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Electromagnetic strong plasma turbulence

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The first large-scale simulations of continuously driven, two-dimensional electromagnetic strong plasma turbulence are performed, for electron thermal speeds 0.01c ≤ v ≤ 0.57c, by integrating the Zakharov equations for coupled Langmuir and transverse (T) waves near the plasma frequency. Turbulence scalings and wave number spectra are calculated, a transition is found from a mix of trapped and free T eigenstates for v ≈ 0.1c to just free eigenstates for v ≲ 0.1c, and wave energy densities are observed to undergo slow quasiperiodic oscillations. © 2007 American Institute of Physics.

Bursts of electromagnetic (EM) radiation at the plasma frequency ωp and harmonics, coexisting with electrostatic (ES) noise and quasineutral density wells, are often observed in beam-plasma1–3 and ionospheric-heating4,5 experiments in the strong-plasma-turbulence (SPT) regime W = ε0 〈|E|^2〉/4n0kBT ≃ (kλD)^2, where 〈|E|^2〉/2 is the rms electric field strength, n0 is the mean electron density, Tp is the electron temperature, kB is the characteristic ES wave number, and λD is the Debye length. One application of EMSPT is to experiments where an ultrashort laser pulse is fired into an underdense plasma, generating slow-moving, long-lived EM wave packets.6–8 It is also relevant to inertial confinement fusion research, e.g., at the National Ignition Facility and Laser Megajoule facility, where Tp ≃ 80 MK.9 Considerable ingenuity is needed to suppress hydrodynamic instabilities at these facilities, and EMSPT can disrupt symmetry of imploding fuel capsules by converting some laser power to free EM radiation at the plasma frequency. These facilities, and EMSPT can disrupt symmetry of imploding fuel capsules by converting some laser power to free EM radiation at the plasma frequency. Previous simulations were limited to a 64^3 grid containing ≤3 packets at a time, and to an undriven case with v ≃ 0.08c.14 Analytic work has been restricted to v ≪ c, ω ≃ 2ωp, where a wave packet radiates as an antenna and the back reaction of T waves is slight,14–16 or to the weak-turbulence regime W ≪ (kλD)^2,20 and always for a single packet rather than a turbulent ensemble.14–16,18,19

In the absence of a zeroth-order magnetic field, EMSPT is approximately described by the EM Zakharov equations10,18,19,22 for the slowly varying L-T electric-field envelope $\mathbf{E}(\mathbf{x}, t)$,

$$i \frac{\partial \mathbf{E}}{\partial t} = \left( \frac{v_T^2}{2\omega_p} \nabla \times \nabla \times - \frac{v_T^2}{2\omega_p} \nabla \cdot + \omega_p \frac{\partial \mathbf{E}}{\partial t} - i \gamma_L \right) \mathbf{E},$$ (1)

defined by $\mathbf{E} = \mathbf{E} e^{-i\omega t}$, and for the slowly varying, ion-sound like (S), quasineutral density perturbation $\mathbf{n}(\mathbf{x}, t)$,

$$\left( \frac{\partial^2}{\partial t^2} + 2i \gamma_S \frac{\partial}{\partial t} - v_S^2 \nabla^2 \right) \mathbf{n} = - \frac{\varepsilon_0}{4\pi n_0} \nabla^2 \langle |\mathbf{E}|^2 \rangle,$$ (2)

with $v_T^2 = (1 + 3v_T^2) (m_e/m_i) v_i^2$, $v_L^2 = 3v_T^2 (1 + \xi/5)/(1 + 5\xi + \xi^2)$, $v_T^2 = c^2 (1 + 6\xi + 6\xi^2)/5/(1 + 5\xi + \xi^2)$, $\omega_p^2 = (n_0 e^2/m_e \varepsilon_0)^2$, $v_S^2 = (n_0 e^2/m_e \varepsilon_0)^2 (2 + \xi)/(2 + 6\xi + 3\xi^2)$, and $\xi = v_L^2/c^2$. We approximate relativistic expressions for $v_L$ and $v_T$ here by Padé formulas,23 but neglect the relativistic ponderomotive terms in (2), which tend to be significantly smaller than the ones retained (the results should be viewed as semiquantitative due to this neglect and the approximations inherent in the Zakharov equations, which neglect second-harmonic effects and thus also require $k^2 v_L^2, k^2 v_T^2 \ll \omega_p^2$), the operators $\gamma_L(\mathbf{x}, t)$ and $\gamma_S(\mathbf{x}, t)$ describe L and S damping and growth, and T waves are not damped/driven directly. We study the regime of weak ion-sound damping ($T_c/T_e=0.01$, $m_e/m_i=183.6$) to avoid suppressing S-T coupling.24,25 Previous EMSPT simulations that did not...
average over fast oscillations in (1) and (2) showed that $T$ power at $2\omega_p$ and $3\omega_p$ has only weak or negligible back reaction on the $\omega_p$ dynamics,\textsuperscript{21} so we ignore $T$ waves at harmonics of $\omega_p$. Neglect of harmonics is also implicit in the derivation of the Zakhrov equations themselves. We solve (1) and (2) using a spectral code with periodic boundary conditions, verifying that it correctly simulates linear wave propagation, ES parametric decays, and ESSPT (Refs. 10, 12, and 13) when the $T$ waves are artificially suppressed.

