Brent spot prices are free on
board, reported in US$ per barrel. The natural gas spot price is the Henry Hub Spot Price captured in US$ per Million BTU. For the futures prices we use the nominal futures contract specifying the earliest delivery date and adjust them using the CPI. In particular, for heating oil, futures contract expires on the last business day of the month preceding the delivery month. Moreover, for the Brent and WTI crude oil, the futures contract expires on the third business day prior to the 25th calendar day of the month preceding the delivery month. A futures contract for natural gas expires three days prior to the first calendar day of the delivery month. If the calendar day is non-business day, trading concludes on the third business day prior to the business day preceding the calendar day.

Panel A in Table 1 reports the summary statistics for the indices, while Panels B and C report spot prices and futures respectively. There is no historical data available for jet fuel futures. Pindyck (2001) explains that heating oil and gasoline futures can help airlines to hedge their exposure to the price of jet fuel. Because the empirical strategy deals with individual series, we do not need to have them in the same measurement units. If our goal were to compare the spot prices across energy commodities rather that testing for explosive behavior, we might have needed to convert all the series to the same units, for example, US$ per gallon.

[Figure 1, about here]

---

6 The crude oil index is traded at the New York Mercantile Exchange (NYMEX), the contract size is 1,000 and the contract unit is in US$ per barrel. The heating oil index is also traded at the NYMEX, with the contract size being 21,000 and the contract unit being in US$ per U.S. gallon. Moreover, the natural gas index is traded at the NYMEX as well with the contract size being 10,000 and the contract unit being in US$ per Million BTU. All indices were obtained from Thomson Reuters.

7 Following definitions from the U.S. EIA, these are all futures contract 1.

8 One U.S. barrel is equal to 42 gallons and 1 Million BTU is equal to 8.0074 gallons.
Figure 1 provides the time-series graphs for the three indices and the five spot prices series that we study. An interesting feature on these graphs occurs during the 2007-2009 recession, where most appear to have experienced a hike.

3. Modeling Bubbles

The idea that asset prices can deviate from their intrinsic values based on market fundamentals because of bubbles is widely accepted. While most of the work on bubbles has focused on asset prices, many models also explain the existence of bubbles in commodity prices. Following Campbell and Shiller’s (1988) model on bubbles for asset prices, we now present a conceptual framework for bubbles using the present value model of rational commodity pricing. The model starts with the following equation:

\[ P_t = \frac{E_t(P_{t+1} + C_{t+1})}{1 + R}, \]  

where \( P_t \) is the real commodity price at time \( t \), \( C_t \) is the convenience yield for the storable commodity, and \( R > 0 \) is the constant discount rate. The convenience yield is a function of implicit and explicit advantages derived from having instant access to the commodity held in inventory, and reflects the ability to speculate in the price appreciation of the underlying asset as well as the benefits stemming from possible alternative uses of the held inventory. Equation (1) is also used in Pindyck (1993) and Gutierrez (2013) to explain the pricing of storable commodities. In case of energy commodities the aggregated storage cost is always positive. Pindyck (1993) argues that

---


10 PWY and PSY used similar setting to derive bubbles for stock markets. Instead of our convenience yield, \( C_t \), they use the real dividend received from owning the asset.
this present value rational commodity pricing model can be viewed as a highly reduced form of a
dynamic supply and demand model.

To explain price exuberance in the commodity price series, we follow Campbell, Lo and
MacKinlay (1998) to obtain the log-linear approximation of equation (1). After taking logs of
both sides of equation (1), we approximate the nonlinear function by using the first-order Taylor
expansion of the arguments. We then obtain the following solution to the difference equation (1)
using the law of iterated expectations:

\[ p_t = p_t^f + b_t. \]  
(2)

Equation (2) illustrates how the logarithm of the commodity price, \( p_t = \log(P_t) \), can be
explained by the fundamental price \( p_t^f \) and a bubble \( b_t \), both expressed in natural logarithms.\(^{11}\)
Campbell and Shiller (1988) derived each of these components as follows:

\[ p_t^f = \frac{\kappa - \gamma}{1 - \rho} + (1 + \rho) \sum_{i=0}^{\infty} \rho^i E_t c_{t+1+i}, \]  
(3)

\[ b_t = \lim_{i \to \infty} \rho^i E_t p_{t+i}, \]  
(4)

\[ E_t(b_{t+1}) = \frac{1}{\rho} b_t = (1 + \exp(c - \bar{p})) b_t, \]  
(5)

where \( c_t = \log(C_t), \gamma = \log(1 + R), \rho = 1/(1 + \exp(c - \bar{p})) \), with \( c - \bar{p} \) being the average
convenience yield–price ratio, \( 0 < \rho < 1 \). Note from (3) that the price of the fundamentals is
exclusively determined by the expected convenience yields. Moreover,

\(^{11}\) Equation (2) is consistent, for example, with Stiglitz (1990), who explains the existence of bubbles as movements
in asset prices that can be based on the self-fulfilling forecasts of the market participants and when “fundamental”
factors do not seem to justify such a price.
\[ \kappa = -\log(\rho) - (1 - \rho) \log \left( \frac{1}{\rho} - 1 \right). \]

Because \( \exp(c - p) > 0 \), the rational bubble \( b_t \) is a submartingale process and is explosive in expectations. From equation (5) we have:

\[ b_t = \frac{1}{\rho} b_{t-1} + \varepsilon_{b,t} \equiv (1 + g) b_{t-1} + \varepsilon_{b,t}, \tag{6} \]

where \( E_{t-1}(\varepsilon_{b,t}) = 0 \), and with \( \varepsilon_{b,t} \) being a martingale difference sequence. Moreover, the growth rate of the logarithm of the bubbles is given by \( g = \frac{1}{\rho} - 1 = \exp(c - p) > 0 \).

