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LOCAL WELL-POSEDNESS OF THE CAUCHY PROBLEM FOR A p -ADIC NAGUMO-TYPE EQUATION

L. F. CHACÓN-CORTÉS, C. A. GARCIA-BIBIANO, AND W. A. ZÚÑIGA-GALINDO¹

ABSTRACT. We introduce a new family of p -adic non-linear evolution equations. We establish the local well-posedness of the Cauchy problem for these equations in Sobolev-type spaces. For a certain subfamily, we show that the blow-up phenomenon occurs and provide numerical simulations showing this phenomenon.

1. INTRODUCTION

Nowadays, the theory of linear partial pseudo-differential equations for complex-valued functions over p -adic fields is a well-established branch of mathematical analysis, see e.g. [1]-[6], [12]-[16], [22]-[25], [27]-[33], and references therein. Meanwhile very little is known about nonlinear p -adic equations. We can mention some semilinear evolution equations solved using p -adic wavelets [1], [24], a kind of equations of reaction-diffusion type and Turing patterns studied in [31], [33], a p -adic analog of one of the porous medium equation [17], [22], the blow-up phenomenon studied in [4], and non-linear integro-differential equations connected with p -adic cellular networks [30].

In this article we introduce a new family of nonlinear evolution equations that we have named as p -adic Nagumo-type equations:

$$u_t = -\gamma \mathbf{D}_x^\alpha u - u^3 + (\beta + 1)u^2 - \beta u + P(\mathbf{D}_x)(u^m), \quad x \in \mathbb{Q}_p^n, \quad t \in [0, T],$$

where $\gamma > 0$, $\beta \geq 0$, \mathbf{D}_x^α , $\alpha > 0$, is the Taibleson operator, m is a positive integer and $P(\mathbf{D}_x)$ is an operator of degree δ of the form $P(\mathbf{D}) = \sum_{j=0}^k C_j \mathbf{D}^{\delta_j}$, where the $C_j \in \mathbb{R}$ and $\delta_k = \delta$. We establish the local well-posedness of the Cauchy problem for these equations in Sobolev-type spaces, see Theorem 1. For a certain subfamily, we show that the blow-up phenomenon occurs, see Theorem 2, and we also provide numerical simulations showing this phenomenon.

The theory of Sobolev-type spaces use here was developed in [34], see also [25], [18]. This theory is based in the theory of countably Hilbert spaces of Gel'fand-Vilenkin [8]. Some generalizations are presented in [9]-[10]. We use classical techniques of operator semigroups, see e.g. [3], [20]. The family of evolution equations studied here contains as a particular case, equations of the form

$$(1.1) \quad u_t = -\gamma \mathbf{D}_x^\alpha u - u^3 + (\beta + 1)u^2 - \beta u,$$

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where $x \in \mathbb{Q}_p^n$, $t \in [0, T]$, \mathbf{D}_x^α is the Taibleson operator, that resemble the classical Nagumo-type equations, see e.g. [21].

In [7], the authors study the equations

$$(1.2) \quad u_t = Du_{xx} - u(u - \kappa)(u - 1) - \varepsilon u_x^m,$$

where $D > 0$, $\kappa \in (0, \frac{1}{2})$, $\varepsilon > 0$, $x \in \mathbb{R}$, $t > 0$. They establish the local well-posedness of the Cauchy problem for these equations in standard Sobolev spaces. There are several crucial differences between (1.1) and (1.2). The operators u_{xx} , u_x^m are local while the operators \mathbf{D}_x^α , $P(\mathbf{D}_x)(\cdot)^m$ are non-local. The p -adic heat equation $u_t = -\gamma \mathbf{D}_x^\alpha u$ has an arbitrary order of pseudo-differentiability $\alpha > 0$ in the spatial variable, while in the classical fractional heat equation $u_t = D \frac{\partial^\mu u}{\partial x^\mu}$, the degree of pseudo-differentiability $\mu \in (0, 2]$. This implies that the Markov processes attached to $u_t = -\gamma \mathbf{D}_x^\alpha u$ are completely different to the ones attached to $u_t = Du_{xx}$. In other words, the diffusion mechanisms in (1.1) and (1.2) are completely different. Notice that our non-linear term involves pseudo-derivatives of arbitrary order $P(\mathbf{D}_x)(u^m)$, while in [7] only of first order u_x^m . Of course, the p -adic Sobolev spaces behave completely different from their real counterparts, but the semigroup techniques are the same in both cases, since time is a non-negative real variable.

The article is organized as follows. In section 2, we review some basic aspects of the p -adic analysis and fix the notation. In section 3, we present some technical results about Sobolev-type spaces and p -adic pseudo-differential operators. In section 4, we show the local well-posedness of the p -adic Nagumo-type equations, see Theorem 1. In section 5, we show a subfamily of p -adic Nagumo-type equations whose solutions blow-up in finite time, see Theorem 2. In section 6, we present a numerical simulation showing the blow-up phenomenon.

2. p -ADIC ANALYSIS: ESSENTIAL IDEAS

In this section, we collect some basic results on p -adic analysis that we use through the article. For a detailed exposition the reader may consult [1], [14], [26], [29].

2.1. The field of p -adic numbers. Along this article p will denote a prime number. The field of p -adic numbers \mathbb{Q}_p is defined as the completion of the field of rational numbers \mathbb{Q} with respect to the p -adic norm $|\cdot|_p$, which is defined as

$$|x|_p = \begin{cases} 0 & \text{if } x = 0 \\ p^{-\gamma} & \text{if } x = p^\gamma \frac{a}{b}, \end{cases}$$

where a and b are integers coprime with p . The integer $\gamma := \text{ord}(x)$, with $\text{ord}(0) := +\infty$, is called the p -adic order of x .

Any p -adic number $x \neq 0$ has a unique expansion of the form

$$x = p^{\text{ord}(x)} \sum_{j=0}^{\infty} x_j p^j,$$

where $x_j \in \{0, \dots, p-1\}$ and $x_0 \neq 0$. By using this expansion, we define *the fractional part of $x \in \mathbb{Q}_p$* , denoted $\{x\}_p$, as the rational number

$$\{x\}_p = \begin{cases} 0 & \text{if } x = 0 \text{ or } \text{ord}(x) \geq 0 \\ p^{\text{ord}(x)} \sum_{j=0}^{-\text{ord}_p(x)-1} x_j p^j & \text{if } \text{ord}(x) < 0. \end{cases}$$

2.2. Topology of \mathbb{Q}_p^n . For $r \in \mathbb{Z}$, denote by $B_r^n(a) = \{x \in \mathbb{Q}_p^n; \|x - a\|_p \leq p^r\}$ *the ball of radius p^r with center at $a = (a_1, \dots, a_n) \in \mathbb{Q}_p^n$* , and take $B_r^n(0) := B_r^n$. Note that $B_r^n(a) = B_r(a_1) \times \dots \times B_r(a_n)$, where $B_r(a_i) := \{x_i \in \mathbb{Q}_p; |x_i - a_i|_p \leq p^r\}$ is the one-dimensional ball of radius p^r with center at $a_i \in \mathbb{Q}_p$. The ball B_0^n equals the product of n copies of $B_0 = \mathbb{Z}_p$, *the ring of p -adic integers*. We also denote by $S_r^n(a) = \{x \in \mathbb{Q}_p^n; \|x - a\|_p = p^r\}$ *the sphere of radius p^r with center at $a = (a_1, \dots, a_n) \in \mathbb{Q}_p^n$* , and take $S_r^n(0) := S_r^n$. We notice that $S_0^n = \mathbb{Z}_p^\times$ (the group of units of \mathbb{Z}_p), but $(\mathbb{Z}_p^\times)^n \subsetneq S_0^n$. The balls and spheres are both open and closed subsets in \mathbb{Q}_p^n . In addition, two balls in \mathbb{Q}_p^n are either disjoint or one is contained in the other.

As a topological space $(\mathbb{Q}_p^n, \|\cdot\|_p)$ is totally disconnected, i.e. the only connected subsets of \mathbb{Q}_p^n are the empty set and the points. A subset of \mathbb{Q}_p^n is compact if and only if it is closed and bounded in \mathbb{Q}_p^n , see e.g. [29, Section 1.3], or [1, Section 1.8]. The balls and spheres are compact subsets. Thus $(\mathbb{Q}_p^n, \|\cdot\|_p)$ is a locally compact topological space.

Since $(\mathbb{Q}_p^n, +)$ is a locally compact topological group, there exists a Haar measure $d^n x$, which is invariant under translations, i.e. $d^n(x + a) = d^n x$. If we normalize this measure by the condition $\int_{\mathbb{Z}_p^n} dx = 1$, then $d^n x$ is unique.

