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Higinio Dominguez

Sandra Crespo

Tamara del Valle

Melissa Adams Corral

Megan Coupe

See next page for additional authors

Authors

Higinio Dominguez, Sandra Crespo, Tamara del Valle, Melissa Adams Corral, Megan Coupe, Gustavo Gonzalez, and Yordhan Ormazabal

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Higinio Dominguez
Michigan State University

Sandra Crespo

Tamara del Valle

Melissa Adams

Megan Coupe

See next page for additional authors

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Authors

Higinio Dominguez, Sandra Crespo, Tamara del Valle, Melissa Adams, Megan Coupe, Gustavo Gonzalez, and Yordhan Ormazabal

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Higiniio Dominguez

Teacher Preparation Program, College of Education, Michigan State University, USA
higinio@msu.edu

Sandra Crespo

Teacher Preparation Program, College of Education, Michigan State University, USA
crespo@msu.edu

Tamara del Valle

Universidad Metropolitana de Ciencias de la Educación, Santiago, CHILE

Melissa Adams

The Ohio State University, USA

Megan Coupe

Sunset Valley Elementary School, Austin, Texas, USA

Gustavo González-García

Universidad Católica Silva Henríquez, Santiago, CHILE

Yordhan Ormazabal

Corporación Educacional Albert Schweitzer, Santiago, CHILE

Abstract

In this paper we contribute an alternative conceptualization of creativity by highlighting movement as creating spatial and temporal dimensions that are important to make sense of creativity in children mathematical thinking. Using data from an international collaboration between two teaching-research teams from

the United States and Chile, we trace how children mobilized their social bodies, materials, tools, images, metaphors, languages, and improvisations in order to make sense of the concept of fractions. Based on these findings we offer a number of discussion points that highlight the importance of creating these kinds of learning spaces, the role of tasks in promoting different kinds of movement, and implications for thinking about alternative conceptualization and operationalization of creativity as responding to issues of cultural and linguistic diversity and inclusivity.

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1. Introduction

In this article we explore two encounters of creative mathematical thinking between a group of Tejanas/os and a group of Chilenas/os children doing mathematics together as part of a one-year collaborative research project that culminated in a teaching exchange. We trace creativity in children's mathematical thinking by following the process of mobilizing the concept of fractions across the past, present, and future of learners [7]. This mobilization brings the mathematical concept into a dialogue with the students' multicultural, multilingual, and academic lives. This dialogue—in its hybrid, dynamic, and fluid form—is the locus of students' mathematical creativity. We argue that the past, present, and future in a learner's life, and the intraspaces across them, are saturated with meanings that are not finite but temporal; not static but fluid and spatial. Exploring these meanings with students can reshape informative learning (learning by repetition) into transformative learning (learning by re-envision) [16]. Traditionally, theories of learning have emphasized connecting students' prior knowledge to their current learning [20], a position that overlooks the fact that the most creative ideas begin by imagining our future [22]. Teaching mathematics to foster students' ability to mobilize concepts across the past, present, and future resonates with Dewey's [6] concern that "if we teach today's students as we taught yesterday's, we rob them of tomorrow". As students mobilize concepts back and forth across time and space, what they learn gets transformed, and that in turn transforms their learning.

2. Creativity Then and Now

While mathematical creativity has not been clearly defined in research, it is often linked to the early writings of mathematicians about the process and methods of mathematical thinking (e.g., [18, 19]). Discussed mainly as individual cognition, mathematical creativity in these historical writings has been construed as a cognitive process that happens in an individual's mind. From this perspective, it is assumed that cognitive processes are responsible for producing an original and extraordinary product. Characteristics of the creative processes have been elaborated as Gestalt-like stages of preparation, incubation, illumination, and verification by research studies that have observed and interviewed mathematicians (e.g., [24]). Much of the research on mathematical creativity has concentrated on the nature of the cognitive processes that contribute to creative thinking (e.g., the transformation of information, application of known strategies to new situations) and on specifying criteria by which a product may be recognized as being creative. Along this stance, for decades an uncritical discourse in education has contributed to a general association between mathematical creativity and giftedness, thus perpetuating a pernicious divide between the normal and the extraordinary [5].

This conceptualization of mathematical creativity has been influential in mathematics education and to the study of this phenomenon. For example, the mathematical creativity of school children has been studied by focusing on children's ability to generate divergent ideas within mathematical situations and on their ability to overcome algorithmic fixations in mathematical problem solving (e.g., [11]). Take for example the unexpected response of a fourth grader to completing the following number sentence: $1/2 + \_\_\_\_ = 1$. The student response was that the answer could be either $2/4$, $3/6$, $4/8$, $5/10$, etc. and in fact she further stated that there are an infinite set of numbers that can make that number sentence equal to 1 but that probably the teacher was expecting the answer of $1/2$ [3]. The production of multiple equivalent fractions to a closed problem can be considered as an example of children's creative thinking in mathematics. Furthermore, the student's recognition that her answer was unusual also suggests an awareness of typical and atypical solutions to given problems. Children's invented algorithms, use of drawings to solve problems, and creation of problems from given situations have also been studied as examples of children's creativity in mathematics.

These studies tend to focus on individual children's responses typically collected in interview settings, and not necessarily within the context of whole class instruction. These studies have been important in recognizing the potential and the existence of children's mathematical creativity, including for very young pre-school children. However, these studies do not consider the dynamic and social nature of mathematical creativity, which is the type of children's creativity in mathematics that we are interested in documenting. Our work aligns more with theories that consider learning as a social process that happens in interaction with others and with the environment, and with ecological theories of learning (e.g., [5]) which consider learning as a natural occurrence within an ecological system, in our case that ecological system being the school classroom. For the most part, classroom research has tended to showcase the ways in which instruction constrain children's creative thinking in mathematics. Our work therefore takes a different stance and instead considers children's interactions among themselves and with their teachers to be part of the ecosystem in which mathematical creativity occurs, not only or because the children are encouraged to be creative but rather because the very act of learning requires creative actions and thinking.