In case where there are no bubbles (i.e., \( b_t = 0, \forall t \)), equation (2) tells us that the price sequence is entirely determined by fundamentals, \( p_t = p_t^f \). Hence, as equation (3) suggests, prices are entirely determined by the discounted expected future convenience yield \( c_t \). From the same equation and under no bubbles we have

\[ c_t - p_t = -\frac{\kappa}{1 - \rho} - \sum_{i=0}^{\infty} \rho^i E_t(\Delta c_{t+1+i}). \tag{7} \]

If each of the left-hand side terms is integrated of order one, the stationarity of the right-hand side suggest that \( p_t \) and \( c_t \) are cointegrated with the cointegrating vector \([1, -1]\). That is, shocks to the difference \( c_t - p_t \) are only transitory.

On the other hand, under the existence of bubble episodes (i.e., \( b_t \neq 0 \)), the combination of equation (2) and the implied explosive behavior of \( b_t \) in equation (6) mean that the pricing sequence \( p_t \) will be explosive as well. This will be the case regardless of whether convenience yield \( c_t \) is stationary or integrated of order one. In addition, the first difference of \( p_t \) cannot be stationary as this difference sequence is also explosive. Diba and Grossman (1988a,b) use this result to test for explosive rational bubbles in stock prices, where the convenience yield in
equation (1) is simply replaced with the dividend series. Diba and Grossman (1988a,b) interpret the rejection of the unit root null in the first differences of $p_t$ as evidence that $p_t$ in levels is not explosive. Hence, they would conclude that there is no bubble.

A periodically collapsing bubble process given that a non-negligible probability of collapse exists, would behave as a process integrated of order one or as a stationary autoregressive process. Under this scenario, Evans (1991) showed with simulations the low power of the standard unit root tests used in Diba and Grossman (1988a). However, with a constant discount rate and given that $c_t$ is not explosive, equations (2) and (6) suggest that evidence of explosive behavior in $p_t$ would be a direct way to test for bubbles. In the following section we detail how recursive unit root tests can allow us to test for explosive behavior in the $p_t$ and $c_t$ series.

To be able to obtain a measure of the convenience yield $C_t$, we use futures prices. Following Pindyck (1993), we know that the convenience yield net of storage costs from date $t$ to $T$ and per unit of commodity, $C_{t,T}$, must satisfy:

$$C_{t,T} = (1 + r_T)P_t - f_{t,T}. \quad (8)$$

As before, $P_t$ is the commodity spot price. In addition, $f_{t,T}$ is the futures price for delivery at $t + T$, and $r_T$ is the risk-free $T$-period interest rate. We obtain the standardized convenience yield $C_t$ by dividing $C_{t,T}$ by the time to delivery. As explained in Lammerding et al. (2013), equation (8) holds under no arbitrage and for commodities with actively traded future contracts. It shows
an equilibrium condition where spot prices adjusted by the opportunity cost are equal to the benefits of holding the commodity.\(^\text{12}\)

4. Empirical Strategy

The empirical strategy follows the methods in PWY and PSY to test for the existence of single and multiple explosive behavior episodes using recurring estimations of the Augmented Dickey-Fuller (ADF) unit root test. These methods further allow us to date stamp the origin and collapse of the explosive behavior episodes. The main idea in these tests is to employ ADF-style regressions that shift the start and end dates of a rolling window. Both, the PWY and the PSY, start with the following ADF regression:

\[
\Delta y_t = a_{r_1,r_2} + \beta_{r_1,r_2} y_{t-1} + \sum_{i=1}^{k} \varphi_{r_1,r_2}^i \Delta y_{t-i} + \epsilon_t, \tag{9}
\]

where the \(y_t\) series will be replaced with either the logarithm of real energy spot price \(p_t\) or the corresponding logarithm of the convenience yield \(c_t\). \(\Delta y_t\) denotes first differences, and the error term \(\epsilon_t\) is expected to follow a normal distribution, i.e., \(\epsilon \sim iidN(0, \sigma_{r_1,r_2}^2)\). The \(k\) lagged difference terms are included to control for serial correlation. The subscripts \(r_1\) and \(r_2\) on the parameters to be estimated are the fractions of the total sample size and represent the starting and ending points of a subsample period. Note that not only the estimates, but also the error term variance depend on \(r_1\) and \(r_2\).

