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DETECTING THE STOCHASTIC GRAVITATIONAL WAVE BACKGROUND USING PULSAR TIMING

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ABSTRACT

The direct detection of gravitational waves is a major goal of current astrophysics. We provide details of a new method for detecting a stochastic background of gravitational waves using pulsar timing data. Our results show that regular timing observations of 40 pulsars each with a timing accuracy of 100 ns will be able to make a direct detection of the predicted stochastic background from coalescing black holes within 5 years. With an improved prewhitening algorithm, or if the background is at the upper end of the predicted range, a significant detection should be possible with only 20 pulsars.

Subject headings: gravitational waves — pulsars: general

1. INTRODUCTION

Analysis of pulsar pulse time-of-arrival (TOA) data shows that pulsars, especially millisecond pulsars (MSPs), are very stable clocks. The measurement of timing residuals, that is, the differences between observed and predicted TOAs, enables the direct detection of gravitational waves (GWs; Estabrook & Wahlquist 1975; Sazhin 1978; Detweiler 1979). The fluctuating TOAs induced by a GW will be correlated between widely spaced pulsars. Hellings & Downs (1983) attempted to detect this correlation by cross-correlating the time derivative of the timing residuals for multiple pulsars. In our work, we have developed a similar cross-correlation technique and have, for the first time, a fully analyzed method for combining multiple pulsar observations in order to make an unambiguous detection of a GW background. We emphasize that, in contrast to Hellings & Downs (1983), our method is based entirely on the measured residuals.

Only the effects of a stochastic background of GWs are considered. Astrophysical sources of such a background include cosmological processes (e.g., Maggiore 2000) and coalescing massive black hole binary systems (Jaffe & Backer 2003; Wyithe & Loeb 2003; Enoki et al. 2004). We show that a direct detection of a stochastic GW background is possible using pulsar timing observations and that the significance of the detection depends on the number of pulsars observed, the rms timing noise achieved, the number of observations, and the power spectrum of the measured timing residuals. The results are applied to the case of the Parkes pulsar timing array (PPTA).

In the next section, the analysis technique is described. In § 3 the significance of detecting a given stochastic background using this method is estimated. The effects of prefiltering the residual time series are also discussed. The results are summarized in § 4.

2. DETECTION TECHNIQUE

As a first step, the power spectra of the pulsar timing residuals are analyzed. If they all show a very red power-law spec-
lation between the data and the known function. Hence, to detect the presence of the GW background, one needs to calculate

$$\rho = \frac{(1/N_p) \sum_{n=0}^{N_p-1} [r(\theta) - \bar{r}][\xi(\theta) - \bar{\xi}]}{\sigma_r \sigma_\xi},$$

(4)

where $\theta_i$ is the angle between the $i$th pair of pulsars and $N_p$ is the number of distinct pairs of pulsars; $\bar{r}$ and $\bar{\xi}$ indicate the mean values over all pairs of pulsars, and $\sigma_r^2$ and $\sigma_\xi^2$ are the variances of $r$ and $\xi$, respectively. For $M$ pulsars, $N_p = M(M-1)/2$.

From the definition of $r(\theta)$ and equation (4), one can show that the expected value of $\rho$ is approximately

$$\rho \approx \frac{\alpha^2 r^2 + \beta^2 \xi^2}{\sqrt{\alpha^2 r^2 + \beta^2 \xi^2}},$$

(5)

$$\sigma_r^2 = \frac{1}{N_p} \sum_{i=0}^{N_p-1} \langle [r(\theta_i) - \langle r(\theta_i) \rangle]^2 \rangle.$$

(6)

