8-2011

Qualitative Analysis to a Nonlinear System

Pengcheng Xiao
University of Texas-Pan American

QUALITATIVE ANALYSIS TO A

NONLINEAR SYSTEM

A Thesis

by

PENGCHENG XIAO

Submitted to the Graduate School of the
University of Texas-Pan American
In partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2011

Major Subject: Mathematics

QUALITATIVE ANALYSIS TO A

NONLINEAR SYSTEM

A Thesis
by
PENGCHENG XIAO

COMMITTEE MEMBERS

Dr. Zhaosheng Feng
Chair of Committee

Dr. Baofeng Feng
Committee Member

Dr. Tim Huber
Committee Member

Dr. Constantin Onica
Committee Member

August 2011

ABSTRACT

Xiao, Pengcheng, Qualitative Analysis to A Nonlinear System. Master of Science (MS), August, 2011, 30 pp., references, 24 titles.

In this thesis, we first present a qualitative analysis to a nonlinear system under certain parametric conditions. Then for a special case, we make a series of variable transformation and apply the Prelle-Singer Method to find the first integrals of the simplified equations without complicated calculations. Through the inverse transformations we get the first integrals of the original equation. Finally, we use the same Prelle-Singer method to get the first integral for an extended nonlinear system.

DEDICATION

The completion of my master studies would not have been possible without the love and support of my family. My mother, Yuxiang Wang, my father, Zhihe Xiao, wholeheartedly inspired, motivated and supported me by all means to accomplish this degree. Thank you for your love and patience.

ACKNOWLEDGMENTS

Foremost, I would like to express my utmost gratitude to my supervisor Dr. Zhaosheng Feng for his continuous support of my Master study and research, for his patient guidance, enthusiastic motivation and tremendous help in making this work possible. I will always take pride in saying that I studied under, and worked with Dr. Zhaosheng Feng.

Besides my supervisor, I am sincerely grateful to the members of my advisory committee, Drs. Baofeng Feng, Tim Huber and Constantin Onica, for their kind service, cordial encouragement and constructive comments on this thesis.

I would like to thank the Department of Mathematics at the University of Texas-Pan American for providing me with a generous teaching assistantship, which greatly supported my study and research work.

I would also like to thank my colleagues at the UTPA library who helped me locate supporting documents for my research. Also, I would like to acknowledge the many volunteers who participated in the focus group research.

I thank my friends Xiaochuan Hu, Jing Tian, Yu Bai, Yubing Wan, Lu Xu and Wei Yin for the inspiring discussions, stimulating study environment and all the fun we have had in the last two years.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

Many nonlinear differential equations arise in physical, chemical and biological contexts. In the past few decades, qualitative analysis together with ingenious mathematical techniques for handling some physical and biological systems has been studied extensively and a considerable number of works have been devoted to qualitative study and solutions of the nonlinear oscillator systems [1–3]. It also has been an interesting subject to find the innovative methods to solve and analyze these equations in the field of differential equations and dynamical systems [1, 4]. For these problems, it is not always possible and sometimes not even advantageous to express exact solutions of nonlinear differential equations explicitly in terms of elementary functions [20]. However, it is possible to find elementary functions that are constant on solutions curves and that is the elementary first integrals. These first integrals allow us to occasionally deduce properties that an explicit solutions would not necessary reveal [1]. In the pioneering work [5], Prelle and Singer introduced a procedure to find the first integrals of the first-order ordinary differential equations (ODEs) of the form $y = P(x, y)/Q(x, y)$, with both $P(x, y)$ and $Q(x, y)$ polynomials whose coefficients lie in the field of complex numbers $\mathbb{C}$. Duarte et al. [5] extended this procedure to second-order ordinary differential equations which is based on a conjecture that is the given second-order ordinary differential equation has an elementary solution, then there exists at least one elementary first integrals $I(x, y, y^{'})$ whose derivatives are all rational functions of x, y and $y^{'}$. It is very important to understand physical, chemical and biological phenomena modelled differential equations through special types of first integrals.

In this thesis, we consider a more general nonlinear oscillator system of the form

$$\ddot{u} + (\delta + \beta u^m)\dot{u} + u - \mu u^n = 0, \tag{1}$$

where an over-dot represents differentiation with respect to the independent variable ξ, and all coefficients δ,β and μ are real.

The organization of this thesis is as follows. In Chapter 2, making use of the qualitative theory of planar systems, we demonstrate a qualitative analysis to a two-dimensional plane autonomous system which is equivalent to equation (1). Some properties for the nonlinear oscillator system are obtained for given parametric choices. In Chapter 3, we summarize the Prelle-Singer procedure for the first-order ODEs and introduce the method developed by Duarte et al. [6] for constructing the first integrals of second-order ODEs. In Chapter 4, we re-produce the first integral of equation (1) under the condition $n = m + 1$. First we will simplify the equation (1) through a series of nonlinear transformations, then by means of the Prelle-Singer method we derive the first integral of the simplified equation without complicated calculations. After the inverse transformations we will get the first integral of the original oscillator equation. In Chapter 5 ,we will use the Prelle-Singer method to find a first integral for a extended nonlinear system $\ddot{u} + (k_1 u^2 + k_2 u)\dot{u} + \frac{k_1^2 u^5}{16} + \frac{k_1 k_2 u^4}{6} + \frac{k_2^2 u^3}{9} = 0$. Chapter 6 is a brief conclusion.

CHAPTER II

QUALITATIVE ANALYSIS

In this thesis, we study the free nonlinear oscillator equation

$$\ddot{u} + (\delta + \beta u^m)\dot{u} + u - \mu u^n = 0, \tag{2}$$

where an over-dot represents differentiation with respect to the independent variable ξ, and all coefficients δ, β, and μ are real constants.