Figure 1 depicts steady-state ESSPT for (a) $v=0.57c$ and (b) $v=0.01c$, via contours of $W_L=T$, via contours of $\Delta n/n_0$, where a typical range from $-0.4$ at burnout to $0.1$ W between packets, as in Ref. 13, $W_L=2,4,6,8,10 \times 10^{-3}$ (yellow contours), and $W_T=2,4,6,8,10 \times 10^{-3}$ (red).

FIG. 1. (Color) EMSPT plots from the central part of the simulation box for (a) $v=0.57$ and (b) $v=0.01$, showing $\Delta n/n_0$ (blue scale; dark negative, light positive, with a typical range from $-0.4$ at burnout to $0.1$ W between packets, as in Ref. 13, $W_L=2,4,6,8,10 \times 10^{-3}$ (yellow contours), and $W_T=2,4,6,8,10 \times 10^{-3}$ (red).

Trapped and free modes in Figs 1(a) and 1(b) approximately satisfy the Schrödinger equation $\dot{\psi}_l=\mathcal{Z}_l\psi_l$, where $\mathcal{Z}_l$ denotes the vector electric field and $\mathcal{Z}_l$ is the operator in parentheses in (1) with $\delta\omega$ fixed and $\gamma_l=0$. As $\mathcal{Z}_l$ does not depend explicitly on $t$, $\psi_l$ can be expanded as a sum $\sum_{\omega_{nm}} e^{-i\omega t} |\mathcal{Z}_l(m)\rangle = e^{-i\omega t} |\mathcal{Z}_l(m)\rangle$. By numerically decomposing trapped modes in Figs 1(a) and 1(b) into 2D multipole harmonics $\Phi_\omega(m)\propto e^{i\omega t}$, we find they are chiefly dipolar, with $|\mathcal{Z}_L(m)|=\mathcal{Z}_L(m)$, $|\mathcal{T}_L(m)|=\mathcal{Z}_L(m)$, $|\mathcal{T}_L(m)|=\mathcal{Z}_L(m)$, having peanut-shaped $W_L$ and $W_T$ contours with perpendicular major axes. The half-widths of the autocorrelation functions of the $L$ and $T$ fields, satisfy $\delta L=\alpha$ and $\delta T=\beta$. We find $\alpha_T=\beta_T=1.0(C(\nu)^{1/2})$, where $C$ is the length scale of the density well. Within uncertainties over the range investigated, these results are consistent with $\delta L=\alpha_T=\beta_T=1.0(C(\nu)^{1/2})$, as expected under subsonic conditions ($\delta r=\nu^{1/2}\delta n/\delta\mathcal{E}_L$, $\delta n \propto \mathcal{E}_L$).

Trapped $T$ states cannot exist for low $\nu$ because $T$ dispersion overcomes refraction into density wells dug by trapped $L$ waves. The (degenerate) eigenvalue of a trapped, dipole ($m=\pm 1$) mode is $\omega_T \propto \omega_0/a_T^2 + \delta\omega/2n_0$. Using the empirical result $\delta\omega/2n_0 = -31\lambda_p^2/a_T^2$, with $\alpha_T$ as above, we find $\omega_T=0$ (detrapping) for $\nu<0.13c$. [The $L$ eigenvalue $\omega_L \propto \omega_0/2a_L^2 + \delta\omega/2n_0$ is negative for all $\nu$.] This explains the delocalized structure of $T$ modes in Figs 1(b); $W_T$ contours meander through the box, slightly favoring centers of density wells. Numerically decomposing the free $T$ modes into 2D multipoles, we find $|\mathcal{Z}_L(m)|=\mathcal{Z}_L(m)$, $|\mathcal{T}_L(m)|=\mathcal{T}_L(m)$, which have amplitudes comparable to the noise; they approximate plane waves ($\omega_T=\omega_0/a_T^2$).