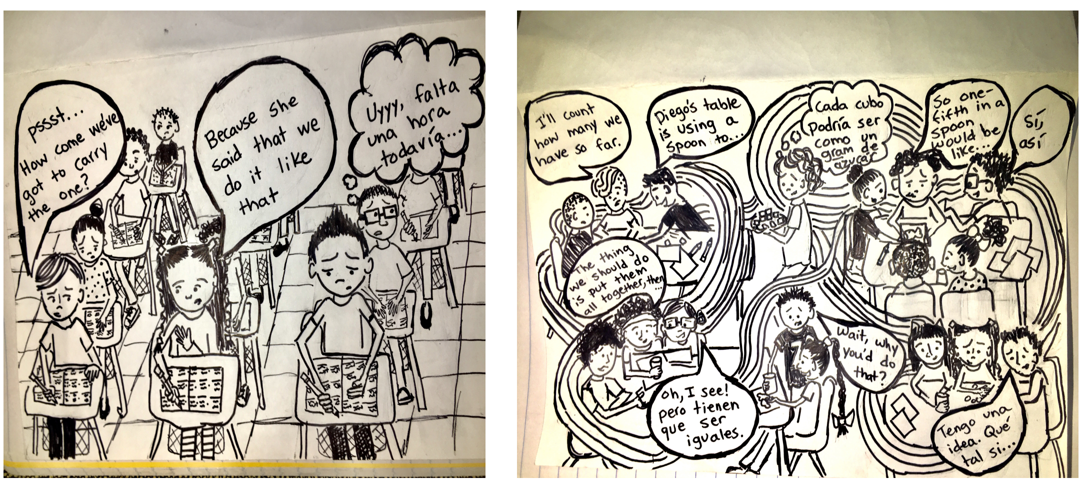
3. Conceptual Framework

We found the conceptions of creativity in the previous literature review to be too limiting. Our work seeks to extend these static, nativist, and normative conceptions of creativity toward considering more socially and interactive forms of mathematical creativity. We see creativity as a socially constructed liberatory force that allows groups to think differently about mathematical concepts. In art, for example, Isabel and Rubén Toledo conceived of creativity as "the perpetual next, the never ending new, the reinvention of inventions" [21]. Rather than imposing a definition of creativity in mathematics as a cognitive, individual, and atemporal ability—which reflects a nativist and potentially segregationist perspective that contributes little to understanding how all students learn important mathematical ideas—we are interested in tracing the creativity that emerges and is nurtured in spaces collectively constituted by non-expert individuals working with challenging tasks, fluid materials, moving bodies, and curious minds. We are, in other words, interested in understanding the collective and improvisational nature of creativity. And because collectives work in a given context, in this case

we are looking at cultural, linguistic, and academic aspects of this context, and we are locating all these aspects across space and time.

We theorize creativity in children's mathematical thinking by drawing from feminist-influenced perspectives on multiple and connected ways of knowing mathematics [23, 9] and theories of embodied cognition [17]. Our tracing of creativity originates in our careful consideration of movement, both physical and intellectual. First, we recognize that "the body does not move into space and time, it creates space and time: There is no space and time before movement" [13]. The movement of learning bodies collectively creates spaces that are temporal in nature, and thus creativity in children's mathematical thinking is necessarily spatial and temporal. Learning bodies do not move aimlessly; they mobilize material and human resources purposefully across time and space as part of mobilizing mathematical concepts.

The following illustrations of two classrooms were created by Megan Coupe, a teacher and co-author in our collaboration who has learned to transform her classroom from A, where restricted movement also restricts creativity, to B, where participants construct social space in which creativity in children's mathematical thinking thrives (Figure 1).



Classroom A Classroom B

Figure 1: Movement in the mathematics classroom.

In classrooms such as Classroom A, movement is highly regulated and directed either toward tasks that are disconnected from past and future expe-

riences. In some instances, these tasks include weak connections to the past. This stance appears in how mathematics curricula are organized, and how instruction highlights students' prior knowledge as building blocks for new learning [20]. This kind of repetitive and regimented movement recreates a space to which children come back each day to do the same. A body that moves the same way again and again is a constricted body often producing what is expected not what is capable of creating. The repetitive movement of completing exercises following teacher explanations, filling in answers to partially completed worksheet tasks, or memorizing steps and formulas, leaves little opportunity for the social arrangement of spaces where creativity in children's mathematical thinking emerges.

Few are the classrooms in which movement is not regulated by an expert, typically a teacher, but by the learning system represented by all the students and the teacher. In classrooms such as Classroom B, all participants coordinate movement to create socially constructed learning spaces. In these spaces, students feel safe and free to revisit their past ways of thinking, but they bring images, constructions, and ideas from that past to be reworked in ways that are new, unexpected, and creative. That is, they revisit their past not to stay there, but to reinvent it because the tasks they are solving requires the kind of thinking that is spatially and temporally connected. Their movement, in other words, oscillates between the past and the future. A body that moves both to the known past and toward the unknown future is a creative body. In this creative movement, the body recognizes that the past and the future are not disconnected dimensions because these bodies, through movement, create both the spatial and temporal quality of learning.