Notation 1. We will use $\Omega(p^{-r}\|x - a\|_p)$ to denote the characteristic function of the ball $B_r^n(a)$. For more general sets, we will use the notation 1_A for the characteristic function of a set A .

2.3. The Bruhat-Schwartz space. A complex-valued function φ defined on \mathbb{Q}_p^n is called *locally constant* if for any $x \in \mathbb{Q}_p^n$ there exist an integer $l(x) \in \mathbb{Z}$ such that

$$(2.1) \quad \varphi(x + x') = \varphi(x) \text{ for any } x' \in B_{l(x)}^n.$$

A function $\varphi : \mathbb{Q}_p^n \rightarrow \mathbb{C}$ is called a *Bruhat-Schwartz function (or a test function)* if it is locally constant with compact support. Any test function can be represented as a linear combination, with complex coefficients, of characteristic functions of balls. The \mathbb{C} -vector space of Bruhat-Schwartz functions is denoted by $\mathcal{D}(\mathbb{Q}_p^n) := \mathcal{D}$. We denote by $\mathcal{D}_{\mathbb{R}}(\mathbb{Q}_p^n) := \mathcal{D}_{\mathbb{R}}$ the \mathbb{R} -vector space of Bruhat-Schwartz functions. For $\varphi \in \mathcal{D}(\mathbb{Q}_p^n)$, the largest number $l = l(\varphi)$ satisfying (2.1) is called *the exponent of local constancy (or the parameter of constancy) of φ* .

We denote by $\mathcal{D}_m^l(\mathbb{Q}_p^n)$ the finite-dimensional space of test functions from $\mathcal{D}(\mathbb{Q}_p^n)$ having supports in the ball B_m^n and with parameters of constancy $\geq l$. We now define a topology on \mathcal{D} as follows. We say that a sequence $\{\varphi_j\}_{j \in \mathbb{N}}$ of functions in \mathcal{D} converges to zero, if the two following conditions hold:

- (1) there are two fixed integers k_0 and m_0 such that each $\varphi_j \in \mathcal{D}_{m_0}^{k_0}$;
- (2) $\varphi_j \rightarrow 0$ uniformly.

\mathcal{D} endowed with the above topology becomes a topological vector space.

2.4. L^ρ spaces. Given $\rho \in [1, \infty)$, we denote by $L^\rho := L^\rho(\mathbb{Q}_p^n) := L^\rho(\mathbb{Q}_p^n, d^n x)$, the \mathbb{C} -vector space of all the complex-valued functions g satisfying

$$\int_{\mathbb{Q}_p^n} |g(x)|^\rho d^n x < \infty.$$

The corresponding \mathbb{R} -vector spaces are denoted as $L_{\mathbb{R}}^\rho := L_{\mathbb{R}}^\rho(\mathbb{Q}_p^n) = L_{\mathbb{R}}^\rho(\mathbb{Q}_p^n, d^n x)$, $1 \leq \rho < \infty$.

If U is an open subset of \mathbb{Q}_p^n , $\mathcal{D}(U)$ denotes the space of test functions with supports contained in U , then $\mathcal{D}(U)$ is dense in

$$L^\rho(U) = \left\{ \varphi : U \rightarrow \mathbb{C}; \|\varphi\|_\rho = \left\{ \int_U |\varphi(x)|^\rho d^n x \right\}^{\frac{1}{\rho}} < \infty \right\},$$

where $d^n x$ is the normalized Haar measure on $(\mathbb{Q}_p^n, +)$, for $1 \leq \rho < \infty$, see e.g. [1, Section 4.3]. We denote by $L_{\mathbb{R}}^\rho(U)$ the real counterpart of $L^\rho(U)$.

2.5. The Fourier transform. Set $\chi_p(y) = \exp(2\pi i\{y\}_p)$ for $y \in \mathbb{Q}_p$. The map $\chi_p(\cdot)$ is an additive character on \mathbb{Q}_p , i.e. a continuous map from $(\mathbb{Q}_p, +)$ into S (the unit circle considered as multiplicative group) satisfying $\chi_p(x_0 + x_1) = \chi_p(x_0)\chi_p(x_1)$, $x_0, x_1 \in \mathbb{Q}_p$. The additive characters of \mathbb{Q}_p form an Abelian group which is isomorphic to $(\mathbb{Q}_p, +)$. The isomorphism is given by $\kappa \rightarrow \chi_p(\kappa x)$, see e.g. [1, Section 2.3].

Given $\xi = (\xi_1, \dots, \xi_n)$ and $y = (x_1, \dots, x_n) \in \mathbb{Q}_p^n$, we set $\xi \cdot x := \sum_{j=1}^n \xi_j x_j$. The Fourier transform of $\varphi \in \mathcal{D}(\mathbb{Q}_p^n)$ is defined as

$$(\mathcal{F}\varphi)(\xi) = \int_{\mathbb{Q}_p^n} \chi_p(\xi \cdot x) \varphi(x) d^n x \quad \text{for } \xi \in \mathbb{Q}_p^n,$$

where $d^n x$ is the normalized Haar measure on \mathbb{Q}_p^n . The Fourier transform is a linear isomorphism from $\mathcal{D}(\mathbb{Q}_p^n)$ onto itself satisfying

$$(2.2) \quad (\mathcal{F}(\mathcal{F}\varphi))(\xi) = \varphi(-\xi),$$

see e.g. [1, Section 4.8]. We will also use the notation $\mathcal{F}_{x \rightarrow \xi} \varphi$ and $\widehat{\varphi}$ for the Fourier transform of φ .

The Fourier transform extends to L^2 . If $f \in L^2$, its Fourier transform is defined as

$$(\mathcal{F}f)(\xi) = \lim_{k \rightarrow \infty} \int_{\|x\|_p \leq p^k} \chi_p(\xi \cdot x) f(x) d^n x, \quad \text{for } \xi \in \mathbb{Q}_p^n,$$

where the limit is taken in L^2 . We recall that the Fourier transform is unitary on L^2 , i.e. $\|f\|_2 = \|\mathcal{F}f\|_2$ for $f \in L^2$ and that (2.2) is also valid in L^2 , see e.g. [26, Chapter III, Section 2].

2.6. Distributions. The \mathbb{C} -vector space $\mathcal{D}'(\mathbb{Q}_p^n) := \mathcal{D}'$ of all continuous linear functionals on $\mathcal{D}(\mathbb{Q}_p^n)$ is called the *Bruhat-Schwartz space of distributions*. Every linear functional on \mathcal{D} is continuous, i.e. \mathcal{D}' agrees with the algebraic dual of \mathcal{D} , see e.g. [29, Chapter 1, VI.3, Lemma]. We denote by $\mathcal{D}'_{\mathbb{R}}(\mathbb{Q}_p^n) := \mathcal{D}'_{\mathbb{R}}$ the dual space of $\mathcal{D}_{\mathbb{R}}$.

We endow \mathcal{D}' with the weak topology, i.e. a sequence $\{T_j\}_{j \in \mathbb{N}}$ in \mathcal{D}' converges to T if $\lim_{j \rightarrow \infty} T_j(\varphi) = T(\varphi)$ for any $\varphi \in \mathcal{D}$. The map

$$\begin{aligned} \mathcal{D}' \times \mathcal{D} &\rightarrow \mathbb{C} \\ (T, \varphi) &\rightarrow T(\varphi) \end{aligned}$$

is a bilinear form which is continuous in T and φ separately. We call this map the pairing between \mathcal{D}' and \mathcal{D} . From now on we will use (T, φ) instead of $T(\varphi)$.

Every f in L^1_{loc} defines a distribution $f \in \mathcal{D}'(\mathbb{Q}_p^n)$ by the formula

$$(f, \varphi) = \int_{\mathbb{Q}_p^n} f(x) \varphi(x) d^n x.$$

Such distributions are called *regular distributions*. Notice that for $f \in L^2_{\mathbb{R}}$, $(f, \varphi) = \langle f, \varphi \rangle$, where $\langle \cdot, \cdot \rangle$ denotes the scalar product in $L^2_{\mathbb{R}}$.

