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Abstract

This paper extends the median voter result of Meltzer and Richard (1981) to the case where a labor economy has any constant returns to scale production function under quasi-linear preferences with constant wage elasticity. Average productivities of the different labor inputs depend on their relative abundance in the economy. Agents are heterogeneous due to their labor type and (given type) due to their relative efficiency. They vote over income tax rates which in turn dictate the level of redistribution. The paper shows that preferences over tax rates are single-peaked and hence the median voter theorem applies. This framework connects the scarcity of inputs to the most preferred tax rates.

Keywords: *Meltzer-Richard, Median Voter, Interdependent Labor, Redistribution*
JEL: *D72, E6, H2, D33*

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1 Introduction

Meltzer and Richard's (1981) (M&R in what follows) celebrated model showed that the median voter theorem can be applied to voting over redistribution under distortionary taxation in labor economies with rational voters. In their model the most preferred tax rate of the voter with median productivity and thus median income is predicted to be the equilibrium political choice under pure majority rule, providing a link between inequality (as captured by the distance between the median and the mean) and an economy's politically chosen level of redistribution; as well as a possible explanation for why rational voters would limit the amount of redistribution in any economy.

A common assumption in the literature is that workers are perfect substitutes in the production process and as a consequence any heterogeneity in labor productivity is just the result of assuming different productivity indexes (i.e. effective units of labor or relative efficiency of agents) for an otherwise homogeneous labor input.

The proof offered by M&R is very general regarding utility functions but restricted to a particular simple production process: output is proportional to the single type of labor input. When more general production functions are considered in the analysis (i.e. production functions with multiple types of interdependent labor inputs), it is not necessarily the case that preferences over redistribution yield a Condorcet-winner and perhaps one of the reasons that the literature has not considered this extension.[1]

This paper shows that the M&R results still apply to the case with any constant returns to scale (CRS) production function with multiple labor inputs, provided that one uses the popular specification of quasilinear preferences that yield a constant wage elasticity of the labor supply, similar to Greenwood-Hercowitz-Huffman (1988) preferences (GHH in what follows) in dynamic settings.

[1] Persson and Tabellini (2002) presents the M&R model with quasilinear utility but a single type of labor input. Some authors have generalized the model to include capital in CRS production function frameworks, with capital but a single type of labor input. Since capital accumulation adds a dynamic dimension to these models, the source of inequality is different than in the current paper. Some examples include Alesina and Rodrik (1994), Krusell and Rios-Rull (1999), Bassetto and Benhabib (2006), Azzimonti, de Francisco and Krusell (2006), Corbae, D'Erasmo and Kuruscu (2009) and Piguillem and Schneider (2013). Meltzer and Richard's (2015) paper is different to the previous literature in that it discusses economic growth and voting over redistribution in a dynamic politico economic model of a labor economy, similar on the production side to their (1981) model.

The study of this framework with heterogeneous labor inputs in rational voter models is of interest for at least two reasons. First, the assumption that workers have different skill types is widely used in other areas in Economics (e.g. studies about skilled versus unskilled labor, studies about wage effects of immigration among others) which use the neoclassical theory of distribution as a building block and which represents a more realistic production process. Second, the proof offered in this paper helps advancing the knowledge of economists on the relevant conditions that support the conclusions of the original M&R model.

2 The Model

The model economy has an aggregate production function where output (Y) is produced with n different labor inputs, given by

$$Y = F\left(L_1, L_{2,...}, L_n\right), \tag{1}$$

where $F(\cdot)$ is assumed CRS with $\frac{dF}{dL_i} > 0$ and $\frac{d^2F}{dL_i^2} < 0$ for all $i = 1, 2, ...n$ labor inputs; L_i is the total amount of labor of type i used and F accommodates complementarities between inputs.

Markets are assumed competitive, so that wages equal marginal productivity:

$$w_i = \frac{dF(\cdot)}{dL_i} \equiv F_i. \tag{2}$$

Agents differ along 2 dimensions, by their skill (or labor) type and within their skill type they differ by their particular efficiency index (i.e. their effective units of labor).

Each labor input denoted by $i \in \{1, 2, 3, ..., n\}$ has a continuum of agents indexed by an element in the interval $[0, H_i]$, where H_i represents the mass of agents with skill type i. An agent is described by the pair (e, i) where $e \in [0, \infty)$ is the number of efficiency units that the agent has, while $i \in \{1, 2, 3, .., n\}$ represents its labor type. For economy of notation, let e_i describe the particular efficiency index of an agent within the $i - th$ labor type. The efficiency index of each of the n labor inputs is described by continuous probability density functions (p.d.f.) labeled as $g_{e_i}(e_i)$.

