3-10-2009

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Recommended Citation

Teviet Creighton, et. al., (2009) Pulsar timing and spacetime curvature.Astrophysical Journal693:21113. DOI: http://doi.org/10.1088/0004-637X/693/2/1113

The Astrophysical Journal, 693:1113–1117, 2009 March 10 doi:10.1088/0004-637X/693/2/1113

PULSAR TIMING AND SPACETIME CURVATURE

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Received 2008 September 12; accepted 2008 November 21; published 2009 March 5

ABSTRACT

We analyze the effect of weak field gravitational waves on the timing of pulsars, with particular attention to gauge invariance, that is, to the effects that are independent of the choice of coordinates. We find (1) the Doppler shift cannot be separated into gauge invariant gravitational wave and kinetic contributions; (2) a gauge invariant separation *can* be made for the time derivative of the Doppler shift in which the gravitational wave contribution is directly related to the Riemann tensor, and the kinetic contribution is that for special relativity; (3) the gauge-dependent effects in the Doppler shift play no role in the program of gravitational wave detection via pulsar timing. The direct connection shown between pulsar timing and the Riemann tensor of the gravitational waves will be of importance in discussions of gravitational waves from alternative (non-Einsteinian) theories of gravitation.

Key words: gravitational waves – pulsars: general

1. INTRODUCTION

The possibility of detecting gravitational waves through pulsar timing, first suggested independently by Sazhin (1978) & Detweiler (1979), is of increasing interest as part of the birth of gravitational wave astronomy, and the details of this technique continue to be advanced by several researchers (Hellings & Downs 1983; Backer & Hellings 1986; Hellings 1986; Kaspi et al. 1994; Lommen & Backer 2001; Jenet et al. 2005; Hobbs 2008). The basis of this detection technique is the effect that a gravitational wave has on the arrival times of pulsar signals. In the analysis of this effect (Hellings 1981; Backer & Hellings 1986), the gravitational wave has been described in terms of perturbations $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$ of the spacetime metric $g_{\mu\nu}$. This description of gravitational waves is analogous to describing electromagnetic waves using the potentials $\{\Phi, \mathbf{A}\}$. In particular, coordinate transformations are analogous to the gauge transformations of electromagnetic theory, and—at least in linearized general relativity—are also called gauge transformations.

If electromagnetic fields are to be described in terms of potentials, then one must choose a particular gauge for the description. Similarly, in relativistic gravity, coordinate conditions ("gauge fixing") must be added. The need for a gauge choice gives rise to the possibility of confusing a gauge effect with a physical effect. This confusion, for relativistic gravity, kept it a controversial question for many years whether gravitational waves were physical or coordinate waves (Kennefick 2007). A further source of confusion has been the distinction between the Doppler shift due to gravitational waves, and the Doppler shift due to relative motion of the emitting pulsar and the receiving telescope. This has the potential to be particularly confusing, since the relative motion is a manifestation of spacetime curvature, as is the gravitational wave.

Our purpose is to clarify the description of pulsar timing, by analyzing pulsar timing with particular attention to what aspects of the pulsar Doppler shift are gauge dependent and what aspects are physical. What we will show is that pulsar timing can be analyzed in a manner in which there is an incontrovertible contribution due to gravitational waves, and a contribution due to relative motion of emitter and receiver. The result is couched completely in terms of quantities that are intuitively appealing, as well as mathematically well defined. The gravitational wave contribution will be based on the Riemann tensor, which is gauge invariant very much as are the electric and magnetic fields in classical electromagnetism. We will see, moreover, that in the literature there are claims about Doppler shift effects that are not gauge invariant, but that these statements refer to intermediate steps in the analysis of pulsar timing and do not affect the overall program of pulsar timing. The technique of gravitational wave detection by pulsar timing is physically valid; it is clearly distinguishable both from coordinate effects and from source/receiver motions.

Besides clarifying what is and is not physically meaningful in pulsar timing, this paper helps to establish a firm background for studying the nonstandard polarization modes of alternative theories of gravitation, modes for which pulsar-timing detection may be particularly useful. Such modes are most easily described in terms of components of the Riemann tensor, not in terms of metric perturbations.

