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PULSAR TIMING AND THE DETECTION OF BLACK HOLE BINARY SYSTEMS IN GLOBULAR CLUSTERS

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ABSTRACT

The possible existence of intermediate-mass binary black holes (IMBBHs) in globular clusters (GCs) offers us a unique geometry in which to detect spacetime oscillations. For certain pulsar-IMBBH configurations possible within a GC, the usual far-field plane wave approximation for the IMBBH metric perturbation severely underestimates the induced pulse time-of-arrival (TOA) fluctuations. In this Letter, the expected TOA fluctuations induced by an IMBBH lying close to the line of sight between a pulsar and the Earth are calculated for the first time. For an IMBBH consisting of 10 and $10^3\ M_\odot$ components, a 10 yr orbital period, and located 0.1 lt-yr from the Earth-pulsar line of sight, the induced TOA fluctuations will be of order 5–500 ns.

Subject headings: black hole physics — gravitational waves — pulsars: general

1. INTRODUCTION

The unique stability of electromagnetic pulses emitted by radio pulsars could allow one to detect the presence of gravitational wave (GW) radiation. It is normally assumed that the pulsar, Earth, and all points along the line of sight are far enough away from the GW source that the spacetime oscillations may be treated as gravitational plane waves (Sazhin 1978; Detweiler 1979). Recently, it has been suggested that intermediate-mass black hole binary systems may form in the centers of globular clusters (GCs; Miller 2002; Will 2004; Miller & Colbert 2004). Given the high density of radio pulsars in GCs together with the relatively small scale length (100 pc) of a GC, it is possible to find a pulsar whose line of sight will pass closer to the binary system than its endpoints (Lommen et al. 2005). It is shown in this Letter that this scenario differs from the plane gravitational wave case in two important ways. First, the amplitude of the induced time-of-arrival (TOA) fluctuations depends on the distance of closest approach, or impact parameter, between the binary system and the Earth-pulsar line of sight. Second, since the closest distance may be well within one gravitational wavelength, the near-field terms will play the dominant role in inducing the observed TOA fluctuations.

In this Letter, the effect of the gravitational field of a binary system on pulsar timing residuals is calculated. This is the first time both near- and far-field terms are included in this calculation, and no assumptions are made about the structure of the emitted gravitational wave front. The results are accurate to order $(v/c)^2$ and $(d/r)^2$, where v is the characteristic relative velocity of the mass-energy within the source, c is the speed of light, d is the length scale of the binary system, and r is the distance between the binary system and the Earth-pulsar line of sight. The general expressions for the timing residuals derived here may be applied to pulsar timing data to constrain the properties of putative black hole binary systems.

It should be noted that the 20 new millisecond pulsars recently discovered in the dense globular cluster Terzan 5 (Ransom et al. 2005) may provide an interesting application of these results. The large number of pulsars together with this cluster's high central density ($\approx 10^6\ L_\odot\ \mathrm{pc}^{-3}$) make it a good candidate to detect or at least limit the existence of an intermediate-mass binary black hole (IMBBH) system in its core.

In the next section, the general effect of a binary system on the arrival times of pulses emitted by a pulsar is calculated. We make use of geometric units where $G = c = 1$. Order-of-magnitude estimates of the induced timing residuals are also made for astrophysically relevant systems. The results are summarized in the last section.

2. CALCULATING THE RESIDUALS

The curvature of spacetime will cause the observed pulsar frequency to vary as a function of time. The timing residuals, $R(t)$, are given by (Detweiler 1979)

$$R(t) = \int_0^t \frac{\nu_o - \nu_e}{\nu_e}\, dt, \tag{1}$$

where t is time, ν_o is the observed frequency, and ν_e is the emitted frequency. The frequency shifts will be calculated using perturbative methods. Given the four-velocity of an observer, V^μ, the observed pulse frequency is given by $\nu = -V^\mu K_\mu$, where K_ν is the dual to the photon four-velocity. Let $V^\mu = \bar{V}^\mu + \delta V^\mu$ and $K_\mu = \bar{K}_\mu + \delta K_\mu$, where the barred quantities are the unperturbed values, and δV and δK are the corresponding perturbations. For the purposes of this discussion, we choose the unperturbed four-velocities of both the emitter and the observer to be 1 in the time direction and zero for the three spatial components. To lowest order, the timing residuals are given by

$$R(t) = \int_0^t \frac{\delta K_0^o - \delta K_0^e + (\delta V^{o\mu} - \delta V^{e\mu})\bar{K}_\mu}{\bar{K}_0}\, dt. \tag{2}$$

The last term on the right-hand side is the standard Doppler shift due to the relative velocity between the source and the emitter. The first two terms are determined by the actual path of the photons as they travel toward the receiver.