Following [22], Letting $\frac{du}{d\xi} = v$, then equation (1) is equivalent to the following two-dimensional autonomous system

$$\begin{cases} \dot{u} = P(u,v) = v, \\ \dot{v} = Q(u,v) = -(\delta + \beta x^m)v - u + \mu u^n. \end{cases} . \tag{3}$$

In this part, using the qualitative theory of differential equations, we will show a qualitative result to the non-linear oscillator equation (1). Specifically, we show that under certain conditions

$$\delta < 0, \quad \beta < 0, \quad \mu > 0, \tag{4}$$

Consider the two-dimensional autonomous system (3) in the Poincare phase plane. Under condition (4) , n is a positive odd integer and greater than 1 and m is a positive even integer, system (3) has three equilibrium points

$$E(-\left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}, 0),\ O(0,0),\ Q(\left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}, 0). \tag{5}$$

The coefficient matrices of the linearizing systems with respect to E, O and Q are as follows, respectively,

(1) Case of Equilibrium point $E(-\left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}, 0)$,

$$M_E = \begin{pmatrix} P_u'(u,v) & P_v'(u,v) \\ Q_u'(u,v) & Q_v'(u,v) \end{pmatrix} |_E \tag{6}$$

$$= \begin{pmatrix} 0, & 1 \\ -1+n, & -(\delta+\beta\left[-\left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}\right]^m) \end{pmatrix}$$

,

and the corresponding eigenvalues of M_E are:

$$\lambda_{1=\frac{1}{2}\left\{-(\delta+\beta\left[-\left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}\right]^m)+\sqrt{\{\delta+\beta\left[-\left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}\right]^m\}^2-4+4n}\right\}},$$

$$\lambda_{2=\frac{1}{2}\left\{-(\delta+\beta\left[-\left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}\right]^m)-\sqrt{\{\delta+\beta\left[-\left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}\right]^m\}^2-4+4n}\right\}}.$$

(2) Case of Equilibrium point $O(0,0)$,

$$M_O = \begin{pmatrix} P_u'(u,v) & P_v'(u,v) \\ Q_u'(u,v) & Q_v'(u,v) \end{pmatrix} |_O \tag{7}$$

$$= \begin{pmatrix} 0, & 1 \\ -1, & -\delta \end{pmatrix},$$

and the corresponding eigenvalues of M_O are:

$$\lambda_1 = \frac{1}{2}\left(-\delta + \sqrt{\delta^2 - 4}\right),$$

$$\lambda_2 = \frac{1}{2}\left(-\delta - \sqrt{\delta^2 - 4}\right).$$

(3) Case of Equilibrium point $Q(\left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}, 0)$,

$$M_Q = \begin{pmatrix} P_u'(u,v) & P_v'(u,v) \\ Q_u'(u,v) & Q_v'(u,v) \end{pmatrix} |_Q \tag{8}$$

$$= \begin{pmatrix} 0, & 1 \\ -1+n, & -(\delta + \beta\left[\left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}\right]^m) \end{pmatrix}$$

.

and the corresponding eigenvalues of M_Q are :

$$\lambda_{1=\frac{1}{2}\left\{-(\delta+\beta\left(\frac{1}{\mu}\right)^{\frac{m}{n-1}})+\sqrt{\left[(\delta+\beta\left(\frac{1}{\mu}\right)^{\frac{m}{n-1}})^2-4+4n\right]}\right\}},$$

$$\lambda_{2=\frac{1}{2}\left\{-(\delta+\beta\left(\frac{1}{\mu}\right)^{\frac{m}{n-1}})-\sqrt{\left[(\delta+\beta\left(\frac{1}{\mu}\right)^{\frac{m}{n-1}})^2-4+4n\right]}\right\}}.$$

Now that when condition (4) holds, we can know that the equilibrium point $O(0,0)$ is

(i)a repeller if $\delta < -2$,n is a positive odd integer and greater than 1,m is a positive even integer.

(ii)a spiral repeller if $-2 < \delta < 0$, n is a positive odd integer and greater than 1 and m is a positive even integer. Since the corresponding eigenvalues for $O(0,0)$ are $\lambda_{1,2} = \frac{1}{2}\left(-\delta \pm i\sqrt{|\ \delta^2 - 4\ |}\right)$.

The equilibrium points $E(-\left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}, 0)$, $Q(\left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}, 0)$ are all saddle points under condition (4).

For

$$\frac{\partial P(u,v)}{\partial u} + \frac{\partial Q(u,v)}{\partial v} = -(\delta + \beta u^m),$$

we obtain the following proposition from the Bendiuson-Dulac criterion[17].

PROPOSITION:$When\ \delta + \beta u^m \neq 0, system(3)\ does\ not\ have\ any\ closed\ orbit\ or$

$singular\, closed\, orbit\, with\, finite\, singular\, points\, on\, the\, (u,v)\, phase\, plane.$

$Consequently, equation (2)\, has\, neither\, periodic\, travelling - wave\, solution\, nor\, bell$

$profile\, solitary - wave\, solution\, as\, \delta + \beta u^m \neq 0.$

From the above results, we will find that the v-coordinates of the points which lie on the orbits except the equilibrium points and the orbits between them are unbounded, so are the corresponding u-coordinates on the same orbits.

This can be seen by the way of contradiction , assume that there exists a positive number α such that $\mid u \mid < \alpha$ as $v \to \infty$. By the Mean-value theorem, $\frac{dv}{du}$ is unbounded.On the other hand, since the slope of the tangent line to each orbit at the point (u, v)can be expressed

$$\frac{du}{dv} = -(\delta + \beta u^m) + \frac{\mu u^n - u}{v} \tag{9}$$

Equation (9) implies that $\frac{dv}{du} \to \mid \delta + \beta \delta^m \mid$ as $v \to \infty$.This yields a contradiction.

Letting $Q(u, v) = 0$, we have

$$v = \frac{\mu u^n - u}{\delta + \beta u^m}, \tag{10}$$

which is the trajectory on which each orbit points to the left or right.Under the condition (4), expression (10) can be rewritten as

$$v = \frac{\mu u(u^{\frac{n-1}{2}} - \sqrt{\frac{1}{\mu}})(u^{\frac{n-1}{2}} + \sqrt{\frac{1}{\mu}})}{\delta + \beta u^m}.$$

Note that the graph of equation (10) is symmetric about the origin and the derivative of (10) is

$$v^{'} = \frac{\delta(n\mu u^{n-1} - 1) + \beta(m-1)u^m + \mu\beta(n-m)u^{m+n-1}}{(\delta + \beta u^m)^2}. \tag{11}$$

Construct two lines $l_1$and l_2. l_1 is the tangent line of the curve of equation (10) at the origin, that is $v = -\frac{1}{\delta}u$. And l_2 passes through $Q(u_1, 0)$ with the slope

$$K = \frac{-\delta + \sqrt{\delta^2 + 16\mu_1^2}}{4},$$

i.e.,

$$l_2 : \ v = \frac{-\delta + \sqrt{\delta^2 + 16\mu_1^2}}{4}(u - u_1). \tag{12}$$

