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# Sharing economy of electric vehicle private charge posts

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## **Recommended Citation**

Hu, X., Yang, Z., Sun, J., Zhang, Y., 2021. Sharing economy of electric vehicle private charge posts. Transportation Research Part B: Methodological 152, 258–275. https://doi.org/10.1016/j.trb.2021.09.001

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## **Sharing Economy of Electric Vehicle Private Charge Posts**

**Abstract:** The increasing popularity of electric vehicles (EVs) leads to heightened demand for the charging infrastructure. More and more EV drivers install private charge posts, which can now be shared with others through certain mobile apps. This emerging phenomenon is becoming a prominent part of the sharing economy. To examine the impacts of post sharing on EV charging market, this study establishes game theory models on consumer choices among private, public, and shared options. Such peer-to-peer sharing and collaborative consumption redistribute the installation and operation costs of private charge posts in proportion to their increased utilization. Numerical analyses suggest that the sharing mode provides a winwin solution for charge post owners and non-owner consumers, as well as electricity distributors and public charging infrastructure operators. In the case of China, the estimated saving for charge post owners is between 20% and 50%, which can be translated into more non-government investment in the EV industry chain. The findings provide supporting evidence for policy-makers to promote private charge post sharing, especially with certain consumer subsidization at a reasonable level.

Keywords: Green travel; electric vehicle charge post; sharing economy; industry chain; game theory.

## 1. Introduction

The public attention to low-carbon emission and green travel promotes the diffusion of electric vehicles (EVs) in many countries and regions. Jumping from two million in 2016, global cumulative sales of EVs hit the milestone of five million in 2018 (IEA, 2017). Among them, two million were in China alone, as the government encourages consumers to purchase and use EVs with favorable policies like what many other countries do (MPS, 2019). The rapid expansion of the EV market, however, is curbed by the limited charging infrastructure. To solve the issue, the Chinese government set a goal of building 12,000 charge stations and 4.8 million charge posts, 500,000 public and 4.3 million private, by 2020 (NDRC, 2018a). As of May 2018, a total of 266,000 public and 304,000 private charge posts were established, indicating that the construction of public charging infrastructure is on track, but the progress of private posts lags far behind (NDRC, 2018b).

Recently, peer-to-peer charge post rental emerged as a new form of sharing economy (Koç, Jabali, Mendoza et al., 2019). An EV driver who owns a private charge post can lease it during idle time through a sharing platform like the Tgood mobile app, as shown in Figure 1. Another non-owner driver enjoys a lower charging price than what the public infrastructure offers. With the additional leasing revenue, the installation of a private charge post becomes less of a burden to a consumer. The increased market competition will largely break down the current monopoly of public charging infrastructure, and enhance service accessibility that is critical to the healthy development of the EV industry chain.



Fig.1. Tgood App for Charge Post Sharing

Facing the competition from charge post sharing, existing market players have to adjust their strategies. For instance, public charging infrastructure operators need to reprice their services, and electricity distributors may adjust their rates. Such decisions must be based on a sound understanding of the new sharing economy. As charge post sharing becomes an indispensable link of the EV industry chain, it is also critical for policy-makers to offer guidance on the healthy development of this emerging market, such as consumer subsidization to speed up its growth. At present, few researchers have investigated the dynamics involved in the phenomenon, and this study attempts to fill in the research gap by examining private charge post sharing with game theory models. They provide insights on optimal pricing strategies of market participants as well as reasonable incentive programs that policy-makers may consider. The findings contribute to the literature of transportation policy with the evidence to support the sharing economy of private charge posts for the sustainable development of EV industry chain and urban transportation.

The rest of this article is organized as follows. It first reviews the literature on EV charging infrastructure and sharing economy. The understanding leads to the establishment of game theory models

that benchmark the sharing economy of private charge posts against the traditional mode of public charging services. Numerical analyses reveal the equilibria in terms of how post sharing impacts consumer choices and pricing strategies. Then, consumer subsidization is incorporated into the models to assess its facilitation of the sharing economy. The theoretical and practical implications of the findings are discussed, followed by the conclusions.

## 2. Research Background

#### 2.1 EV Charging Infrastructure

By replacing fuel cars, EVs can effectively improve the air quality of metropolitan areas (Chemama, Cohen, Lobel et al., 2018). However, many consumers are hesitant to purchase and use EVs due to the lack of charging infrastructure and lengthy service stops, especially for long-distance driving (Kuppusamy, Magazine, & Rao, 2017). Extant studies explore different approaches to the facilitation of EV charging, such as demand forecasting (Moon, Park, Jeong et al., 2018), economic performance (Tong, Henickson, Biehler et al., 2017), location optimization (Wu & Sioshansi, 2017a, 2017b), power distribution (Kabli, Quddus, Nurre et al., 2019), charging strategy (Ucer, Kisacikoglu, Yuksel et al., 2019), and battery swap (Sun, Sun, Tsang et al., 2019).

In addition to the construction of public charging infrastructure, the government plays an important role in promoting private post installation with consumer subsidies (Li, Zhan, Jong et al., 2016). Many developed countries like the USA, UK, Netherlands, Canada, France, Italy, and Norway have implemented such incentive programs (Rietmann & Lieven, 2019). Having the largest number of EVs, China also encourages EV owners to install private charge posts with an action plan, yet local administrations are still waiting for more concrete policy guidelines (NDRC, 2018a). Although consumer subsidization is helpful at the beginning, sufficient government funding may not be available to support large-scale construction of charging infrastructure (Li, Zhan, Jong et al., 2016). For the sustainable development of the EV industry chain, more innovative approaches are proposed (Helveston, Liu, Feit et al., 2015). For instance, researchers explore crowdfunding for public charging infrastructure to supplement governmental funding (Zhu, Zhang,

Lu et al., 2017). EV charging infrastructure public-private partnership (EVCI-PPP) is another approach to attract private capital with a lucrative return (Yang, Long, & Li, 2018; Zhang, Zhao, Xin et al., 2018). Both crowdfunding and EVCI-PPP provide an opportunity for individual investors to enter the arena of public charging infrastructure (NDRC, 2018a). By July 2017, 13 EVCI-PPP projects had been successfully implemented in China (Yang, Long, & Li, 2018).

Meanwhile, the number of private charge posts in China exceeded that of public ones in 2018 (NDRC, 2018b). Under this trend, mobile app platforms like Tgood emerged recently for owners to lease charge posts to others during idle periods. The sharing mode helps more EV consumers meet their charging demand by strengthening equipment utilization. As potential leasing revenues make the installation of private charge posts more attractive, consequential widespread ownership will largely solve the insufficient infrastructure issue. Compared with the investment in public charge infrastructure, private post sharing provides a more direct avenue for consumers to participate in the EV charging market for utilization and monetary benefits.

Due to its novelty, there have been only a few publications on the sharing of private charge posts. Based on the business model of peer-to-peer sharing and collaborative consumption (P2P SCC), one study proposed the use of new service development and provider assessment methods for process design (Plentera, Chasina, Hoffena et al., 2018). As for the utilization of shared posts, non-deterministic polynomial modeling was used to match potential demand and supply to the greatest extent (Gong, Tang, Buchmeister et al., 2019). Due to the dynamic nature of this emerging market, however, it is not clear what the optimal pricing strategies for market participants, especially the owners who share their charge posts.

In recent years, game theory modeling is becoming notable in the field of transportation research (Avraham, Raviv, & Khmelnitsky, 2017; Shao, Xu, Yang et al., 2020; Xu & Huang, 2014). So far, two publications have applied this methodology on the phenomenon of charge post sharing. One established a bilateral bargaining game for optimal pricing strategy based on the relationships among charge post owners, electricity distributors, and EV drivers, yet leaving out the major market players of public charging service

providers (Zhao, Zhang, Zhu et al., 2020). The other set up a non-cooperative two-matrix game on the competition between private charge post leasers and public service providers for EV drivers who do not have their own posts (Zhao, Zhang, Yang et al., 2020).