The geodesic equations will be used to solve for both δK

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and δV. To lowest order, these equations take the following form:

$$\frac{d\delta K_\alpha}{d\lambda} - \frac{1}{2} g_{\mu\nu,\alpha}\bar{K}^\mu \bar{K}^\nu = 0, \tag{3}$$

$$\frac{d\delta V_\alpha}{d\tau} - \frac{1}{2} g_{\mu\nu,\alpha}\bar{V}^\mu \bar{V}^\nu = 0, \tag{4}$$

where λ is the affine parameter along the light-ray path, τ is the proper time of the observer, and $g_{\mu\nu}$ is the spacetime metric. The metric is assumed to be of the form $\eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is the flat Minkowski metric and $h_{\mu\nu}$ is a small perturbation given, at spacetime location $(t, \boldsymbol{r})$, by

$$h_{\mu\nu}(t, \boldsymbol{r}) = \int \frac{\bar{T}_{\mu\nu}(t - |\boldsymbol{r} - \boldsymbol{r}'|, \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}\, d^3\boldsymbol{r}', \tag{5}$$

where $\bar{T}_{\mu\nu}$ is the trace-reversed stress energy tensor of the region generating the spacetime perturbation.

The timing residual due to ray-path propagation will be calculated to first order in h using equation (3) rewritten in integral form as

$$\frac{\delta K_0^o - \delta K_0^e}{K_0} = \int_{z_e}^{z_o} H(t + z, b\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}})\, dz, \tag{6}$$

where $H(t, \boldsymbol{r}) = (h_{00,0} + h_{zz,0} + 2h_{0z,0})/2$. This is given in the black hole barycentered coordinate system shown in Figure 1, with $\hat{\boldsymbol{z}}$ pointing parallel to the light ray from the radio emitter to the observer (so that $z = K^0\lambda$) and with $\hat{\boldsymbol{x}}$ pointing from the black hole binary barycenter to the closest point on that ray. The emitter and observer positions z_e and z_o, and the impact parameter b, are defined in Figure 1.

Using equation (5), $h_{\mu\nu}$ can be calculated using standard perturbative techniques to order $(v/c)^2$ and $(d/r)^2$. Remarkably, H can be written to this order as $H = dF/dz$, where F is given by

$$F(t, \boldsymbol{r}) = \frac{\dot{Q}_{rr} - \dot{Q}_i^i}{(r - z)r} + \frac{\dot{Q}_{rr} + \dot{Q}_{zz} - 2\dot{Q}_{rz}}{(r - z)^2} + \frac{\ddot{Q}_{rr} + \ddot{Q}_{zz} - 2\ddot{Q}_{rz}}{r - z}. \tag{7}$$

$Q_{ij} = \int T^{00} r_i' r_j' d^3\boldsymbol{r}'$ is the second moment of the mass-energy distribution, and the overdot represents the time derivative. Given that the mass-energy distribution is usually specified in the $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}})$-coordinate system, it is useful to note that $Q_{rr} = Q_{zz}\cos^2\theta + 2Q_{zx}\cos\theta\sin\theta + Q_{xx}\sin^2\theta$, $Q_{rz} = Q_{zz}\cos\theta + Q_{zx}\sin\theta$, and $Q_i^i = Q_{xx} + Q_{yy} + Q_{zz}$, where $r = (z^2 + b^2)^{1/2}$ and $\theta = \arccos z/r$.