An agent's labor supply is the product of its raw labor $l_{e_i}^*$ (whose determinants are soon discussed) times its efficiency index e_i. Then the $i-th$ aggregate labor input (L_i) is the product of the mass of agents of that type (H_i) and their respective average labor supply, given by

$$L_i = H_i \cdot \int_0^{\infty} e_i l_{e_i}^* g_{e_i}(e_i)\, de_i = H_i \cdot E\left[e_i l_{e_i}^* | i\right] \text{ for } i = 1, 2, .., n. \tag{3}$$

One could alternatively think of L_i as "intermediate goods", where the production of each of these goods is proportional to the amount of labor used in the production process (given by (3)), so that the total intermediate good of type i would be given by L_i. Then a "bundler" would combine these intermediate goods according to (1) in order to produce the final good Y. For brevity, the alternative interpretation is not discussed further.

Preferences over consumption and leisure are assumed quasilinear with a constant wage elasticity of the labor supply

$$u(c_{e_i}, l_{e_i}) = c_{e_i} - \frac{[l_{e_i}]^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}, \tag{4}$$

where c_{e_i} is consumption of agent e_i, l_{e_i} is the individual labor supply, and ν is the elasticity of labor effort with respect to wages. These preferences, known as GHH in macroeconomics, are popular among other reasons because there are no income effects on the demand for leisure.

Consumption for agent e_i is given by income net of taxes plus transfers B,

$$c_{e_i} = (1-\tau) e_i w_i l_{e_i} + B \tag{5}$$

where $\tau \in [0, 1)$ is the income tax rate and $e_i w_i l_{e_i}$ is gross income.

Maximizing utility (4) subject to (5) taking wages, taxes and transfers as given yields the optimal labor decision by the agent

$$l_{e_i}^* = e_i^{\nu} w_i^{\nu} (1-\tau)^{\nu}. \tag{6}$$

Average labor supply per agent of input type i is therefore

$$E\left[e_i l^*_{e_i}|i\right] = E\left[(1-\tau)^{\nu} e_i^{1+\nu} w_i^{\nu}|i\right] = w_i^{\nu}(1-\tau)^{\nu} E\left[e_i^{1+\nu}|i\right]$$

where the term $w_i^{\nu}(1-\tau)^{\nu}$ can be taken out of the expectation since $(1-\tau)^{\nu}$ is constant and w_i^{ν} is constant for all workers of the same type i. Replacing the above equation in (3) defines aggregate labor of the $i-th$ type

$$L_i^* = H_i \cdot E\left[e_i^{1+\nu}|i\right] \cdot (w_i)^{\nu} \cdot (1-\tau)^{\nu}. \tag{7}$$

Agents pay proportional taxes on their income, which are returned to all agents via the universal transfer B. These are defined by $B = \frac{\tau \sum_i^n w_i \cdot L_i}{\sum_i^n H_i}$, where by the CRS assumption, output Y can be rewritten as $Y = \sum_i^n F_i \cdot L_i = \sum_i^n w_i \cdot L_i$ from (2). Then transfers can be written as

$$B = \frac{\tau Y}{\sum_i H_i}. \tag{8}$$

The results are put in the form of propositions.

Proposition 1. *Under quasilinear utility with a constant wage elasticity and any CRS production function $F(\cdot)$ that satisfies $\frac{dF}{dL_i} > 0$ and $\frac{d^2F}{dL_i^2} < 0$, equilibrium wages per effective unit of labor are independent of the tax rate and depend only on ratios of the different aggregate labor inputs.*

Proof. See appendix.

The appendix shows that even though a higher tax rate discourages labor effort, it does not distort equilibrium labor *ratios*. Since labor ratios are unaffected, wages of labor input j only depend on $\left\{\frac{E\left[e_1^{1+\nu}\right]H_1}{E\left[e_j^{1+\nu}\right]H_j}, ..., \frac{E\left[e_n^{1+\nu}\right]H_n}{E\left[e_j^{1+\nu}\right]H_j}\right\}$. This does not imply that gross income is unaffected by tax rates but rather that *wages per effective units of labor* are unaffected.

Define the maximum feasible level of output in this economy by $Y^{\max} \equiv F\left(L_1^{\max}, L_2^{\max}, ..., L_n^{\max}\right) > 0$, where $L_i^{\max}$ is the maximum level of equilibrium labor of input type i which is obtained when τ is set to 0 (using (7), $L_i^{\max} \equiv L_i^*(\tau = 0)$). Similarly, define maximum income per person as $y^{\max} = Y^{\max}/\sum H_i$. Proposition 2 follows.