In reality, many EV drivers who install private charge posts still have the need to rent others' or use public charging services when they are away from home. Thus, it is necessary to consider all three options that EV drivers have in game theory modeling: installing (and sharing) private posts, renting private posts, and using public posts. The incorporation of such a comprehensive market structure distinguishes this study from the aforementioned studies that focus on the first two and last two options, respectively. It will develop a game theory model for a market where public charging infrastructure and private post sharing co-exist, while consumers can decide whether to build their own posts or rent from others in addition to the public option.

### 2.2 Sharing Economy

Enabled by platforms like Tgood, charge post sharing is essentially a new form of sharing economy. There are generally three types of sharing economy: on-demand rental network, on-demand service platform, and peer-to-peer resource sharing (Benjaafar & Hu, 2019). An on-demand rental network features an organization's leasing of products, such as bikes (Kabra, Belavina, & Girotra, 2018) and cars (Lu, Chen, & Shen, 2017), to be shared with a large population. An on-demand service characterizes an online platform matching freelance workers and target customers, as instantiated by Uber and Lyft (Cohen, Fiszer, & Kim, 2018). Peer-to-peer resource sharing is enabled by an online platform matching buyers and sellers (a seller at one time can be a buyer at another) for repeat-use resources, and Airbnb is an example (Fradkin, Grewal, Holtz et al., 2015).

Private charge post sharing is a form of peer-to-peer resource sharing as owners may use others' posts to charge their EVs. It allows a non-owner consumer to enjoy a lower charging rate than a public service provider's and helps an owner consumer cover post construction cost. More specifically, private charge post sharing is a P2P SCC service as owners are also consumers on platforms like Tgood (in contrast

to Airbnb), enabling collaborative consumption along with resource sharing. As an attempt to investigate this new form of sharing economy under the P2P SCC framework, this study refers to relevant publications in transportation research as well as other fields. For peer-to-peer car rental, a dynamic model based on transaction-level data suggests that P2P SCC increases consumer surplus (Fraiberger & Sundararajan, 2015). For the optimal outcome of car sharing, it is necessary for the automakers that play the role of participating OEMs to pursue a clear dominant strategy by providing consumers fuel-efficient vehicles (Bellos, Ferguson, & Toktay, 2017).

In more general P2P SCC settings, researchers explore the behaviors of market participants with different modeling approaches. An overlapping-generations model compares product pricing and consumer surplus before and after P2P SCC is introduced to a retail market, and the results suggest that the new mode benefits both retailers and consumers when the marginal production cost is relatively high, whereas retailers suffer profit loss with low-cost products for sufficiently impatient consumers (Weber, 2016). Similarly, an analytical framework of a P2P SCC market comprising a monopolist manufacturer and consumers shows that the sharing of high-cost products provides a win-win solution for them (Jiang & Tian, 2018).

On the demand side, an equilibrium model of a P2P SCC market in which consumers can choose to own or rent products finds that the sharing of high-cost products boosts ownership rate and usage level (Benjaafar, Kong, Li et al., 2019). Another study explores the equilibria of P2P rental markets in the short run, in which ownership decisions are fixed, as well as in the long run, in which ownership decisions can be changed, and suggests that such a sharing economy always expands consumption and increases surplus, but ownership may increase or decrease (Filippas, Horton, & Zeckhauser, 2020). On the supply side, manufacturers as OEMs tend to enhance product quality and increase optimal price in the presence of the sharing market (Jiang & Tian, 2018). In a P2P SCC market comprising an OEM, a retailer and consumers, P2P product sharing is likely to increase the retailer's share of the gross profit margin in the distribution channel (Tian & Jiang, 2018).

Based on different modeling approaches, extant studies provide valuable insights on the market in which consumers purchase products (for self-use or leasing) or rent products from others. In a typical SCC market, however, P2P faces the business-to-consumer (B2C) competition, which is less represented in the literature. As for the sharing economy of private charge posts, consumers always have public charging infrastructure as an option in the decision-making regarding whether to install (and share) private posts, rent private posts, or use public posts. Thus, it is necessary to model the competition between B2C public charging and P2P post sharing together in the market.

Two recent studies examined the B2C-P2P competition indirectly and directly, respectively, based on various assumptions. One established three separate models corresponding to the possible roles that an OEM plays as the seller in a P2P market, the renter in a B2C market, or both in a B2C-P2P market, and numerical analyses hinted at the positive interaction between P2P and B2C on OEM profitability if consumer heterogeneity and usage rate are sufficiently high (Abhishek, Guajardo, & Zhang, 2020). Though such an OEM-centered study suggests that the introduction of P2P does not necessarily conflict with B2C, it is up to the direct modeling of B2C-P2P competition to provide more conclusive and insightful findings. As an attempt, another study set up a "product-sharing market" comprising an OEM's own platform for B2C and a third-party platform for P2P to determines the equilibrium rental price that is assumed to be uniform on different platforms eventually (Jiang, Tian, & Xu, 2021).

Unlike regular products that are largely homogenous in a sharing market, public and private charge posts employ distinct technologies that lead to incompatible pricing mechanisms. This study adopts a truly heterogeneous market structure to free up the locking constraint of uniform pricing between B2C service and P2P rental in modeling. Thus, it is possible to incorporate the market competition between two distinct sources of supplies (rather than an OEM that provides the same products for B2C and P2P) in the assessment of their optimal pricing strategies respectively. Such an approach is crucial for the investigation of how the introduction of private post sharing will impact the EV charging market originally based on public charging infrastructure. Instead of OEMs, therefore, this study includes public charging infrastructure operators and electricity distributors as B2C players.

#### 3. Benchmark Model

By promoting peer-to-peer resource sharing in the EV charging market, the government can make less investment in the public infrastructure but meet consumer demand with enhanced private equipment utilization. This study focuses on how to encourage more EV drivers to install, share and/or use private charge posts with Stackelberg game models considering the dual identities of consumers as leasers and renters. In particular, it explores optimal pricing strategies for post rental and electricity distribution, along with consumer subsidization as critical success factors of private charge post sharing.

#### 3.1 Charge Post Installation and Operation

The assumptions required for mathematical modeling are established below based on some simplifications of real-world scenarios. As shown in Figure 2, there are two types of charge posts: public (denoted as *P*) and private (denoted as *I*). Public posts use faster direct current (DC) charging at a higher equipment cost, whereas private posts use slower alternating current (AC) charging at a lower equipment cost. The varied charging speeds lead to different service volumes and values. Most EVs just need 1-2 hours to recharge with public posts, but over 7 hours with private posts. For example, it takes only 80 minutes to charge a Tesla Model S at a super charging station using DC, whereas 8 hours and 19 minutes with a private charge post using AC (Levinson & West, 2018).

When consumers charge EVs with public and private posts, the main difference lies in charging durations. Accordingly, the value provided by a public post can be denoted as  $v_P$ , and the value provided by a private post can be denoted as  $v_I$ . The service volumes of public and private posts are about 18 and three times per day in theory, respectively, but the actual volumes are lower considering peak and off hours at different locations. Based on the battery capacity and charging speed of most common EVs, this study specifies average service volume, denoted as  $\theta_P$  for public posts and  $\theta_I$  for private posts, in numerical analyses. The charging price, denoted as  $P_P$  for public posts and  $P_I$  for private posts, includes electricity rate and service fee.



Fig.2. Charging supply chain in the benchmark case.

The costs of running a charge post cover construction (equipment and labor), electricity, and operation. As DC charging equipment is more expensive than AC equipment, the construction cost of a public post is higher than that of a private post:  $C_P > C_I$ . In terms of electricity, public posts use industrial rates whereas private posts use household rates, the former typically higher than the latter. In China, for example, the average industrial rate is about CNY0.8 per kWh, and the average household rate is about CNY0.5 per kWh (Zhao, Cai, & Ma, 2018). Correspondingly, the wholesale rate for public posts is higher than that for private ones:  $W_P > W_I$ . Finally,  $O_P$  and  $O_I$  denote the operation costs for public and private posts respectively. Table 1 lists all the notations in this study.