PWY discuss both, the left-side unit root tests and the right-sided unit root tests; however, we primarily focus in testing the unit root null hypothesis against the alternative of mildly explosive behavior in \(y_t\) using right-sided unit root tests. As discussed in PWY, right-sided unit

\(^{12}\) Pindyck (1993), Pindyck (2001), and Lammerding et al. (2013) explain the importance of incorporating the convenience yield in price formation for storable commodities such as crude oil or its derivatives, while Wei and Zhu (2006) considers the convenience yield for natural gas.
root tests are informative about mildly explosive behavior in the data and hence are useful as a form of market warning alert against mispricing. We are interested in the following test statistics:

\[ ADF_{r_1}^{r_2} = \frac{\hat{\beta}_{r_1, r_2}}{s.e(\hat{\beta}_{r_1, r_2})}. \]  

(10)

Note that when we set \( r_1 = 0 \) and \( r_1 = 1 \), we obtain the well-known standard form of the ADF test statistics. PWY propose a recursive procedure on the estimation of \( ADF_{r_1}^{r_2} \) using different subsamples of data to detect the occurrences of explosive behavior. The proposed test statistic is then the supremum value of the \( ADF_{0}^{r_2} \) on the forward recursive regression. This one is defined as:

\[ SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_{0}^{r_2}. \]  

(11)

The unit root null hypothesis is rejected in favor of explosive behavior when the SADF test statistic, as presented in equation (11), surpasses the right tale critical value. Homm and Breitung (2012) compared various econometric approaches similar in nature to find that this SADF test has greater power than that of methods proposed in Bhargava (1986), the modified Kim (2000), and the modified Busetti-Taylor statistics (Busetti and Taylor, 2004). In addition, Homm and Breitung (2012) and PSY argue that the PWY procedure works reasonably well against recursive procedures for structural breaks and is significantly effective as a method to detect explosive behavior in real-time. Notably, this technique can detect exuberance that may arise from various sources, such as mildly explosive behavior that may be prompted by altering fundamentals such as time preferences.

One concern with the SADF is that even though it performs well to identify a single boom and bust in a series, it may not consistently identify the origination and termination when
multiple episodes of exuberance are present. PSY propose the Generalized SADF (GSADF) to
deal with multiple events of boom and bust in a single series. The GSADF procedure follows the
idea of repeated ADF test regression on subsamples of data in a recursive fashion, covering a
broader number of subsamples than the SADF test. Unlike the SADF method, the GSADF not
only changes the initial observation of the subsample \((r_1)\), but also changes the end point \((r_2)\).
PSY describe the GSADF statistics to be the largest ADF statistic in this double recursion over
all feasible ranges of \(r_1\) and \(r_2\). The GSADF statistic is given by:

\[
GSADF(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} \sup_{r_2 \in [r_0, 1]} ADF_{r_1}^{r_2}.
\]

In Equation (12), if the GSADF\((r_0)\) statistic is greater than the right tail critical value, we then
reject the null in favor of the explosive alternative hypothesis.

It is possible that the data may include multiple bubbles episodes within a series, so the
ADF test, like earlier unit root and cointegration-based tests for explosive behavior, may find a
pseudo stationary behavior and is typically less successful in identifying subsequent bubbles
after the first (Evans, 1991). Therefore, to date stamp the origination and termination of the
bubble, we follow the PSY methodology where the proposed strategy relies on obtaining the
following Backward SADF (BSADF) statistic,

\[
BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}.
\]

The distributions of the \(GSADF(r_0)\) and the \(BSADF_{r_2}(r_0)\) test statistics in equations (12) and
(13) are non-standard. This means that we will perform Monte Carlo simulations to obtain the
critical values. Moreover, the BSADF enhances the bubble identification accuracy when
allowing for a flexible window in the double recursion. We define the initiation date of the
bubble as the first observation in which the BSADF statistic exceeds its corresponding critical value. This is given by,

\[
\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \{r_2: \text{BSADF}_{r_2} (r_0) > \text{scv}_{r_2}^\alpha \}. \tag{14}
\]

Likewise, using \( T \) to denote the total sample size, the termination date of a bubble is calculated as the first observation after \( \hat{r}_e + \frac{12}{T} \) in which the BSADF falls below its critical value,

\[
\hat{r}_f = \inf_{r_2 \in [\hat{r}_e + \frac{12}{T}, 1]} \{r_2: \text{BSADF}_{r_2} (r_0) < \text{scv}_{r_2}^\alpha \}. \tag{15}
\]

In equation (15) we have that \( \text{scv}_{r_2}^\alpha \) represents the \( 100(1 - \alpha)\% \) critical value of the SADF based on \( [r_2 T] \) observations and at a significance level \( \alpha \). The notation \( \lfloor . \rfloor \) is the floor function that gives the integer part of \( r_2 T \). Note that \( \frac{12}{T} \) in equation (15) is selected arbitrarily to make sure that explosive episodes last at least twelve weeks.

5. Results

Using equations (11) and (12) we report in Panel A of Table 2 the SADF and GSADF statistics for the three value-weighted indices. Moreover, Panels B and C report the same statistics for the five spot prices series and the four implied convenience yields, respectively. The construction of the implied convenience yields follow equation (8), where we use the three-month U.S. Treasury bill as a proxy for the risk-free interest rate \( r_T \). The critical values for both tests were obtained via Monte Carlo simulations with 2,000 replications. Different series have different sets of critical values as these ones depend on the sample size.