2.7. The Fourier transform of a distribution. The Fourier transform $\mathcal{F}[T]$ of a distribution $T \in \mathcal{D}'(\mathbb{Q}_p^n)$ is defined by

$$(\mathcal{F}[T], \varphi) = (T, \mathcal{F}[\varphi]) \text{ for all } \varphi \in \mathcal{D}(\mathbb{Q}_p^n).$$

The Fourier transform $T \rightarrow \mathcal{F}[T]$ is a linear (and continuous) isomorphism from $\mathcal{D}'(\mathbb{Q}_p^n)$ onto $\mathcal{D}'(\mathbb{Q}_p^n)$. Furthermore, $T = \mathcal{F}[\mathcal{F}[T](-\xi)]$.

3. SOBOLEV-TYPE SPACES

The Sobolev-type spaces used here were introduced in [34], [25]. We follow here closely the presentation given in [18, Sections 10.1, 10.2].

We set $[\xi]_p := \max\{1, \|\xi\|_p\}$ for $\xi = (\xi_1, \dots, \xi_n) \in \mathbb{Q}_p^n$. Given $\varphi, \varrho \in \mathcal{D}(\mathbb{Q}_p^n)$ and $s \in \mathbb{R}$, we define the scalar product:

$$\langle \varphi, \varrho \rangle_s = \int_{\mathbb{Q}_p^n} [\xi]_p^s \widehat{\varphi}(\xi) \overline{\widehat{\varrho}(\xi)} d^n \xi,$$

where the bar denotes the complex conjugate. We also set $\|\varphi\|_s^2 = \langle \varphi, \varphi \rangle_s$, and denote by $\mathcal{H}_s := \mathcal{H}_s(\mathbb{Q}_p^n, \mathbb{C}) = \mathcal{H}_s(\mathbb{C})$ the completion of $\mathcal{D}(\mathbb{Q}_p^n)$ with respect to $\langle \cdot, \cdot \rangle_s$. Notice that if $r, s \in \mathbb{R}$, with $r \leq s$, then $\|\cdot\|_r \leq \|\cdot\|_s$ and $\mathcal{H}_s \hookrightarrow \mathcal{H}_r$ (continuous embedding). In particular,

$$\cdots \supset \mathcal{H}_{-2} \supset \mathcal{H}_{-1} \supset \mathcal{H}_0 \supset \mathcal{H}_1 \supset \mathcal{H}_2 \cdots,$$

where $\mathcal{H}_0 = L^2$. We set

$$\mathcal{H}_{\infty}(\mathbb{Q}_p^n, \mathbb{C}) = \mathcal{H}_{\infty} := \bigcap_{s \in \mathbb{N}} \mathcal{H}_s.$$

Since $\mathcal{H}_{[s]+1} \subseteq \mathcal{H}_s \subseteq \mathcal{H}_{[s]}$ for $s \in \mathbb{R}_+$, where $[\cdot]$ is the integer part function, then $\mathcal{H}_{\infty} = \bigcap_{s \in \mathbb{R}_+} \mathcal{H}_s$. With the topology induced by the family of seminorms $\{\|\cdot\|_l\}_{l \in \mathbb{N}}$, \mathcal{H}_{∞} becomes a locally convex space, which is metrizable. Indeed,

$$d(f, g) := \max_{l \in \mathbb{N}} \left\{ 2^{-l} \frac{\|f - g\|_l}{1 + \|f - g\|_l} \right\}, \text{ with } f, g \in \mathcal{H}_{\infty},$$

is a metric for the topology of \mathcal{H}_{∞} considered as a convex topological space. The metric space $(\mathcal{H}_{\infty}, d)$ is the completion of the metric space $(\mathcal{D}(\mathbb{Q}_p^n), d)$, cf. [18,

Lemma 10.4]. Furthermore, $\mathcal{H}_\infty \subset L^\infty \cap C^{\text{unif}} \cap L^1 \cap L^2$, and $\mathcal{H}_\infty(\mathbb{Q}_p^n, \mathbb{C})$ is continuously embedded in $C_0(\mathbb{Q}_p^n, \mathbb{C})$. This is the non-Archimedean analog of the Sobolev embedding theorem, cf. [18, Theorem 10.15].

Lemma 1. *If $s_1 \leq s \leq s_2$, with $s = \theta s_1 + (1 - \theta)s_2$, $0 \leq \theta \leq 1$, then $\|f\|_s \leq \|f\|_{s_1}^\theta \|f\|_{s_2}^{(1-\theta)}$.*

Proof. Take $f \in \mathcal{H}_s$, then by using the Hölder inequality for the exponents $\frac{1}{\theta} = \theta, \frac{1}{\theta'} = 1 - \theta$,

$$\begin{aligned} \|f\|_s^2 &= \int_{\mathbb{Q}_p^n} [\xi]_p^s |\widehat{f}(\xi)|^2 d^n \xi = \int_{\mathbb{Q}_p^n} [\xi]_p^{\theta s_1 + (1-\theta)s_2} |\widehat{f}(\xi)|^{2(\theta + (1-\theta))} d^n \xi \\ &= \int_{\mathbb{Q}_p^n} \left([\xi]_p^{s_1} |\widehat{f}(\xi)|^2 \right)^\theta \left([\xi]_p^{s_2} |\widehat{f}(\xi)|^2 \right)^{1-\theta} d^n \xi \\ &\leq \left(\int_{\mathbb{Q}_p^n} [\xi]_p^{s_1} |\widehat{f}(\xi)|^2 d^n \xi \right)^\theta \left(\int_{\mathbb{Q}_p^n} [\xi]_p^{s_2} |\widehat{f}(\xi)|^2 d^n \xi \right)^{1-\theta} d^n \xi. \end{aligned}$$

□

The following characterization of the spaces \mathcal{H}_s and \mathcal{H}_∞ is useful:

Lemma 2 ([18, Lemma 10.8]). *(i) $\mathcal{H}_s = \{f \in L^2; \|f\|_s < \infty\} = \{T' \in \mathcal{D}; \|T'\|_s < \infty\}$,
(ii) $\mathcal{H}_\infty = \{f \in L^2; \|f\|_s < \infty \text{ for any } s \in \mathbb{R}_+\} = \{T' \in \mathcal{D}; \|T'\|_s < \infty \text{ for any } s \in \mathbb{R}_+\}$.
The equalities in (i)-(ii) are in the sense of vector spaces.*

Proposition 1. *If $s > n/2$, then \mathcal{H}_s is a Banach algebra with respect to the product of functions. That is, if $f, g \in \mathcal{H}_s$, then $fg \in \mathcal{H}_s$ and $\|fg\|_s \leq C(n, s) \|f\|_s \|g\|_s$, where $C(n, s)$ is a positive constant.*

Proof. By the ultrametric property of $\|\cdot\|_p$, $\|\xi\|_p \leq \max\{\|\xi - \eta\|_p, \|\eta\|_p\}$ for $\xi, \eta \in \mathbb{Q}_p^n$, we have $\max\{1, \|\xi\|_p\} \leq \max\{1, \|\xi - \eta\|_p, \|\eta\|_p\}$, which implies that

$$\left[\max\{1, \|\xi\|_p\} \right]^s \leq \max\{1, \|\xi - \eta\|_p^s, \|\eta\|_p^s\} = \max\{1, \|\xi - \eta\|_p, \|\eta\|_p\}^s$$

for $s > 0$. Therefore

$$(3.1) \quad [\xi]_p^s \leq [\xi - \eta]_p^s + [\eta]_p^s.$$

Now, for $f, g \in L^2$, by using (3.1),

$$\begin{aligned} [\xi]_p^{\frac{s}{2}} |\widehat{fg}(\xi)| &= \left| [\xi]_p^{\frac{s}{2}} \int_{\mathbb{Q}_p^n} \widehat{f}(\xi - \eta) \widehat{g}(\eta) d^n \eta \right| \\ &\leq \int_{\mathbb{Q}_p^n} [\xi - \eta]_p^{\frac{s}{2}} |\widehat{f}(\xi - \eta)| |\widehat{g}(\eta)| d^n \eta + \int_{\mathbb{Q}_p^n} [\eta]_p^{\frac{s}{2}} |\widehat{g}(\eta)| |\widehat{f}(\xi - \eta)| d^n \eta \\ &= [\xi]_p^{\frac{s}{2}} |\widehat{f}(\xi)| * |\widehat{g}(\xi)| + |\widehat{f}(\xi)| * [\xi]_p^{\frac{s}{2}} |\widehat{g}(\xi)|. \end{aligned}$$

Then

$$\begin{aligned} \|fg\|_s &\leq \left\| [\xi]_p^{\frac{s}{2}} \widehat{f}(\xi) * |\widehat{g}(\xi)| + \widehat{f}(\xi) * [\xi]_p^{\frac{s}{2}} |\widehat{g}(\xi)| \right\|_2 \\ &\leq \left\| [\xi]_p^{\frac{s}{2}} \widehat{f}(\xi) * |\widehat{g}(\xi)| \right\|_2 + \left\| \widehat{f}(\xi) * [\xi]_p^{\frac{s}{2}} |\widehat{g}(\xi)| \right\|_2. \end{aligned}$$