Symbol	Definition
Charge post operator:	
$C_P/C_I$	Construction cost of a public/private charge post
C <sub>S</sub>	Upgrade cost for sharing a private charge post
$O_P/O_I$	Operation cost of a public/private charge post
$P_P/P_S$	Charging price of a public/private charge post
$N_P/N_I$	Number of public/private charge posts
$ heta_P/ heta_I$	Average charging service volume
$\pi_0$	Operator profit
Electricity distributor:	
С	Marginal cost of electricity distributor
$W_P/W_I$	Electricity rate for a public/private charge post
$\pi_D$	Distributor profit
Consumer:	
$v_k$	Value of a post k based on charging speed, $k \in \{P, I, S, IS\}$
$U_k$	Service utility of a post k considering user convenience, $k \in \{P, I, S, IS\}$

Table 1. Model notations.

## 3.2 Consumer Behavior in the Benchmark Case

Before the emergence of private post sharing, an EV driver has two charging options: the first is to use a public charge post that is faster but more expensive per service, and the second is to install a private charge post that is slower but cheaper. These two options are considered in the benchmark model, and the service values provided by a public post and a private post can be formulated as  $v_P$  and  $v_I$  respectively. Consumers' valuation of charge posts relies more on their speeds than locations (Wolff & Madlener, 2019). Almost all city residents in China live in condos, and they have to install private charge posts in community parking lots (underground or surface). To EV drivers in metropolitan areas, therefore, private and public charge posts are not that different in accessibility. Rather, charging speeds vary significantly between the two options due to their distinct technologies and power ratings (Dong, Liu, & Lin, 2014; IEA, 2020; Leea, Chakrabortyb, Hardmanb et al., 2020). Thus, the service value of a public charge post is higher than that of a private one:  $v_P > v_I$ . Even if EV drivers use their own posts, quicker charging still makes a difference by giving them more flexibility in travel planning and trip making.

In addition to charging speed, service utility also depends on consumer charging service preference, x, which is uniformly distributed between 0 and 1. A larger value indicates a lower perceptional "discount" on the valuation of a charge post to a consumer.

Utility from using a public charge post:

$$U_P = v_P x - P_P. \tag{1}$$

Utility from using a private charge post:

$$U_{I} = v_{I}x - C_{I} - (W_{I} + O_{I}).$$
<sup>(2)</sup>

Assuming that the percentage of consumers installing private charge posts is  $q_I$ , and the percentage of consumers choosing public charge posts is  $q_P$ , and  $q_P = 1 - q_I$ . An indifferent consumer derives the equal utility from both charging options:  $U_P(x) = U_I(x)$ . Solving the equation, the cutoff consumer preference between using private and public posts is  $x = \frac{P_P - C_I - W_I - O_I}{v_P - v_I}$ . Based on the uniform distribution of consumer preference, the percentage of consumers who decide to install private charge posts is  $q_I = x = \frac{P_P - C_I - W_I - O_I}{v_P - v_I}$ .

#### 3.3 Stackelberg Game of the Benchmark Case

Among game theory models, Stackelberg game is able to handle the EV charging market as a multiagent system in which players make decisions sequentially based on heterogeneous strategies (i.e., leader vs. follower). In the benchmark model, there are three players: electricity distributor, public charging infrastructure operator, and EV consumers. As shown in Figure 3, the distributor decides the wholesale electricity rate first due to its strong bargaining power (Zhu, Zhang, Lu et al., 2017). Then the operator decides the construction quantity of public charge posts and charging service price. Finally, each consumer chooses a charge post between public and private options.



Fig.3. The timeline of Stackelberg game in the benchmark case.

The electricity distributor only decides the wholesale rate for the public charging infrastructure operator,  $W_P$ , whereas the rate for private charge posts  $W_I$  is fixed as an exogenous variable. Facilitating model solution, such a setup also reflects what is in the practice. Public posts are directly connected to the grid, allowing the electricity distributor to set the wholesale rate based on the demand. Meanwhile, most private posts are indirectly connected to the grid through residential circuits, and their usage is charged at the same electricity rate as other household appliances. Distributor profit can be formulated as:

$$\pi_D = W_I q_I + W_P q_P - c, \tag{3}$$

where c represents the marginal cost of electricity for the distributor.

The operator decides the number of public charge posts to be constructed,  $N_P$ , and the charging service price,  $P_P$ . Operator profit can be formulated as:

$$\pi_{0} = P_{P}q_{P} - C_{P}N_{P} - (W_{P}q_{P} + O_{P}N_{P}), \tag{4}$$

where  $N_P = \frac{q_P}{\theta_P}$ .

Solving the first-order conditions for the profit-maximizing operator and distributor yields the equilibrium prices and sales as summarized in *Lemma 1* (see Appendix for all proofs).

*Lemma 1.* In the benchmark case without considering charge post sharing, the equilibrium price of public charging infrastructure operator is  $P_P^* = W_I + \frac{3}{4}(v_P - v_I + C_I + O_I) + \frac{C_P + O_P}{4\theta_P}$ , and the equilibrium price

of electricity distributor is  $W_P^* = W_I + \frac{1}{2} \left( v_P - v_I + C_I - \frac{C_P + O_P}{\theta_P} + O_I \right)$ . The equilibrium sales are  $q_P^* = \frac{1}{4} - \frac{C_P + O_P - \theta_P (C_I + O_I)}{4\theta_P (v_P - v_I)}$  for public charge posts, and  $q_I^* = \frac{3}{4} + \frac{C_P + O_P - \theta_P (C_I + O_I)}{4\theta_P (v_P - v_I)}$  for private ones.

Table 2 reports a sensitivity analysis for a better understanding of *Lemma 1*. There exists a negative relationship between  $C_I$  and  $q_I^*$ : the increase of private charge post construction costs will incur the decrease of private post equilibrium sales. An increase in any private charge post costs, including construction cost  $(C_I)$ , operation cost  $(O_I)$  and electricity wholesale cost  $(W_I)$ , or the improvement of public charge post value  $(v_P)$  will trigger both public charging infrastructure operator and electricity distributor to raise their service price  $(P_P^*)$  and wholesale rate  $(W_P^*)$ . Similarly, an increase in private charge post value  $(v_I)$  will trigger public charging infrastructure operator and electricity distributor to reduce their service price  $(P_P^*)$  and wholesale rate  $(W_P^*)$ . Similarly, an increase in private charge post value  $(v_I)$  will trigger public charging infrastructure operator and electricity distributor to reduce their service price  $(P_P^*)$  and wholesale rate  $(W_P^*)$ . Meanwhile, a decrease in public charge post operation cost  $(O_P)$ , construction cost  $(C_P)$ , or an increase in average public post service volume  $(\theta_P)$  will trigger the operator to reduce the charge service price  $(P_P^*)$  and the distributor to raise the electricity wholesale rate  $(W_P^*)$ . Finally, an increase in private charge post construction cost  $(C_I)$ , or average service volume  $(\theta_I)$  or a decrease in public post operation cost  $(O_P)$  will incur more public post charge sales  $(q_P^*)$ . but less private post charge sales  $(q_I^*)$ .

	C <sub>I</sub>	$C_P$	<i>OI</i>	$O_P$	$W_I$	$v_I$	$v_P$	$ heta_P$
$P_P^*$	+	+	+	+	+	_	+	_
$W_P^*$	+	—	+	_	+	_	+	+
$q_P^*$	+	—	+	—	N/A	_	+	+
$q_I^{*}$	—	+	—	+	N/A	+	—	—

Table 2. Sensitivity analysis of benchmark model

Note: N/A - not applicable (i.e., no/weak relationship)

## 4. Sharing Model

## 4.1 Private Charge Post Sharing

This study assumes that all owners choose to share their private charge posts. Before joining the sharing platform, an owner needs to upgrade post construction to payment and security standards. The sharing upgrade construction cost is denoted as  $C_S$ . Sharing a private charge post reduces the utility to its owner in terms of convenience, who needs to pre-arrange his/her charging schedule without much flexibility. The value of a shared private post to its owner is denoted as  $v_{IS}$ , and  $v_{IS} < v_I$ . Once private charge posts are shared, consumers have the option to rent them for EV charging. The value of shared posts to non-owner consumers is lower than that of owners,  $v_S < v_{IS}$ , due to less convenience. Although the sharing of charge posts reduces their values to owners, it generates revenue at the price of  $P_S$  per charging service.