Finally, the space and time that moving bodies create serves as a social, dialogical, and collective system for learning. The tasks that teachers and researchers bring into these systems are delivered to students without prescriptions as to how to solve them because we trust them to draw from their cultural, linguistic, and academic pasts, and also to engage their curiosity to imagine, predict, hypothesize, revise, and reinvent their ways of thinking about mathematical concepts. Our commitment to this conceptualization of creativity in children's mathematical thinking is rooted in our belief that more creativity may escape us the more we try to see it through rigid conceptualizations.

4. Methodology and Participants

Our US-Chile collaboration began as the two research teams visited each other's countries in order to better understand the contexts of our classroom-based educational research. This international collaboration allowed us to cross physical, cultural, linguistic, and contextual borders, reminding us of the centrality of movement for locating human learning across spatial and temporal dimensions. During our mutual visits, the two teams agreed that a powerful way of enhancing and unifying our research agenda would be to develop and explore together a teaching exchange with teachers whose practice each research team knew well. Thus, Higinio invited two teachers who were participants in one of his research projects, Megan Coupe and Melissa Adams, to participate in this exchange. Tamara del Valle also invited a teacher, Yordhan Ormazabal, a former student of hers to participate in our collaboration. Prior to and in order to plan the teaching exchange, we hosted several teleconferences with all the teachers and researchers.

The first visit occurred when the US research team, Higinio and Sandra, traveled to Santiago to work with the Chile team, Tamara, Gustavo and teacher Yordhan. During that visit, the collaborating team co-planned and implemented a coherent set of tasks focused on providing students with multiple opportunities to recognize, create, and justify unconventional halves. This exchange continued with a second visit to Chile, when the US team brought two teachers, Melissa and Megan, to co-plan and implement a second set of tasks focused on representing fractions by contextualizing them in student-selected experiences such as sharing bread, or finding equivalent fractions in familiar objects like their country flag. During this visit, the teachers and researchers took advantage of being together to co-plan the third and final visit of this teaching exchange. In this visit, the Chilean team visited Austin, Texas to co-plan and implement another set of tasks focused on fractions as measurement by exploring children's typical daily sugar consumption. The Chilean students (in fifth grade, ages 10-11) had sent their Texan counterparts (in third grade, ages 8-9) paper flags where they had identified various fractions. Taking advantage of the similarity between the Chilean and Texan flags, the teachers decided to explore the task of comparing fractions using these flags.

The teachers in our collaboration were influential in the design of the tasks described above. We used these tasks to explore "the images, illustrations,

examples, analogies, metaphors, gestures, exercises and other experiences that come up in the mathematics classroom" [4]. As researchers with close connections with and robust understanding of the teaching practice, we appropriated the teachers' predictions that once in the students' hands, the students would take the mathematical tasks to mobilize the concept of fractions with physical and intellectual movement that is relational, spatial, and temporal.

Accordingly, our methodology responded by following students—sometimes individually, sometimes in groups—as they moved around the room. The purpose was to judge the appropriateness of tools and materials, share ways of thinking, engage in translanguaging—a dialogical space where two languages freely flow toward creativity and criticality [10]—and gesturing, pause to reflect on their creative processes, and invent and reinvent ways of measuring, drawing, and diagramming as part of enacting their creative thinking across space and time. Our focus on understanding movement socially enacted guided the design of our methodology with its focus on video recording of children mobilizing the concept of fractions.

5. Nontraditional Analysis: How We Traced Creativity

Sources of data included videotaping of the implementation of the various sets of tasks. Both teams of researchers and one graduate research assistant followed with recording cameras the dynamic mobilization of the concept of fraction. Researchers, teachers, and even students shared this process of video recording the fluid development of the concept of fraction. Other sources of data that enhanced our analysis included pictures, student work, and memos generated during and after the task implementation.

Although analysis is a mainstream term in research, we prefer to view the work we did in this project as tracing creativity in children's mathematical thinking and this is why. To analyze means to take apart, to separate a whole into elements. Our focus on movement does not allow us to separate it because to make sense of movement one needs to understand its purpose, which often requires the opposite: to synthesize all that is captured by movement in order to fulfill its purpose. To make sense of movement, what we did was "to read widely at first and deeply once immersed in the data" [1]. In terms of our data this meant engaging all participants in the data in the

moment that they were being generated [12]. This collective involvement of teachers, students, and researchers promoted "the creativity that emerges in the spaces and times between the *doing* of research, the coding of data and its subsequent writing up" [12]. This is how we consider our *analysis* as tracing: what we have in addition to the traditional concept of data are the traces of our work together, of our "being there" [**?**] in the form of moments filled with collective interpretations awaiting our assembling of them in a way that communicates a coherent experience understanding creativity in children' mathematical thinking. By tracing movement this way, we recognized creativity as a fluid, social, and complex phenomenon that emerges in spaces and across time where people—students, teachers, and researchers in this case—come together to understand both a and with a mathematical concept. Also, since seven of us—three teachers and four researchers—did this analysis together, we had enough perspectives to synthesize rather than to try to take apart the phenomenon we were making sense of.

6. Tracing Creativity in Children's Mathematical Thinking

In the following two subsections, we present two examples of how students mobilized the concept of fractions. For each example we describe the mathematical task first, what children did to make sense of the task and eventually solve the task. We illustrate this part with photographs that show key moments of the students' work. Finally, we describe how we traced creativity in these children's mathematical thinking.