Next, the contribution due to the 4-velocity of the observer and emitter will be calculated. Equation (4) determines the acceleration of the body in question. Under the assumption that the body does not move appreciably under the influence of the metric perturbation so that the time derivatives of r may be ignored, we have $\delta V^\mu K_\mu / K_0 = G(t, \boldsymbol{r}) - G(0, \boldsymbol{r})$, where

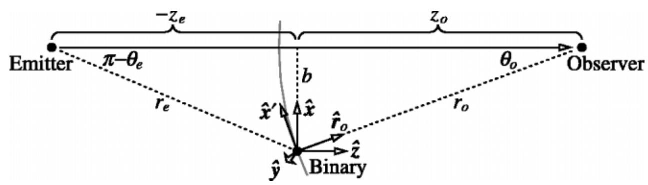

FIG. 1.—System configuration and coordinate system.

$$\begin{aligned} G(t, \boldsymbol{r}) = {} & \frac{2\ddot{Q}_{rz} - (1 + z/r)\ddot{Q}_{rr}/2 - (1 + z/r)\ddot{Q}_i^i/2}{r} \\ & + \frac{3\dot{Q}_{rz} - 3\dot{Q}_{rr}(1 + z/r) - \dot{Q}_i^i/2}{r^2} \\ & + \frac{3Q_{rz} - \frac{3}{2}(1 + 5z/r)Q_{rr} + (1 + 3z/r)Q_i^i/2}{r^3} \\ & + \int_0^t \left[\frac{3Q_{rz} + (3z/2r)Q_i^i - (15z/2r)Q_{rr}}{r^4}\right] dt'. \end{aligned} \tag{8}$$

Using equations (2), (6), and (8), the complete residual may be written as

$$R(t) = \int_0^t [F(t', \boldsymbol{r}_o) - F(t_{\rm ret}, \boldsymbol{r}_e) + G(t', \boldsymbol{r}_o) - G(t_{\rm ret}, \boldsymbol{r})]dt', \tag{9}$$

where $t_{\rm ret} = t' - (z_o - z_e)$. Note that the terms that will ultimately give rise to secular terms [i.e., $G(0, \boldsymbol{r})$] are not included in the above expression.

The above derivation assumes that the pulsar and observer are stationary in the barycentric frame of the binary system. However, for velocities $\ll c$, equations (7)–(9) give the correct leading-order behavior of the residual, provided we allow z and r to be functions of t.

For the case when z_e goes to negative infinity and z_o goes to positive infinity, one can show that the expected residual does not go to zero. Instead, it limits to

$$R(t) = 2\int_0^t \frac{\dot{Q}_{xx}(t' - r_o) - \dot{Q}_{yy}(t' - r_o)}{b(t')^2}\, dt'. \tag{10}$$

Now when the source of the gravitational perturbation is a massive binary system, we can give an explicit form for the quadrupole moment in the barycentric frame: $Q_{ij} = \mu s_i s_j$, where $\boldsymbol{s} = \boldsymbol{s}_2 - \boldsymbol{s}_1$ is the separation vector and $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass of the binary system with component masses m_1 and m_2. Using standard astrometric notation (see Fig. 2 and Smart 1977), we can write Q_{ij} in terms of the inclination i of the orbit to the plane of the sky, the position angle Ω of the ascending node, the argument of periapse ω, and the true anomaly v. Taking $\hat{\boldsymbol{L}}$ to be the direction of angular momentum of the system, $\hat{\boldsymbol{u}}$ to be ascending node of the orbit, and $\hat{\boldsymbol{w}} = \hat{\boldsymbol{L}} \times \hat{\boldsymbol{u}}$, the motion of the binary in that coordinate system is

$$\boldsymbol{s} = a\,\frac{1 - e^2}{1 + e\cos v}\,[\cos(\omega + v)\hat{\boldsymbol{u}} + \sin(\omega + v)\hat{\boldsymbol{w}}], \tag{11}$$