Fig.4. The charging supply chain in the sharing case.

When owner consumers share their private charge posts, there exists an average charging service volume, denoted as  $\theta_I$ . How frequent a charge post can serve its owner and other consumers depends on the charging speed and the sharing platform's matching efficiency. Owner consumers play double roles,

buyers, and sellers. As shown in Figure 4, when owner consumers use private posts to charge EVs, they play the role of buyers; when they lease their posts to others, they play the role of sellers.

#### 4.2 Consumer Behavior in the Sharing Case

Three charging options are available for a consumer in the sharing case: using a public post, using one's own post, using a shared post. Assuming that the number of owners sharing their posts is  $q_{IS}$ , and the number of non-owner consumers using the shared posts is  $q_S = (\theta_I - 1)q_{IS}$ . The utility that a consumer derives from using a public post is the same as that in the benchmark case. The utilities concerning private post sharing can be formulated as follows.

When a consumer uses a shared post as a non-owner:

$$U_S = v_S x - P_S. \tag{5}$$

When an owner shares a post:

$$U_{IS} = v_{IS}x - (C_I + C_S) + P_S(\theta_I - 1) - (W_I\theta_I + O_I).$$
(6)

The volume of consumers using the public charging infrastructure is  $q_P = 1 - q_{IS} - q_S$ . The choice-making of each consumer among three charging options for utility maximization can be determined with two indifference points  $x_1$  and  $x_2$ . Consumers within the range from 0 to  $x_1$  tend to rent shared private charge posts, consumers within the range from  $x_1$  to  $x_2$  are likely to own and share posts, and consumers within the range from  $x_2$  to 1 prefer public charge posts. Indifferent consumers at  $x_1$  obtain the same utility from renting shared posts and owning private posts, but suffer a loss using public charge posts, but suffer a loss renting shared private posts.

The indifferent consumer who derives equal utility from sharing his/her own post and using a shared post can be located by solving  $U_S(x_1) = U_{IS}(x_1)$ , where  $x_1$  denotes the cutoff preference between using private and shared posts. Another indifferent consumer who derived equal utility from sharing his/her own post and using a public post can be located by solving  $U_{IS}(x_2) = U_P(x_2)$ , where  $x_2$  denotes the

cutoff preference between using private and public posts. The cutoff values are respectively  $x_1 = (\theta_I - 1) \frac{C_I + C_S + O_I + \theta_I(W_I - P_P)}{K_1 K_2}$  and  $x_2 = \theta_I \frac{C_I + C_S + O_I + \theta_I(W_I - P_P)}{K_1 K_2}$ . Then the sales of shared posts are  $q_{IS} = \frac{C_I + C_S + O_I + \theta_I(W_I - P_P)}{K_1 K_2}$ , with the rental price  $P_S = \frac{(\theta_I - 1)(v_{IS} - v_S)}{K_1 K_2} P_P + \frac{C_I + C_S + O_I + \theta_I W_I}{K_1}$ , where  $K_1 = \theta_I - 1 + \frac{\theta_I(v_{IS} - v_P)}{K_2}$  and  $K_2 = v_{IS} - v_S - \theta_I(v_P - v_S)$ .

#### 4.3 Stackelberg Game of the Sharing Case

In the sharing model, the electricity distributor first decides the wholesale rate of electricity. Then the public charging infrastructure operator decides the volume of charging infrastructures and charging service price. Finally, each consumer makes a choice among three charging options.



Fig.5. The timeline of Stackelberg game in the sharing case.

The electricity distributor only decides the wholesale rate for the public charging infrastructure operator,  $W_P$ . The rate for private posts  $W_I$  is a constant. Distributor profit can be formulated as:

$$\pi_D = W_I(q_S + q_{IS}) + W_P q_P - c, \tag{7}$$

where c represents the marginal cost of the distributor to obtain electricity.

The public charging infrastructure operator decides the construction number of posts,  $N_P$ , and charging service price,  $P_P$ . Operator profit can be formulated as:

$$\pi_{O} = P_{P}q_{P} - (W_{P}q_{P} + O_{P}N_{P}) - C_{P}N_{P},$$
(8)

where  $N_P = \frac{q_P}{\theta_P}$ . Solving the first-order conditions for the profit-maximizing operator and distributor yields the equilibrium prices and sales, as summarized by *Lemma 2*. In the benchmark case without considering charge post sharing, the equilibrium charging price of public posts is  $P_P^* = W_I + \frac{3}{4}(v_P - v_I + C_I + O_I) + \frac{C_P + O_P}{4\theta_P}$ , the equilibrium rate of the distributor is  $W_P^* = W_I + \frac{1}{2}(v_P - v_I + C_I - \frac{C_P + O_P}{\theta_P} + O_I)$ , and the equilibrium sales of public and private posts are  $q_P^* = \frac{1}{4} - \frac{C_P + O_P - \theta_P (C_I + O_I)}{4\theta_P (v_P - v_I)}$  and  $q_I^* = \frac{3}{4} + \frac{C_P + O_P - \theta_P (C_I + O_I)}{4\theta_P (v_P - v_I)}$ , respectively.

Lemma 2. In the sharing case considering charge post sharing, the equilibrium charging price of public posts is  $P_P^{**} = W_I + \frac{3}{4} \left( \frac{C_I + C_S + O_I}{\theta_I} - \frac{K_1 K_2}{\theta_I^2} \right) + \frac{C_P + O_P}{4\theta_P}$ , the equilibrium rate of the distributor is  $W_P^{**} = W_I + \frac{1}{2} \left( -\frac{K_1 K_2}{\theta_I^2} + \frac{C_I + C_S + O_I}{\theta_I} - \frac{C_P + O_P}{\theta_P} \right)$ , and the equilibrium rental price of shared posts is  $P_S^{**} = \frac{(\theta_I - 1)(v_{IS} - v_S)}{K_1 K_2} P_P^{**} + \frac{C_I + C_S + O_I + \theta_I W_I}{K_1}$ . The equilibrium sales of public and private posts are  $q_P^{**} = \frac{1}{4} - \theta_I \frac{\theta_P (C_I + C_S + O_I) - \theta_I (C_P + O_P)}{4\theta_P K_1 K_2}$  and  $q_{IS}^{**} = \frac{3}{4\theta_I} + \frac{\theta_P (C_I + C_S + O_I) - \theta_I (C_P + O_P)}{4\theta_P K_1 K_2}$ , respectively, where  $K_1 = \theta_I - 1 + \frac{\theta_I (v_{IS} - v_P)}{K_2}$  and  $K_2 = v_{IS} - v_S - \theta_I (v_P - v_S)$ .

Table 3 gives the sensitivity analysis for a better understanding of *Lemma 2*. There exists a negative relation between  $C_I$  and  $q_{IS}^{**}$ : the increase in private post construction will incur a decrease in average private post charging volume. An increase in any private post costs, including construction cost  $(C_I)$ , sharing upgrade cost  $(C_S)$ , operation cost  $(O_I)$  and electricity wholesale cost  $(W_I)$ , or the improvement of charging service value  $(v_P, v_{IS} \text{ and } v_S)$  will trigger public charging infrastructure operator and electricity distributor to raise service price  $(P_P^{**})$  and wholesale rate  $(W_P^{**})$  respectively. In terms of public posts, a decrease in operation cost  $(O_P)$  or construction cost  $(C_P)$  or an increase in average charging service volume  $(\theta_P)$  will trigger the operator to reduce the service price  $(P_P^{**})$  but the electricity distributor to raise the wholesale rate  $(W_P^{**})$ . Any increases in private post costs and average charging service volume or decreases in public post costs will incur more public post sales  $(q_P^{**})$  but less private post sales  $(q_{IS}^{**})$ .