The SADF results in Panel A show strong evidence that each of our price indices has at least one episode of explosive behavior. Moving to the GSADF statistics we further observe that the results are consistent with multiple episodes of explosive behavior in each of the indices series.
For example, for the real crude oil index, both SADF and GSADF exceed the corresponding 1% right-tail critical value (SADF: 3.936>1.912 and GSADF: 3.988>2.778). The same is true for the heating oil index, where we reject the null of a unit root at a 5% for the SADF and at 1% for the GSADF. For the natural gas index both statistics are above the corresponding 1% right-tail critical values.

When looking at the spot prices statistics reported in Panel B, we observe that the results are very similar for the general oil categories WTI and Brent crude oil. For both, the WTI spot price (SADF: 3.293>1.912 and GSADF: 3.833>2.778) and for the Brent spot price (SADF: 3.594>2.021 and GSADF: 5.148>2.705), we have significance at the 1% right-tail critical values.

When considering additional energy categories, our results in the lower part of Panel B suggest that heating oil, natural gas, and jet fuel spot price, they all present strong evidence of multiple episodes of explosive behavior. Following the interpretation in PSY and PWY, the SADF statistics shows evidence that there exists at least one episode of explosive behavior, while the GSADF statistics provide evidence of multiple episodes of explosive behavior.

As suggested in equation (2) along with the derivation of equation (6), evidence of explosive behavior in the spot prices as presented in Panel B is not necessarily evidence of bubble periods. We further need a constant discount rate and a non-explosive logarithm of the convenience yield, $c_t$. The statistics presented in Panel C test for explosive behavior in the logarithm of the convenience yield for all the series in which futures prices are available. The construction of the implied convenience yields follow equation (8). We read the relatively small

---

13 The assumption of a constant discount rate is relatively common in the literature (see, e.g., Shiller, 1981; Campbell and Shiller, 1987; Campbell and Deaton, 1989).
SADF and GSADF statistics for the WTI, Brent, and natural gas implied convenience yields as evidence that these series are not explosive. Hence, we can further interpret the explosive behavior in the WTI, Brent and natural gas spot prices as evidence of bubble episodes. The relatively large SADF and GSADF statistics for the heating oil implied convenience yield is evidence against bubbles in the heating oil spot price. However, one benefit of the methods in PSY is that we can additionally analyze whether the periods of explosive behavior in the spot match with episodes of explosive behavior in the implied convenience yield. This allows us to identify which episodes of explosive behavior in the spot price series can be interpreted as evidence of bubbles.

[Figure 2, about here]

To study the timing of explosive behavior periods in the crude oil, heating oil, and natural gas indices, Figures 2, 3 and 4 plot the corresponding recursive BSADF statistics calculated using equation (13) along their 95% critical value sequences. Figure 2 shows evidence of six statistically significant bubbles, which lasts at least twelve weeks each (April 1989 to March 1990; February 1996 to March 1997; January 2000 to November 2000; March 2004 to August 2006; March 2008 to August 2008; and November 2014 to March 2015). These periods, represented in Figure 2 as the shaded areas, correspond to the beginning and end of explosive behavior episodes as identified by equations (14) and (15). The volatility in crude oil price before 1990 is due to conflict between Gulf countries. Moreover, the relatively short phase early in the 1990 is associated with the Gulf War as well. Similarly, just before the 1997-1998 Asian Crisis, the explosive behavior in crude oil prices is evident. In the following years, and just before the

---

14 We are only referencing the most probable cause of explosive behavior in price series; there can be additional factors that might have influenced the explosive behavior in energy price series during same period.
2000s recession that affected the European Union and United States, there is a statistically significant additional explosive behavior episode. One of the most prominent among all GSADF identified explosive behavior episodes is observed before the 2007-2009 global recession; the crude oil price showed a strong hike that remained consistently high for more than a year. The most recent shaded area in Figure 2, from December 2014 to March 2015, would be classified as a price implosion. This significant drop was likely caused by a supply glut and the economic slowdown in China. Energy Analyst speculate that continued growth in U.S. shale production and increase in non-OPEC nations oil exports have led to excess capacity.

[Figure 3, about here]

Figure 3 shows the explosive behavior episodes of the inflation-adjusted heating oil value-weighted index. Because heating oil is a lower viscosity derivative of crude oil, it follows closely crude oil prices. Notice that from Figure 3 we observe a single sudden explosion in prices at the end of 2007. We label the shaded area between November 2014 and February 2015 as an implosive episode that is likely to come as a response to the continuous drop in crude oil prices during the previous three to four years.

[Figure 4, about here]

As we move to Figure 4 to study the inflation-adjusted natural gas index, we notice that there is only one short-lived episode of explosive behavior between November 1996 and December 1997. Note that the quick jumps in the BSADF around December 2000 and around January of 2003 are not labeled as explosive behavior as they fail to comply with the 12-weeks retention criteria presented in equation (15). One interesting aspect peculiar to the natural gas price series is that there is no statistically significant explosive behavior during the 2007-2009
global recession. The justification may be rooted in the natural gas pricing series which has remained stable and relatively low compared to other energy sources.