Since $[\xi]_p^{\frac{s}{2}} \widehat{f}(\xi)$, $[\xi]_p^{\frac{s}{2}} |\widehat{g}(\xi)| \in L^2$, by using the Cauchy-Schwarz inequality with $s > n/2$, we have $\|\widehat{g}(\xi)\|_1 \leq A(n, s) \|g\|_s$, $\|\widehat{f}(\xi)\|_1 \leq A(n, s) \|f\|_s$, i.e. $|\widehat{g}(\xi)|$, $|\widehat{f}(\xi)| \in L^1$. Now, by the Young inequality, we obtain that

$$\|fg\|_s \leq \|f\|_s \|\widehat{g}\|_1 + \|g\|_s \|\widehat{f}\|_1 \leq 2A(n, s) \|f\|_s \|g\|_s.$$

□

3.1. The Taibleson operator. Let $\alpha > 0$, the Taibleson operator is defined as

$$(\mathbf{D}^\alpha \varphi)(x) = \mathcal{F}_{\xi \rightarrow x}^{-1}(\|\xi\|_p^\alpha (\mathcal{F}_{x \rightarrow \xi} \varphi)),$$

for $\varphi \in \mathcal{D}(\mathbb{Q}_p^n)$. This operator admits the extension

$$(\mathbf{D}^\alpha f)(x) = \frac{1 - p^\alpha}{1 - p^{-\alpha-n}} \int_{\mathbb{Q}_p^n} \|y\|_p^{-\alpha-n} \{f(x-y) - f(x)\} d^n y$$

to locally constant functions satisfying

$$\int_{\|x\|_p > 1} \|x\|_p^{-\alpha-n} |f(x)| d^n x < \infty.$$

The Taibleson operator \mathbf{D}^α is the p -adic analog of the fractional derivative. If $n = 1$, \mathbf{D}^α agrees with the Vladimirov operator. The operator \mathbf{D}^α does not satisfy the chain rule neither Leibniz formula. We also use the notation \mathbf{D}_x^α , when the Taibleson operator acts on functions depending on the variables $x \in \mathbb{Q}_p^n$ and $t \geq 0$.

Given $0 = \delta_0 < \delta_1 < \dots < \delta_{k-1} < \delta_k = \delta$, we define

$$P(\mathbf{D}) = \sum_{j=0}^k C_j \mathbf{D}^{\delta_j}, \text{ where the } C_j \in \mathbb{R}.$$

Lemma 3 ([18, Lemma 10.13 and Theorem 10.15]). *For $s \in \mathbb{R}_+$, the mapping $P(\mathbf{D}) : \mathcal{H}_{s+2\delta} \rightarrow \mathcal{H}_s$ is a well-defined continuous mapping between Banach spaces.*

Lemma 4. *Take $s - 2\delta > n/2$ and $f, g \in \mathcal{H}_{s+2\delta}$. Then*

$$\|P(\mathbf{D})(fg)\|_s \leq C(n, s, \delta) \|f\|_{s+2\delta} \|g\|_{s+2\delta},$$

where $C(n, s, \delta)$ is a positive constant that depends of n , s and δ .

Proof. Since $s > n/2$ and $f, g \in \mathcal{H}_{s+2\delta}$, by Proposition 1, $fg \in \mathcal{H}_{s+2\delta}$, and by Lemma 3, $P(\mathbf{D})(fg) \in \mathcal{H}_s$. Now by using Proposition 1,

$$\begin{aligned}
\|P(\mathbf{D})(fg)\|_s &\leq \sum_{j=0}^k |C_j| \left\| \mathbf{D}^{\delta_j}(fg) \right\|_s \\
&= \sum_{j=0}^k |C_j| \left(\int_{\mathbb{Q}_p^n} [\xi]_p^s \|\xi\|_p^{2\delta_j} |\widehat{fg}(\xi)|^2 d^n \xi \right)^{\frac{1}{2}} \leq \sum_{j=0}^k |C_j| \left(\int_{\mathbb{Q}_p^n} [\xi]_p^{s+2\delta_j} |\widehat{fg}(\xi)|^2 d^n \xi \right)^{\frac{1}{2}} \\
&= \sum_{j=0}^k |C_j| \|fg\|_{s+2\delta_j} \leq \sum_{j=0}^k |C_j| C(n, s, \delta_j) \|f\|_{s+2\delta_j} \|g\|_{s+2\delta_j} \\
&\leq \left(\sum_{j=0}^k |C_j| C(n, s, \delta_j) \right) \|f\|_{s+2\delta} \|g\|_{s+2\delta}.
\end{aligned}$$

□

4. LOCAL WELL-POSEDNESS OF THE p -ADIC NAGUMO-TYPE EQUATIONS

4.1. Some technical remarks. Let X, Y Banach spaces, $T_0 \in (0, \infty)$ and let $F : [0, T_0] \times Y \rightarrow X$ a continuous function. The Cauchy problem

$$(4.1) \quad \begin{cases} \partial_t u(t) = F(t, u(t)) \\ u(0) = \phi \in Y \end{cases}$$

is locally well-posed in Y , if the following conditions are satisfied.

(i) There is $T \in (0, T_0]$ and a function $u \in C([0, T]; Y)$, with $u(0) = \phi$, satisfying the differential equation in the following sense:

$$\lim_{h \rightarrow 0} \left\| \frac{u(t+h) - u(t)}{h} - F(t, u(t)) \right\|_X = 0,$$

where the derivatives at $t = 0$ and $t = T$ are calculated from the right and left, respectively.

(ii) The initial value problem (4.1) has at most one solution in $C([0, T]; Y)$.

(iii) The function $\phi \rightarrow u$ is continuous. That is, let $\{\phi_n\}$ be a sequence in Y such that $\phi_n \rightarrow \phi_\infty$ in Y and let $u_n \in C([0, T_n]; Y)$, resp. $u_\infty \in C([0, T_\infty]; Y)$, be the corresponding solutions. Let $T \in (0, T_\infty)$, then the solutions u_n are defined in $[0, T]$ for all n big enough and

$$\lim_{n \rightarrow \infty} \sup_{t \in [0, T]} \|u_n(t) - u_\infty(t)\|_Y = 0.$$

4.2. Main result. Consider the following Cauchy problem:

$$(4.2) \quad \begin{cases} u \in C([0, T], \mathcal{H}_s) \cap C^1([0, T], \mathcal{H}_s); \\ u_t = -\gamma \mathbf{D}_x^\alpha u - u^3 + (\beta + 1)u^2 - \beta u + P(\mathbf{D}_x)(u^m), \quad x \in \mathbb{Q}_p^n, t \in [0, T]; \\ u(0) = f_0 \in \mathcal{H}_s, \end{cases}$$

where $T, \gamma, \alpha, \beta > 0$, and m is a positive integer. The main result of this work is the following:

Theorem 1. *For $s > n/2 + 2\delta$, the Cauchy problem (4.2) is locally well-posed in \mathcal{H}_s .*

4.3. Preliminary results. We denote by $\mathbf{V}(t) = e^{-(\gamma \mathbf{D}^\alpha + \beta \mathbf{I})t}$, $t \geq 0$, the semigroup in L^2 generated by the operator $\mathbf{A} = -\gamma \mathbf{D}^\alpha - \beta \mathbf{I}$, that is,

$$\mathbf{V}(t)f(x) = \mathcal{F}_{\xi \rightarrow x}^{-1} \left(e^{-(\gamma \|\xi\|_p^\alpha + \beta)t} \mathcal{F}_{x \rightarrow \xi} f \right), \text{ for } f \in L^2, t \geq 0.$$

Lemma 5. *$\{\mathbf{V}(t)\}_{t \geq 0}$ is a C^0 -semigroup of contractions in \mathcal{H}_s , $s \in \mathbb{R}$, satisfying $\|\mathbf{V}(t)\|_s \leq e^{-\beta t}$ for $t \geq 0$. Moreover, $u(x, t) = \mathbf{V}(t)f_0(x)$ is the unique solution to the following Cauchy problem:*

$$(4.3) \quad \begin{cases} u \in C([0, T], \mathcal{H}_s) \cap C^1([0, T], \mathcal{H}_s); \\ u_t = -\gamma \mathbf{D}^\alpha u - \beta u, t \in [0, T]; \\ u(x, 0) = f_0(x) \in \mathcal{H}_s, \end{cases}$$

where T is an arbitrary positive number.