The paper is organized as follows. In Section 2, the mathematical heart, we derive the expressions that form the basis of our analysis, and clearly show what is and what is not gauge invariant in pulsar timing. To help focus on the distinction between kinematic and gravitational effects, in Section 2 we assume that the motions of the emitter and the receiver of pulses are driven by nongravitational forces. We remedy this unphysical assumption in Section 3, where we add gravity as the source of the astrophysical motions of the emitter and receiver. In Section 4, we discuss the implications of these results for the way in which pulsar-timing gravitational wave detection is to be carried out, and is to be viewed. In the Appendix, we fill in some details of Section 2 that we thought would divert too much attention from the main point of that section.

We have chosen to present these results with a minimum of unnecessary generality that would make the mathematics more elegant, but more obscure. Except as noted, we use the notation and conventions of the text by Misner et al. (1973).

2. DOPPLER SHIFT AND GAUGE INVARIANCE

The Doppler shift from emitter to observer can be considered to have two sources: (1) general relativistic: the effects of gravitational waves, and (2) special relativistic: the relative motion of emitter and receiver. The second effect will be much larger than the first in astrophysical situations. The relative velocity of astrophysical bodies is on the order of $v \sim 10^{-3}$ (in $c = 1$ units), while the characteristic magnitude of gravitational waves is smaller than 10^{-20}, usually much smaller. In our development, we keep special relativistic effects to all orders of v, but we will ignore effects of order $v|h_{\mu\nu}|$. The justification is that gravitational effects will be at the limit of detectibility; effects that are smaller by 10^{-3} are not of immediate interest. Here and below, we will use v to denote an astrophysical velocity, and repeatedly use the fact that $v|h_{\mu\nu}|$ can be ignored.

As pointed out in Section 1, here we will make the artificial assumption that the emitter and receiver are point particles that are being driven in accelerated motions by nongravitational forces, such as rocket engines. In this section, the only curvature of spacetime is due to gravitational waves.

We will denote our emitter worldline by E, and receiver worldline by R, as shown in Figure 1. We choose Minkowski background coordinates t, x, y, z for the background so that they are appropriate for the approximations just discussed. That is, in these coordinates, the E and R worldlines are at rest aside from velocities of order v, and the metric perturbations $h_{\mu\nu}$ due to gravitational waves are extremely small. The 4-momentum of a photon from emission to reception is written as $P^\mu = P^0(1, \vec{n})$, so that $\vec{n}$ plays the role of a unit vector pointing (in the Minkowski background) from the emitter to the receiver.

Now, we consider the following expression:

$$\begin{aligned}\mathrm{Dopp} = &-\int_{\mathrm{E}}^{\mathrm{R}} \left(\tfrac{1}{2}\, h_{tt,t} + n^j h_{tj,t} + \tfrac{1}{2} n^j n^k\, h_{jk,t}\right)\, d\lambda \\ &+ [U^t - n_k U^k]^R - [U^t - n_k U^k]^E\,. \end{aligned} \tag{1}$$

Here, U^μ is the 4-velocity of the emitter (E) and receiver (R), at the events of emission and reception; $d\lambda$ indicates integration along the photon worldline, with $dt = d\lambda$ and $dx^j = n^j d\lambda$; Latin indices are spatial (referring to the x, y, z components of the coordinate basis).

In the Appendix, we show that the expression in Equation (1) represents the Doppler shift of the photon, that is, the fractional difference by which the photon energy observed by the receiver is greater than that observed by the emitter. Here, we focus on the gauge property of the expression, the changes induced in the expression by a coordinate transformation $x^{\mu\ \mathrm{new}} = x^\mu + \xi^\mu$ in which the gauge vector $\vec{\xi}$ is of the order of the metric perturbations $h_{\mu\nu}$.

The standard gauge transformations of the perturbations and of the components of the 4-velocities are

$$h^{\mathrm{new}}_{\mu\nu} = h_{\mu\nu} + \xi_{\mu,\nu} + \xi_{\nu,\mu} \qquad\qquad U^{\mu\ \mathrm{new}} = \xi^\alpha U^\mu_\alpha - U^\alpha \xi^\mu_\alpha\,. \tag{2}$$

We note that the 4-velocities have the components $U^\mu = \delta^\mu_t$ aside from corrections of order v. With terms of order $v|\vec{\xi}|$ ignored, we are left with

$$U^{\mu\ \mathrm{new}} = U^\mu - \xi^\mu_{,t}\,. \tag{3}$$

We also note that in Equation (1), a gauge change in $\vec{n}$ would give terms of order $v|\vec{\xi}|$ and $h_{\mu\nu}|\vec{\xi}|$, so no gauge transformation of $\vec{n}$ is carried out.