[Figure 5, about here]

Following a similar approach, Figures 5, 6, 7 and 8 focus on the inflation-adjusted spot prices for the first four series of Panel B of Table 2 (i.e., WTI spot price, Brent spot price, heating oil spot price, and the natural gas spot price). In addition to showing the spot prices, the 95% critical value sequences, and the BSADF sequences for the corresponding spot price series, these four figures also present the BSADF sequences of the corresponding tests of explosive behavior in the implied convenience yield series. These latter BSADF sequences appear on the lower part of the figures as dashed lines and are measured on the left-hand side axes.

[Figure 6, about here]

For the WTI spot price (Figure 5) and Brent spot price (Figure 6), we observe similar dynamics as in crude oil series reported in Figure 2. Consistent with the SADF and the GSADF statistics of Panel C of Table 2, where no explosive behavior evidence is found on the implied convenience yield series, the corresponding BSADF for the same test consistently lies below the 95% critical values. Hence, we interpret the evidence of explosive behavior as evidence of bubbles. For the Brent spot, a major bubble is observed during late 1999 and early 2000. This matches the global optimism and bullish markets of early 2000. The bubble occurring before the 2007-2009 recession is consistent with the observed behavior in the crude oil index series. The implosive behavior close to the end of 2014 is likely to be the result of two types of oil related economics shocks, i.e., positive oil supply shock reflecting unexpected surge in production of crude oil, and a negative shock to the demand for oil inventories reflecting prospects of higher
future oil production (see, Kilian and Murphy, 2014). This shows the importance of various potential factors affecting the oil price dynamics. There is previous work that has analyzed the dynamics between energy series and other asset classes such as stocks (see, e.g., Sadorsky, 1999; Mollick and Assefa, 2013) and other commodities (Nazlioglu and Soytas, 2012). In addition, others have explicitly looked into the relationship between oil prices and the macroeconomic factors (see, e.g., Kilian, 2008; Hamilton, 1983; Lee et al., 1995). Kilian (2008) shows that the impact of exogenous oil supply shocks on the U.S. real GDP growth and inflation were comparatively small, and that supply shocks did matter for particular historical episodes such as the Persian Gulf War. Furthermore, Kilian (2009) shows that controlling for the reverse causality between macro aggregates and oil prices is essential to analyze structural oil supply and aggregate demand shocks because these underlying shocks may have very different effects on the real price of oil. Our approach to identify explosive episodes and bubbles is consistent with previous work that focuses on explaining the factors that alter the dynamics of these energy series.

[Figure 7, about here]

When looking at the heating oil spot price, a salient feature in Figure 7 is the large jump in the BSADF statistics for the convenience yield series (dashed line) early in the year 2000. The magnitude is measured on the left-hand side axis and it reaches a maximum value of 13.96, the same as the GSADF statistic for the heating oil implied convenience yield reported in Panel C of Table 2. This short-lived jump, as well as the jumps on December 1989 and March 1996 coincide with jumps in the BSADF statistics for the heating oil spot price series. This means that even though the BSADF statistics for the spot price is above its critical values, we cannot label those periods as bubbles. In addition to being short lived and failing to comply with the
definition in equation (15), it is not complying with the assumption of non-explosive \( c_t \) either. However, we interpret the shaded area between February and August of 2008 as a bubble because the spot price’s BSADF lies above the critical values, while the implied convenience yield’s BSADF lies below the critical values. The shaded areas around 2015 are price implosions consistent with the findings in Figures 5 and 6.

[Figure 8, about here]

[Figure 9, about here]

Figures 8 and 9 present the results for the natural gas spot price and the jet fuel spot price. Because the implied convenience yield shows no evidence of explosive behavior (as reported in Panel C of Table 2), we interpret the single shaded area in Figure 8 as bubble. It runs from May 2000 to January 2001. The explosive behavior episodes in the jet fuel spot prices presented in Figure 9 are similar to the ones reported for WTI and Brent spot prices.

Our findings of multiple bubble periods in a given energy series supplements the vast literature on short-run and long-run oil price behavior. For example, oil prices are presumed to follow either deterministic (Lee et al., 2006) or stochastic trends (Slade, 1988) in the long-run. On the other hand, some recent studies provide evidence of jumps in oil prices in the short-run (see, e.g., Gronwald, 2012). Moreover, some prior work shows that fundamentals are the major influencer while justifying the energy price movement or explosive behavior (e.g., Kilian and Murphy, 2014; Knittel and Pindyck, 2016). Many analysts support the concept of financialization of commodities such as oil futures as a major driver behind 2004-2008 energy price hikes. However, Sanders and Irwin (2014) find no empirical support for financialization.
Excluding the natural gas index and natural gas spot price, the rest of the energy sector series showed evidence of explosive behavior during the months prior to and at the beginning of the 2007-2009 recession. In addition, these series also showed a price implosion around 2015. The dynamics of the natural gas series appeared to be relatively different from the rest of the indices and spot series in our study. From the correlation table in the appendix we can observe that there is a relatively weak correlation between oil and both of the natural gas series. The relatively weak correlation may be justified given the competition and substitutable characteristics between these two fuel sources. Our findings are consistent with Villar and Joutz (2006), who use vector error correction models on crude oil and natural gas prices to find statistical evidence that the oil price may influence the natural gas price, but the impact of natural gas price on the oil price is negligible.