Figure 2(a) displays $W_L$ and $W_T$ versus $t$ for $\nu=0.1c$. We initialize $\mathcal{E}(x,0)$ with curl-free noise with a $k^{-2}$ spectrum to mimic the $k=0$ condensate of a parametric cascade, and set $\delta\omega_0=0$.\textsuperscript{10} Initial rises in $W_L$ and $W_T$ mark a plane-wave modulational instability, in which the condensate converts into a single, large wave packet that fills the simulation then collapses. The $e$-folding time of $W_L$ is twice that of $W_T$ (second-order process); growth takes place on the ponderomotive time scale $(W_L\omega_L)^{-1}$ and even faster if $\delta\omega<0$ initially (more $k$-matched decay channels available).\textsuperscript{1–3} Similar behavior is shown in undriven simulations for similar parameters ($\nu=0.079c$) but in different initial conditions (e.g., single packet).\textsuperscript{21}
Once the initial modulational instability saturates, at \( t = 400 \omega_p^{-1} \) in Fig. 2(a), memory of the condensate is lost and steady-state turbulence is maintained by a negative damping driver at \( (\omega_p, k_p) = (\omega_p, 0) \), of width \( \Delta k_p = 2 k_{\text{max}}/N \) and peak growth given by \( \gamma_L = -10^{-3} \omega_p \), where \( k_{\text{max}} \) is the maximum \( k \) and \( N \) is the number of grid units along each axis. Previous EMSPT simulations were undriven and did not attain a steady state.\(^{17,21}\) We choose \( k_{\text{max}} = 0.31 \lambda_p^{-1} \) and time step \( \Delta t = 2.1 \omega_p^{-1} (v/c)^2 \) to follow the fast \( \nabla \times \nabla \times \) term in (1). Damping occurs at a rate \( \gamma_L = \gamma_0(k) + a[(k/k_c)^2 - 1] \), with \( a = 0.04 \omega_p \) and \( k_c = 0.8 k_{\text{max}} \); the first term is linear Landau damping, the second a proxy for coherent transit-time damping, which is set to zero for \( k < k_c \).\(^{10}\)

A prominent feature of Fig. 2(a), absent from ESSPT driven at \( k = 0 \), is the existence of quasiperiodic, long-period cycles in \( W_L \) and \( W_T \), with period \( T_{LT} = 1.2 \times 10^3 \omega_p^{-1} \), and similar fractional amplitudes (factors of up to \( \sim 2 \) and \( \sim 4 \), peak to trough, for \( L \) and \( T \), respectively, with this \( \sim 2:1 \) ratio consistent with nonlinear generation of \( T \) waves). They intermittently die out and re-emerge, persisting for \( >50 T_{LT} \). (These global cycles are not the same as the nucleation cycles seen at individual collapse sites,\(^{10,12,24}\) which are also observed here.) On cross-correlating the time series, one finds that \( W_L \) lags \( W_T \) by \( \Delta T_{LT} = (7.7 \pm 1.1) \times 10^2 \omega_p^{-1} \), comparable to the ponderomotive (i.e., modulational) time scale \( (W_p \omega_p)^{-1} \). The period \( T_{LT} \) varies by less than 20% over the range \( 0.01 c \approx v \lesssim 0.57 c \) [i.e., \( v_T \approx v_L \) at the upper bound, where \( c/ \sqrt{3} \approx 0.57 c \) is the sound speed in a relativistic gas; the lower bound is set by the point where \( k^2 v_T^2 \) becomes large, as seen in Fig. 3(a)], does not depend on box size, and

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**FIG. 2.** (a) Wave-energy density: total \( W \) (dashed), \( W_L \) (top solid curve), and \( W_T \) (lower solid curve). Left panel shows initial modulational instability; right shows steady state EMSPT. (b) \( W_T/W_L \) (squares) and \( \alpha_T/\alpha_L \) (triangles).

**FIG. 3.** Angle-averaged spectra (arbitrary units) for \( v/c = 0.01, 0.03, 0.1, 0.3, 0.57 \). (a) \( \langle |\xi_L(k)|^2 \rangle \), (b) \( \langle |\delta n(k)|^2 \rangle \), and (c) \( \langle |\xi_T(k)|^2 \rangle \). Spectra are averaged over a time \( 5 \times 10^4 \omega_p^{-1} \).
is ≃ 20 times longer than the growth times of (i) ES three-wave decay,10 (ii) EM modulational instability $L + L \rightarrow T + T$,26,27 and (iii) sloshing between the mixed $L-T$ normal modes of an inhomogeneous plasma. As these cycles are absent from ESSPT, they likely involve 3- or 4-wave coupling of $L$ and $T$ modes.