For the case in which there is no correlation in the data, the statistics of $\rho$ will be Gaussian with zero mean and variance given by $\sigma_r^2 = 1/N_p = 2/(M^2 - M)$. Hence, the significance of a measured value of $\rho$ may be defined as $S = \rho/\sigma_r$. The probability of measuring a correlation greater than or equal to $\rho$ when no actual correlation is present is given by $1 - \text{erf}(\rho/2)$. If the number of distinct pairs of pulsars; and indicate the $\bar{r}$ and $\bar{\xi}$, expected value of $\rho$ of a measured value of $\rho$. Thus, accounting the effects of subtracting linear and quadratic terms between and the GW frequency, $f$, the total rms fluctuation induced of the GW background propose a power-law dependence be-

3. ESTIMATING THE DETECTION SIGNIFICANCE

In order to estimate the expected detection significance, $S$, one needs to estimate $\sigma_r$ and $\sigma_\xi$. It is assumed that the timing residuals, $R(t, \hat{k})$, are stationary Gaussian random variables that are sampled at regular intervals denoted by $\Delta t$. It is also assumed that terms proportional to $t$ and $t^2$ (i.e., the period and period-derivative terms) have been subtracted from $R(t, \hat{k})$.

The spacetime fluctuations induced by a stochastic GW background are described by a quantity known as the characteristic strain spectrum denoted by $h_s$ (e.g., Maggiore 2000). Models of the GW background propose a power-law dependence between $h_s$ and the GW frequency, $f$: $h_s(f) = Af^{\gamma}$ (Jaffe & Backer 2003; Wyithe & Loeb 2003; Maggiore 2000; Enoki et al. 2004). Using this form of the characteristic strain spectrum, the power spectrum of the induced residuals is given by $P_t(f) = \langle |R(f)|^2 \rangle = (A^2/4\pi^2) f^{2\gamma-3}$, where $R(f)$ is the Fourier transform of $R(t)$. Given $P_t(f)$, the total rms fluctuation induced by the stochastic GW background is given by

$$\sigma_r^2 = \int_{f_t}^{f_h} P_t(f) df$$

(7)

$$= \frac{A^2}{2\pi^2 (2 - 2\alpha)} (f_t^{2\gamma-2} - f_h^{2\gamma-2}),$$

(8)

where $f_t$ is the lowest detectable frequency given by $1/T$ and $f_h$ is the highest detectable frequency typically given by $\frac{1}{2}\Delta t$. $T$ is the total time span of the data set. Since $\alpha < 0$ for backgrounds of interest (Maggiore 2000), the term containing $f_h$ is negligible.

Estimating $\sigma_r$ is slightly more complicated. To take into account the effects of subtracting linear and quadratic terms from the residuals, a semianalytic approach was adopted. As outlined below, an estimate for $\sigma_r$ is made analytically, but with one free parameter $\beta$. For a given value of $\beta$, $S$ is calculated as a function of $A$ for a given set of pulsars and timing parameters. $S(A)$ is compared to Monte Carlo simulations in order to determine the correct value of $\beta$. This showed that the value of $\beta$ is insensitive to the values $\alpha$, $N$, $M$, $\sigma_r$, and the rms residual noise level.

Using equation (1) together with the assumption that $R(t, \hat{k})$ is a Gaussian random variable, one can show that

$$\sigma_r^2 \approx \frac{1}{N_p} \sum_{i=0}^{N_p-1} \sum_{j=0}^{N_p-1} c_{ij}(\hat{k}_i)c_{ij}(\hat{k}_j),$$

(9)

where $c_{ij}(\hat{k}) = \langle R(t + i\Delta t, \hat{k})R(t + j\Delta t, \hat{k}) \rangle$. The bar above equation (9) represents an average over all pairs of pulsars. As the autocorrelation function and the power spectrum are Fourier transforms of one another, one can estimate $\sigma_r^2$ from the expected power spectrum of the residuals. The statistics of the residuals are assumed to be stationary so that $c_{ij}(\hat{k})$ depends only on $i - j$. The expected discrete power spectrum of $R(t, \hat{k})$, which includes both a GW component and a white-noise component, is given by

$$P_i(i, \hat{k}) = \begin{cases} P_e(i) + 2\sigma_e(\hat{k})^2 i/N & \text{for } i > 0, \\ 0 & \text{for } i = 0, \end{cases}$$