6.1. Fractions and Children's Sugar Consumption

The meaning of fractions as measurement is underemphasized in the elementary mathematics curricula where the meaning of part-whole is favored. The emphasis on this meaning has been associated with children's difficulty understanding fractions greater than one [25]. Other countries, such as Japan, emphasize this meaning when the concept of fractions is first introduced to students [26]. The teachers fused the importance of teaching this meaning of fractions with the importance of examining access to healthy foods in the students' community. As Melissa Adams stated, "the goal of the unit was to make 'visible' what can often be obscured in discussions urging children to make healthy choices." As part of the unit, students learned to read nutrition

labels and analyze advertisements. In order to make sense of the measure of sugar in grams, students converted grams to teaspoons and filled bags with sugar to model the quantities in their favorite beverages. They used maps to explore the distribution of grocery stores near their community and compared them with maps revealing the city's racial segregation. They also analyzed the ways in which healthy and unhealthy foods are priced in ways that affect access.

6.1.1. Example A: "Sweeping" Fourths into Fifths

One of the tasks included in the unit on children's sugar consumption consisted of measuring grams of sugar in various foods that students liked. To make the measurement aspect flow without interfering with the focus on fractions, the teacher gave students an approximation of one spoon = 5 grams. Faced with the task of measuring 76 grams of sugar, one student, Victoria, decided to empty one spoonful of sugar (5 grams) onto the table. She first passed the tip of the spoon though the middle of the sugar, thus creating two parts. Then she split the first half into half again and proceeded to do the same with the second half, thus creating fourths. As a first attempt at making fifths, Victoria mobilized the concept of fractions to a past experience: repeated halving, which has been interpreted in research as a potential precursor for multiplicative thinking [8]. Obviously, mobilizing the concept of fifths to the experience of repeated halving did not help Victoria to make fifths. Not until the teacher, Megan, and the student began to think creatively. This moment began with a teacher's question.

MEGAN: ¿Cómo hacemos para sacar quintos? [How do we go about making fifths?]

VICTORIA: Uh, so maybe, we could grab a little bit of each one.

Megan's question prompted Victoria to give a strategy from the past a different use by taking sugar from each fourth in order to make a fifth part (Figure 2).

Victoria took care to find a 1/4 teaspoon measure to "grab a little bit of each one." The movable material in this task (sugar), the flat surface of the table, the spoon, and the teacher's question facilitated the mobilization of fourths into fifths. But there was yet another level at which creativity emerged:

Figure 2: Transforming fourths into fifths.

language. This is evident in the translanguaging that characterized the interaction between Megan and Victoria. Translanguaging is fundamentally a creative phenomenon that is about "pushing and breaking the boundaries between the old and the new, the conventional and the original, and the acceptable and the challenging [28]. In this case, there are no boundaries between Megan and Victoria's languages, allowing their conversation to move fluidly toward challenging other boundaries (e.g., can we take a little from each fourth? Can we use measuring tools to repartition fractions?). Moreover, "a translanguaging space has its own transformative power because it is forever ongoing and combines and generates new identities, values and practices" [10]. Also, both the student and teacher managed enacted creative metaphors through language. This occurred when Victoria was trying to move sugar from each forth to form a fifth part. Noticing that she was struggling trying to pick up sugar with the small tip of the spoon, Megan suggested:

MEGAN: You may just want to slide it across, like if you're sweeping the floor, como si fuera una escoba. [as if it were a broom]. No tiene que ser exacto, pero más o menos [It doesn't have to be exact, but more or less].

VICTORIA: (She begins to notice that some of the little mounds have more sugar than others so she starts sliding sugar from one to another in a continuous process of adjusting the size of the five parts).

The metaphoric language that Megan and Victoria used to construct their learning space connects the familiar experience of sweeping to the student's creative strategy of making fifths out of fourths. Interestingly, Victoria continued "sweeping" grains of sugar both with the spoon and her fingers as she seemed not fully convinced of the equal size of the five parts. Victoria's unfinished and always perfectible fifths suggest how she and her teacher collectively mobilized the concept of fraction across time and space.

6.1.2. Example B: Elongating a Rectangle of Sugar to Make Fifths

This episode was recorded with two cameras, one of which came in the middle of the interaction once it had finished recording the episode in Example A above. Different groups of students and teachers followed along the movement of these camera shifts, attracted by what their classmates were doing. Together, the eye of the camera and the eyes of the visitors would re-energize the physical, intellectual, and social movement that was happening in these learning spaces. Someone in the group would catch up these "visitors" with what the group was doing.

In this case, Higinio (a coauthor of this article) was interacting with a small group of students led by Giselle and José Luis. This group was measuring 56 grams of sugar, the content in one of their favorite soft drinks. Like Victoria in Example A, Giselle and her group had measured 11 spoonfuls of sugar (55 grams) and they had emptied the sugar of a 12th spoon onto the table in order to split it into fifths. Aided by the straightedge of a ruler, Giselle began squeezing the sugar into a row. Seeing this row of sugar forming mobilized members of the group to anticipate the shape of a rectangle, a familiar shape that students see when they are introduced to fractions. When Sandra (a coauthor of this article) arrived to visit this group, a student explained what they had just done.

JOSÉ LUIS: We made it into a rectangle and then we put a line to get it.

SANDRA: Deja ver, ¿cómo lo van a hacer? [Let me see. How are you going to do it?] (Using the spoon handle, Giselle marks 3 lines on the rectangle of sugar).

HIGINIO: Those are fourths.

SANDRA: ¿Cómo vamos a hecer los quintos? [How are we going to make fifths?]

JOSÉ LUIS: We need to make it longer! We need to make it longer! (Giselle elongates the rectangle of sugar and this time marks 4 lines, thus creating fifths)

HIGINIO: Oh, I like that!