A straightforward calculation shows the gauge invariance of Dopp,

$$\begin{aligned}\delta\mathrm{Dopp} \equiv \mathrm{Dopp}^{\mathrm{new}} - \mathrm{Dopp} &= -\int_{\mathrm{E}}^{\mathrm{R}} (\xi_{t,t,t} + n^j \xi_{t,j,t} + n^j \xi_{j,t,t} \\ &\quad + n^j n^k \xi_{j,k,t})\, d\lambda + [\xi_{t,t} + n^j \xi_{j,t}]^{\mathrm{R}} - [\xi_{t,t} + n^j \xi_{j,t}]^{\mathrm{R}} \\ &= -\int_{\mathrm{E}}^{\mathrm{R}} \left(\frac{\partial}{\partial t} + n^p \frac{\partial}{\partial x^p}\right)(\xi_{t,t} + n^q \xi_{q,t})\, d\lambda \\ &\quad + [\xi_{t,t} + n^j \xi_{j,t}]^{\mathrm{R}} - [\xi_{t,t} + n^j \xi_{j,t}]^{\mathrm{R}} = 0\,. \end{aligned} \tag{4}$$

(It is worth noting here that time derivatives of $\vec{n}$ can be ignored since these are of order v and would be multiplied by terms of order $|\vec{\xi}|$.)

Since the Doppler shift, as defined, refers to an objective physical measurement, the fact that it is gauge invariant is simply a consistency requirement. The details of the gauge transformation, however, underscore an important point: neither the integral nor the 4-velocity contributions to Equation (1) are separately gauge invariant. Thus, the temptation to identify the integral in Equation (1) as the gravitational wave contribution and the 4-velocity terms as the kinetic contribution must be avoided, since that identification has no invariant meaning.

To arrive at a more physically useful expression, we take the derivative of Equation (1) with respect to the coordinate time t. (In doing this, we again note that $d\vec{n}/dt$ is of order v, and hence that time differentiation of $\vec{n}$ in the integral can be ignored.) The result of time differentiation is

$$\begin{aligned}\frac{d\,\mathrm{Dopp}}{dt} = &-\int_{\mathrm{E}}^{\mathrm{R}} \left(\tfrac{1}{2}\, h_{tt,tt} + n^j h_{tj,tt} + \tfrac{1}{2}\, n^j n^k h_{jk,tt}\right)\, d\lambda \\ &+ \left[\frac{dU^t}{dt} - n_j \frac{dU^j}{dt} - \frac{dn_j}{dt} U^j\right]^R \\ &- \left[\frac{dU^t}{dt} - n_j \frac{dU^j}{dt} - \frac{dn_j}{dt} U^j\right]^E\,. \end{aligned} \tag{5}$$

A few comments on the time differentiation are appropriate. The total time derivative of the integral should, in principle, include the change in the integral due to the change of the time of the endpoints of the integral. But this change involves v, and hence terms of order $v|h_{\mu\nu}|$ which we ignore. The time-changing endpoints, on the other hand, cannot be ignored in the 4-velocity terms, since these terms are not multiplied by metric perturbations. The time derivatives d/dt in the 4-velocity terms are therefore understood to be the derivatives along the worldlines, i.e., $d/dt = \partial_t + v^k \partial_{x^k}$.

Next, we consider the components of the 4-acceleration a^μ of the E and R worldlines

$$a^j = \gamma\, \frac{dU^j}{dt} + (U^t)^2 \Gamma^j_{tt} = \gamma\, \frac{dU^j}{dt} + h_{jt,t} - \tfrac{1}{2} h_{tt,j}, \tag{6}$$

$$a^t = \gamma\, \frac{dU^t}{dt} + (U^t)^2 \Gamma^t_{tt} = \gamma\, \frac{dU^t}{dt} - \tfrac{1}{2} h_{tt,t}, \tag{7}$$

where we have ignored terms of order $v|h_{\mu\nu}|$ in the Γ term, and where γ, as usual, represents $1/\sqrt{1 - v^2}$. When these