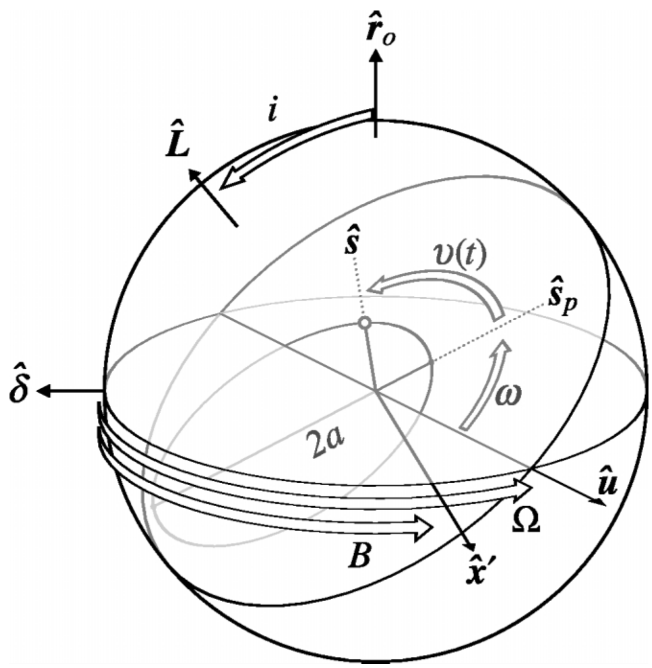

FIG. 2.—Binary orbital parameters.

where a is the orbital semimajor axis and e is the eccentricity. Noting that $\hat{x}' = \hat{x}\cos\theta_o - \hat{z}\sin\theta_o$ is the projection of $\hat{x}$ into the plane of the sky, and defining its position angle B in the same sense as Ω, the separation vector in the $(\hat{x}, \hat{y}, \hat{z})$-coordinate system is

$$\begin{aligned} \boldsymbol{s} = {} & \hat{x}[s_u \cos(\Omega - B)\cos\theta_o - s_w \sin(\Omega - B)\cos\theta_o \cos i \\ & + s_w \sin\theta_o \sin i] + \hat{y}[s_u \sin(\Omega - B) \\ & + s_w \cos(\Omega - B)\cos i] + \hat{z}[-s_u \cos(\Omega - B)\sin\theta_o \\ & + s_w \sin(\Omega - B)\sin\theta_o \cos i + s_w \cos\theta_o \sin i]. \end{aligned} \tag{12}$$

Here we have taken υ to be our independent parameter, which must be evaluated at appropriate retarded times. The time derivatives can be computed analytically using Kepler's law $\dot{\upsilon} = (2\pi/P)(1 + e\cos\upsilon)^2/(1 - e^2)^{3/2}$, where $P = 2\pi(a^3/M_t)^{1/2}$ is the orbital period and $M_t = m_1 + m_2$ the total mass of the system.

Solving explicitly for the case in which $b \ll r_e, r_o$ and is effectively constant over the time of observation, equation (10) becomes

$$\begin{aligned} R(t) = {} & 2\frac{\mu a^2}{b^2}\left(\frac{1 - e^2}{1 + e\cos\upsilon}\right)^2 \{\sin 2(\omega + \upsilon)\sin 2(\Omega - B)\cos i \\ & + [\sin^2(\omega + \upsilon)\cos^2 i - \cos^2(\omega + \upsilon)]\cos 2(\Omega - B)\}. \end{aligned} \tag{13}$$

Next, order-of-magnitude estimates are made for the amplitude of the induced residuals using the above results. Two cases will be considered. In case I, z_e and z_o are infinite, but b is finite. In case III, z_o is infinite, but $|z_e| \sim b \ll P$ (i.e., the pulsar is in the near zone of the gravitational field).

In case I, the amplitude of the induced residuals can be estimated using equation (13): $R_{\rm I} \sim 2\mu a^2/b^2 = 2\mu M_t^{2/3}(P/2\pi)^{4/3}/b^2$. For the nominal case of a binary system with 10 and $10^3\ M_\odot$ components, a 10 yr orbital period, and an impact parameter of 0.1 lt-yr, one obtains the following order-of-magnitude estimates:

$$R_{\rm I} \sim 5\ {\rm ns}\left(\frac{\mu}{10\ M_\odot}\right)\left(\frac{M_t}{10^3\ M_\odot}\right)^{2/3}\left(\frac{b}{0.1\ \text{lt-yr}}\right)^{-2}\left(\frac{P}{10\ \text{yr}}\right)^{4/3}. \tag{14}$$