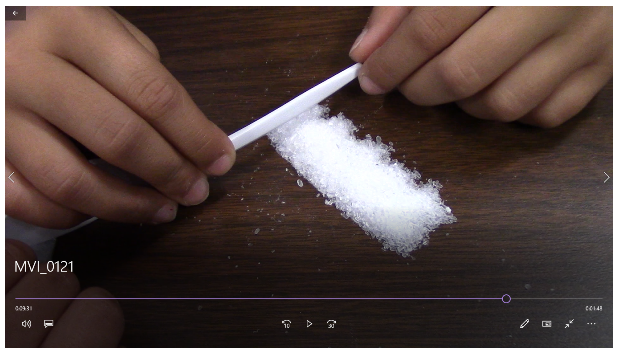


Figure 3: Splitting a rectangle of sugar into fifths.

The details of this strategy were different from Victoria's, yet the process of mobilizing the concept of fractions across time and space was similar. First, Giselle's reshaping of the mound of sugar into a row and then into a rectangle was a collective construction that was possible through movement. As her hands moved the ruler against the sugar, a new shape emerged before the eyes of all the participants, moving them to evoke the shape of a rectangle. As Giselle moved the handle of the spoon by marking lines along the rectangle, participants visually followed her actions by counting 1, 2, 3, 4, 5 for each part she was making (Figure 3).

This group's ability to self-regulate their collective activity was evident in how some members of the group volunteered to record on a notebook the process of counting eleven whole spoons of sugar for the first 55 grams. When they were trying to represent fifths by drawing and partitioning a circle into five parts, José Luis exclaimed: "It's like the tire of a car!" Mobilized into this learning space were tools that included a ruler, a spoon, a notebook opened with a student diagramming the work the group was doing, and the sugar. At least two of these tools—the ruler used to level the sugar in each spoon and later used to form the rectangle and the spoon used to mark lines on

the rectangle—were used creatively as part of the improvisational nature of creative thinking. Translanguaging also continued being used as a creative dialogical interaction.

6.2. Fractions as Children See Them

The instructional tasks for this part of our collaboration emerged from our conversations with Yordhan, the teacher in Chile. The US teaching-research team met Yordhan during his first year of professional practice. In various conversations, Yordhan expressed a deep interest in learning to transform his teaching practice early in his career to make it more immune to the highly restrictive educational context in his country. Our first visit coincided with his teaching of fractions, so together we designed tasks intended to promote deeper understanding of this concept. While some tasks focused on the meaning of fractions as measurement, another set of tasks focused on exploring how students saw fractions represented in unusual ways generated the most creative thinking. These tasks were so impactful for Yordhan's students that next year, when he visited Melissa's classroom in Texas, his students sent the Texan students drawings of where and how they were seeing fractions. In Example A below, students were studying how their Chilean counterparts saw fractions in the Chilean flag, mobilizing them to compare this flag to their own Texas flag.

6.2.1. Example A: "Sin Tercios No Puedes Hacer Sextos." [Without Thirds You Can't Make Sixths]

In small groups, the Texan students were exploring the ideas that the Chilean students had shared with them regarding where and how they were seeing fractions in the Chilean flag. Teachers Yordhan and Melissa asked students to respond with their own thinking. In this example, two students, Christian and Yoldi, were working with Higinio (a coauthor of this article). Christian's focus on the concept of equivalence guided this discussion. He was drawing with a pencil and tracing vertical lines with his fingers. The first line traced was an extension from the small square inside the Texas flag, and the second line was a complete vertical line, thus creating thirds. Then he drew a horizontal line across the flag, thus splitting each third in half and therefore creating sixths. For each partition, Higinio asked him to write the fractions he was making, 3/3 and 6/6. Then he asked Christian:

HIGINIO: ¿Y qué notas de estas dos fracciones, 3/3 y 6/6? [And what do you notice about these two fractions, 3/3 and 6/6?]

CHRISTIAN: Que si le agregas tres a estos (señala 3/3) es lo mismo que esto (6/6). [That if you add three to these (points to 3/3) it's the same as these (6/6)]

HIGINIO: Es lo mismo que estos. [It's the same as these.]

CHRISTIAN: Sí, nomás, le pones dos, porque lo cortas [Yes, you just, you just put two, because you cut it] (makes sound as if cutting the flag while gesturing halving the flag).

HIGINIO: Um, ¿y es lo mismo a qué? [Um, and that's the same as what?]

CHRISTIAN: Esto, es decir, así todo, pero (señala rápidamente varios puntos sobre la bandera), pero sin, sin eso (3/3) no puedes hacer esto (6/6). [This, I mean, like the whole, but (points to several places on the flag very fast), but without, without that (3/3) you can't make this (6/6)]

HIGINIO: Sin eso no puedes hacer esto. ¡Uh! ¡Qué interesante! O sea que tienes que tener tercios para poder hacer sextos. [Without that you can't make this. Uh! How interesting! So you have to have thirds in order to make sixths] (Christian nods both with his head and also with his whole body as he moves up and down on his chair.)

HIGINIO: ¿Habría manera de que si yo te doy 6/6 que tú me dieras 3/3? [Would there be a way that, if I gave you 6/6, that you would give me 3/3?]