Denote the intersection point of $l_1$and l_2 by T ,and the $u-$coordinate of T by u_T. Immediately we have the following

$$u_T = \frac{-\delta^2 + \delta\sqrt{\delta^2 + 16u_1^2}}{4 - \delta^2 + \delta\sqrt{\delta^2 + 16u_1^2}}u_1. \tag{13}$$

In addition, at the each point of the line segment OT, we have

$$\frac{dv}{du} \mid_{(u,v)\in OT} = -(\delta + \beta u^m) + \frac{\mu u^n - u}{(-\frac{1}{\delta}u)} \mid_{(u,v)\in OT}$$

$$= -\beta u^m - \delta\mu u^{n-1}$$

Let$G(u) = -\beta u^m - \delta\mu u^{n-1}$, then $G(u)^{'} = -m\beta u^{m-1} - (n-1)\delta\mu u^{n-2}$. If we want to prove that all orbits at each point on the line segment OT point outward. Then we must have $G(u)^{'} > 0$,so we can get $G(u) > -\frac{1}{\delta}$.

Since m is a positive integer and under condition (4), so we have the term $-m\beta u^{m-1} > 0$.Similarly we can get the term $(n-1)\delta\mu u^{n-2} < 0$.So $G(u)^{'} > 0$, we can get

$$-m\beta u^{m-1} > (1-n)\delta\mu u^{n-2}$$

$$\frac{-m\beta}{(1-n)\delta\mu} > u^{n-m-1} \tag{14}$$

Also we have the condition for $u-$coordinate of each point on the line segment OT ,that is

$$0 < u < u_T = \frac{-\delta^2 + \delta\sqrt{\delta^2 + 16u_1^2}}{4 - \delta^2 + \delta\sqrt{\delta^2 + 16u_1^2}} u_1,$$

where $u_1 = \left(\frac{1}{\mu}\right)^{\frac{1}{n-1}}$.

Similarly, on the line segment QT ,using (9), we have

$$\frac{du}{dv} \mid_{(u,v)\in l_2} = -(\delta + \beta u^m) + \frac{\mu u^n - u}{v} \mid_{(u,v)\in QT}$$

$$= -(\delta + \beta u^m) + \frac{\mu u^n - u}{\frac{-\delta + \sqrt{\delta^2 + 16\mu_1^2}}{4}(u - u_1)} \mid_{(u,v)\in QT}$$

$$= -\delta - \beta u^m + \frac{4(\mu u^n - u)}{(-\delta + \sqrt{\delta^2 + 16\mu_1^2})(u - u_1)} \mid_{(u,v)\in QT}$$

Since $u_T \leq u \leq u_1$, we can get that

$$\frac{du}{dv} \mid_{(u,v)\in QT} > K.$$

This implies that except at Q, all orbits at each point of line segment QT point outward.

CHAPTER III

PRELLE-SINGER PROCEDURE

3.1 Prelle-Singer Procedure for First-Order ODEs

There are many methods to solve nonlinear differential equations, but most of them only work for a limited class. Despite its effectiveness in solving FOODEs, the Prelle-Singer (PS) procedure is not very well known outside mathematical circles. This is probably due to its non-standard approach, coupled with the fact that a computer is almost essential to realize its full efficiency. Hence we present a brief overview of the main ideas of the Prelle-Singer procedure [5–14].

Consider the autonomous system of ODEs [5]:

$$\dot{x} = Q(x,y), \qquad \dot{y} = P(x,y), \qquad P,\, Q \in \mathbb{C}[x,y],$$

where an overdot represents a derivative with respect to the independent variable t. This system is equivalent to the class of FOODEs which can be written as

$$y' = \frac{dy}{dx} = \frac{P(x,y)}{Q(x,y)}, \tag{15}$$

in other words those FOODEs which can be isolated in y', leaving a rational function of x and y on the right-hand side.

Prelle and Singer [5] showed that, if an elementary first integral of (15) exists, there exists an integrating factor R with $R^n \in \mathbb{C}[x,y]$ for some integer n, such that

$$\frac{\partial RQ}{\partial x} + \frac{\partial RP}{\partial y} = 0. \tag{16}$$

The key to the success of the PS procedure is that, given the particular form of the FOODE, we know the most general form that the integrating factor can take. We can then realize a computer-assisted exhaustive search for the correct integrating factor. With the integrating factor determined, the ODE can be solved by quadrature. From (16) we see that

$$Q\frac{\partial R}{\partial x} + R\frac{\partial Q}{\partial x} + P\frac{\partial R}{\partial y} + R\frac{\partial P}{\partial y} = 0. \tag{17}$$

Thus, defining the differential operator

$$D \equiv Q\frac{\partial}{\partial x} + P\frac{\partial}{\partial y}, \tag{18}$$

we have that

$$\frac{D[R]}{R} = -\left(\frac{\partial Q}{\partial x} + \frac{\partial P}{\partial y}\right). \tag{19}$$

Now let $R = \prod_i f_i^{n_i}$ where f_i are monic irreducible polynomials and n_i are non-zero rational numbers [5].

From (18) we have

$$\begin{aligned} \frac{D[R]}{R} &= \frac{D[\prod_i f_i^{n_i}]}{\prod_k f_k^{n_k}} = \frac{\sum_i f_i^{n_i-1} n_i D[f_i] \prod_{j\neq i} f_j^{n_j}}{\prod_k f_k^{n_k}}, \\ &= \sum_i \frac{f_i^{n_i-1} n_i D[f_i]}{f_i^{n_i}} = \sum_i \frac{n_i D[f_i]}{f_i}. \end{aligned} \tag{20}$$

From (16), plus the fact that P and Q are polynomials, we conclude that $D[R]/R$ is a polynomial. Therefore, from (20), we see that $f_i | D[f_i]$. Written in the form

$$D[f_i] = f_i g_i, \tag{21}$$

for some polynomial g_i, we see that the equation for the f_i has aspects similar to an eigenvalue equation, and for that reason f_i are sometimes called eigenpolynomials. However current usage seems to prefer the term Darboux polynomials, and we shall refer to the f_i as such in this paper.