Proof. We just verify the strongly continuity of the semigroup. Since

$$\begin{aligned} & \left\| \mathcal{F}_{\xi \rightarrow x}^{-1} \left(e^{-(\gamma \|\xi\|_p^\alpha + \beta)t} \mathcal{F}_{x \rightarrow \xi} f \right) - f(x) \right\|_s^2 \\ &= \int_{\mathbb{Q}_p^n} [\xi]_p^s \left| \widehat{f}(\xi) \right|^2 \left\{ 1 - e^{-(\gamma \|\xi\|_p^\alpha + \beta)t} \right\}^2 d^n \xi \leq \|f\|_s^2, \end{aligned}$$

it follows from the dominated convergence theorem that

$$\lim_{t \rightarrow 0^+} \|\mathbf{V}(t)f - f\|_s = 0.$$

The existence and uniqueness of a solution for the Cauchy problem (4.3) follows from a well-known result, see e.g. [20, Theorem 4.3.1]. \square

Lemma 6. *Let $f_0 \in \mathcal{H}_s$, $s \in \mathbb{R}$, $\lambda \geq 0$. Then, there exists a positive constant $C(\lambda, \alpha)$ that depends of λ and α such that*

$$(4.4) \quad \|\mathbf{V}(t)f_0\|_{s+\lambda} \leq e^{-\beta t} \left(1 + C(\lambda, \alpha) \left(\frac{\lambda}{2\alpha\gamma t} \right)^{\frac{\lambda}{2\alpha}} \right) \|f_0\|_s \text{ for } t > 0.$$

Proof. We first notice that

$$\begin{aligned} \|\mathbf{V}(t)f_0\|_{s+\lambda}^2 &= \int_{\mathbb{Q}_p^n} [\xi]_p^{s+\lambda} e^{-2(\gamma \|\xi\|_p^\alpha + \beta)t} |f_0(\xi)|^2 d^n \xi \\ &\leq e^{-2\beta t} \left(\sup_{\xi \in \mathbb{Q}_p^n} [\xi]_p^\lambda e^{-2\gamma \|\xi\|_p^\alpha t} \right) \|f_0\|_s^2 \leq e^{-2\beta t} \left(1 + \sup_{\xi \in \mathbb{Q}_p^n \setminus \mathbb{Z}_p^n} \|\xi\|_p^\lambda e^{-2\gamma \|\xi\|_p^\alpha t} \right) \|f_0\|_s^2 \\ &\leq e^{-2\beta t} \left(1 + \sup_{\xi \in \mathbb{Q}_p^n} \|\xi\|_p^\lambda e^{-2\gamma \|\xi\|_p^\alpha t} \right) \|f_0\|_s^2. \end{aligned}$$

We now set $y = \|\xi\|_p$ and $h(y) = y^\lambda e^{-2\gamma y^\alpha t}$. By using the fact that $h(y)$ reaches its maximum at $y_{\max} = \left(\frac{\lambda}{2\alpha\gamma t}\right)^{\frac{1}{\alpha}}$, we conclude that

$$\sup_{\xi \in \mathbb{Q}_p^n} \|\xi\|_p^\lambda e^{-2\gamma \|\xi\|_p^\alpha t} \leq \left(\frac{\lambda}{2\alpha\gamma t}\right)^{\frac{\lambda}{\alpha}} e^{-\frac{\lambda}{\alpha}} \leq C(\lambda, \alpha) \left(\frac{\lambda}{2\alpha\gamma t}\right)^{\frac{\lambda}{\alpha}}.$$

□

Proposition 2. *Let $s > n/2 + 2\delta$ and $F(u) = (\beta + 1)u^2 - u^3 + P(\mathbf{D})(u^m)$. Then $F : \mathcal{H}_s \rightarrow \mathcal{H}_{s-2\delta}$ is a continuous function satisfying*

$$(4.5) \quad \|F(u) - F(w)\|_{s-2\delta} \leq L(\|u\|_s, \|w\|_s) \|u - w\|_s,$$

for $u, w \in \mathcal{H}_s$, here $L(\cdot, \cdot)$ is a continuous function, which is not decreasing with respect to each of their arguments. In particular,

$$(4.6) \quad \|F(u)\|_{s-2\delta} \leq L(\|u\|_s, 0) \|u\|_s.$$

Proof. We first notice that

$$\begin{aligned} F(u) - F(w) &= (\beta + 1)(u^2 - w^2) - (u^3 - w^3) + P(\mathbf{D})(u^m - w^m) \\ &= (\beta + 1)(u - w)(u + w) - (u - w)(u^2 + uw + w^2) + P(\mathbf{D})((u - w)q(u, w)), \end{aligned}$$

where $q(u, w) = \sum_{k=0}^{m-1} u^k w^{m-1-k}$. By using Proposition 1 and Lemma 4, the condition $s > n/2$ implies that if $u, w \in \mathcal{H}_s$, then any polynomial function in u, w belongs to \mathcal{H}_s , and

$$\begin{aligned} \|F(u) - F(w)\|_{s-2\delta} &\leq C \{(\beta + 1)\|u - w\|_{s-2\delta} \|u + w\|_{s-2\delta} + \\ &\quad \|u - w\|_{s-2\delta} \|u^2 + uw + w^2\|_{s-2\delta} + \|u - w\|_s \|q(u, w)\|_s \}, \end{aligned}$$

where $C = C(n, s, \delta)$. Then

$$\|F(u) - F(w)\|_{s-2\delta} \leq A(\|u\|_s, \|w\|_s) \|u - w\|_s,$$

where

$$\begin{aligned} A(\|u\|_s, \|w\|_s) &= C \{(\beta + 1)\|u + w\|_s + \|u^2 + uw + w^2\|_s + \|q(u, w)\|_s\} \\ &\leq C \left\{ (\beta + 1)\|u\|_s + (\beta + 1)\|w\|_s + \|u^2\|_s + \|uw\|_s + \|w^2\|_s + \sum_{k=0}^{m-1} \|u^k w^{m-1-k}\|_s \right\} \\ &\leq C(\beta + 1)\|u\|_s + C(\beta + 1)\|w\|_s + C^2\|u\|_s^2 + C^2\|u\|_s\|w\|_s + C^2\|w\|_s^2 + \\ &\quad C^{m+1} \sum_{k=0}^{m-1} \|u\|_s^k \|w\|_s^{m-1-k} =: L(\|u\|_s, \|w\|_s). \end{aligned}$$

□

For $M, T > 0$ and $f_0 \in \mathcal{H}_s$, we set

$$\mathcal{X}(M, T, f_0) := \left\{ u(t) \in C([0, T]; \mathcal{H}_s); \sup_{t \in [0, T]} \|u(t) - V(t)f_0\|_s \leq M \right\}.$$

We endow $\mathcal{X}(M, T, f_0)$ with the metric $d(u(t), v(t)) = \sup_{t \in [0, T]} \|u(t) - v(t)\|_s$. The resulting metric space is complete.

Proposition 3. *Take $f_0 \in \mathcal{H}_s$ with $s > n/2 + 2\delta$, $\delta > 0$. Then, there exists $T = T(\|f_0\|_s, M) > 0$ and a unique function $u \in C([0, T]; \mathcal{H}_s)$ satisfying the integral equation*

$$(4.7) \quad u(t) = \mathbf{V}(t)f_0 + \int_0^t \mathbf{V}(t-\tau)F(u(\tau))d\tau,$$

such that $u(0) = f_0$. Here $F(u) = (\beta + 1)u^2 - u^3 + P(\mathbf{D})(u^m)$ as before.

Remark 1. *Since $F(u)$ is not a locally Lipschitz function because inequality (4.6) involves two different norms, the existence of mild solutions of type (4.7) does not follow directly from standard results in semigroup theory, see e.g. [20, Theorem 5.2.2].*

Proof. Given $u \in \mathcal{X}(M, T, f_0)$, we set

$$\mathbf{N}u(t) = \mathbf{V}(t)f_0 + \int_0^t \mathbf{V}(t-\tau)F(u(\tau))d\tau.$$

Claim 1. $\mathbf{N} : \mathcal{X}(M, T, f_0) \rightarrow C([0, T]; \mathcal{H}_s)$.