CHRISTIAN: Quitar la linea de enmedio. [Remove the line in the middle] (hand gestures erasing the line in the middle)

The ability to imagine or ignore partition lines has been described as key for developing understanding of fraction equivalence [2]. While the dimensions of the design in the Texas flag do not coincide exactly with the thirds or sixths that Christian created, what is important here is how Christian mobilized the flag's design by tracing both with his finger and a pencil projected lines from the interior rectangle of the Texas flag, thus creating thirds

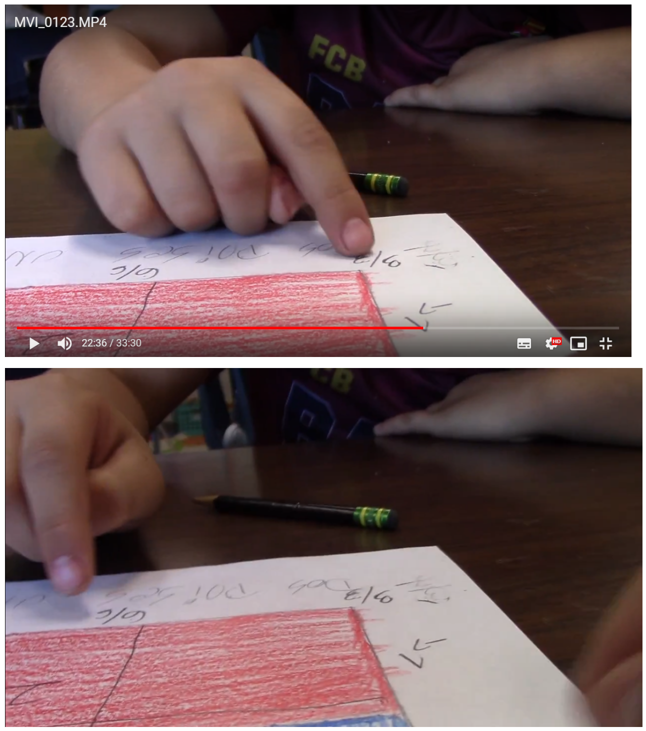


Figure 4: Mobilizing the multiplicative structure of fractions.

first and then sixths. The order in which this mobilization occurred—by imagining, air tracing, and then drawing lines to create thirds followed by sixths—supported Christian to arrive at an important conclusion: "without thirds you can't make sixths" (Figure 4). The importance of this realization is far greater than the inexactness of Christian's thirds or sixths as it suggests the development of multiplicative thinking [8]. This evidence is supported by my continued interaction with Christian and Yoldi when, following Christian's realization, I gave them multiple examples of possible and not possible fraction equivalence. For example, when I asked whether they could make 3/3 out of 4/4, Yoldi exclaimed and explained:

YOLDI: It's not possible! This is possible (points to $3/3 = 6/6$) because this 3 is a half of 6, but 3 is not a half of 4, so that's why it's not possible.

HIGINIO: OK, I want to hear that idea again.

YOLDI: Because this (points to 3/3) is not the half of this (points to 4/4).

The creative work of imagining, tracing, and removing projected lines—all of which was possible through movement—served to mobilize fraction representations, allowing Christian and Yoldi to move to a next level of mathematical thinking: judging whether two fractions, expressed symbolically, were equivalent or not. In this dialogical movement we saw an important precursor to using multiplicative thinking as their primary reasoning regarding fraction equivalence.

6.2.2. Example B: Making "Unusual" Halves Less Unusual

For this task, teacher Yordhan gave students a worksheet with fractions represented in various shapes. Some fractions were halves and others were not, and many were represented in unusual ways. Yordhan asked students to work collaboratively in small groups to discriminate between halves and non-halves and to justify their thinking. We found the creativity of two small groups interesting to trace because they exemplify different levels of mobilizing the concept of fractions. In the first group, Jimmy explained to his tablemates Isadora and Kevin how he was moving—mentally and dialogically—the various shaded sections of the unusual halves (Figure 5). When Jimmy said, "Si pudieramos mover éste para acá" [if we could move this one over here], to make an unusual half look more conventional, Higinio (a coauthor of this article) encouraged Jimmy to write that down and to draw an arrow to depict how he was imagining moving one section of the half up to make it look like a more conventional half.

HIGINIO: Qué piensas, Kevin, Jimmy dice que si moviéramos esta hacia arriba se convierte en una mitad. ¿Qué dices? [What do you think, Kevin, Jimmy says that if we could move this one up, it would become a half. What do you say?] (Kevin nods and continues to agree with the "half-ness" of other squares split into unusual halves, except for one square that is split into two triangles. So Higinio asks Jimmy to explain to Kevin.)

JIMMY: Si porque, mira, de todo acá (pasa la mano sobre toda la hoja de trabajo), este está volteado, y este está volteado (compara las mitades rectangulares que aparecen verticalmente con las que

aparecen horizontalmente), y éste (las mitades triangulares), si movemos este por aquí, tendría, quedaría volteado por la mitad igual que estos (indica con gestos cómo colorearía una mitad rectangular completa). [Yes because, look, of all these (moves hand over the entire worksheet), this one is turned, and this one is turned (comparing the rectangular halves that are positioned vertically and those that are positioned horizontally), and this one (the triangular halves), if we move this one over here, it would have, it would end up turned around across its half just like these ones (hand gestures coloring a complete rectangular half).]

ISADORA: (Turns to look at the triangle shape) Y si corremos ese pa' allá... [And if we scoot this one over there...]

JIMMY: Ese si lo movemos pa' acá, nos quedaría igual (a una mitad) porque este (empty one) quedaría sombreado (indica con gestos cómo colorearía la mitad) [That one if we move it over here, it would turn out to be equal (to a half) because this one (empty one) would be shaded in (hand gestures coloring in one half)].