Given an upper bound, B, on the degree of the Darboux polynomials, f_i, we thus have a criterion for finding them. We can, for example, construct all possible polynomials of degree up to B with monic leading term and arbitrary complex coefficients, construct equation (21) and see if there are non-trivial solutions for the arbitrary coefficients. With this in mind the PS procedure works as follows [5]:

(1) Set the current degree bound, $N = 1$.

(2) Find all Darboux polynomials f_i such that deg $f_i \leq N$ and $f_i | D[f_i]$.

(3) Let $D[f_i] = f_i g_i$. If there exist constants n_i, not all zero, such that

$$\sum_{i=1}^{m} n_i g_i = 0, \tag{22}$$

then from (16) $D[R]/R = 0$ and the ODE is exact. The solution is $w = c$, where c is an arbitrary constant and $\prod_{i=1}^{m} f_i^{n_i}$.If (22) has no solution then

(4) if there exist constants n_i, not all zero, such that

$$\sum_{i=1}^{m} n_i g_i = -\left(\frac{\partial Q}{\partial x} + \frac{\partial P}{\partial y}\right), \tag{23}$$

then return the solution $w = c$, where c is an arbitrary constant and w is either of

$$\int RPdx - \int \left(RQ + \frac{d}{dy}\int RPdx\right) dy,$$

or

$$-\int RQdy + \int \left(RP + \frac{d}{dx}\int RQdy\right) dx.$$

(5) Set $N = N + 1$. If $N > B$ then exit with no result. Else go to 2.

3.2 Prelle-Singer Procedure for Second-Order ODEs

In this section, we follow [5-14] to modify the techniques developed by Prelle and Singer and apply them to second-order ODEs (SOODEs) with the following rational form. This modified technique was also developed by Chandrasekar et al. [7-9].

Consider the second-order ODE:

$$y'' = \frac{P(x, y, y')}{Q(x, y, y')}, \qquad P,\, Q \in \mathbb{C}[x, y, y']. \tag{24}$$

We restrict ourselves for the time being to SOODEs which have elementary solutions, i.e. which can be written in the form

$$f(x, y) = 0,$$

where f is an arbitrary combination of exponentials, logarithms and polynomials in its arguments. Since we are working over a complex field, this includes standard trigonometric functions. Our goal is to find elementary first integrals of (24) when such elementary first integrals exist. We believe, given the conditions above, that these first integrals have a very particular form, described later, which permits us to construct a semi-decision procedure analogous to the PS method to find them. Once such a first integral is found, if y' can be isolated, then the PS method (or any other solution method for FOODEs) can then be applied to obtain the full solution.

In this section, in order to present our results in a straightforward way, we start our study by briefly reviewing the Prelle–Singer procedure for solving second-order ODEs developed by Duarte *et al.* [6] and Chandrasekar *et al.* [7-9].

Consider the second-order ODE of the rational form

$$\frac{d^2y}{dx^2} = \phi(x,\, y,\, y') = \frac{P(x,\, y,\, y')}{Q(x,\, y,\, y')}, \qquad P,\, Q \in \mathbb{C}[x,\, y,\, y'], \tag{25}$$

where y' denotes differentiation with respect to x, P and Q are polynomials in x, y and y' with coefficients in the complex field. Suppose that equation (25) admits a first integral $I(x,\, y,\, y') = C$, with C constant on the solutions, so we have the total differential

$$dI = I_x dx + I_y dy + I_{y'} dy' = 0, \tag{26}$$

where the subscript denotes partial differentiation with respect to the corresponding variable. On the solution, since $y'dx = dy$ and equation (25) is equivalent to $\frac{P}{Q}dx = dy'$, adding a null term $S(x,\, y,\, y')y'dx - S(x,\, y,\, y')dy$ to both sides yields

$$\left(\frac{P}{Q} + Sy'\right) dx - Sdy - dy' = 0. \tag{27}$$

Comparing (26) and (27), one can see that on the solutions, the corresponding coefficients of (26) and (27) should be proportional. There exists an integrating factor $R(x,\, y,\, y')$ for (27), such that on the solutions

$$dI = R(\phi + Sy')dx - SRdy - Rdy' = 0. \tag{28}$$

From (26) and (28), we have

$$I_x = R(\phi + Sy'), \quad I_y = -SR, \quad I_{y'} = -R, \tag{29}$$

and the compatibility conditions $I_{xy} = I_{yx},\ I_{xy'} = I_{y'x}$ and $I_{yy'} = I_{y'y}$, which are equivalent to

$$D[S] = -\phi_y + S\phi_{y'} + S^2,\ \ D[R] = -R(S + \phi_{y'}),\ \ R_y = R_{y'}S + S_{y'}R, \tag{30}$$

where D is an differential operator

$$D = \frac{\partial}{\partial x} + y'\frac{\partial}{\partial y} + \phi\frac{\partial}{\partial y'}.$$

For the given expression of ϕ, one can solve the first equation of (30) for S. Substituting S into the second equation of (30) one can get an explicit form for R by solving it. Once a compatible solution R and S satisfying the extra constraint (the third equation of (30)) is derived, integrating (29), from (26) one may obtain a first integral of motion as follows [6-9]:

$$
\begin{aligned}
I(x,\, y,\, y') = &\int R(\phi + Sy')dx - \int \left[RS + \frac{\partial}{\partial y}\int R(\phi + Sy')dx\right] dy \\
&- \int \left\{R + \frac{\partial}{\partial y'}\left(\int R(\phi + Sy')dx - \int \left[RS + \frac{\partial}{\partial y}\int R(\phi + Sy')dx\right] dy\right)\right\} dy'.
\end{aligned}
$$

CHAPTER IV

FIRST INTEGRAL OF NONLINEAR SYSTEM

4.1 Nonlinear Transformations

In this subsection, we use the transform method and the Prelle-Singer Procedure to re-produce the first integrals of the Duffing-van de Pol equation which have been presented in [23, 24].

We will make a series of nonlinear transformations to equation

$$\ddot{u} + (\delta + \beta u^m)\dot{u} + u - \mu u^n = 0, \tag{31}$$

where an over-dot denotes differentiation with respect to ξ.