Take $u \in \mathcal{X}(M, T, f_0)$, then

$$(4.8) \quad \begin{aligned} \|\mathbf{N}u(t_1) - \mathbf{N}u(t_2)\|_s &\leq \|(\mathbf{V}(t_1) - \mathbf{V}(t_2))f_0\|_s \\ &+ \left\| \int_0^{t_1} \mathbf{V}(t_1 - \tau)F(u(\tau))d\tau - \int_0^{t_2} \mathbf{V}(t_2 - \tau)F(u(\tau))d\tau \right\|_s. \end{aligned}$$

Since $\{\mathbf{V}(t)\}_{t \geq 0}$ is a C_0 -semigroup in \mathcal{H}_s , cf. Lemma 5, the first term on the right-hand side of the inequality (4.8) tends to zero when $t_2 \rightarrow t_1$. To study the second term, we assume without loss of generality that $0 < t_1 < t_2 < T$. Then

$$\begin{aligned} &\left\| \int_0^{t_1} \mathbf{V}(t_1 - \tau)F(u(\tau))d\tau - \int_0^{t_2} \mathbf{V}(t_2 - \tau)F(u(\tau))d\tau \right\|_s \\ &\leq \int_0^{t_1} \|\{\mathbf{V}(t_1 - \tau) - \mathbf{V}(t_2 - \tau)\}F(u(\tau))\|_s d\tau + \int_{t_1}^{t_2} \|\mathbf{V}(t_2 - \tau)F(u(\tau))\|_s d\tau. \end{aligned}$$

By using Lemma 6 with $\lambda = \alpha$ and Proposition 2,

$$\begin{aligned} &\|(\mathbf{V}(t_1 - \tau) - \mathbf{V}(t_2 - \tau))F(u(\tau))\|_s \\ &\leq \|\mathbf{V}(t_1 - \tau)F(u(\tau))\|_s + \|\mathbf{V}(t_2 - \tau)F(u(\tau))\|_s \\ &\leq \left\{ 2 + C_0 \left(\frac{1}{2\gamma(t_1 - \tau)} \right)^{\frac{1}{2}} + C_0 \left(\frac{1}{2\gamma(t_2 - \tau)} \right)^{\frac{1}{2}} \right\} \|F(u(\tau))\|_{s-\alpha} \\ &\leq 2 \left\{ 1 + C_0 \left(\frac{1}{2\gamma(t_1 - \tau)} \right)^{\frac{1}{2}} \right\} \sup_{\tau \in [0, T]} \|F(u(\tau))\|_{s-\alpha} \\ &= A(T, s, \alpha) \left\{ 1 + C_0 \left(\frac{1}{2\gamma(t_1 - \tau)} \right)^{\frac{1}{2}} \right\} \in L^1([0, t_1]). \end{aligned}$$

Now, by applying the dominated convergence theorem,

$$\lim_{t_2 \rightarrow t_1} \int_0^{t_1} \|(\mathbf{V}(t_1 - \tau) - \mathbf{V}(t_2 - \tau))F(u(\tau))\|_s d\tau = 0.$$

By a similar argument, one shows that

$$\|\mathbf{V}(t_2 - \tau)F(u(\tau))\|_{s-2\delta} \leq 1 + C_0 \left(\frac{1}{2\gamma(t_2 - \tau)} \right)^{\frac{1}{2}} L(\|u(\tau)\|_s, 0) \|u(\tau)\|_s,$$

and since

$$(4.9) \quad \|u(\tau)\|_s \leq \|u(\tau) - \mathbf{V}(\tau)f_0\|_s + \|\mathbf{V}(\tau)f_0\|_s \leq M + \|f_0\|_s, \text{ for all } \tau \in [0, T],$$

we have

$$(4.10) \quad \begin{aligned} & \int_{t_1}^{t_2} \|\mathbf{V}(t_2 - \tau)F(u(\tau))\|_s d\tau \\ & \leq L(M + \|f_0\|_s, 0)(M + \|f_0\|_s) \left(\int_{t_1}^{t_2} \left(1 + C_0 \left(\frac{1}{2\gamma(t_2 - \tau)} \right)^{\frac{1}{2}} \right) d\tau \right) \\ & = L(M + \|f_0\|_s, 0)(M + \|f_0(\cdot)\|_s) \left((t_2 - t_1) + C_0 \left(\sqrt{\frac{2(t_2 - t_1)}{\gamma}} \right) \right), \end{aligned}$$

and consequently, by applying the dominated convergence theorem,

$$\lim_{t_2 \rightarrow t_1} \int_{t_1}^{t_2} \|\mathbf{V}(t_2 - \tau)F(u(\tau))\|_s d\tau = 0.$$

Claim 2. There exists T_0 such that $\mathcal{N}(\mathcal{X}(M, T_0, f_0)) \subseteq \mathcal{X}(M, T_0, f_0)$.

By using a reasoning similar to the one used to established inequality (4.10), one gets

$$\begin{aligned} \|\mathbf{N}u(t) - \mathbf{V}(t)f_0\|_s & \leq \int_0^t \|\mathbf{V}(t - \tau)F(u(\tau))\|_s d\tau \\ & \leq L(M + \|f_0\|_s, 0)(M + \|f_0\|_s) \left(\int_0^t \left(1 + C_0 \left(\frac{1}{2\gamma(t - \tau)} \right)^{\frac{1}{2}} \right) d\tau \right) \\ & \leq L(M + \|f_0\|_s, 0)(M + \|f_0\|_s) \left(T + C_0 \left(\sqrt{\frac{2T}{\gamma}} \right) \right). \end{aligned}$$

Now taking T_0 such that

$$(4.11) \quad L(M + \|f_0\|_s, 0)(M + \|f_0\|_s) \left(T_0 + C_0 \left(\sqrt{\frac{2T_0}{\gamma}} \right) \right) \leq M,$$

we conclude that $\mathbf{N}u \in \mathcal{X}(M, T_0, f_0)$, for all $u(t) \in \mathcal{X}(M, T_0, f_0)$.

Claim 3. There exists T'_0 such that \mathbf{N} is a contraction on $\mathcal{X}(M, T'_0, f_0)$.

Given $u(t), v(t) \in \mathcal{X}(M, T_0, f_0)$, by using Proposition 2, with

$$C'_0 = L(M + \|f_0\|_s, M + \|f_0\|_s),$$

see (4.9), we have

$$\begin{aligned}
\|\mathbf{N}u(t) - \mathbf{N}v(t)\|_s &\leq \int_0^t \|\mathbf{V}(t-\tau)[F(u(\tau)) - F(v(\tau))]\|_s d\tau \\
&\leq \int_0^t \left(1 + C_0 \left(\frac{1}{2^\gamma(t-\tau)}\right)^{\frac{1}{2}}\right) \|F(u(\tau)) - F(v(\tau))\|_{s-\alpha} d\tau \\
&\leq C'_0 \int_0^t \left(1 + C_0 \left(\frac{1}{2^\gamma(t-\tau)}\right)^{\frac{1}{2}}\right) \|u(\tau) - v(\tau)\|_s d\tau \\
&\leq C'_0 \left(\sup_{\tau \in [0, T_0]} \|u(\tau) - v(\tau)\|_s\right) \int_0^t \left(1 + C_0 \left(\frac{1}{2^\gamma(t-\tau)}\right)^{\frac{1}{2}}\right) d\tau \\
&\leq C'_0 \left(T_0 + C_0 \left(\sqrt{\frac{2T_0}{\gamma}}\right)\right) d(u(t), v(t)).
\end{aligned}$$

Thus, taking T'_0 such that

$$(4.12) \quad C := C'_0 \left(T'_0 + C_0 \left(\sqrt{\frac{2T'_0}{\gamma}}\right)\right) < 1,$$

we obtain that $d(\mathbf{N}u(t), \mathbf{N}v(t)) \leq Cd(u(t), v(t))$, that is, \mathbf{N} is a strict contraction in $\mathcal{X}(M, T'_0, f_0)$. We pick T such that the inequalities (4.11) and (4.11) hold true, and apply the Banach Fixed Point Theorem to get $u(t) \in \mathcal{X}(M, T, f_0)$ a unique fixed point of \mathbf{N} , which satisfies the integral equation (4.7), where $T = T(\|f_0\|_s, M) > 0$. \square

Remark 2. Let \mathcal{X} be a Banach space and let $\mathbf{A} : \text{Dom}(\mathbf{A}) \rightarrow \mathcal{X}$ be an operator with dense domain such that \mathbf{A} is the infinitesimal generator of a contraction semigroup $(\mathbf{S}t)_{t \geq 0}$. Fix $T > 0$ and let $f : [0, T] \rightarrow \mathcal{X}$ be a continuous function. Consider the Cauchy problem:

$$(4.13) \quad \begin{cases} u \in C([0, T], \text{Dom}(\mathbf{A})) \cap C^1([0, T], \mathcal{X}); \\ u_t = \mathbf{A}u + f(t), \quad t \in [0, T]; \\ u(0) = u_0 \in \mathcal{X}. \end{cases}$$

Then

$$(4.14) \quad u(t) = \mathbf{S}(t)u_0 + \int_0^t \mathbf{S}(t-\tau)f(\tau)d\tau,$$

for $t \in [0, T]$, see e.g. [3, Lemma 4.1.1]. Conversely, if $u_0 \in \text{Dom}(\mathbf{A})$, $f \in C([0, T], \mathcal{X})$,

$$\int_{(0, T)} \|f(\tau)\|_{\mathcal{X}} d\tau < \infty,$$

then a solution of (4.14) is a solution of the Cauchy problem (4.13), see e.g. [3, Proposition 4.1.6].