Proposition 2. *Aggregate output (income) can be written as* $Y = (1-\tau)^{\nu} Y^{\max}$ *and average income per person is given by* $y = (1-\tau)^{\nu} y^{\max}$ *for any arbitrary tax rate* $\tau \in [0,1)$.

Proof. By substituting (7) into the production function (1) and since from proposition 1 taxes do not affect wages, the result follows from the CRS assumption, which yields $Y = (1-\tau)^{\nu} \cdot F(L_1^{\max}, ..L_n^{\max})$. Dividing by total population yields income per person $y = (1-\tau)^{\nu} y^{\max}$.

From the equations in proposition 2, it follows that output and average income per worker are decreasing in the tax rate. Using this result into transfers in equation (8) yields

$$B(\tau) = \tau (1-\tau)^{\nu} y^{\max}. \tag{9}$$

Equation (9) represents the Laffer curve for transfers taking into account general equilibrium effects: one can easily verify that transfers initially increase for $0 \leq \tau < \frac{1}{1+\nu}$, reach a maximum when $\tau = \frac{1}{1+\nu}$ and decrease for $\tau > \frac{1}{1+\nu}$.

In order to derive preferences over redistribution, it will be convenient to define "undistorted income" of agent e_i as $y_{e_i} \equiv e_i^{1+\nu} w_i^{1+\nu}$, which by proposition (1) is unaffected by τ. Then for any $\tau \in [0,1)$, gross income can be written as $e_i w_i l_{e_i}^* = (1-\tau)^{\nu} y_{e_i}$, while net-of-taxes income is given by $(1-\tau) e_i w_i l_{e_i}^* = (1-\tau)^{1+\nu} y_{e_i}$.

Substituting the budget constraint (5) into utility (4), then substituting optimal labor supply (6), transfers (9) and finally using the definition of undistorted income yields indirect utility of agent e_i

$$u_{e_i}(\tau) = \frac{(1-\tau)^{1+\nu} y_{e_i}}{(1+\nu)} + \tau (1-\tau)^{\nu} y^{\max}. \tag{10}$$

The most preferred tax rate for an agent with skill e_i is obtained as the tax rate that maximizes the above expression. Let this particular tax rate be defined as $\tau_{e_i}^* \equiv \underset{\tau \in [0,1)}{\arg\max} \{u_{e_i}(\tau)\}$. Maximizing the above function (where y_{e_i} and $y^{\max}$ do not depend on τ) yields the following first order condition

$$-\left(1-\tau_{e_i}^*\right)^{\nu} y_{e_i} + y^{\max}\left[\left(1-\tau_{e_i}^*\right)^{\nu} - \nu\tau_{e_i}^*\left(1-\tau_{e_i}^*\right)^{\nu-1}\right] \leq 0; = 0 \text{ if } \tau_{e_i}^* > 0. \tag{11}$$

Solving for the tax rate and taking into account the possibility of the corner yields an analytic expression for the most preferred tax rate of agent e_i, given by

$$\begin{aligned}\tau^*_{e_i} &= 1 - \frac{\nu}{(1+\nu) - \frac{y_{e_i}}{y^{\max}}} \quad \text{if } y_{e_i} < y^{\max}; \\ &= 0 \quad \text{if } y_{e_i} \geq y^{\max}.\end{aligned} \tag{12}$$

In order to relate the above tax rate with typical expressions in this class of models, notice that $y^{\max}$ is also the unconditional expectation of undistorted income ($y^{\max} \equiv E[y_{e_i}]$). Hence as in a M&R economy, an agent prefers a positive tax rate (and thus redistribution) if $y_{e_i} < E[y_{e_i}]$, and a zero tax rate otherwise. Agents who do not work ($y_{e_i} = 0 \leftrightarrow e_i = 0$) prefer the tax rate that maximizes the transfer (9), given by $\frac{1}{1+\nu}$. From equation (12), it can be seen that agents with identical undistorted income (irrespective of their skill type) prefer the same tax rate.

Proposition 3. *Preferences over the tax rate are single-peaked and therefore satisfy the conditions for the median voter theorem.*

Proof. See appendix.

The proof is straightforward. Alternatively, that the median voter applies can be established from the fact that (10) has the form of "intermediate preferences", which in turn satisfy the "single-crossing condition" (see for example Persson and Tabellini (2002)).