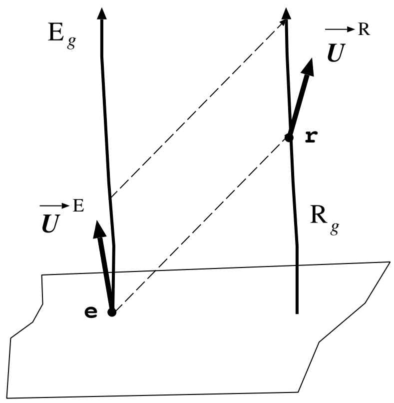

Figure 1. Construction of the timelike geodesic worldlines E_g and R_g, and the Minkowski-like coordinates.

equations are used in Equation (5), we get

$$\frac{d\,\mathrm{Dopp}}{dt} = \int_{\mathrm{E}}^{\mathrm{R}} \left(-\tfrac{1}{2} n^j n^k h_{tt,jk} + n^j n^k h_{tj,tk} - \tfrac{1}{2} n^j n^k h_{jk,tt}\right) d\lambda + \left[\frac{a^t - n_j a^j}{\gamma}\right]^R - \left[\frac{a^t - n_j a^j}{\gamma}\right]^E + \left[-U_j \frac{dn^j}{dt}\right]^R - \left[-U_j \frac{dn^j}{dt}\right]^E \tag{8}$$

$$= \int_{\mathrm{E}}^{\mathrm{R}} n^j n^k R_{tjtk} d\lambda + \left[\frac{a^t - n_j a^j}{\gamma}\right]^R - \left[\frac{a^t - n_j a^j}{\gamma}\right]^E + \left[-U_j \frac{dn^j}{dt}\right]^R - \left[-U_j \frac{dn^j}{dt}\right]^E, \tag{9}$$

where R_{tjtk} represents the components of the Riemann tensor.

The expression in Equation (9) is, of course, gauge invariant, but unlike the gauge-invariant expression in Equation (1), the individual contributions are now gauge invariant. The integrand contains only a projection of the manifestly gauge-invariant Riemann tensor. The 4-velocity and 4-acceleration terms are all of first or higher order in v, so their gauge changes would be of order $v|h_{\mu\nu}|$, and hence ignorable. Unlike the expression in Equation (1) for the Doppler shift, the expression in Equation (9) for the *time derivative* of the Doppler shift contains contributions that have physical gauge-invariant meaning; the integral gives the effect of gravitational waves and the remaining terms give the special relativistic contributions due to acceleration (the 4-acceleration terms) and due to the relative geometry of the worldlines (the $d\vec{n}/dt$ terms).

3. GRAVITATIONALLY DRIVEN ORBITS

In Section 2, we assumed that the motions of the emitter and receiver were driven by nongravitational forces ("rocket engines"), so that the role of spacetime curvature lay solely in the perturbations $h_{\mu\nu}$ identified as gravitational waves. In reality, of course, astrophysical orbital motions are driven by gravity. In this section, we explain how to incorporate other aspects of gravity, in particular orbital forces, into the considerations of Section 2.

We imagine that the emitter and receiver are on astrophysical bodies that are moving under the influence of other astrophysical bodies. As assumed in Section 2, orbital velocities are small compared to c. This means that the gravitational interactions among all bodies are weak-field interactions. More precisely, for our system of gravitationally interacting bodies, the Newtonian potential Φ due to one body, at the position of the other body, must be small compared to $c^2 = 1$. We now make the additional assumption that the gravitational field is weak ($|\Phi| \ll 1$) everywhere along the photon path. This allows us to treat all non-gravitational-wave fields relevant to the problem as being adequately described by a post-Newtonian (pN) approximation (Will 1993). Though we may consider a higher order pN approximation, we do not consider orders high enough (order (v^7)) for the orbiting bodies themselves to become sources of gravitational radiation.

In a pN approximation, a background Minkowski-like coordinate system is used, and metric perturbations in terms of these potentials are required to have "post-Newtonian" character (Will 1993). This character requires, among other things, that the potentials be functions only of the coordinate separation of source point and field point. The equations determining the metric perturbations are computed from Einstein's equations truncated to the order of the pN approximation being used. Gravitationally driven motions are then taken to be the geodesics of the pN metric, and may be interpreted as having accelerations in the flat background.

Due to the nature of the pN approach, a pN gauge vector ξ_μ would have to have pN character. This very strong constraint turns out to leave almost no gauge freedom. Moreover, there is a standard pN coordinate system (Will 1993) in which even the small gauge freedom is removed. When this gauge is adopted, the question of pN gauge transformations ceases to exist.