	$C_I$	C <sub>S</sub>	$C_P$	<i>O</i> <sub><i>I</i></sub>	$O_P$	$W_I$	$v_{IS}$	$v_P$	$v_S$	$\theta_P$
${P_{P}}^{**}$	+	+	+	+	+	+	+	+	+	_
$W_{P}^{**}$	+	+	—	+	_	+	+	+	+	+
$q_{P}^{**}$	+	+		+	—	N/A	—		+	+
$q_{IS}^{**}$	—	—	+	—	+	N/A	+	—	+	—

Table3. Sensitivity analysis of sharing model.

Note: N/A – not applicable (i.e., no/weak relationship)

#### 5. Observations

The equilibria obtained in *lemma 1* and *lemma 2* can be compared to assess the impacts of private charge post sharing on public post service price, electricity wholesale rate and consumer charging choice. The results are summarized in the following observations.

**Observation 1.** The lower the sharing upgrade cost ( $C_S$ ), the lower the public post service price ( $P_P^{**}$ ) and electricity wholesale rate ( $W_P^{**}$ ).

Case 1. When the upgrading cost is relatively high,  $C_S > (\theta_I - 1)(C_I + \theta_I) + \theta_I(v_P - v_I) + \frac{K_1K_2}{\theta_I}$ , the operator may raise the charging service price of public posts  $(P_P^* < P_P^{**})$ , and the electricity distributor may raise the wholesale rate for public posts  $(W_P^* < W_P^{**})$ , as R1 and R3 depict in Fig. 6.

Case 2. When the sharing upgrade cost is relatively low,  $C_S < (\theta_I - 1)(C_I + \theta_I) + \theta_I(v_P - v_I) + \frac{K_1K_2}{\theta_I}$ , the public charging infrastructure operator may reduce the service price  $(P_P^* > P_P^{**})$ , and the electricity distributor may raise the wholesale rate for the operator  $(W_P^* > W_P^{**})$ , as R2 and R4 depict in Fig. 6.

*Observation 2.* The lower the sharing upgrade cost, the lower the profits of public charging infrastructure operator ( $\pi_0^{**}$ ) and electricity distributor ( $\pi_0^{**}$ ).

Case 1. When the upgrading cost is relatively high  $C_S > K_5 C_I + K_6 \left(\frac{O_I}{\theta_I} - \frac{C_P + O_P}{\theta_P} - \frac{\sqrt{(v_I - v_P)K_1K_2}}{\theta_I}\right)$ , where  $K_5 = \sqrt{\frac{K_1K_2}{v_I - v_P}} - 1$  and  $K_6 = \sqrt{\frac{K_1K_2}{v_I - v_P}} - \theta_I$ , the profits of public charging infrastructure operator and electricity distributor increase  $(\pi_0^{**} > \pi_0^* \text{ and } \pi_D^{**} > \pi_D^*)$ , as R1 and R2 depict in Fig. 6.

Case 2. When the upgrading cost is relatively low  $C_S < K_5 C_I + K_6 \left(\frac{O_I}{\theta_I} - \frac{C_P + O_P}{\theta_P} - \frac{\sqrt{(v_I - v_P)K_1K_2}}{\theta_I}\right)$ , the profits of both public charging infrastructures operator and electricity distributor decrease ( $\pi_0^{**} < \pi_0^*$  and  $\pi_D^{**} < \pi_D^*$ ), as R3 and R4 depict in Fig. 6.



Fig.6. Price and profit shifts from benchmark to sharing. ( $C_I = 10, O_I = 6$ )

Price hiking does not always lift profitability. In Fig. 6, for example, R3 indicates that when the public charging infrastructure operator raises the service price, its profit may suffer due to the loss of market share to private post sharing. As for the electricity distributor, R2 suggests that the cut in wholesale rate leads to lower public post service price, which brings more profit to the operator as well as the distributor. When the sharing upgrade cost decreases or the average service volume increases, as shown in R4, more consumers rent private charge posts, making the public charging infrastructure operator less profitable.

Observation 3. The sharing upgrade cost of private posts influences the choice-making of consumers.

Case 1. When the sharing upgrade cost is lower than a certain threshold,  $C_S < MIN$ , fewer consumers choose public posts,  $N_P^* > N_P^{**}$ . Accordingly, the demand for shared private posts increases,  $N_I^* < N_I^{**}$ . Case 2. When the sharing upgrade cost is higher than a certain threshold,  $C_S > MAX$ , more consumers choose public posts,  $N_P^* < N_P^{**}$ . Accordingly, the demand for private posts decreases,  $N_I^* > N_I^{**}$ .

Case 3. When the sharing upgrade cost is moderate,  $MIN < C_S < MAX$ . If  $K_3 \left(\frac{C_P + O_P}{\theta_P} - C_I - O_I\right) > K_3(C_I + O_I) - K_4 \frac{C_P + O_P}{\theta_P} + 3K_1K_2\frac{\theta_I - 1}{\theta_I}$ , the demand for public posts decreases,  $N_P^* > N_P^{**}$ , but the demand for private posts increases,  $N_I^* > N_I^{**}$ . If  $K_3 \left(\frac{C_P + O_P}{\theta_P} - C_I - O_I\right) < K_3(C_I + O_I) - K_4 \frac{C_P + O_P}{\theta_P} + 3K_1K_2\frac{\theta_I - 1}{\theta_I}$ , the demand for public posts increases,  $N_P^* < N_P^{**}$ , but the demand for private posts decreases,  $N_P^* < N_P^{**}$ , but the demand for private posts decreases,  $N_P^* < N_P^{**}$ .

Where 
$$MAX = max \left\{ K_3(C_I + O_I) - K_4 \frac{C_P + O_P}{\theta_P} + 3K_1 K_2 \frac{\theta_I - 1}{\theta_I}, K_3 \left( \frac{C_P + O_P}{\theta_P} - C_I - O_I \right) \right\}, MIN = min \left\{ K_3(C_I + O_I) - K_4 \frac{C_P + O_P}{\theta_P} + 3K_1 K_2 \frac{\theta_I - 1}{\theta_I}, K_3 \left( \frac{C_P + O_P}{\theta_P} - C_I - O_I \right) \right\}, K_3 = \frac{K_1 K_2}{v_I - v_P} + \theta_I, K_4 = \frac{K_1 K_2}{v_I - v_P} + 1$$
  
and  $K_3 > K_4 > 0.$ 

Fig. 7 explores how the demands for public and private posts shift from the benchmark case to the sharing case under the influence of private post service volume  $\theta_I$  and upgrade cost  $C_S$ . R1 suggests that a higher  $C_S$  reduces the demand for private posts. Meanwhile, R2 shows that a higher  $\theta_I$  reduces the demand for public posts. Typically, an increase in the demand for private posts typically leads to a decrease in the demand for public posts, as R4 depicts. However, there is an exception as R3 indicates: when  $\theta_I$  is very low, the demands for private and public posts increase simultaneously.



Fig.7. Market demand shifts from benchmark to sharing. ( $C_I = 5, O_I = 6$ )

## 6. Numerical Analyses

To further explore the critical success factor of private charge post sharing, a numerical study is conducted. The first step is to determine the constant values of exogenous variables based on real-world data. For demonstration purposes, this study adopts relevant values from China, as listed in Table 4.

Exogenous variable	Value (CNY)	Residual lifetime (year)
Cost of a 40 kW DC public charge post	120,000	15
Cost of an 11 kWAC private charge post	10,000	5
Upgrade cost for sharing an AC private charge post	5,000	5
Unit electricity rate for residents (1 kWh)	0.4983	-
Unit electricity marginal cost (1 kWh)	0.27	-
Operation cost of a public charge post	27,000	1
Operation cost of a private charge post	1000	1

Table 4. Exogenous variables in the case of China.

Source: Zhu, Zhang, Lu et al. (2017); Zhao, Cai, and Ma (2018); State Grid in China (2017); Zhang, Li, Zhu et al. (2018).

Each EV has a range of 300 km and a battery of 45 kWh capacity. Considering the energy conversion efficiency, 50 kWh is needed to fully charge the battery. Assuming all EVs are recharged every day, model parameters are estimated in Table 5.

