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Abstract

The role of dissipation with respect to a microscopic superposition of quantum states is investigated by studying master equations. This has implications for the study of the emergence of classicality from the quantum level. In particular, it illustrates why it is difficult to observe a macroscopic quantum state. The role of the environment is assumed by the measuring apparatus. A pure state is reduced to a mixture in the pointer basis of the system by means of the interaction with the apparatus. It is the intention that this type of analysis will have applications to experiments which are designed to better understand the environmental assisted invariance formulation of quantum mechanics.

Keywords: probability, momentum, operator, quantum mechanics

1 Introduction

Symmetry plays a very important role in physics and this is true in particular in the study of the quantum world [**1-4**]. A symmetry which applies to quantum systems in the large and is more recent has come to be known as environmental assisted invariance or more simply envariance [**5-7**]. It applies to systems in which a quantum system can be thought of as composite in the sense that it is composed of a part called the system S and a part referred to as the environment [**8-9**] which may include the apparatus.

Suppose an operator given by a unitary transformation U_S is applied to the part of the combination called the system. The state is said to be envariant under transformation U_S if another transformation exists, U_E such that, when U_E is applied to the environment component, the initial state is restored. Mathematically stated, when the state $|\Psi_{SE}\rangle$ of a pair of systems S, E can be transformed by operator $U_S = u_S \otimes \mathbf{1}_E)$

$$U_S|\Psi_{SE}\rangle = (u_S \otimes \mathbf{1}_E)|\Psi_{SE}\rangle = |\zeta_{SE}\rangle. \tag{1.1}$$

The effect of U_S can be reversed by acting solely on E with an appropriatly chosen U_E

$$U_E|\zeta_{SE}\rangle = (\mathbf{1}_S \otimes u_E)|\zeta_{SE}\rangle = |\Psi_{SE}\rangle. \tag{1.2}$$

then $|\Psi_{SE}\rangle$ is called envariant under U_S. Once the system is transformed by some unitary operator U_S, it may be reset to its original form by another operation on a physically distinct system we call the environment. Envariance can be referred to as an assisted symmetry for this reason. This is certainly a quantum symmetry. In as much as a pure quantum state represents complete knowledge of the quantum system, in an entangled quantum state, the complete knowledge of the whole system does not imply complete knowledge of all the parts as happens classically. It is possible that an operation on one part of a quantum state can alter the global state. Local effects are covered over by incomplete knowledge of what was the case before. The effect on the global state can be undone by an action on a different part.

Complete knowledge of a composite classical system implies complete knowledge of each of its parts. This means transforming one part of a classical system cannot be covered up by incomplete knowledge or reversed by some transformation applied to another part. On the other hand, quantum mechanics presents a different view. For example, decoherence transforms amplitudes in coherent superposition of states to probabilities in mixtures and is central to the emergence of the classical world from the quantum domain. The mixture mathematically appears in the reduced density operator of the system which is extracted from the global wavefunction by a partial trace [**10**].

In quantum mechanics, the environment plays a vastly different role than in classical physics. Referred to as monitoring of the environment, it is responsible for the destabilization of the vast majority of the states in the relevant Hilbert spaces of the open systems [**4**]. These types of processes leave what are called preferred pointer states. This is what is intended by the term environment induced superselection, also called einselection. Picking up some second hand information about the system by measuring fragments of the environment makes just a selection of states of the system accessible. These states happen to be called the preferred pointer states of S. These pointer states are not only the best at surviving the ambient environment, but also in propagating by way of the environment information about themselves. This also permits many observers to learn about the pointer states without perturbing them any more than decoherence has already done. The new and remarkable feature of envariance is that it leads to the definitions of probabilities. It then raises the possibility of a completely quantum derivation of Born's rule.

A density matrix can be regarded as a projector or linear combination of them, and their evolution can be described by the Lindblad equation. The Lindblad equation can be microscopically derived under certain approximations, such as the Born-Markov and weak coupling approximation.

It is the intention here to study some quantum systems in which some of this physics is made apparent and mathematical work can be carried out to illustrate some of these ideas. Any experimental developments this might lead to would be very worth while to the study of this interpretation. One can say that the observers aquire information concerning measured systems, or the state of the apparatus pointer indirectly by monitoring the environment. It correlates with the system as a result of decoherence, which in turn is caused by the environment monitoring either the apparatus, the system or both [**11-14**].

2 Oscillator Model with Damped Amplitude

A coupling to a reservoir or bath can be considered such that the amplitude of the harmonic oscillator is damped over time. The measuring apparatus can play the role of the environment right here. In what follows $a^\dagger$, a represent boson creation and annihilation operators which satisfy $[a, a^\dagger] = 1$. Using these a description of this can be given by a Hamiltonian of the form

$$H = \hbar\omega\, a^\dagger\, a + a\Gamma_R^\dagger + a^\dagger\, \Gamma_R. \tag{2.1}$$