6. Conclusion

This paper sets to study explosive behavior and bubbles in eight energy sector series using the recursive flexible window right-tailed ADF-based procedure proposed in PWY (2011) and further extended in PSY (2015). In addition to testing for the existence of episodes of explosive behavior, these methods allow us to identify the beginning and the end of each of these episodes. We present a simple commodity pricing theoretical framework that allows us to understand the conditions under which explosive behavior in a series can be interpreted as a bubble. In particular, the main condition is that the convenience yield of the commodity is not explosive. Using data on futures we construct an implied convenience yield to test if this condition non-explosiveness holds.
The empirical approach uses 28 years of weekly data for most of our eight energy sector series. Three of our energy series are indices (crude oil, heating oil, and natural gas), while five are spot prices (WTI, Brent, heating oil, natural gas, and jet fuel). The results for the indices show strong statistical evidence of multiple episodes of explosive behavior along with some periods of implosive prices. Some of these periods can be explained by the Gulf War, the years leading to the Asian crisis, and the years leading to the 2007-2009 financial crisis.

The explosive behavior results for the spot prices are consistent with the energy indices categories. Moreover, following a simple energy commodity pricing theoretical framework we can interpret our results as evidence of bubbles if convenience yields are not explosive. After constructing implied convenience yields using futures prices, we find that for the WTI, Brent, and natural gas, there is strong evidence of non-explosive convenience yields. This is true throughout our period of study. However, for the heating oil we found short periods of explosiveness in the convenience yield.

Our results are likely to be valuable for energy analyst. PSY argue that this analysis can act as an early warning alert system for investors, economists, and regulators. Identification of explosive behavior and bubbles is of further importance in light of the links between energy prices and the overall economic activity, including stock prices. Moreover, they are additionally important given the common agreement that the most recent financial crisis was originated from a bubble burst. Timely identification of bubbles can provide policy makers (e.g., the Federal Energy Regulatory Commission or the Fed) with a window of opportunity if they decide to act. While the identification of bubbles can provide valuable information, the analysis of potential steps by policy makers in the presence of bubbles is beyond the scope of this article. Alan Greenspan and Ben Bernanke are known to be against targeting bubbles with monetary policy.
On the other hand, Roubini (2006) argues that the Fed’s practice of refuting bursting bubbles and episodes of systemic risk has contributed to the asset bubbles, low savings, and the large current account deficit.
References


### Table 1. Descriptive statistics.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sample Period</th>
<th>Obs.</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Crude Oil Index</td>
<td>May 22, 1987 – Dec 25, 2015</td>
<td>1,493</td>
<td>414.3104</td>
<td>328.8164</td>
<td>45.1500</td>
<td>1772.6000</td>
</tr>
<tr>
<td>Real Heating Oil Index</td>
<td>Jan 20, 2006 – Dec 25, 2015</td>
<td>519</td>
<td>91.2716</td>
<td>21.9970</td>
<td>42.4060</td>
<td>186.3200</td>
</tr>
<tr>
<td>Real Natural Gas Index</td>
<td>Jan 06, 1995 – Dec 25, 2015</td>
<td>1,095</td>
<td>195.7652</td>
<td>192.0020</td>
<td>2.4800</td>
<td>1065.0200</td>
</tr>
<tr>
<td><strong>Panel B:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Heating Oil Spot</td>
<td>May 22, 1987 – Dec 25, 2015</td>
<td>1,493</td>
<td>0.7346</td>
<td>0.4063</td>
<td>0.1990</td>
<td>2.0596</td>
</tr>
<tr>
<td>Real Natural Gas Spot</td>
<td>Jan 10, 1997 – Dec 25, 2015</td>
<td>990</td>
<td>2.5880</td>
<td>1.2762</td>
<td>0.0000</td>
<td>8.2654</td>
</tr>
<tr>
<td>Real Jet Fuel Spot</td>
<td>Apr 6, 1990 – Dec 25, 2015</td>
<td>1,343</td>
<td>0.7730</td>
<td>0.4328</td>
<td>0.2021</td>
<td>2.1200</td>
</tr>
<tr>
<td><strong>Panel C:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Heating Oil Futures</td>
<td>May 22, 1987 – Dec 25, 2015</td>
<td>1,493</td>
<td>0.7393</td>
<td>0.4123</td>
<td>0.2072</td>
<td>2.0669</td>
</tr>
<tr>
<td>Real Natural Gas Futures</td>
<td>Jan 10, 1997 – Dec 25, 2015</td>
<td>990</td>
<td>2.6340</td>
<td>1.2960</td>
<td>0.8580</td>
<td>8.2494</td>
</tr>
</tbody>
</table>