In case II, the residuals induced by the motion of the binary system will be dominated by the last term in equation (8). Hence, an estimate for the residual amplitude is given by $R_{\rm II} \sim (3/4)\mu M_t^{2/3}(P/2\pi)^{10/3}/b^4$. For the same system, one obtains

$$R_{\rm II} \sim 500\ {\rm ns}\left(\frac{\mu}{10\ M_\odot}\right)\left(\frac{M_t}{10^3\ M_\odot}\right)^{2/3}\left(\frac{b}{0.1\ \text{lt-yr}}\right)^{-4}\left(\frac{P}{10\ \text{yr}}\right)^{10/3}. \tag{15}$$

In order to understand the above scaling, note that b is the only external scale factor in the problem and that the residuals are proportional to the quadrupole moment $Q \sim \mu a^2$. A simple dimensional argument therefore gives R scaling as $\mu a^2/b^2$, times P/b for every time integral of Q in the leading-order term. Case I involves no time integrals, while case II has two. Kepler's law is then used to write a in terms of the orbital period.

TABLE 1
RESIDUALS INDUCED IN GLOBULAR CLUSTER PULSARS BY A $10 + 10^3\ M_\odot$, 10 yr BINARY

Globular Cluster	Pulsar	b (lt-yr)	$R_{\rm I}$ (ns)	$R_{\rm II}$ (ns)	Reference
47 Tuc	J0024−7204O	0.26	0.8	11	1
	J0024−7204W	0.34	0.5	4	2
NGC 6266	J1701−3006B	0.18	1.6	50	3
NGC 6624	B1820−30A	0.37	0.4	3	4
M28 (NGC 6626)	B1821−24	0.12	4	200	5
NGC 6752	J1910−5959B	0.38	0.4	2	6
	J1910−5959E	0.49	0.2	0.9	6
M15 (NGC 7078)	B2127+11D	0.19	1.4	40	7
	B2127+11H	0.37	0.4	3	7

NOTE.—The binary is assumed to be at the core of the GC. The pulsars listed here all have periods of less than 10 ms. The following URL was very useful in constructing this table: http://www.naic.edu/~pfreire/GCpsr.html.

REFERENCES.—(1) Freire et al. 2003; (2) Camilo et al. 2000; (3) Possenti et al. 2003; (4) Biggs et al. 1994; (5) Rutledge et al. 2004; (6) D'Amico et al. 2002; (7) Anderson 1993.

3. APPLICATION AND DISCUSSION

General expressions for the periodic timing residuals induced by a binary system were calculated. It was shown that systematic variations in the pulsar timing residuals depend not only on the location of the pulsar and the observer, but also on how close the binary system is to the pulsar observer line of sight. As long as the line-of-sight impact parameter is finite, a nonzero residual amplitude can still occur even if both the pulsar and the observer are infinitely far away from the binary system. For a given impact parameter, the residuals calculated using case I and case II represent the range of possible residual amplitudes provided that $z_e \lesssim 0$.

Globular clusters present an interesting opportunity to discover intermediate-mass binary black holes using pulsar timing. Table 1 shows the timing residuals, for cases I and II, that a $10 + 10^3\ M_\odot$ binary system with a 10 yr period would induce on known pulsars whose lines of sight pass near the cores of their respective clusters. By way of comparison, the rms timing noise for millisecond pulsars is approaching the level of 100 ns or better (van Straten et al. 2001). New efforts like the Parkes Pulsar Timing Array project are actively working to improve the rms noise level.[5] The proposed Square Kilometer Array project will provide timing precisions as low as 10 ns in the next 10–20 years and will also uncover all pulsars beamed at Earth within globular clusters (Cordes et al. 2004; Kramer et al. 2004).

Of course, a single pulsar could never definitely detect a binary system, although it could be suggestive. In order to make a strong case, timing residual oscillations must be seen in two or more pulsars, and these oscillation must be consistent with the same binary system. The 20 millisecond pulsars in Terzan 5 may offer us such an opportunity (Ransom et al. 2005).

[5] See http://www.atnf.csiro.au/research/pulsar/psrtime.

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