First we make the natural logarithm transformation:

$$\xi = -\frac{1}{\delta} \ln \tau, \tag{32}$$

that is

$$\frac{\partial \tau}{\partial \xi} = -\delta e^{-\xi\delta} = -\delta\tau. \tag{33}$$

After substituting the following two derivatives into equation (31):

$$\frac{\partial u}{\partial \xi} = \frac{\partial u}{\partial \tau} \cdot \frac{\partial \tau}{\partial \xi} = -\delta\tau \frac{\partial u}{\partial \tau},$$

$$\frac{\partial^2 u}{\partial \xi^2} = \delta^2 \tau \frac{\partial u}{\partial \tau} + \delta^2 \tau^2 \frac{\partial^2 u}{\partial \tau^2},$$

then it becomes

$$\delta^2\tau^2\frac{\partial^2 u}{\partial\tau^2} - \beta\delta\tau u^m\frac{\partial u}{\partial\tau} + u - \mu u^n = 0. \tag{34}$$

Furthermore, we take the variable transformation as:

$$q = \tau^k, \quad u = \tau^{-\frac{1}{2}(\kappa-1)}H(q), \tag{35}$$

then a direction calculation gives

$$\frac{\partial u}{\partial\tau} = -\frac{1}{2}(\kappa-1)q^{-\frac{\kappa+1}{2\kappa}}H(q) + \kappa q^{\frac{\kappa-1}{2\kappa}}\frac{\partial H}{\partial q},$$

$$\frac{\partial^2 u}{\partial\tau^2} = \frac{1}{4}(\kappa^2-1)q^{-\frac{\kappa+3}{2\kappa}}H(q) + \kappa^2 q^{\frac{3(\kappa-1)}{2\kappa}}\frac{\partial^2 H}{\partial q^2}.$$

Then substitute the above equations into the equation (34), we can get

$$\frac{1}{4}\delta^2\tau^2(\kappa^2-1)q^{-\frac{\kappa+3}{2\kappa}}H + \delta^2\tau^2\kappa^2 q^{\frac{3(\kappa-1)}{2\kappa}}\frac{\partial^2 H}{\partial q^2} + \frac{1}{2}\beta\delta(\kappa-1)\tau^{-\frac{m(\kappa-1)}{2}+1}q^{-\frac{\kappa+1}{2\kappa}}H^{m+1}$$

$$-\beta\delta\kappa\tau^{-\frac{m(\kappa-1)}{2}+1}q\frac{\kappa-1}{2\kappa}\frac{\partial H}{\partial q} + \tau^{-\frac{1}{2}(\kappa-1)}H - \mu\tau^{-\frac{n(\kappa-1)}{2}}H^n = 0. \tag{36}$$

After careful observation, we find that in the equation (36), the two terms involved with H can be eliminated when

$$\frac{1}{4}\delta^2\tau^2(\kappa^2-1)q^{-\frac{\kappa+3}{2\kappa}} + \tau^{-\frac{1}{2}(\kappa-1)} = 0. \tag{37}$$

Then we can simplify equation (37) and get the condition

$$\kappa^2 = -\frac{4}{\delta^2} + 1. \tag{38}$$

Therefore, in the condition (38), equation (36) becomes

$$\delta\tau^2\kappa^2 q^{\frac{3(\kappa-1)}{2\kappa}}\frac{\partial^2 H}{\partial q^2}+\frac{1}{2}\beta\delta(\kappa-1)\tau^{-\frac{m(\kappa-1)}{2}+1}q^{-\frac{\kappa+1}{2\kappa}}H^{m+1}$$

$$-\beta\delta\kappa\tau^{-\frac{m(\kappa-1)}{2}+1}q\frac{\kappa-1}{2\kappa}\frac{\partial H}{\partial q}-\mu\tau^{-\frac{n(\kappa-1)}{2}}H^n=0. \tag{39}$$

From equation (39), we can get

$$\frac{\partial^2 H}{\partial q^2}=\frac{\beta\tau^{-\frac{m(\kappa-1)}{2}-1}q^{-\frac{\kappa-1}{\kappa}}}{\delta\kappa}H^m\frac{\partial H}{\partial q}+\frac{\mu\tau^{-\frac{n(\kappa-1)}{2}-2}}{\delta^2\kappa^2 q^{\frac{3(\kappa-1)}{2\kappa}}}H^n$$

$$-\frac{\beta(\kappa-1)\tau^{-\frac{m(\kappa-1)}{2}-1}q^{-\frac{(\kappa+1)}{2\kappa}-\frac{3(\kappa-1)}{2\kappa}}}{2\delta\kappa^2}H^{m+1}, \tag{40}$$

Also from equation (35), we know that $q=\tau^k$. Through this equation , we can simplify equation (40) into

$$\frac{\partial^2 H}{\partial q^2}=\frac{\beta}{\delta\kappa}q\frac{m-\kappa(m+2)}{2\kappa}H^m\frac{\partial H}{\partial q}+\frac{\mu}{\delta^2\kappa^2}q^{\frac{-(n+3)(\kappa-1)-4}{2\kappa}}H^n$$

$$-\frac{1}{2}\frac{\beta(\kappa-1)}{\delta\kappa^2}q^{\frac{m-\kappa(m+4)}{2\kappa}}H^{m+1}, \tag{41}$$

with the condition (38).

4.2 Special Case: When $n=m+1$

In this subsection, we use the Prelle-Singer method to find the first integral for the oscillator equation in the case of $n=m+1$,that is

$$\ddot{u}+(\delta+\beta u^m)\dot{u}+u-\mu u^{m+1}=0, \tag{42}$$

where an over-dot denotes differentiation with respect to ξ. Note that when $n=m+1$, equation