Proposition 4. The problem (4.2) is equivalent to the integral equation (4.7). More precisely, if $s > n/2 + 2\delta$, and $u(t) \in C([0, T]; \mathcal{H}_s) \cap C^1((0, T]; \mathcal{H}_{s-2\delta})$ is a solution of (4.2), then $u(t)$ satisfies the integral equation (4.7). Conversely,

if $s > n/2 + 2\delta$, and $u(t) \in C([0, T]; \mathcal{H}_s)$ is a solution of (4.7), then $u(t) \in C^1([0, T]; \mathcal{H}_{s-2\delta})$ and it satisfies (4.2).

Proof. It follows from Remark 2, Propositions 3, 2, by taking $\mathbf{A} = -\gamma \mathbf{D}_x^\alpha - \beta \mathbf{I}$, $\text{Dom}(\mathbf{A}) = \mathcal{H}_s$, $\mathcal{X} = \mathcal{H}_{s-2\delta}$, $f(t) = F(u(t))$. We first recall that $\mathcal{D} \hookrightarrow \mathcal{H}_s \hookrightarrow \mathcal{H}_{s-2\delta}$, where \hookrightarrow means continuous embedding, and that \mathcal{D} is dense in $\mathcal{H}_{s-2\delta}$. If $u(t)$ is a solution of (4.2), then, since $F(u(t)) \in C([0, T]; \mathcal{H}_{s-2\delta})$, by Proposition 2, $u(t)$ is a solution of (4.7). Conversely, if $u(t)$ is a solution of (4.7), since

$$\int_{(0, T)} \|F(u(\tau))\|_{s-2\delta} d\tau < \infty,$$

by Proposition 2, $u(t)$ is a solution of (4.2). \square

Lemma 7 ([20, Theorem 5.1.1]). *If $h \in L^1(0, T)$, with $T > 0$, is real-valued function such that. If*

$$h(t) \leq a + b \int_0^t h(s) ds,$$

for $t \in (0, T)$ a.e., where $a \in \mathbb{R}$ and $b \in [0, \infty)$ then $h(t) \leq ae^{bt}$ for almost all t in $(0, T)$.

Proposition 5. *Let $f_0, f_1 \in \mathcal{H}_s$ and $u(t), v(t) \in C[0, T]; \mathcal{H}_s$ be the corresponding solutions of equation (4.7) with initial conditions $u(0) = f_0$ and $v(0) = f_1$, respectively. If $s > n/2 + 2\delta$, then*

$$\|u(t) - v(t)\|_s \leq e^{L(W, W)} \|f_0 - f_1\|_s,$$

where L is given in Proposition 1 and

$$W := \max \left\{ \sup_{t \in [0, T]} \|u(t)\|_s, \sup_{t \in [0, T]} \|v(t)\|_s \right\}.$$

Proof. By using (4.7), we have

$$u(t) - v(t) = \mathbf{V}(t)(f_0 - f_1) + \int_0^t \mathbf{V}(t - \tau) \{F(u(\tau)) - F(v(\tau))\} d\tau.$$

By using Proposition 1, we get

$$\begin{aligned} \|u(t) - v(t)\|_s &\leq \|f_0 - f_1\|_s + \int_0^t \|\mathbf{V}(t - \tau) \{F(u(\tau)) - F(v(\tau))\}\|_s d\tau \\ &\leq \|f_0 - f_1\|_s + \int_0^t \|F(u(\tau)) - F(v(\tau))\|_{s-\alpha} d\tau \\ &\leq \|f_0 - f_1\|_s + L(W, W) \int_0^t \|u(\tau) - v(\tau)\|_s d\tau. \end{aligned}$$

Now the result follows from Lemma 7, by taking $h(t) = \|u(t) - v(t)\|_s$, $a = \|f_0 - f_1\|_s$, $b = L(W, W)$. \square

Proposition 6. *Let $s > n/2 + 2\delta$ and $\delta \geq 0$. Then, the map $f_0 \mapsto u(t)$ is continuous in the following sense: if $f_0^{(n)} \rightarrow f_0$ in \mathcal{H}_s and $u_n(t) \in C([0, T_n]; \mathcal{H}_s)$, with $T_n = T \left(\left\| f_0^{(n)} \right\|_s, M \right) > 0$, are the corresponding solutions to the Cauchy*

problem (4.2) with $u_n(0) = f_0^{(n)}$. Then, there exist $T > 0$ and a positive integer $N = N(\gamma, f_0, T)$ such that $T_n \geq T$ for all $n \geq N$ and

$$(4.15) \quad \lim_{n \rightarrow \infty} \sup_{t \in [0, T]} \|u_n(t) - u(t)\|_s = 0.$$

Proof. By Proposition 3, the $T_n = T \left(\left\| f_0^{(n)} \right\|_s, M \right) > 0$ are continuous functions of $\left\| f_0^{(n)} \right\|_s$, then, given $T^* > 0$ there exists $N \in \mathbb{N}$ such that $T^* \leq T_n$ for all $n \geq N$. We set $\bar{T} := \min \{T^*, T_1, T_2, \dots, T_{N-1}\}$. Therefore, all the $u_n(t)$ are defined on $[0, \bar{T}]$, furthermore, $u \in \mathcal{X} \left(M, \bar{T}, f_0^{(n)} \right)$ for all n , and

$$\|u_n(t)\|_s \leq \left\| f_0^{(n)} \right\|_s + M \leq \delta + M,$$

where $\delta = \sup_{n \in \mathbb{N}} \left\| f_0^{(n)} \right\|_s$. Now

$$\sup_{t \in [0, \bar{T}]} \|u_n(t)\|_s \leq \delta + M \text{ for all } n, \text{ and } \sup_{t \in [0, \bar{T}]} \|u(t)\|_s \leq \delta + M.$$

On the other hand, by reasoning as in the proof of Proposition 5, we have

$$\|u_n(t) - u(t)\|_s \leq \left\| f_0^{(n)} - f_0 \right\|_s + L(\delta + M, \delta + M) \int_0^t \|u_n(\tau) - u(\tau)\|_s d\tau,$$

and by applying Lemma 7

$$\|u_n(t) - u(t)\|_s \leq e^{TL(\delta+M, \delta+M)} \left\| f_0^{(n)} - f_0 \right\|_s,$$

which in turns implies (4.15). \square

4.4. Proof of the Main result. The local well-posedness of the Cauchy problem (4.2) in \mathcal{H}_s , $s > n/2 + 2\delta$, follows from Propositions 3, 5, 6.

5. THE BLOW-UP PHENOMENON

In this section, we study the blow-up phenomenon for the solution of the equation

$$(5.1) \quad \begin{cases} u_t = -\gamma \mathbf{D}_x^\alpha u + F(u) + \mathbf{D}_x^{\alpha_1} u^3, & x \in \mathbb{Q}_p^n, t \in [0, T]; \\ u(0) = f_0 \in \mathcal{H}_\infty, \end{cases}$$

where $F(u) = -u^3 + (\beta + 1)u^2 - \beta u$. We will say that a non-negative solution $u(x, t) \geq 0$ of (5.1) blow-up in a finite time $T > 0$, if $\lim_{t \rightarrow T^-} \sup_{x \in \mathbb{Q}_p^n} u(x, t) = +\infty$. This limit makes sense since $\mathcal{H}_\infty(\mathbb{Q}_p^n, \mathbb{C})$ is continuously embedded in $C_0(\mathbb{Q}_p^n, \mathbb{C})$, [18, Theorem 10.15].

5.1. p -adic wavelets and pseudo-differential operators. We denote by $C(\mathbb{Q}_p, \mathbb{C})$ the \mathbb{C} -vector space of continuous \mathbb{C} -valued functions defined on \mathbb{Q}_p .