Jimmy's movement of the unusual halves was not a physical movement as in Victoria's case with the sugar or a projected movement as in Christian's case with the flags. Rather, his movement was imagined, mental, and highly dialogical. He supported the expression of this movement with gestures such as pretending that he was pushing one shaded section with the tip of his pencil or air tracing the coloring in of the receiving section. Higinio also supported Jimmy to communicate his mental movement by asking him to write what he had said and to diagram the movement with an arrow. All of this work mobilized Kevin who began to see the halves that Jimmy was seeing, and Isadora who joined Jimmy in mentally, gesturally, and dialogically mobilizing an unusual half depicted in a triangular shape.

In the second group, Julissa and her tablemate Esperanza were working on the same worksheet as Jimmy's group. Both groups were creative in the sense that their thinking involved imagining the movement of shaded pieces that they could not physically move because they were drawn on paper. These imaginary movements were described gesturally—with fingers "jumping" across the drawings as if lifting one piece from one place to another—and linguistically—by explaining to partners these movements using "if-then" statements. However, unlike Jimmy's focus on making unusual halves look

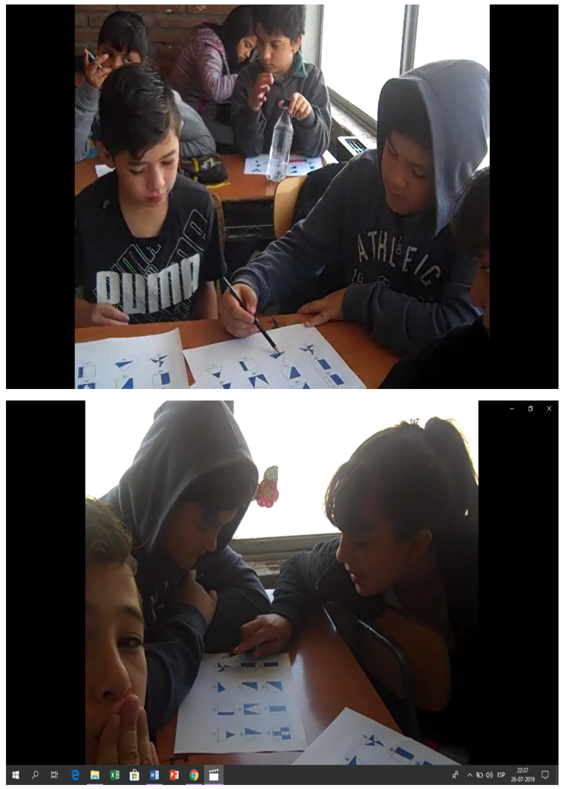


Figure 5: Students mobilizing fractions with their imagination.

more conventional, Julissa was focused on identifying fractions that were not halves. While Jimmy's creativity consisted of imagining the rearranging of parts to look like more obvious images of halves, Julissa's creativity consisted of applying her invented criteria of "same space" in order to visually discriminate between halves and not halves. Although Jimmy was able to mobilize the collective imagination of his tablemates, Julissa struggled to promote the same kind of imagination in her tablemate Esperanza (Figure 6). One of our cameras arrived when Julissa was explaining the following to Esperanza.

JULISSA: Ese (señala una fracción), no sería igual porque mira, las fracciones son así, si tienen el mismo espacio, sí, pero mira, esta es rectangular y esta triangular (mira a su compañera), sería lo mismo (continúa mirando a su compañera), ¿entendiste? [That one (points to a fraction) wouldn't be equal because look, frac-

tions are like this, if they have the same space, yes, but look, this one is rectangular and this one is triangular (looks at tablemate), it would be the same (continues looking at tablemate), did you understand?]

ESPERANZA: No.

JULISSA: (Grabs her forehead. Sandra, a coauthor of this article, reminds Esperanza to request a better explanation from Julissa)

JULISSA: Es que mira, tienen que tener partes iguales (retraza con el lápiz la fracción con dos partes iguales). [It's cause, look, they have to have equal parts (retraces with pencil the fraction with two equal parts)]

ESPERANZA: Eso te lo entendí, pero lo otro no lo entendí. [That part I got it, but the other part I didn't understand.] (Julissa continues discriminating those fractions that are not equivalent to one half. Esperanza yawns.)

Engaging one's imagination and inviting others to do so is both a cognitively demanding and highly creative process. In the picture above, Julissa employed multiple artefacts—e.g., her red pencil parallel to her index finger and the worksheet with multiple drawings of fractions—to visually compare distances and areas in the figures drawn on paper. She did this as she was mobilizing the concept of fractions off the static representations on paper. Our research-teaching team conjectured whether her focus on categorizing halves and non-halves, however important, may have prevented her from engaging Esperanza's imagination as successfully as Jimmy did. The tablemates in Jimmy's group were focusing their attention on one shape at a time, mentally rearranging pieces to see if they could make the shape look more like a conventional half. Julissa's imaginative attention, on the other hand, was moving back and forth between halves and not halves. We included this task in our analysis to highlight different instantiations of creativity, not to suggest that one instantiation is better than the other [14]. Instead, these two ways of interpreting the same task, of invoking and mobilizing different materials, are unique assemblages that illuminate the larger, dynamic process we have called creative thinking.