(41) will be simplified into the form

$$\frac{\partial^2 H}{\partial q^2} = \frac{\beta}{\delta\kappa} q^p H^m \frac{\partial H}{\partial q} + (\frac{\mu}{\delta^2\kappa^2} - \frac{\beta(\kappa-1)}{2\delta\kappa^2}) q^{p-1} H^{m+1}, \tag{43}$$

where

$$p = \frac{m - \kappa(m+2)}{2\kappa}.$$

For the notational convenience, we donote that

$$A = \frac{\beta}{\delta\kappa}, \qquad B = \frac{\mu}{\delta^2\kappa^2} - \frac{\beta(\kappa-1)}{2\delta\kappa^2},$$

then equation (43) becomes

$$\ddot{H} = Aq^p H^m \dot{H} + BH^{m+1} q^{p-1}. \tag{44}$$

Choosing $\phi(q, H, H') = Aq^p H^m \dot{H} + BH^{m+1} q^{p-1}$and following the procedure in Section 2, we obtain three determining equations:

$$S_q + \dot{H} S_H + \phi S_{\dot{H}} = -mAq^p H^{m-1} \dot{H} + (ASq^p - B(m+1)q^{p-1}) H^m + S^2, \tag{45}$$

$$R_q + R_H \dot{H} + \phi R_{\dot{H}} = -RS - RAq^p H^m, \tag{46}$$

$$R_H = R_{\dot{H}} S + S_{\dot{H}} R. \tag{47}$$

In general, it is not easy to solve system (30) and get exact solutions $(S,\ R)$ in the explicit forms. But in our case of (45)-(47) we may seek an ansatz for S and R of the forms as suggested in the Duarte et al [6]. and Chandrasekar et al [7-9] paper:

$$S = \frac{a(q,H) + b(q,H)\dot{H}}{c(q,H) + d(q,H)\dot{H}}, \quad R = e(q,H) + f(q,H)\dot{H}, \tag{48}$$

where a,b,c,d and e,f are functions of q, H to be determined. Substituting S into equation (45) , we can get the equation system:

$$\left[\dot{H}\right]^0 : -(m+1)Bc^2H^mq^{p-1} + Aacq^pH^m + a^2 = a_qc - ac_q + bcBH^{m+1}q^{p-1} - adBH^{m+1}q^{p-1},$$

$$\left[\dot{H}\right]^1 : -mAc^2q^pH^{m-1} - 2(m+1)BcdH^mq^{p-1} + Aadq^pH^m + Aq^pH^mbc + 2ab$$

$$= a_qd + b_qc - ad_q - bc_q + a_Hc - ac_H + bcAq^pH^m - adAq^pH^m, \tag{49}$$

$$\left[\dot{H}\right]^2 : -2mAcdq^pH^{m-1} - (m+1)Bd^2H^mq^{p-1} + Abdq^pH^m + b^2 = b_qd - bd_q + a_Hd + b_Hc - ad_H - bc_h,$$

$$\left[\dot{H}\right]^3 : -mAd^2q^pH^{m-1} = b_Hd - bd_H.$$

Substituting S and R into equation (46), we can get another equation system:

$$\left[\dot{H}\right]^0 : \ e_qc + BcfH^{m+1}q^{p-1} = -ae - Aceq^pH^m,$$

$$\left[\dot{H}\right]^1 : \ f_qc + e_Hc + 2Afcq^pH^m + e_qd + BfdH^{m+1}q^{p-1} = -be - Adeq^pH^m - af,$$

$$\left[\dot{H}\right]^2: \ f_H c + f_q d + e_H d + 2Afdq^p H^m = -bf, \tag{50}$$

$$\left[\dot{H}\right]^3: \ f_H d = 0$$

We solve the above two nonlinear system (49) and (50) for a nontrivial solution with the aid of the Mathematica, we can first find that $b = 0$, $d = 0$ and $f = 0$.

Then from the first equation of system (49) we can get

$$-(m+1)Bc^2 q^{p-1} H^m + Aacq^p H^m + a^2 = a_q c - ac_q,$$

that is

$$S_q = \frac{a_q c - ac_q}{c^2} = -(m+1)Bq^{p-1}H^m + ASq^p H^m + S^2. \tag{51}$$

From the second equation of system (49) we can get

$$-mAc^2 q^p H^{m-1} = a_H c - ac_H,$$

that is

$$S_H = \frac{a_H c - ac_H}{c^2} = -mAq^p H^{m-1}. \tag{52}$$

Thus , from equation (52) we can get

$$S = -Aq^p H^m + F(q). \tag{53}$$

where $F(q)$ is a function of q to be determined.

Next we plug equation (53) in to equation (51) , we can get

$$-(m+1)Bq^{p-1}H^m + A(-Aq^p H^m + F(q))q^p H^m$$

$$+(A^2q^{2p}H^{2m}+F^2(q)-2Aq^pH^mF(q))=-Apq^{p-1}H^m+F_q(q). \tag{54}$$

From equation (54) we can get the equation system:

$$[H^m]^0:\ F^2(q)=F_q(q),$$

$$[H^m]^1:-(m+1)Bq^{p-1}-Aq^pF(q)=-Aq^{p-1}. \tag{55}$$

Through the two equations we can get

$$F(q)=-\frac{1}{q}. \tag{56}$$

Based on the above results , we can obtain that under the parametric conditions

$$m=\frac{4\beta\delta\kappa}{2\mu-\delta\beta(\kappa-1)}-1,\quad \kappa^2=-\frac{4}{\delta^2}+1, \tag{57}$$

the three determining equations (45)-(47) have the solutions of the form

$$S=-\frac{\beta}{\delta\kappa}q^{\frac{m-\kappa(m+2)}{2\kappa}}H^m-\frac{1}{q},\qquad R=q. \tag{58}$$

After substitution of the solution set (57) into formula

$$\begin{aligned}I(x,\,y,\,y')&=\int R(\phi+Sy')dx-\int\left[RS+\frac{\partial}{\partial y}\int R(\phi+Sy')dx\right]dy\\&-\int\left\{R+\frac{\partial}{\partial y'}\left(\int R(\phi+Sy')dx-\int\left[RS+\frac{\partial}{\partial y}\int R(\phi+Sy')dx\right]dy\right)\right\}dy',\end{aligned}$$

we derive the first integral of equation (42) as follows

$$\delta\kappa H-\delta\kappa q\dot{H}+\frac{2\mu-\delta\beta(\kappa-1)}{4\delta\kappa}q^{\frac{(1-\kappa)m}{2\kappa}}H^{m+1}=I, \tag{59}$$

where I is an arbitrary integration constant. By virtue of inverse transformations (32) and (35), and changing to the original variables, we obtain the followings under parametric condition (57):

$$H = uq^{\frac{\kappa-1}{2\kappa}}$$

$$\dot{H} = \frac{\partial u}{\partial q}q^{\frac{\kappa-1}{2\kappa}} + u\frac{\kappa-1}{2\kappa}q^{\frac{-1-\kappa}{2\kappa}} = \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial q}q^{\frac{\kappa-1}{2\kappa}} + u\frac{\kappa-1}{2\kappa}q^{\frac{-1-\kappa}{2\kappa}}$$

$$= -\dot{u}\frac{1}{\delta\kappa q}q^{\frac{\kappa-1}{2\kappa}} + u\frac{\kappa-1}{2\kappa}q^{\frac{-1-\kappa}{2\kappa}}. \tag{60}$$