We fix a function $\mathbf{a} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and define the pseudo-differential operator

$$\mathcal{D} \rightarrow C(\mathbb{Q}_p, \mathbb{C}) \cap L^2$$

$$\varphi \rightarrow \mathbf{A}\varphi,$$

where $(\mathbf{A}\varphi)(x) = \mathcal{F}_{\xi \rightarrow x}^{-1} \left\{ \mathbf{a} \left(|\xi|_p \right) \mathcal{F}_{x \rightarrow \xi} \varphi \right\}$.

The set of functions $\{\Psi_{rnj}\}$ defined as

$$(5.2) \quad \Psi_{rnj}(x) = p^{-\frac{r}{2}} \chi_p(p^{-1}j(p^r x - n)) \Omega(|p^r x - n|_p),$$

where $r \in \mathbb{Z}$, $j \in \{1, \dots, p-1\}$, and n runs through a fixed set of representatives of $\mathbb{Q}_p/\mathbb{Z}_p$, is an orthonormal basis of $L^2(\mathbb{Q}_p)$ consisting of eigenvectors of operator \mathbf{A} :

$$(5.3) \quad \mathbf{A}\Psi_{rnj} = \mathfrak{a}(p^{1-r})\Psi_{rnj} \text{ for any } r, n, j,$$

see e.g. [18, Theorem 3.29], [1, Theorem 9.4.2]. Notice that

$$\widehat{\Psi}_{rnj}(\xi) = p^{\frac{r}{2}} \chi_p(p^{-r}n\xi) \Omega(|p^{-r}\xi + p^{-1}j|_p),$$

and then

$$\mathfrak{a}(|\xi|_p) \widehat{\Psi}_{rnj}(\xi) = \mathfrak{a}(p^{1-r}) \widehat{\Psi}_{rnj}(\xi).$$

In particular, $\mathbf{D}_x^\alpha \Psi_{rnj} = p^{(1-r)\alpha} \Psi_{rnj}$, for any r, n, j and $\alpha > 0$, and since $p^{(1-r)\alpha}$,

$$\mathbf{D}_x^\alpha \operatorname{Re}(\Psi_{rnj}) = p^{(1-r)\alpha} \operatorname{Re}(\Psi_{rnj}), \quad \mathbf{D}_x^\alpha \operatorname{Im}(\Psi_{rnj}) = p^{(1-r)\alpha} \operatorname{Im}(\Psi_{rnj}).$$

And,

$$\begin{aligned} \{\Psi_{rn1}(x)\}^2 &= p^{-r} \chi_p(2p^{-1}(p^r x - n)) \Omega(|p^r x - n|_p) \\ &= p^r \{\Psi_{rn1}(x)\}^2 = p^{\frac{r}{2}} \Psi_{rn2}(x), \end{aligned}$$

then

$$\mathbf{D}_x^\alpha \operatorname{Re}(\{\Psi_{rn1}(x)\}^2) = p^{\frac{r}{2}} p^{(1-r)\alpha} \operatorname{Re}(\Psi_{rn2}(x)) = p^{(1-r)\alpha} \operatorname{Re}(\{\Psi_{rn1}(x)\}^2).$$

5.2. The blow-up. In this section, we assume that $u(x, t)$ is real-valued non-negative solution of the Cauchy problem (4.2) in \mathcal{H}_∞ . We set $w(x) := \operatorname{Re}(\{\Psi_{rn1}(x)\}^2)$, so $\mathbf{D}_x^\alpha w(x) = p^{(1-r)\alpha} w(x)$. Thus $w(x)dx$ defines a (positive) measure. We also set $G(t) := \int_{\mathbb{Q}_p} u(x, t)w(x)dx$, where $u(x, t)$ is a positive solution of (5.1), then

$$(5.4) \quad \begin{aligned} G'(t) &= \int_{\mathbb{Q}_p} u_t(x, t)w(x)dx = -\gamma \int_{\mathbb{Q}_p} (\mathbf{D}_x^\alpha u)(x, t)w(x)dx \\ &+ \int_{\mathbb{Q}_p} F(u(x, t))w(x)dx + \int_{\mathbb{Q}_p} (\mathbf{D}_x^{\alpha_1} u^3)(x, t)w(x)dx. \end{aligned}$$

Now, by using that $\mathbf{D}_x^\alpha u(\cdot, t)$, $w \in L^2$, and $F(u(\cdot, t))$, $\mathbf{D}_x^{\alpha_1} u^3(\cdot, t) \in L^2$ since for $s > n/2$, \mathcal{H}_s is a Banach algebra contained in L^2 cf. Proposition 1, and applying the Parseval-Steklov theorem, we get (5.4) can be rewritten as

$$G'(t) = \int_{\mathbb{Q}_p} \left(-\gamma p^{(1-r)\alpha} u(x, t) + F(u(x, t)) + p^{(1-r)\alpha_1} u^3(x, t) \right) w(x)dx.$$

Since the function $H(y) = -\gamma p^{(1-r)\alpha} y + F(y) + p^{(1-r)\alpha_1} y^3$ is convex because

$$H''(y) = -6y + 2(\beta + 1) + p^{(1-r)\alpha_1} 6y = 6y(p^{(1-r)\alpha_1} - 1) + 2(\beta + 1) \geq 0,$$

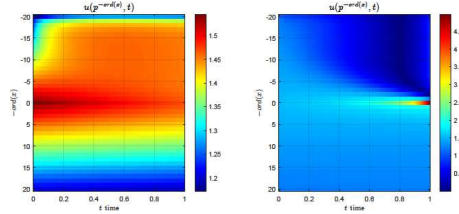
for $y \geq 0$, and $r \leq 0$, we can use the Jensen's inequality to get $G'(t) \geq H(G(t))$, then the function $G(t)$ can not remain finite for every $t \in [0, \infty)$. Then there exists

$T \in (0, \infty)$ such that $\lim_{t \rightarrow T^-} G(t) = +\infty$, hence $u(x, t)$ blow ups at the time T . Then we have established the following result:

Theorem 2. *Let $u(x, t)$ be a positive solution of (5.1). Then there $T \in (0, +\infty)$ depending on the initial datum such that $\lim_{t \rightarrow T^-} \sup_{x \in \mathbb{Q}_p^n} u(x, t) = +\infty$.*

6. NUMERICAL SIMULATIONS

In this section, we present two numerical simulations for the solution of problem (5.1) (in dimension one) for a suitable initial datum. We solve and visualize (using a heat map) the radial profiles of the solution of (5.1). We consider equation (5.1) for radial functions $u(x, \cdot)$. In [15], Kochubei obtained a formula for $\mathbf{D}_x^\alpha u(x, t)$ as an absolutely convergent real series, we truncate this series and then we apply the classic Euler Forward Method (see e.g. [23]) to find the values of $u(p^{-ord(x)}, t)$, when $-20 \leq ord(x) \leq 20$ (vertical axis) and when $t = \{t_k : t_k = 1/k, k = 1, \dots, 300\}$ (horizontal axis). In Figure 1, on the left, the heat map of the numerical solution of the homogeneous equation $u_t(x, t) = -\mathbf{D}_x^\alpha u(x, t)$ with initial data $u(x, 0) = 4e^{-p^{|ord(x)|/100}}$ (Gaussian bell type), and parameters $p = 3, \alpha = 0.2, \gamma = 1$. On the right side, we have the numerical solution of the equation $u_t(x, t) = -\mathbf{D}_x^\alpha u(x, t) - u^3(x, t) + (\beta + 1)u^2(x, t) - \beta u(x, t) + \mathbf{D}_x^{\alpha_1} u^3(x, t)$, with $p = 3, \alpha = 0.2, \alpha_1 = 0.1$, and $\beta = 0.7$.



On the left side of the Figure 1, we observe that the solution u is uniformly decreasing with respect to the variable t . This behavior is typical for solutions of diffusion equations. These equations have been extensively studied, see e.g. [18], [35] and the references therein.

On the right side of Figure 1, we see that the evolution of $u(x, t)$ is controlled by the diffusion term $-\mathbf{D}_x^\alpha u(x, t)$, up to a time T (blow-up time), this behavior is similar to that described above. When $t > T$, the reactive term $-u^3(x, t) + (\beta + 1)u^2(x, t) - \beta u(x, t) + \mathbf{D}_x^{\alpha_1} u^3(x, t)$ takes over and $u(x, t)$ grows rapidly towards infinity.

The method converges quite fast, but still lacks a mathematical formalism to support it, for this reason we refer to it as a numerical simulation of the solution.

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