Let y_{med} denote the median undistorted income, then from proposition 3 and expression (12), the most preferred tax rate by the median voter is

$$\begin{aligned}\tau^*_{med} &= 1 - \frac{\nu}{(1+\nu) - \frac{y_{med}}{E\left[y_{e_i}\right]}} \text{ if } y_{med} < E[y_{e_i}] \\ &= 0 \text{ if } y_{med} \geq E[y_{e_i}].\end{aligned} \tag{13}$$

Proposition 4. *Holding constant median undistorted income, the tax rate of the median voter is increasing in average undistorted income of the population and holding constant the average undistorted income, decreasing in median undistorted income. Holding constant the ratio* $\frac{y_{med}}{E\left[y_{e_i}\right]}$, *the tax rate is decreasing in the wage elasticity ν.*

Proof. Immediate from differentiating expression (13).

3 Discussion and Conclusions

The results in this note are very general on the production side in the sense that the proof admits any CRS production function and thus any degree of complementarity/substitution between multiple inputs, but they only apply to the popular but specialized case of GHH preferences and which imply that there are no income effects in the demand for leisure, only substitution effects. For other utility functions (e.g. Cobb-Douglas) changes in the tax rate produce income effects which might interact with the different degrees of substitution/complementarity in CRS production functions, and this implies that the wage structure is in general a function of the tax rate. In such cases, computational solutions could still be used to verify whether preferences over redistribution yield a Condorcet-winner, but at the cost of generality as in those cases one would also have to specify the particular functional form of the production function, as well as the particular parameterization.

An interesting implication of this model is that the median voter is endogenous and by the assumption of CRS production functions, its identity depends on the relative scarcity of the different inputs, as well as on the particular shape of the production function. Its implications are left for future research.

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A Appendix

A.1 Proof of Proposition 1

In order to simplify the proof, let $E\left[e_i^{1+\nu}|i\right] \equiv A_i$.

Since a CRS production function is homogeneous of degree one, the marginal productivities and wages are homogeneous of degree zero. Thus $F_i\left(L_1, L_{2,}...L_n\right) = w_i = F_i\left(\pi L_1, \pi L_{2,}..., \pi L_n\right)$. Setting $\pi = \frac{1}{L_i}$, one can also write

$$w_i = F_i\left(\frac{L_1}{L_i}, \frac{L_2}{L_i}, ..., \frac{L_i}{L_i}, ..., \frac{L_n}{L_i}\right) \tag{14}$$

Replacing (7) into the above equation and assuming that $\tau \in [0, 1)$ yields

$$\begin{aligned} w_i &= F_i\left(\tfrac{(1-\tau)^\nu A_1 w_1^\nu H_1}{(1-\tau)^\nu A_i w_i^\nu H_i}, \tfrac{(1-\tau)^\nu A_2 w_2^\nu H_2}{(1-\tau)^\nu A_i w_i^\nu H_i} .., \tfrac{(1-\tau)^\nu A_i w_i^\nu H_i}{(1-\tau)^\nu A_i w_i^\nu H_i}, ., \tfrac{(1-\tau)^\nu A_n w_n^\nu H_n}{(1-\tau)^\nu A_i w_i^\nu H_i}\right) \\ &= F_i\left(\left(\tfrac{w_1}{w_i}\right)^\nu \tfrac{A_1 H_1}{A_i H_i}, .., \left(\tfrac{w_i}{w_i}\right)^\nu \tfrac{A_i H_i}{A_i H_i}, ., \left(\tfrac{w_n}{w_i}\right)^\nu \tfrac{A_n H_n}{A_i H_i}\right) \end{aligned}$$

Thus one can rewrite (14) as

$$w_i = F_i\left(\left(\frac{w_1}{w_i}\right)^\nu \frac{A_1 H_1}{A_i H_i}, .., \left(\frac{w_i}{w_i}\right)^\nu \frac{A_i H_i}{A_i H_i}, ., \left(\frac{w_n}{w_i}\right)^\nu \frac{A_n H_n}{A_i H_i}\right). \tag{15}$$

It immediately follows that relative wages between any inputs $i, j \in \{1, 2, .., n\}$ are given by

$$\left(\frac{w_i}{w_j}\right) = \frac{F_i\left(\left(\frac{w_1}{w_i}\right)^\nu \frac{A_1 H_1}{A_i H_i}, .., \left(\frac{w_i}{w_i}\right)^\nu \frac{A_i H_i}{A_i H_i}, ., \left(\frac{w_n}{w_i}\right)^\nu \frac{A_n H_n}{A_i H_i}\right)}{F_j\left(\left(\frac{w_1}{w_j}\right)^\nu \frac{A_1 H_1}{A_j H_j}, .., \left(\frac{w_j}{w_j}\right)^\nu \frac{A_i H_i}{A_j H_j}, ., \left(\frac{w_n}{w_j}\right)^\nu \frac{A_n H_n}{A_j H_j}\right)} \tag{16}$$