**Notes:** The weekly energy indices, spot prices and futures prices were obtained from Thomson Reuters. This database provides prices for individual series as traded on exchanges. We obtained real values by adjusting nominal values using the U.S. CPI (obtained from the Federal Reserve Bank of St. Louis). The sample periods vary based on data availability.
Table 2. SADF and GSADF statistics for the real energy indices, real spot prices, and implied convenience yields.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>SADF Critical Values</th>
<th>GSADF Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>99%</td>
<td>95%</td>
</tr>
<tr>
<td>SADF</td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Real Crude Oil Index</td>
<td>3.9362***</td>
<td>3.9884***</td>
</tr>
<tr>
<td>Real Heating Oil Index</td>
<td>1.9809**</td>
<td>3.0435***</td>
</tr>
<tr>
<td>Real Natural Gas Index</td>
<td>2.5444***</td>
<td>3.0434***</td>
</tr>
<tr>
<td>Real WTI Spot Price</td>
<td>3.2933***</td>
<td>3.8329***</td>
</tr>
<tr>
<td>Real Brent Spot Price</td>
<td>3.5942***</td>
<td>5.1479***</td>
</tr>
<tr>
<td>Real Heating Oil Spot Price</td>
<td>4.2669***</td>
<td>5.6609***</td>
</tr>
<tr>
<td>Real Natural Gas Spot Price</td>
<td>4.0739***</td>
<td>5.0872***</td>
</tr>
<tr>
<td>Real Jet Fuel Price</td>
<td>2.4403***</td>
<td>4.0711***</td>
</tr>
</tbody>
</table>

Panel C:

<table>
<thead>
<tr>
<th>Statistics</th>
<th>SADF</th>
<th>GSADF</th>
<th>99%</th>
<th>95%</th>
<th>90%</th>
<th>99%</th>
<th>95%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTI Convenience Yield</td>
<td>-2.6938</td>
<td>1.6056</td>
<td>1.9122</td>
<td>1.5151</td>
<td>1.2822</td>
<td>2.7777</td>
<td>2.3726</td>
<td>2.1480</td>
</tr>
<tr>
<td>Brent Convenience Yield</td>
<td>-1.6068</td>
<td>-1.8654</td>
<td>2.0207</td>
<td>1.5166</td>
<td>1.2997</td>
<td>2.7048</td>
<td>2.3625</td>
<td>2.1627</td>
</tr>
<tr>
<td>Heating Oil Convenience Yield</td>
<td>4.4919***</td>
<td>13.9601***</td>
<td>1.9122</td>
<td>1.5151</td>
<td>1.2822</td>
<td>2.7777</td>
<td>2.3726</td>
<td>2.1480</td>
</tr>
<tr>
<td>Natural Gas Convenience Yield</td>
<td>-2.9888</td>
<td>1.6432</td>
<td>1.9988</td>
<td>1.4914</td>
<td>1.2798</td>
<td>2.7937</td>
<td>2.3181</td>
<td>2.0885</td>
</tr>
</tbody>
</table>

Notes: The real energy sector indices, real spot prices and real futures prices were obtained by adjusting the nominal value-weighted series (obtained from Thomson Reuters) using the U.S. CPI (obtained from the Federal Reserve Bank of St. Louis). The sample periods are different for different series based on data availability (refer to Table 1 for the details). Implied convenience yields are constructed following equation (8). The Supremum Augmented Dickey-Fuller (SADF) is proposed in PWY, while the Generalized SADF (GSADF) is proposed in PSY. Critical values of both tests were obtained using Monte Carlo simulations with 2,000 replications. *** significant at 1%, ** significant at 5%, and * significant at 10%.
Figure 1. Time series graphs of each of our eight series under analysis.

Notes: These are the time series graphs of the three inflation adjusted indices (i.e., crude oil, heating oil, and natural gas) and the five spot prices (i.e., heating oil, jet fuel, Brent, natural gas, and WTI). The weekly energy indices and spot prices were obtained from Thomson Reuters. This database provides prices for individual series as traded on national exchanges. We obtained the real values using the U.S. CPI, as obtained from the Federal Reserve Bank of St. Louis. The sample period varies by series depending on data availability.
Figure 2. GSADF: Explosive behavior periods in the real crude oil value-weighted index.

Notes: The real Crude Oil index was obtained by adjusting the nominal Crude Oil price value-weighted index (obtained from Thomson Reuters) using the U.S. CPI (obtained from the Federal Reserve Bank of St. Louis). The sample spans from May 22, 1987 to December 25, 2015 with the total number of observations being 1,493. The Backward Supremum Augmented Dickey-Fuller (BSADF) follows PSY with the 95% critical values coming from Monte Carlo simulations with 2,000 replications.
Figure 3. GSADF: Explosive behavior periods in the real heating oil value-weighted index.

Notes: The real Heating Oil index was obtained by adjusting the nominal Heating Oil price value-weighted index (obtained from Thomson Reuters) using the U.S. CPI (obtained from the Federal Reserve Bank of St. Louis). The sample spans from January 20, 2006 to December 25, 2015 with the total number of observations being 519. The Backward Supremum Augmented Dickey-Fuller (BSADF) follows PSY with the 95% critical values coming from Monte Carlo simulations with 2,000 replications.
**Figure 4.** GSADF: Explosive behavior periods in the real natural gas value-weighted index.