(10)

where $P_e(i)$ is the discrete power spectrum of the GW-induced timing residuals, $i$ is the discrete frequency bin number corresponding to frequency $i/\Delta f$; and $\sigma_e(\hat{k})$ is the rms value of the residual fluctuations caused by all non-GW sources for the pulsar in the $\hat{k}$-direction. It is assumed that all noise sources have a flat spectrum. This assumption is consistent with most observations of MSPs. $P_e(i)$ is given by

$$P_e(i) = \frac{A^2 T^{2-2\alpha}}{(2\pi)^2 (2 - 2\alpha)} m(i),$$

(11)

where

$$m = \begin{cases} 0 & \text{for } i = 0, \\ \beta^{2\alpha-2} - (1.5)^{2\alpha-2} & \text{for } i = 1, \\ (i - 0.5)^{2\alpha-2} - (i + 0.5)^{2\alpha-2} & \text{for } i > 1. \end{cases}$$

Effectively, $\beta$ is the lowest frequency used to calculate the correlation function $c_{ij}$. Monte Carlo simulations show that $\beta = 0.97$.

For the case in which all pulsars have the same noise level, the detection significance becomes

$$S = \sqrt{\frac{M(M-1)/2}{1 + \chi(1 + \frac{\chi}{\lambda}) + 2(\sigma_e/\sigma_r)^2 + (\sigma_e/\sigma_r)^4}/\lambda \sigma_r^2},$$

(12)

Here $\chi = (1/\sigma_e^2 N) \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} c_{ij}^2$, where $c_{ij}$ is the correlation function for the GW-induced component of the timing residuals; $\chi$ is a measure of the “whiteness” of the residuals.

The solid curve in Figure 1a plots the detection significance versus power-law amplitude for $\alpha = -\frac{5}{2}$, the expected value for a background generated by an ensemble of supermassive black hole binaries (Jaffe & Backer 2003).
power equal amplitude and set higher components to zero. In frequency bin, we give each Fourier component with significant series a flat spectrum before correlation. This will act to reduce cance. This method involves both low-pass filtering and a tech-
uals will be dominated by low frequencies, one can apply a law spectrum. Hence, low-pass filtering will not increase the effect. It also has the effect of increasing when is a red power-
effectively reduces while keeping relatively unchanged.

For purposes of this discussion, was set to . The dot-
dashed line in Figure 1 includes the effect of low-
Curve was generated using different noise levels and a number of pulsars. See text for further details.

Since the power spectrum of the GW-induced timing resid-
uals will be dominated by low frequencies, one can apply a low-pass filter to each of the residual time series before corre-
ating. This is similar to fitting a low-order polynomial to the data and then correlating the resulting fits. To estimate the significance for this technique, one evaluates and using equations (8) and (10), but with a high-frequency cutoff . For purposes of this discussion, was set to . The dot-
dashed line in Figure 1a shows the effect of using a low-pass filter on the residuals. All the other parameters are the same as those used to generate the solid line. Low-pass filtering effectively reduces while keeping relatively unchanged. It also has the effect of increasing when is a red power-
low spectrum. Hence, low-pass filtering will not increase the maximum attainable significance, but it will lower the value of where the roll-off starts to occur.