Figure 2(b) shows the mean steady-state value of $W_{TT}/W_L$ versus $v$ (excluding the $k = 0$ mode, which contains ≃ 2% of the total energy), with $\alpha_T/\alpha_L$ also plotted for reference. Least-squares fits yield $W_{TT}/W_L \propto v^{4.0\pm 0.1}$ ($v \leq 0.1c$) and $W_{TT}/W_L \propto v^{1.2\pm 0.1}$ ($v \geq 0.1c$); the proportionality constants are functions of the driver and damping. However, when underestimates of the low-$k$ part of the spectrum [see Fig. 3(c), where the lower bound to each curve is determined by the box size] are taken into account, the results are consistent with $W_{TT}/W_L \propto v^{1.8}$ over the whole range investigated. We also obtain $W_L \propto v^{-0.67\pm 0.03} \propto (\Delta k_\parallel)^{0.67}$, in accord with Eq. (6.20) of Ref. 10 for 2D ESSPT.

Wave number spectra $W_{TT}(k) = (|E_{TT}(k)|^2)$ and $\bar{\delta n}(k)^2 = \langle |\delta n(k)|^2 \rangle$, averaged over time and angle, are shown in Fig. 3. For $v \leq 0.1c$, we find power-law inertial ranges at $k_n \approx k_c$, with $k_n = \pi/\alpha_L = 0.66v^{1/2}\lambda_p^{-1}$, $W_T(k) \propto k^{3.9\pm 0.4}$, $\bar{\delta n}(k)^2 \propto k^{-2.3\pm 0.5}$, and $W_L(k) \propto k^{5.8\pm 0.8}$. [For our parameters, the minimum nonzero $k$ at the left end of each curve in Fig. 3(a) is at $k_{\text{min}} \approx \alpha_L \omega_p = 0.003c/v$, the beginning of the inertial range at the right end of the shallowly sloped part of each curve is at $k_c \approx \omega_p / v \approx 0.04c/v$, the upper bound of each curve is at $k_{\text{max}} \approx \omega_p / v = 0.35c/v \approx 1.25k_c \omega_p / \omega_p$, where $k_c$ is the upper bound of the Langmuir inertial range.] The latter scaling is new, while the first two agree with the two-component theory of 2D ESSPT, which predicts $W_L(k) \propto k^{-4}$ and $\bar{\delta n}(k)^2 \propto k^{-2}$.10,12,13 Although we have $W_T \ll W_L$ overall, we find $W_T(k) \sim W_L(k)$ for $k_n \leq k \leq k_n$, where $T$ waves are generated by weak turbulence processes that partition energy roughly equally between modes.

For $v \geq 0.1c$, a slight bump interrupts the inertial range of $W_L(k)$ at $k_r = \pi/\alpha_L = 0.2v/c = 0.33\omega_p/c$. (Most $T$ waves are generated near when collapse arrests at $\alpha_L = 15\lambda_p$.13) We find $k_n < k_r < k_c$ for $v \leq 0.1c$. The bump also exists for $v \geq 0.1c$, but falls outside the inertial range ($k_T < k_n$). It may mark enhanced focusing of $L$ waves into density wells, deepened by the additional ponderomotive force of the $T$ waves, but it is unclear why there is no bump in $W_T(k)$. $T$ waves are not damped directly, but are converted to damped $L$ and $S$ waves via 3- and 4-wave processes. Indirect $T$ damping explains the (i) absence of a sharp cutoff in $W_T(k)$ at $k_c$, because the damping operates at all $k$, and (ii) sensitivity of $W_T$, but not $W_L$, to $\gamma_S$, which affects the steady-state level of $S$ waves. The gradual decline of $W_T(k)$ above $k_c$ is due to the shortage of $k$-matched $L$ waves converting to $T$ waves.

Several potential applications are flagged above. One is to interpret particle-in-cell (PIC) simulations of ultrashort, relativistic laser pulses propagating into an underdense plasma, where 30%–40% of the incident energy is transformed into “solitons” behind the pulse.6–8 The group velocity of most solitons is negligible, consistent with wave packets in Fig. 1 and ESSPT (Ref. 10) (although in some PIC simulations7 solitons drift backwards at 0.3–0.8c). Solitons nucleate in laser-generated channels, then collapse. After collapse is arrested, the fields relax to fill slowly expanding, ring-like density cavities which trap $T$ modes.9,28 This behavior closely matches that of the density depressions in Fig. 1(b).

Good prospects also exist to test our predictions in other laboratory situations; e.g., when packets containing coupled $L$ and $T$ modes [see Fig. 1(a)] approach a plasma-vacuum interface, they radiate a burst of EM waves, whose spectrum reflects the field structure of the packet and hence $W_T(k)$. As this structure changes at $v \geq 0.1c$, it is possible in principle to detect the delocalization transition with this diagnostic. In addition, we only expect to see ring-like postcollisions for $v \geq 0.1c$. This ability to infer some parameters of SPT (e.g., $v$, $W_T$) from remote measurements is also potentially useful in astrophysics14–17 and laboratory experiments demanding noninvasive diagnostics.

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