Then plug equation (60) into equation (59) , we can get the nonlinear oscillator equation (42) has the first integral of the form

$$[\dot{u} + \frac{\delta(\kappa+1)}{2}u + \frac{2\mu - \delta\beta(\kappa-1)}{4\delta\kappa}u^{m+1}]e^{\frac{1}{2}\delta\xi(1-\kappa)} = I_{2.} \tag{61}$$

CHAPTER V

FIRST INTEGRAL FOR AN EXTENDED NONLINEAR SYSTEM

Now we want use the same Prelle-Singer method to find the first integral for an extended nonlinear equation

$$\ddot{u} + (k_1 u^2 + k_2 u)\dot{u} + \frac{k_1^2 u^5}{16} + \frac{k_1 k_2 u^2}{6} + \frac{k_2^2 u^3}{9} = 0, \tag{62}$$

where an over-dot denotes differentiation with respect to ξ. k_1 and k_2 are arbitrary parameters. From the PS method showed in section 2, we know that

$$\phi = -(k_1 u^2 + k_2 u)\dot{u} - (\frac{k_1^2 u^5}{16} + \frac{k_1 k_2 u^2}{6} + \frac{k_2^2 u^3}{9}). \tag{63}$$

Since

$$\dot{u} d\xi = du,$$

we add a null term $S(\xi, u, \dot{u})\dot{u} d\xi - S(\xi, u, \dot{u}) du$,then we will get the equation$(\phi + S\dot{u})d\xi - Sdu - d\dot{u} = 0$.

Define the differential operator

$$D \equiv \frac{\partial}{\partial \xi} + \dot{u}\frac{\partial}{\partial u} + \phi \frac{\partial}{\partial \dot{u}},$$

then we can get

$$S_\xi + \dot{u} S_u + \phi S_{\dot{u}} = -\phi_u + S\phi_{\dot{u}} + S^2, \tag{64}$$

$$R_\xi + \dot{u}R_u + \phi R_{\dot{u}} = -R(S + \phi_{\dot{u}}), \tag{65}$$

$$R_u = R_{\dot{u}}S + S_{\dot{u}}R. \tag{66}$$

Also we know that

$$\phi_u = -(2k_1u + k_2)\dot{u} - (\frac{5k_1^2u^4}{16} + \frac{4k_1k_2u^3}{6} + \frac{3k_2^2u^2}{9}),$$

$$\phi_{\dot{u}} = -(k_1u^2 + k_2u).$$

Assume that

$$S = \frac{a(\xi,u) + b(\xi,u)\dot{u}}{c(\xi,u) + d(\xi,u)\dot{u}},\ R = e(\xi,u) + f(\xi,u)\dot{u},$$

where a, b, c, d, e and f are functions of (ξ, u) to be determined.

Put S into equation (64), we can get

$$\frac{(a_\xi + b_\xi\dot{u})(c + d\dot{u}) - (c_\xi + d_\xi\dot{u})(a + b\dot{u})}{(c + d\dot{u})^2} + \dot{u}\frac{(a_u + b_u\dot{u})(c + d\dot{u}) - (c_u + d_u\dot{u})(a + b\dot{u})}{(c + d\dot{u})^2}$$

$$-[(k_1u^2 + k_2u)\dot{u} + (\frac{k_1^2u^5}{16} + \frac{k_1k_2u^4}{6} + \frac{k_2^2u^3}{9})]\frac{b(c + d\dot{u}) - d(a + b\dot{u})}{(c + d\dot{u})^2}$$

$$= (2k_1u + k_2)\dot{u} + (\frac{5k_1^2u^4}{16} + \frac{4k_1k_2u^3}{6} + \frac{3k_2^2u^2}{9}) - \frac{a + b\dot{u}}{c + d\dot{u}}(k_1u^2 + k_2u) + \frac{\dot{(a + bu)}^2}{(c + d\dot{u})^2}.$$

After simplifying we can get the equation system:

$$[\dot{u}]^0:\ (a_\xi c - ac_\xi) - (\frac{k_1^2u^5}{16} + \frac{k_1k_2u^4}{6} + \frac{k_2^2u^3}{9})(bc - da)$$

$$= (\frac{5k_1^2u^4}{16} + \frac{4k_1k_2u^3}{6} + \frac{3k_2^2u^2}{9})c^2 - ac(k_1u^2 + k_2u) + a^2,$$

$$[\dot{u}]^1:\ (a_\xi d - ad_\xi + b_\xi c - bc_\xi) + (a_u c - ac_u) - (bc - da)(k_1u^2 + k_2u)$$

$$= (2k_1u + k_2)c^2 + 2cd(\frac{5k_1^2u^4}{16} + \frac{4k_1k_2u^3}{6} + \frac{3k_2^2u^2}{9}) - (ad + bc)(k_1u^2 + k_2u) + 2ab, \tag{67}$$

$$[\dot{u}]^2:\ (b_\xi d - bd_\xi) + (a_u d + b_u c - ad_u - bc_u)$$

$$= 2cd(2k_1u + k_2) + (\frac{5k_1^2u^4}{16} + \frac{4k_1k_2u^3}{6} + \frac{3k_2^2u^2}{9})d^2 - bd(k_1u^2 + k_2u) + b^2,$$

$$[\dot{u}]^3:\ (b_u d - d_u b) = (2k_1u + k_2)d^2.$$

Then put R and S into equation (65), we can get

$$e_\xi + f_\xi\dot{u} + (e_u + f_u\dot{u})\dot{u} - [(k_1u^2 + k_2u)\dot{u} + (\frac{k_1^2u^5}{16} + \frac{k_1k_2u^2}{6} + \frac{k_2^2u^3}{9})]f$$

$$= -(e + f\dot{u})[\frac{a + b\dot{u}}{c + d\dot{u}} - (k_1u^2 + k_2u)].$$

then we can get the equation system as follows:

$$[\dot{u}]^0:\ e_\xi c - (\frac{5k_1^2u^4}{16} + \frac{4k_1k_2u^3}{6} + \frac{3k_2^2u^2}{9})fc = -ea + ec(k_1u^2 + k_2u),$$