Expression (16) defines a system of $(n-1)$ equations in $(n-1)$ unknowns ratios $\left(\frac{w_i}{w_j}\right)$ for $i \neq j$. Hence equilibrium wage ratios are an implicit function of the relative ratios $\left\{\frac{A_1 H_1}{A_i H_i}, \frac{A_2 H_1}{A_i H_i}, ..., \frac{A_n H_1}{A_i H_i}\right\}$ and independent of the tax rate.

Finally, using equation (16) into (15) shows that the level of wages w_i for all $i \in \{1, 2, ..., n\}$ depends only on the ratios $\left\{\frac{A_1 H_1}{A_i H_i}, \frac{A_2 H_2}{A_i H_i}, ..., \frac{A_n H_n}{A_i H_i}\right\}$ as claimed in the text.

Q.E.D.

A.2 Proof of Proposition 3: Preferences over Tax Rates are Single-Peaked

Given any pair of incomes $y \equiv (e_i w_i)^{1+\nu}$, $y' \equiv \left(e'_j w_j\right)^{1+\nu}$ (for all inputs $i, j \in \{1, 2, ...n\}$) with implied optimal choices from expression (12) given by $\tau(y)$, $\tau(y')$, we verify that $y < y'$ implies that $\tau(y) \geq \tau(y')$.

Consider first the case in which $y < y' \leq E[y]$ which implies positive tax rates from (12). Then posit

$$\tau = 1 - \frac{\nu}{(1+\nu) - \frac{y}{E[y]}} > 1 - \frac{\nu}{(1+\nu) - \frac{y'}{E[y]}} = \tau'$$

iff

$$\frac{1}{(1+\nu) - \frac{y}{E[y]}} < \frac{1}{(1+\nu) - \frac{y'}{E[y]}}$$

iff

$$(1+\nu) - \frac{y'}{E[y]} < (1+\nu) - \frac{y}{E[y]}$$

iff

$$-\frac{y'}{E[y]} < -\frac{y}{E[y]}$$

iff $\quad y < y'$.

The second case to consider has $y < E[y] \leq y'$, which implies that $\tau' = 0$ and therefore $\tau > \tau' = 0$ also holds.

The final case is when $E[y] \leq y < y'$, which implies that $\tau = \tau' = 0$.

Putting together all 3 cases, for all $y < y'$, then $\tau(y) \geq \tau(y')$.

Finally, in order to verify that preferences are single-peaked, we show that the optimal choice given by (12) indeed is a maximum by verifying that $\frac{d^2 u_i}{d\tau_i^2} < 0$.

From equation (11), the first order condition can be written as

$$\frac{d\nu}{d\tau} = (1-\tau)^{\nu} \left\{ -(e_i w_i)^{1+\nu} + y^{\max} \left(\frac{1 - \tau(1+\nu)}{(1-\tau)} \right) \right\} \leq 0$$

The second order condition for a maximum is computed as

$$\frac{d^2\nu}{d\tau^2} = (1-\tau)^{\nu} y^{\max} \frac{d\left\{ \left(\frac{1-\tau(1+\nu)}{(1-\tau)} \right) \right\}}{d\tau} + \left\{ -(e_i w_i)^{1+\nu} + y^{\max} \left(\frac{1-\tau(1+\nu)}{(1-\tau)} \right) \right\} \frac{d(1-\tau_i)^{\nu}}{d\tau}$$

Now using the fact that at an interior solution it is the case that $\left\{ -(e_i w_i)^{1+\nu} + y^{\max} \left(\frac{1-\tau(1+\nu)}{(1-\tau)} \right) \right\} = 0$, then one obtains

$\frac{d^2\nu}{d\tau^2}|_{\tau^*} = (1-\tau)^\nu \, y^{\max} \cdot \left(\frac{-(1+\nu)+\tau(1+\nu)+1-\tau(1+\nu)}{(1-\tau)^2}\right)$

Simplifying the above expression yields the desired result

$\frac{d^2\nu}{d\tau^2}|_{\tau^*} = \frac{-\nu y^{\max}}{(1-\tau)^{2-\nu}} < 0.$

Q.E.D.