Parameter	Estimate	Parameter	Estimate
$C_P$	21.92	$ heta_P$	8
$C_I$	5.49	$ heta_I$	2
$C_S$	2.75	$v_P$	100
$O_P$	73.97	$v_I$	80
$O_I$	2.74	$v_{IS}$	75
С	13.50	$v_S$	70
$W_I$	24.92		

Table 5. Model parameter estimates.

A public charge post can handle up to eight EVs per day (State Grid in China, 2017), and a shared private post takes care of two EVs on average. Service value is estimated by comparing the cost of EV charging to the fueling of a regular car as an alternative means of transportation. In China, the latest 2016 standard for fuel consumption of a passenger vehicle is 6.7 L per 100 km on the highway. Considering the braking and traffic in local driving, 10 L per 100 km is a realistic estimate for commuters. Based on the average 2019 fuel price in China, CNY225 is needed to travel 300 km for regular cars. Due to the relative sparseness of charging facilities, EV drivers are often more concerned about the travel range. Thus, this study gives CNY100 and CNY80 as conservative estimates of charge service values for public and private posts, respectively. Consumers are sensitive to charging time, and the service value of a private post is relatively low due to its slower speed. Different levels of inconvenience cost will incur when consumers share their own posts or rent others' posts. Correspondingly, the service value of a shared post is CNY75 for an owner.

Based on the estimated parameters, the results of the numerical study are obtained. First, they confirm the aforementioned observations, as shown in Table 6.

Deverseter	Equilibrium Value			
Parameter -	Benchmark	Sharing		
$\pi_{O}$	0.8246	0.9288		
$\pi_D$	5.0641	5.2727		
$P_P$	49.0800	51.7100		
$N_P$	0.0254	0.0235		
$N_I$	0.7970	0.4059		
$q_P$	20.31%	18.81%		
$q_I(q_{IS})$	79.69%	39.10%		
$W_P$	34.7900	33.0400		
$P_S$	-	29.3900		
$q_S$	-	40.59%		

Table 6. Equilibria of benchmark and sharing cases.

Fig. 8 compares the market shares in the absence and presence of private post sharing. As for the benchmark case, 20.31% of consumers use public charge posts, while the others install private posts. In the sharing case, 18.81% of consumers remain loyal to the public charging infrastructure, whereas 40.6% share their posts with the rest. Thus, numerical analyses suggest that in the sharing economy, fewer consumers have to install private charge posts but rent from others through P2P, which does not affect B2C much.



Fig.8. Consumer choice-making in benchmark and sharing cases.

In the sharing case, 7.36% fewer consumers choose public posts, but the public charging infrastructure operator still sees a profit increase by 12.65%. The loyal consumers of public posts have a relatively high preference for the faster charging speed. Even the operator increases the service price by 5.36%, and such consumers still remain loyal. Beyond that, they turn to shared posts, which brings down the profit of public charging infrastructure operators. To avoid such situations, the public operator should increase the service value by using different strategies to expedite charging (Ucer, Kisacikoglu, Yuksel et al., 2019) or improving consumer experiences (e.g. food and entertainment services).

Private charge post sharing is also attractive to consumers. In the numerical study, the service price of shared posts is 43.17% lower than that of public ones, and the profit per service is CNY3.10, 62.83% of the public ones. As the sharing of charge posts helps their owners cover installation costs, it encourages more private capital investment in the EV charging market (private: 49.06%; public: 7.35%; overall: 44.37%).

Fig. 9 compares how critical success factors concerning private post sharing, including the value of sharing (based on charging speed), sharing upgrade cost, average service volume, electricity wholesale rate, construction cost, and operation cost, affect the EV charging market comprising B2C and P2P. As shown in Fig. 9 (a) and (c), the increases in the value of sharing and average service volume of private charge posts (i.e.,  $v_{IS}$  and  $\theta_I$ ) lead to P2P expansion but B2C contraction. As shown in Fig. 9 (b), (e) and (f), the increases in the upgrade, construction and operation costs of private charge posts (i.e.,  $C_S$ ,  $C_I$  and  $\theta_I$ ) lead to P2P expansion. Fig. 9 (d) suggests that the increase in the electricity rate for private charge posts (i.e.,  $W_I$ ) does not make a big difference in the charging market. To promote P2P growth in the charging market, therefore, it is vital to increase the value and volume of private post sharing but reduce its costs. The average service volume is impactful on consumer choice at the onset from 1 to 2, but the curve flats out later on. While the charging technology will advance in the long run, a new sharing platform may improve supply-demand matching efficiency (e.g., with the help of sophisticated algorithms and consumer incentives) to increase the average service volume quickly.



Fig.9. Effects of critical success factors on private charge post sharing.

## 7. Consumer Subsidization

This section considers how the government is supposed to subsidize consumers for installing and sharing private charge posts. The purpose is to use the subsidy case as a baseline to the sharing case to quantify the effect of private charge post sharing. First, the subsidy case is modeled base on the benchmark case. In the subsidy case, the government subsidizes consumers for installing private charge posts, and the subsidy is denoted as  $S_I$ . The utility a consumer derives from using a public post is the same as in the benchmark case, and the utility a consumer derives from using the private post can be formulated as follow.

$$U_{I} = v_{I}x - C_{I} - (W_{I} + O_{I}) + S_{I}.$$
(9)

The profit function stays the same, and solving the first-order conditions for the profit-maximizing operator and distributor yields the equilibria as summarized by *Lemma 3*.

Lemma 3. When a consumer subsidy is provided by the government and no private post sharing is considered, the equilibrium price of public charging infrastructures is  $P_P^{***} = W_I + \frac{3}{4}(v_P - v_I + C_I + O_I - S_I) + \frac{C_P + O_P}{4\theta_P}$ , the equilibrium rate of electricity distributor is  $W_P^{***} = W_I + \frac{1}{2}(v_P - v_I + C_I - \frac{C_P + O_P}{\theta_P} + O_I - S_I)$ , and the equilibrium sales of public and private posts are  $q_P^{***} = \frac{1}{4} - \frac{\theta_P S_I + C_P + O_P - \theta_P (C_I + O_I)}{4\theta_P (v_P - v_I)}$  and  $q_I^{***} = \frac{3}{4} + \frac{\theta_P S_I + C_P + O_P - \theta_P (C_I + O_I)}{4\theta_P (v_P - v_I)}$ , respectively.





Fig.10. Effects of sharing and subsidization at  $\theta_I = 2$ .

Fig.11. Effects of sharing and subsidization at  $\theta_I = 3$ .



Fig.12. Effects of sharing and subsidization at  $\theta_I = 4$ .

To examine how consumer subsidization interacts with post sharing in promoting the growth of EV charging market, five scenarios are compared based on the parameter estimates in Section 5. As shown in Fig. 10-12, extra attention is paid to the average charging service volume of private posts at three levels ( $\theta_I$  = 2, 3 and 4). The black dash-dotted line refers to the benchmark case in which there is neither sharing nor subsidization. The blue solid line depicts that the government subsidizes the installation of private charge posts for no sharing at the variable rate corresponding to the horizontal axis of Subsidy Level. The green dashed line shows that there is no subsidization in the case of post sharing. Finally, the yellow dash-dotted line and purple dotted line indicate consumer subsidi at 20% and 30%, respectively, in the context of post

sharing. For comparison across benchmark and sharing cases, the calculation of subsidy amount is based on private post installation cost  $C_I$ , but the government may as well subsidize the upgrade cost for post sharing with the same amount.

In all three figures, the number of private charge posts installed in the presence of sharing economy is greater than the benchmark case of no sharing. In the sharing case, the number of private charge posts installed at the 30% subsidy level exceeds that at the 20% level. As for no sharing but subsidization, the number of private charge posts installed will rise as the incentive increases, and gradually exceed that in all other scenarios. The average charging service volume of private posts has a positive impact on the number of private charge posts installed in the sharing economy. The number of private charge posts installed in the sharing case without subsidization is equal to that in the no-sharing case at the subsidy level of 21% in Fig. 10 ( $\theta_I = 2$ ), 43% in Fig. 11 ( $\theta_I = 3$ ), and 50% in Fig. 12 ( $\theta_I = 4$ ). Therefore, when the average service volume of private charge posts is relatively low, as in the case of China, consumer subsidization is still effective in accelerating their installation and sharing.