$$[\dot{u}]^1: \ (e_\xi d + f_\xi c) + e_u c - (k_1 u^2 + k_2 u) f c - (\frac{5k_1^2 u^4}{16} + \frac{4k_1 k_2 u^3}{6} + \frac{3k_2^2 u^2}{9}) f d$$

$$= -(eb + fa) + (k_1 u^2 + k_2 u)(ed + fc), \tag{68}$$

$$[\dot{u}]^2: \ f_\xi d + e_u d + f_u c - (k_1 u^2 + k_2 u) f d = -fb + (k_1 u^2 + k_2 u) f d,$$

$$[\dot{u}]^3: \ f_u d = 0.$$

With the help of Mathematica, we find the formulas of R and S. After substitution of the solution set R and S into the formula showed in Prelle-Singer Procedure. We derive the first integral of the equation and get the results as follows:

$$\frac{\dot{u} + \frac{k_1 u^3}{4} + \frac{k_2 u^2}{3}}{\xi(\dot{u} + \frac{k_1 u^3}{4} + \frac{k_2 u^2}{3}) - u} = I_{3.} \tag{69}$$

CHAPTER VI

CONCLUSION

Through the qualitative analysis to the nonlinear system, we find that equation (1) has a bounded non-trivial solution under certain parametric conditions. By using the Prelle-Singer method, we re-produce the first integral for the nonlinear system in the case of $n = m + 1$ under some parametric conditions. Also we find the first integral for an extended nonlinear system through the same method.

In the future , we want to analyze the property of proper solutions by using the obtained first integrals.

REFERENCES

[1] Jordan DW, Smith P. Nonlinear ordinary differential equations. Oxford: Clarendon Press; 1977.

[2] Thompson JMT, Stewart HB. Nonlinear dynamics and chaos. New York: Jhon Wiley & Sons; 1986.

[3] Hilborn R. Chaos and nonlinear dynamics. New York: Oxford University Press; 1994.

[4] A.D. Polyanin and V.F. Zaitsev, Handbook of Exact solutions for Ordinary Differential Equations, 2nd edition, London:CRC Press,2003.

[5] M.Prelle and M. Singer, Elementary first integrals of differential equations, *Trans. Am. Math. Soc*. 279 (1983), 215-229.

[6] L.G.S. Duarte, S.E.S. Duarte, A.C.P da Mota and J.E.F. Skea, *Solving the second-order ordinary differential equations by extending the Prelle-Singer method*, *J. Phys. A (Math. Gen.*) 34 (2001), 3015-3024.

[7] V.K. Chandrasekar, M. Senthilvelan, & M. Lakshmanan, *On the complete integrability and linearization of certain second-order nonlinear ordinary differential equations*, Proc. R. Soc. Lond. Ser. A 461 (2005),2451-2476.

[8] V.K. Chandrasekar, M. Senthilvelan, & M. Lakshmanan, *On the complete integrability and linearization of nonlinear ordinary differential equations-(II) Thrid-order equations*, Proc.R. Soc. Lond. Ser. A 462 (2006),1831-1852.

[9] V.K. Chandrasekar, M. Senthilvelan, & M. Lakshmanan, *Extended Prelle-Singer method and integrability/solvablity of a class of nonlinear nth order differential equations*, J. Nonlin. Diff. Equs. 12 (2005), 184-201.

[10] Y.K. Man, *Computing closed form solutions of first-order ODEs using the Prelle-Singer procedure*, J. Symb. Comput. 16 (1993), 423-443.

[11] Y.K. Man, *First integrals of autonomous systems of differential equations and the Prelle-Singer procedure*, J. Phys. A. (Math. Gen.) 27 (1994),L329-L332.

[12] R.Shtokhamer, *Solving first-order differential equations using the Prelle-Singer algorithm*, em Technical Report, 88-09 Center for Mathematical Computation,,University of Delaware, 1988.

[13] M. Singer, *Formal solutions of differential equations*, J. Symb. Comput. 10 (1990), 59-94.

[14] M. Singer, *Liouvillian first integrals of differential equations*, Trans. Am. Math. Soc. 333 (1992), 673-688.

[15] Ferris, M.C., Pang, J.S, Engineering and economic applications of complementarity problems. SIAM Rev. 39 (1997), 669-713.

[16] Pang, J.S, Complementarity problems. In; Horst, R., Padalos, P. (eds.) Handbook in Global Optimization. , pp. 669. Kluwer, Boston (1994)

[17] Zhang, J.Y, Geometrical Analysis and Bifurcations of Ordinary Differential Equations. Peking University Press, Peking (1987)

[18] A. Canada, P. Drabek and A. Fonda, Handbook of Differential Equations: Ordinary Differential Equations, Volumes 2-3, Elsevier, 2005.

[19] M. Gitterman, The Noisy Oscillator: the First Hundred Years, from Einstein until Now, World Scientific Publishing Co. Pte. Ltd. Singapore, 2005.

[20] J. Guckenheimer and P. Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, Springer Verlag, New York, 1983.

[21] M. Lakshmanan and S. Rajasekar, Nonlinear Dynamics: Integrability , Chaos and Patterns, Springer Verlag, New York, 2003.

[22] Z. Feng, S.Z. Zheng and D.Y. Gao, Traveling wave solutions to a reaction-diffusion equation, *Z. Angew. Math. Phys.* 60 (2009), 756–773.

[23] G. Gao and Z. Feng, First integrals for the Duffing-van der Pol-type oscillator, E. J. Differential Equations, 19 (2010), 123-133.

[24] Z. Feng, G. Gao and J. Cui, Duffing-van der Pol-type oscillator system and its first integrals, Communications on Pure and Applied Analysis, 10 (2011), 1377—1391.

BIOGRAPHICAL SKETCH

Pengcheng Xiao, the son of Zhihe Xiao and Yuxiang Wang, was born in China in 1986. He received his bachelor degree in Information Security from China University of Mining and Technology, Xuzhou, Jiangsu, China in June of 2009. In August of 2009, he joined the Mathematical Master's Program at the University of Texas-Pan American, Edinburg, Texas. His main research interests were in Nonlinear Analysis of Differential Equations and Dynamic Systems. His permanent mailing address is: 1239 Fuqian Street, 8th Building, Section 2, Apt 404, Xintai City, Shandong, P.R.China, 271200.