We next try to increase the maximum achievable signi-
cance. This method involves both low-pass filtering and a tech-
nique called “whitening.” When correlating two time series that each have a steep power-law spectrum, an optimal signal-to-
oise ratio is obtained if filters are applied to give each time series a flat spectrum before correlation. This will act to reduce in equation (12). In practice, starting from the lowest nonzero frequency bin, we give each Fourier component with significant power equal amplitude and set higher components to zero. In this way, we are correlating only that part of the signal that has a high signal-to-noise ratio and adjusting the power spectrum to optimize the measurement of the correlation function. and need to be calculated in order to estimate using the whitening method. After whitening, , where is the rms of the residual data from the th pulsar. The whitening also affects . In the general case in which the pulsars have different noise levels, will depend on the pulsar. The expression for then becomes

$$\rho \approx \frac{\sigma^2_\theta - \sigma^2_\mu}{\sqrt{\sigma^2_\theta^2 - (\sigma_\mu^2 \mu^2 + \sigma^2_\mu)}},$$

(13)

with given by

$$\sigma^2_\theta = \frac{2}{N} \sigma_\theta(\mathbf{k}_i) \sigma_\theta(\mathbf{k}_j) \times \sqrt{\sum_{i=0}^{N_{max}} P(i) / P(i, \mathbf{k}_j)} \left(\sum_{i=0}^{N_{max}} P(i) / P(i, \mathbf{k}_j)\right)^2,$$

(14)

where is the largest frequency bin used based on the criterion discussed above. The solid line in Figure 1b plots the significance using the whitening versus . The same parameters were used for this case as in the previous cases.

The above discussion assumes that the noise levels were the same for all pulsars. Next, the case in which the pulsars have different noise levels will be considered. All curves in Figure 1b were generated using the whitening technique. Unless specified, 250 observations were taken on each pulsar over 5 years. The dashed line corresponds to pulsars, 10 with and 10 with . The dot-dashed line has pulsars each with and 500 observations. The triple-dot-dashed line has 20 pulsars with and 500 observations over 10 years.

When given a choice between observing a large sample of pulsars with different noise levels and observing only those pulsars with the lowest noise levels but for a longer time, the above curves demonstrate that one should actually observe the larger sample of pulsars. This is not a general statement, but rather it depends on the level of the GW background and the noise level. However, the levels chosen above are relevant to the PPTA (Jaffe & Backer 2003; Wyithe & Loeb 2003; Hobbs 2005). Note that for large , the significance scales as . Hence, doubling the number of pulsars will double the expected significance.

4. SUMMARY

The main goal of this work is to determine the effectiveness of an array of pulsars, such as the PPTA, for detecting a stochastic background of GWs. Using a simple correlation tech-
nique, the detection significance was calculated given the number of pulsars, the location of each pulsar, the TOA precision, the number of observations, the total time span of the data, and the amplitude and power-law index of the GW background. For the case in which all pulsars have the same white-noise spectrum, equation (12) may be used to calculate the detection significance. For the case of the PPTA, it was found that the maximum achievable significance will be about 3 for a background with spectral index and .
of supermassive binary black holes in galaxies (Jaffe & Backer 2003; Wyithe & Loeb 2003; Enoki et al. 2004). Note that lowering the rms noise level will only decrease the minimum detectable value of $A$ and not increase the maximum attainable significance.

Low-pass filtering the timing residuals, or, equivalently, fitting low-order polynomials (i.e., cubic terms) to the residuals and correlating the coefficients, does not increase the maximum attainable significance. The significance level is increased by prewhitening of the timing residuals. Using whitening, it is estimated that the PPTA could obtain a detection significance greater than 4 for $A \geq 3 \times 10^{-15} \text{ yr}^{-2/3}$ provided that efficient whitening filters can be designed and implemented. This is an area of further study and will be addressed in a later paper. With the same qualifications, increasing the total time span of the PPTA to 10 years would yield a significance greater than 4 for $A \geq 10^{-15} \text{ yr}^{-2/3}$. Since the significance scales as the number of pulsars, doubling that number will double the expected significance. Hence, using the simple correlation technique described here without any prefiltering, a stochastic background with $A \geq 10^{-15} \text{ yr}^{-2/3}$ will be detectable at a significance of about 5.5 using 40 pulsars observed 250 times over 5 years and each having 100 ns timing precision.

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