#### 8. Conclusions and Implications

This study establishes a Stackelberg game model for the sharing economy of private charge posts in which the emerging P2P among dual-identity consumers competes with the existing B2C. It examines the impacts of charge post sharing on the pricing strategies of public charging infrastructure operator and electricity distributor, as well as how it affects post construction and is affected by consumer subsidization. The findings suggest that such a new form of P2P SCC attracts non-governmental investment in the EV industry chain for green travel, similar to the crowdfunding on public charging infrastructure.

This study contributes to the literature of sharing economy in the context of EV charging with the modeling of a market structure comprising competing B2C and P2P components. It establishes benchmark and sharing models to compare service pricing and the number of charge posts installed in the absence and presence of private post sharing. The findings enrich the research on P2P SCC in the field of transportation policy by providing theoretical support for the promotion of private charge post sharing. Policy-makers can

assess the extent to which the emerging sharing economy helps reduce market dependence on governmental investment, and understand how to support the healthy development of this critical link in the EV industry chain with appropriate consumer subsidization.

This study yields useful implications for both researchers and practitioners. First of all, private charge post sharing benefits consumers, public charging infrastructure operators, electricity distributors, and the government. For owner consumers, sharing their charge posts generates rental income. For non-owner consumers, using shared posts provides a cheaper charging option. For public charging infrastructure operators and electricity distributors, the existence of loyal consumers provides a possibility to actually enhance profitability. For the government, private post sharing accelerates the expansion of EV charging market for green travel.

The sharing economy brings about significant behavioral changes of EV market stakeholders. When private charge posts are shared, the enhanced equipment utilization reduces the need for the government to build public charging infrastructure. Rather, it may encourage EV drivers to install and share charge posts with consumer subsidization that is relatively affordable and controllable. By playing the additional role of suppliers, consumers enjoy more options and make the best choice considering cost, speed, and convenience. Accordingly, public charging infrastructure operators and electricity distributors need to adjust their pricing strategies. Furthermore, the operator may improve service values (e.g., faster charging speed, complimentary snack/drink) to attract and retain customers as their loyalty is critical to its market share and profitability. The distributor can adjust power grid configurations to cope with the increasing electricity demands from private posts.

The average service volume plays an important role in the success of private charge post sharing. To increase the volume quickly, a P2P SCC platform must improve the efficiency of supply-demand matching through the use of sophisticated information technologies (e.g., artificial intelligence) or even cross-platform cooperation. In the long run, the improvement of charging speed will gradually lift the average service volume. To accelerate this trend, the government can support the research on quick charging technology, and the electricity distributor may modernize power grids in needed areas.

Though numerical analyses are based on the values collected from China, the general inference and methodology are applicable to other settings. To promote private charge post sharing, for instance, the results suggest that consumer subsidization at a reasonable level is worth considering by policy-makers. Therefore, a well-formulated incentive program such as the partial or full subsidization of the upgrade cost for sharing private charge posts is likely to encourage owners to lease them. Stackelberg game based on the dual identity of consumers as buyers and sellers can also be used to model other forms of peer-to-peer resource sharing economy, such as parking lot sharing.

This study has limitations that point to future research directions. It assumes that all consumers enjoy the same opportunity to install private charge posts, but it is often not true in the real world. For example, private posts are prohibited in many residential communities in China due to powerline conditions. In future research, it is necessary to pay attention to such a barrier to private charge post sharing, as a significant percentage of consumers may not have the access to shared posts due to their sparsity in certain areas. In addition, the assumption of a fixed electricity rate for shared private charge posts  $W_I$  is conducive to model solution, but electricity distributors may provide floating rates based on the usage and time of charging (e.g., rate can be lower after midnight till morning). Freeing the constraint on  $W_I$  help establish more flexible decision-making models for consumers as well as electricity distributors, as an increasing number of private charge posts are being directly connected to the grid.

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#### Appendix

*Proof of Lemma 1.* Using the reverse induction method, first solving the operator's profit maximization problem

$$\pi_0 = P_P q_P - C_P N_P - (W_P q_P + O_P N_P), \tag{10}$$

where  $N_P = \frac{q_P}{\theta_P}$ ,  $q_P = 1 - \frac{P_P - C_I - W_I - O_I}{v_P - v_I}$ . And the optimal  $P_P$  can be formulated as a function of  $W_P$ .

$$2P_P = W_I + W_P + v_P - v_I + C_I + O_I + \frac{C_P + O_P}{\theta_P}.$$
(11)

Subsites eq. (11) into the distributor's profit maximization problem

$$\pi_D = W_I q_I + W_P q_P - c. \tag{12}$$

Solving the previous problem by first order condition, the optimal  $W_P$  can be formulated as

$$W_P^* = W_I + \frac{1}{2} \Big( v_P - v_I + C_I - \frac{C_P + O_P}{\theta_P} + O_I \Big).$$
(13)

Subsites eq. (13) into eq. (11), and the optimal  $P_P$  can be formulated as

$$P_P^* = W_I + \frac{3}{4}(v_P - v_I + C_I + O_I) + \frac{C_P + O_P}{4\theta_P}.$$
(14)

Subsites eq. (13) and (14) into the demand functions, and the optimal sales can be formulated as

$$q_P^* = \frac{1}{4} - \frac{C_P + O_P - \theta_P (C_I + O_I)}{4\theta_P (v_P - v_I)},$$
(15)

$$q_I^* = \frac{3}{4} + \frac{C_P + O_P - \theta_P (C_I + O_I)}{4\theta_P (v_P - v_I)}.$$
(16)

Proof of Lemma 2. Similar to the proof of lemma 1.

Proof of Lemma 3. Similar to the proof of lemma 1.

Proof of Observation 1. Conduct difference comparison

$$P_P^* - P_P^{**} = \frac{3}{4\theta_I} \left( (\theta_I - 1)(C_I + \theta_I) - C_S + \theta_I (v_P - v_I) + \frac{K_1 K_2}{\theta_I} \right), \tag{17}$$

$$W_P^* - W_P^{**} = \frac{1}{2\theta_I} \left( (\theta_I - 1)(C_I + \theta_I) - C_S + \theta_I(v_P - v_I) + \frac{K_1 K_2}{\theta_I} \right).$$
(18)

If 
$$C_S < (\theta_I - 1)(C_I + \theta_I) + \theta_I(v_P - v_I) + \frac{K_1 K_2}{\theta_I}$$
, then  $P_P^* > P_P^{**}$  and  $W_P^* > W_P^{**}$ .

## Proof of Observation 2. Similar to the proof of observation 1.

*Proof of Observation 3. Similar to the proof of observation 1.* 

Proof of Table 2. Derivatives of different variables can be solved by the demand and price function.