Gauge transformations can still be made, of course. The $h_{\mu\nu}$ gravitational wave metric perturbations do *not* have a pN character. So long as our gauge field ξ_μ is not pN in character, it is clearly separable from any choice of a pN gauge, and does not affect the construction of pN coordinates. Again, we note that this presumes that we ignore any contribution to the $h_{\mu\nu}$ field of gravitational waves from the pN-modeled bodies themselves.

With these considerations, we can conclude that the gauge transformations of Section 2 can be repeated in the pN plus gravitational wave spacetime, with the accelerations taken as those (relative to the Minkowski background) coming from a pN theory. One might worry that the gravitational wave gauge transformations in Section 2 were made relative to a flat background, while the pN spacetime is not flat. But this simply means that we are ignoring terms of order $|\Phi||\xi|$, which are analogous to the terms of order $v^2|\xi|$ that we ignored in Section 2.

Though we are primarily interested in gauge questions, the matter of the pN metric perturbations raises an important separate issue. The photon's Doppler shift will be affected by the surface gravitational field of the emitter and receiver. In addition, if the photon happens to pass close to another astrophysical body, the pN fields of that other body will affect pulsar time-of-arrivals by altering photon path lengths, and through the Shapiro time delay (Shapiro 1966). These effects can be separately evaluated (as part of a pN calculation) and added to the Doppler shift calculations of Section 2. Since the effects are small, they do not "mix" with the $h_{\mu\nu}$ terms.

We close this short section by pointing out that there are limits to the clean separation of kinematic and gravitational wave terms. If the gravitational fields are strong, orbital velocities are relativistic, or gravitational potentials along the photon path are of order unity, then the gauge-invariance demonstration in Section 2 fails to hold. Indeed, it is intuitively appealing that in such a case, it should not be possible to make a general distinction between small perturbations of spacetime and motions of astrophysical bodies. It would still be possible, of course, to make a practical distinction between the two, if the gravitational waves had a significantly higher frequency than the timescale of the kinematics.

4. CONCLUSIONS

A physical measurement, such as the Doppler shift of a photon, is not changed by the changes in our mathematical choices, so the gauge invariance of the expression in Equation (1) is simply a check of consistency. What *is* subject to our mathematical choices is the interpretation of the terms in Equation (1), in particular the interpretation of the integral term as the gravitational wave contribution. This interpretation is clearly incorrect since we could, for example, choose coordinates that make the integral vanish for any photon, and put all of the Doppler shift into the kinetic term.

Though integrals like that in Equation (1) have appeared in the literature (Detweiler 1979; Mashhoon 1982; Bertotti et al. 1983) in discussions of pulsar timing, in practice, this causes no real difficulty. The Doppler shift can be understood to be the time integral of the gauge-invariant expression in Equation (5). That time integral will contain an integration constant that cannot be meaningfully separated into gravity and kinematics. The observed phase of the arriving pulses, the raw data of pulsar timing, will be the next time integral, and will contain a term $A + Bt$, where A and B are the integration constants. These integration-constant terms play no role in the actual analysis of timing residuals. The program of gravity wave detection by pulsar timing, therefore, has a solid physical foundation based on the Riemann tensor.

This relationship of timing residuals to the Riemann tensor has a useful secondary benefit. As pointed out in the introduction, pulsar timing has the potential to be a particularly sensitive probe of non-Einsteinian polarization modes of gravitational waves (Lee et al. 2008). The description of these modes uses the components of the Riemann tensor. The expression in Equation (5) gives a clear and unequivocal description of how these nonstandard gravitational waves affect pulsar-timing residuals.

We gratefully acknowledge support by the National Science Foundation under grants AST0545837 and PHY0554367. We also thank the NASA Center for Gravitational Wave Astronomy at University of Texas at Brownsville. We thank Kejia Lee for useful discussions of this work.

APPENDIX

DERIVATION OF THE GAUGE-INVARIANT EXPRESSION FOR THE DOPPLER SHIFT

We now turn to a proof, needed in Section 2, that the expression in Equation (1) is the Doppler shift. Since we have shown that the expression in Equation (1) is gauge invariant, it suffices to show that it is equal to Doppler shift in any one gauge.