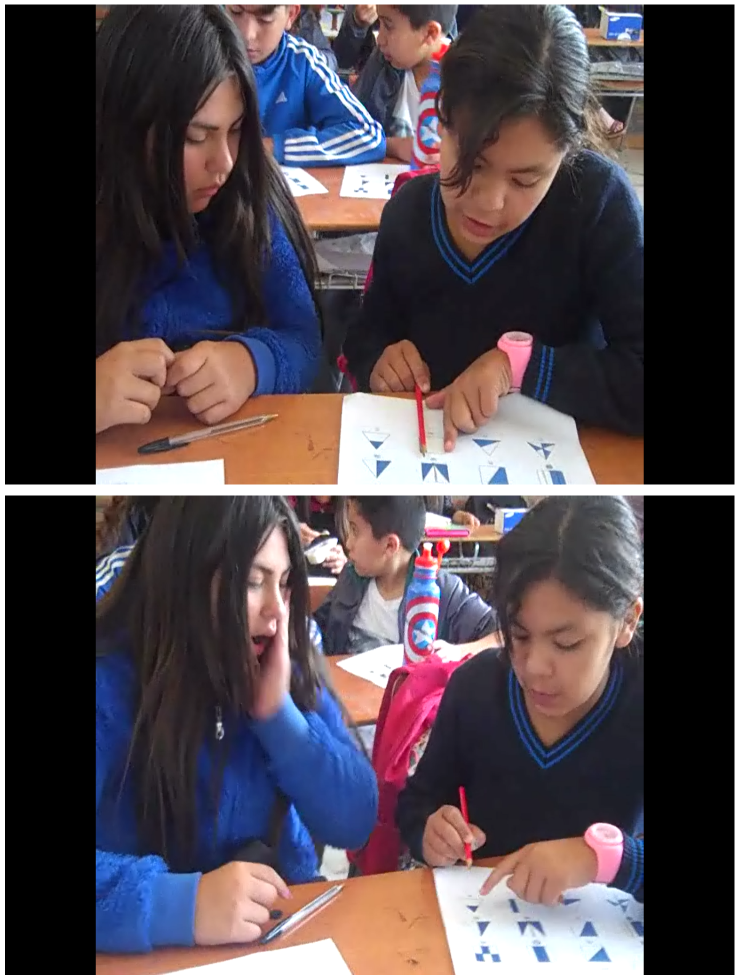


Figure 6: Trying to engage a partner's imagination.

7. Discussion

The purpose of our paper was to provide an alternative conceptualization of creativity in children's mathematical thinking. Rather than following individualistic, nativist, and static conceptualizations that are common in the literature reviewed, we offered a view of creativity rooted in the social, emerging across space and time, and mobilized by the collective. The evidence drawn from multiple examples suggests that creativity emerges from

a group's socially concerted movement. This movement—of bodies, materials, tools, improvisations, images, and linguistic metaphors—creates spaces that exist across time and in which learners are able to mobilize mathematical concepts. An important feature of these learning spaces that enabled mobilizing a mathematical concept was the existence of no boundaries: between languages (translanguaging); between past, present, and future experiences; and between the statuses of students, teachers and researchers who see each other as knowers. In our literature review we found these dimensions of creativity—namely movement and the boundary-free space and time it creates—either absent or weakly articulated.

Our conceptualization of creativity prompted us to think about making sense of our data in a way that cohered with creativity itself, instead of trying to take it apart through more traditional analysis. Our recognition that creativity in children's mathematical thinking leaves a trace of their thinking assisted us in this process. We traced students' creativity in multiple modalities, including drawings, gestures, translanguaging, metaphors, images, and improvisations. In our international collaboration, children populated our video material with images filled with movement as they gestured, rose from their chairs to look for tools and materials, mobilized materials both physically—as in the sugar consumption task—, graphically—as in the flags task—, and mentally/imaginatively—as in the unusual halves task. This physical, conceptual, and social movement was simultaneously characterized by a dialogical movement in which translanguaging, metaphors, and gestures flowed across space and time, thus permitting the socialization of innovative perspectives on the concept of fractions.

The selection of tasks across our examples was purposeful in order to illustrate the different kinds of movement promoted by these tasks. The physical movement of the sugar to create fifths, the graphic movement of the flags to create equivalent fractions, and the imaginary movement of the unusual halves to sort out fractions were all influenced by how the task was set up and how learners interpreted the task. The mobilization of fourths into fifths was possible by Victoria's creative thinking but it was also supported by the movable material, sugar, that she was working with. This movable material contrasts with the paper flags, a more rigid material that Christian and Yoldi were working with. At the same time, this material allowed these students to mobilize the concept in a different way, toward a multiplicative thinking that is foundational for understanding fraction equivalence.

The unusual halves that Jimmy and Julissa worked with in their groups were the most static of these tasks since they were printed on a worksheet. Yet, in various ways, these two groups of students engaged their creativity to mobilize these static fraction representations by engaging their imagination in ways that highlight difference as illuminating children's creative thinking.

These various kinds of movement—physical, material, social, dialogical, and imaginary—promoted a very important movement for student and teacher learning: the mobilization of children' mathematical reasoning. We traced this temporal development of movement particularly in the case of Christian and Yoldi as they first mobilized the lines inside the flag and how this mobilization supported the emergence of multiplicative thinking. An important aspect of mobilizing a mathematical concept this way is how it invites all students to participate in a collective movement. As we followed students' movement in the classrooms in Chile and Texas, we often found students explaining their thinking to others, reminding us that "the person who learns most from an explanation is usually the explainer" [15]. We see this as a positive advance toward enhancing and operationalizing inclusivity in linguistically and culturally diverse classrooms.

Finally, our tracing of creativity in children's mathematical thinking supports arguments for (a) the importance of designing creative and inclusive spaces for exploring mathematical concepts, (b) the need to develop innovative methodologies and more nuanced analyses of students' capacity to take ownership of their own learning as they construct via movement dynamic learning spaces where they decide what counts as valuable for their mathematical learning, and (c) the transformative learning generated in and by these spaces.

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