Notes: The real Natural Gas index was obtained by adjusting the nominal Natural Gas price value-weighted index (obtained from Thomson Reuters) using the U.S. CPI (obtained from the Federal Reserve Bank of St. Louis). The sample spans from January 6, 1995 to December 25, 2015 with the total number of observations being 1,095. The Backward Supremum Augmented Dickey-Fuller (BSADF) follows PSY with the 95% critical values coming from Monte Carlo simulations with 2,000 replications.
Figure 5. GSADF: Bubble periods in the real WTI spot price.

*Notes:* The real WTI spot price was obtained by adjusting the nominal WTI spot price value-weighted index (obtained from Thomson Reuters) using the U.S. CPI (obtained from the Federal Reserve Bank of St. Louis). The sample spans from May 22, 1987 to December 25, 2015 with the total number of observations being 1,493. The Backward Supremum Augmented Dickey-Fuller (BSADF) follows PSY with the 95% critical values coming from Monte Carlo simulations with 2,000 replications.
Figure 6. GSADF: Bubble periods in the real crude Brent spot price.

Notes: The real Crude Brent spot price was obtained by adjusting the nominal Crude Brent spot price value-weighted index (obtained from Thomson Reuters) using the U.S. CPI (obtained from the Federal Reserve Bank of St. Louis). The sample spans from Jun 24, 1988 to December 25, 2015 with the total number of observations being 1,436. The Backward Supremum Augmented Dickey-Fuller (BSADF) follows PSY with the 95% critical values coming from Monte Carlo simulations with 2,000 replications.
Figure 7. GSADF: Bubble periods in the real heating oil spot price.

Notes: The real Heating Oil spot price was obtained by adjusting the nominal Heating Oil price value-weighted index (obtained from Thomson Reuters) using the U.S. CPI (obtained from the Federal Reserve Bank of St. Louis). The sample spans from May 22, 1987 to December 25, 2015 with the total number of observations being 1,493. The Backward Supremum Augmented Dickey-Fuller (BSADF) follows PSY with the 95% critical values coming from Monte Carlo simulations with 2,000 replications.
**Figure 8.** GSADF: Bubble periods in the real natural gas spot price.

Notes: The real Natural Gas spot price was obtained by adjusting the nominal Natural Gas spot price value-weighted index (obtained from Thomson Reuters) using the U.S. CPI (obtained from the Federal Reserve Bank of St. Louis). The sample spans from January 10, 1997 to December 25, 2015 with the total number of observations being 990. The Backward Supremum Augmented Dickey-Fuller (BSADF) follows PSY with the 95% critical values coming from Monte Carlo simulations with 2,000 replications.
Figure 9. GSADF: Explosive behavior periods in the real jet fuel spot price.

Notes: The real Jet Fuel spot price was obtained by adjusting the nominal Jet Fuel price value-weighted index (obtained from Thomson Reuters) using the U.S. CPI (obtained from the Federal Reserve Bank of St. Louis). The sample spans from April 6, 1990 to December 25, 2015 with the total number of observations being 1,343. The Backward Supremum Augmented Dickey-Fuller (BSADF) follows PSY with the 95% critical values coming from Monte Carlo simulations with 2,000 replications.
Appendix

Table A1. Correlations

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Real Crude Oil Index</th>
<th>Real Heating Oil Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Heating Oil Index</td>
<td>0.8735</td>
<td></td>
</tr>
<tr>
<td>Real Natural Gas Index</td>
<td>0.6423</td>
<td>0.3342</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B</th>
<th>Real WTI Spot</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2) Real Brent Spot</td>
<td>0.9836</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) Real Heating Oil Spot</td>
<td>0.9831</td>
<td>0.9921</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) Real Natural Gas Spot</td>
<td>0.2571</td>
<td>0.1525</td>
<td>0.2139</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5) Real Jet Fuel Spot</td>
<td>0.9837</td>
<td>0.9877</td>
<td>0.9952</td>
<td>0.2409</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6) Real WTI Futures</td>
<td>0.9999</td>
<td>0.9838</td>
<td>0.9834</td>
<td>0.257</td>
<td>0.984</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7) Real Brent Futures</td>
<td>0.9842</td>
<td>0.9984</td>
<td>0.9921</td>
<td>0.1561</td>
<td>0.9883</td>
<td>0.9846</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(8) Real Heating Oil Futures</td>
<td>0.9836</td>
<td>0.9925</td>
<td>0.9984</td>
<td>0.2039</td>
<td>0.9949</td>
<td>0.9839</td>
<td>0.9928</td>
<td></td>
</tr>
<tr>
<td>(9) Real Natural Gas Futures</td>
<td>0.2589</td>
<td>0.1511</td>
<td>0.2135</td>
<td>0.9739</td>
<td>0.2456</td>
<td>0.2588</td>
<td>0.1553</td>
<td>0.2043</td>
</tr>
</tbody>
</table>