We construct a convenient gauge for this proof as follows. We let **e** and **r** be the emission and reception events for a pulsar photon, as shown in Figure 1. We choose a timelike geodesic worldline E_g through **e** to be almost tangent (i.e., tangent to order v), at **e**, to the emitter worldline. Next we choose a timelike geodesic worldline R_g through **r** in such a way that it is parallel to the geodesic worldline through **e** in the following sense. Near the emission event, we construct a spacelike surface orthogonal to the emitter 4-velocity and we construct this surface to have extrinsic curvature with a vanishing trace. We generate (much in the manner of Gaussian normal coordinates) a congruence of timelike geodesic worldlines normal to this surface. We assume that the spacetime curvature is small enough, and/or the emitter–receiver distance is small enough that this congruence fills spacetime, with no crossings, in the neighborhood of the reception event. We choose R_g to be the curve in that congruence that goes through **r**.

The next step in the construction is to generate null geodesics (photon worldlines) from E_g to R_g and define our Minkowski coordinates t, x, y, z by the following steps: (1) the surface spanned by the null geodesics is taken to be a surface of constants x and y; (2) z is set to zero along E_g, and along R_g we set z equal to another constant, the length of the spatial geodesic between E_g and R_g on the spatial hypersurface; (3) z is taken to be an affine parameter along the null geodesics; (4) t is taken to be proper time along E_g; and (5) the coordinate t is propagated through the t, z surface by requiring $t-z$ to be constant along the null geodesics. We note that a tangent to the null geodesics is given by

$$\partial_t|_{z=\mathrm{const}} + \partial_z|_{t=\mathrm{const}} \ , \tag{A1}$$

in the coordinate system we have defined.

We now note that the covariant t component of the photon 4-momentum satisfies the geodesic equation

$$\frac{dP_t}{d\lambda} = P_\alpha \frac{dx^\beta}{d\lambda}\Gamma^\alpha_{\beta t} = \tfrac{1}{2} P^\alpha \tfrac{dx^\beta}{d\lambda} h_{\alpha\beta,t} \tag{A2}$$

and

$$\Delta P_t = \int_{\mathrm{E}}^{\mathrm{R}} P^\alpha dx^\beta h_{\alpha\beta,t} = P^t \int_{\mathrm{E}}^{\mathrm{R}} \left(\tfrac{1}{2}h_{tt,t} + h_{tz,t} + \tfrac{1}{2}h_{zz,t}\right) d\lambda, \tag{A3}$$

where

$$\frac{d}{d\lambda} = \frac{\partial}{\partial t}\bigg|_{z=\mathrm{const}} + \frac{\partial}{\partial z}\bigg|_{t=\mathrm{const}} . \tag{A4}$$

From this expression, we have that the difference between the photon energies at **r** and **e** is

$$[\text{Energy measured on } \mathrm{R}_g - \text{Energy measured on } \mathrm{E}_g]/P^t = -\int_{\mathrm{E}}^{\mathrm{R}} \left(\tfrac{1}{2}h_{tt,t} + h_{tz,t} + \tfrac{1}{2}h_{zz,t}\right) d\lambda\,. \tag{A5}$$

We now generalize this to the case in which the photon is not confined to the plane with x and y constant. Since the expression on the right in Equation (A5) is already first order in the metric perturbations, we need to consider only a more general photon direction in the Minkowski background. If we denote by $\vec{n}$ the unit vector pointing in the spatial direction in which the photon moves, then the generalization of Equation (A5) is

$$[\text{Energy measured on } \mathrm{R}_g - \text{Energy measured on } \mathrm{E}_g]/P^t = -\int_{\mathrm{E}}^{\mathrm{R}} \left(\tfrac{1}{2}h_{tt,t} + n^j h_{tj,t} + \tfrac{1}{2}h_{zz,t}\right) d\lambda\,. \tag{A6}$$

To get the full expression for the Doppler shift, we must consider the fractional energy changes from the geodesic worldlines to the observer worldlines. An observer with 4-velocity U^μ observes a photon with 4-momentum P^μ to have energy

$-P_\mu U^\mu$. By construction, our geodesic worldlines have components $U^\mu = \{1, 0, 0, 0\}$. Thus, the energy observed by the receiver is $P^t\,[U^t - n^j U_j]^R$, where P^t is the energy observed at the reception event by the geodesic observer. With this and the similar expression for the emission event, we get Equation (1).

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