$$\begin{aligned} \frac{\partial P_{P}^{*}}{\partial C_{l}} &= \frac{\partial P_{P}^{*}}{\partial 0_{l}} = \frac{\partial P_{P}^{*}}{\partial v_{p}} = \frac{3}{4}, \quad \frac{\partial P_{P}^{*}}{\partial 0_{p}} = \frac{\partial P_{P}^{*}}{\partial C_{p}} = \frac{1}{4\theta_{p}}, \quad \frac{\partial P_{P}^{*}}{\partial v_{l}} = -\frac{3}{4}, \quad \frac{\partial P_{P}^{*}}{\partial \theta_{p}} = -\frac{C_{P} + O_{P}}{4\theta_{P}^{2}}. \end{aligned}$$

$$\begin{aligned} \frac{\partial W_{P}^{*}}{\partial C_{l}} &= \frac{\partial W_{P}^{*}}{\partial 0_{l}} = \frac{\partial W_{P}^{*}}{\partial v_{p}} = \frac{1}{2}, \quad \frac{\partial W_{P}^{*}}{\partial \theta_{p}} = \frac{C_{P} + O_{P}}{2\theta_{P}^{2}}, \quad \frac{\partial W_{P}^{*}}{\partial v_{l}} = -\frac{1}{2}, \quad \frac{\partial W_{P}^{*}}{\partial C_{p}} = \frac{\partial W_{P}^{*}}{\partial \theta_{p}} = -\frac{1}{2\theta_{P}}. \end{aligned}$$

$$\begin{aligned} \frac{\partial q_{P}^{*}}{\partial C_{l}} &= \frac{\partial q_{P}^{*}}{\partial 0_{l}} = \frac{1}{4(v_{P} - v_{l})}, \quad \frac{\partial q_{P}^{*}}{\partial C_{P}} = \frac{\partial q_{P}^{*}}{\partial O_{P}} = -\frac{1}{4\theta_{P}(v_{P} - v_{l})}, \quad \frac{\partial q_{P}^{*}}{\partial \theta_{P}} = \frac{C_{P} + O_{P}}{4\theta_{P}^{2}(v_{P} - v_{l})}, \quad \frac{\partial q_{P}^{*}}{\partial v_{P}} = \frac{\partial q_{l}^{*}}{\partial v_{l}} = \\ \frac{C_{P} + O_{P} - \theta_{P}(O_{l} + C_{l})}{4\theta_{P}(v_{P} - v_{l})^{2}}. \end{aligned}$$

$$\begin{aligned} \frac{\partial q_{I}^{*}}{\partial C_{l}} &= \frac{\partial q_{I}^{*}}{\partial O_{l}} = -\frac{1}{4(v_{P} - v_{l})}, \quad \frac{\partial q_{I}^{*}}{\partial C_{P}} = \frac{\partial q_{I}^{*}}{\partial O_{P}} = \frac{1}{4\theta_{P}(v_{P} - v_{l})}, \quad \frac{\partial q_{I}^{*}}{\partial \theta_{P}} = -\frac{C_{P} + O_{P}}{4\theta_{P}^{2}(v_{P} - v_{l})}, \quad \frac{\partial q_{P}^{*}}{\partial v_{P}} = \frac{\partial q_{I}^{*}}{\partial v_{P}} = \\ -\frac{C_{P} + O_{P} - \theta_{P}(O_{l} + C_{l})}{4\theta_{P}(v_{P} - v_{l})^{2}}. \end{aligned}$$

$$\begin{aligned} \frac{\partial q_{P}^{*}}{\partial V_{I}} &= \frac{\partial q_{I}^{*}}{\partial O_{I}} = -\frac{1}{4(v_{P} - v_{l})}, \quad \frac{\partial q_{I}^{*}}{\partial C_{P}} = \frac{\partial q_{I}^{*}}{\partial O_{P}} = \frac{1}{4\theta_{P}(v_{P} - v_{l})}, \quad \frac{\partial q_{I}^{*}}{\partial \theta_{P}} = -\frac{C_{P} + O_{P}}{4\theta_{P}^{2}(v_{P} - v_{l})}, \quad \frac{\partial q_{P}^{*}}{\partial v_{I}} = \frac{\partial q_{I}^{*}}{\partial v_{P}} = \\ -\frac{C_{P} + O_{P} - \theta_{P}(O_{I} + C_{I})}{4\theta_{P}(v_{P} - v_{l})^{2}}. \end{aligned}$$

Where  $C_P + O_P - \theta_P (O_I + C_I) > 0$  and  $v_P - v_I > 0$ .

Proof of Table 3. Derivatives of different variables can be solved by the demand and price function.

$$\frac{\partial P_P^{**}}{\partial C_I} = \frac{\partial P_P^{**}}{\partial C_S} = \frac{\partial P_P^{**}}{\partial O_I} = \frac{3}{4\theta_I}, \quad \frac{\partial P_P^{**}}{\partial C_P} = \frac{\partial P_P^{**}}{\partial O_P} = \frac{1}{4\theta_P}, \quad \frac{\partial P_P^{**}}{\partial v_P} = \frac{3}{4}, \quad \frac{\partial P_P^{**}}{\partial v_{IS}} = \frac{3}{4\theta_I^2}, \quad \frac{\partial P_P^{**}}{\partial v_S} = \frac{3(\theta_I - 1)^2}{4\theta_I^2}, \quad \frac{\partial P_P^{**}}{\partial \theta_P} = \frac{1}{4\theta_I^2}.$$

$$\frac{\partial W_{P}^{**}}{\partial C_{l}} = \frac{\partial W_{P}^{**}}{\partial C_{S}} = \frac{\partial W_{P}^{**}}{\partial O_{l}} = \frac{1}{2\theta_{l}}, \quad \frac{\partial P_{P}^{**}}{\partial C_{P}} = \frac{\partial P_{P}^{**}}{\partial O_{P}} = -\frac{1}{2\theta_{P}}, \quad \frac{\partial W_{P}^{**}}{\partial v_{P}} = \frac{1}{2}, \quad \frac{\partial W_{P}^{**}}{\partial v_{IS}} = \frac{\partial W_{P}^{**}}{\partial \theta_{P}} = \frac{1}{2\theta_{l}^{2}}, \quad \frac{\partial W_{P}^{**}}{\partial v_{S}} = \frac{(\theta_{l}-1)^{2}}{2\theta_{l}^{2}}.$$

$$\frac{\partial q_{P}^{**}}{\partial C_{l}} = \frac{\partial q_{P}^{**}}{\partial C_{S}} = \frac{\partial q_{P}^{**}}{\partial O_{l}} = -\frac{\theta_{l}}{4K_{1}K_{2}}, \quad \frac{\partial q_{P}^{**}}{\partial C_{P}} = \frac{\partial q_{P}^{**}}{\partial O_{P}} = \frac{\theta_{l}^{2}}{4\theta_{P}K_{1}K_{2}}.$$

$$\frac{\partial q_{IS}^{**}}{\partial C_{l}} = \frac{\partial q_{IS}^{**}}{\partial C_{S}} = \frac{\partial q_{IS}^{**}}{\partial O_{l}} = \frac{\theta_{l}}{4K_{1}K_{2}}, \quad \frac{\partial q_{IS}^{**}}{\partial C_{P}} = \frac{\partial q_{IS}^{**}}{\partial O_{P}} = -\frac{\theta_{l}^{2}}{4\theta_{P}K_{1}K_{2}}.$$

$$\frac{1}{\theta_{l}} \frac{\partial q_{P}^{**}}{\partial V_{IS}} = \frac{1}{\theta_{l}^{2}(\theta_{l}-1)} \frac{\partial q_{P}^{**}}{\partial v_{P}} = \frac{1}{\theta_{l}^{2}-\theta_{l}+1} \frac{\partial q_{IS}^{**}}{\partial v_{S}} = -\frac{\theta_{P}(C_{l}+C_{S}+O_{l})-\theta_{l}(C_{P}+O_{P})}{4\theta_{P}(K_{1}K_{2})^{2}} < 0.$$

$$\frac{1}{\theta_{l}(\theta_{l}^{2}-\theta_{l}+1)} \frac{\partial q_{P}^{**}}{\partial v_{S}} = \frac{\partial q_{IS}^{**}}{\partial v_{IS}} = \frac{1}{\theta_{l}(\theta_{l}-1)} \frac{\partial q_{IS}^{**}}{\partial v_{P}} = \frac{\theta_{P}(C_{l}+C_{S}+O_{l})-\theta_{l}(C_{P}+O_{P})}{4\theta_{P}(K_{1}K_{2})^{2}} > 0.$$

$$\frac{\partial P_{P}^{**}}{\partial W_{l}} = \frac{\partial W_{P}^{**}}{\partial W_{l}} = 1, \quad \frac{\partial q_{IS}^{**}}{\partial W_{l}} = \frac{\partial q_{IS}^{**}}{\partial W_{l}} = 0.$$

Where  $C_P + O_P - \theta_P(O_I + C_I) > 0$ ,  $K_1 K_2 < 0$  and  $v_P - v_I > 0$ .