The operator Γ_R represents the coupling of oscillator to this reservoir. In the Born and Markov approximation, the reduced density operator for the harmonic oscillator in the interaction picture obeys the equation

$$\frac{\partial \rho}{\partial t} = \frac{\lambda}{2}\big(2a\rho a^\dagger - a^\dagger\, a\, \rho - \rho\, a^\dagger a\big), \tag{2.2}$$

where λ is the coupling constant. The temperature of the reservoir is assumed to be zero, so $T = 0$. The solution for $\rho(t)$ will have the form

$$\rho(t) = \sum_{n=0}^{\infty} R_m(t)\, \rho(0). \tag{2.3}$$

The coefficients $R_m(t)$ in (**??**) are determined by

$$R_m(t) = \int_0^t dt_m \int_0^{t_m} dt_{m-1} \cdots \int_0^{t_2} dt\, S_{t-t_m} J S_{t_m - t_{m-1}} S_{t_{m-1}-t_{m-2}} J \cdots J S_{t_1}\, \rho(0), \tag{2.4}$$

where $J\,\rho$ and $S_t\,\rho$ are defined by

$$J\,\rho = \lambda\, a\, \rho\, a^\dagger, \qquad S_t\,\rho = e^{-\frac{\lambda}{2} t a^\dagger a} \cdot \rho \cdot e^{-\frac{\lambda}{2} t a^\dagger a}. \tag{2.5}$$

Suppose $|\alpha\rangle$, $|\beta\rangle$ represent coherent states. The exponential of the operator $a^\dagger\, a$ will have the following action on a state $|\alpha\rangle$,

$$e^{-\frac{\lambda}{\hbar}\, a^\dagger a}|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}(1-e^{-\lambda t})}|\alpha e^{-\frac{\lambda}{2}t}\rangle. \tag{2.6}$$

Using (**??**) and (**??**), it follows that

$$S_t\,|\alpha\rangle\langle\beta| = \exp(-\frac{\lambda}{2} t a^\dagger a)|\alpha\rangle\langle\beta| \exp(-\frac{\lambda}{2} t\, a^\dagger\, a)$$

$$= \exp(-\frac{|\alpha|^2}{2}(1-e^{-\lambda t}) - \frac{|\beta|^2}{2}(1-e^{-\lambda t}))|\alpha e^{-\lambda t/2}\rangle\langle\beta e^{-\lambda t/2}|. \tag{2.7}$$

To work out the time evolution of an arbitrary initial system density matrix of the form $|\alpha\rangle\langle\beta|$, the general term in the integrand is evaluated of (**??**) as follows

$$S_{t_1}\,\rho = \exp(-\frac{\lambda}{2}t_1\,a^\dagger a)\,\rho\,\exp(-\frac{\lambda}{2}t_1\,a^\dagger a),$$

$$S_{t_2-t_1}J\,S_{t_1}\,\rho = \exp(-\frac{\lambda}{2}(t_2-t_1)a^\dagger a)\cdot\exp(-\frac{\lambda}{2}t_1 a^\dagger a)\cdot\rho\cdot\exp(-\frac{\lambda}{2}t_1 a^\dagger a)$$
$$\cdot\exp\big(-\frac{\lambda}{2}(t_2-t_1)\,a^\dagger\,a\big)(\lambda\alpha\beta^\dagger)\cdot e^{-\lambda\,t_1}. \tag{2.8}$$

Iterating this process, in general, the integrand of (**??**) takes the form

$$S_{t-t_m}J\,S_{t_m-t_{m-1}}\,J\;\cdots S_{t_4-t_3}J\,S_{t_3-t_2}J\,S_{t_2-t_1}JS_{t_1}\,\rho = \exp(-\frac{\lambda}{2}ta^\dagger a)\cdot\rho\cdot\exp(-\frac{\lambda}{2}ta^\dagger a)$$
$$(\lambda\alpha\beta^*)\cdot e^{-\lambda t_m}\cdot e^{-\lambda t_{m-1}}\cdots e^{-\lambda t_1}. \tag{2.9}$$

Thus the time development of an arbitrary initial state $|\alpha\rangle\langle\beta|$ is

$$(|\alpha\rangle\langle\beta|)_t = \exp(-\frac{|\alpha|^2}{2}(1-e^{-\lambda t})\cdot\frac{|\beta|^2}{2}(1-e^{-\lambda t}))|\alpha e^{-\lambda t/2}\rangle\langle\beta e^{-\lambda t/2}|$$
$$\cdot\sum_{m=0}^{\infty}(\lambda\alpha\beta)^m\int_0^t dt_m\,e^{-\lambda t_m}\int_0^{t_m}dt_{m-1}e^{-\lambda t_{m-1}}\cdots\int_0^{t_2}dt_1 e^{-\lambda t_1}. \tag{2.10}$$

Upon time-ordering, this becomes,

$$(|\alpha\rangle\langle\beta|)_t = \exp\big(-\frac{|\alpha|^2}{2}(1-e^{-\lambda t})-\frac{|\beta|^2}{2}(1-e^{-\lambda t})\big)\cdot|\alpha e^{-\lambda t/2}\rangle\langle\beta e^{-\lambda t/2}|\sum_{m=0}^{\infty}\frac{(\lambda\alpha^*\beta)^m}{m!}(\int_0^t e^{-\lambda s}\,ds)^m. \tag{2.11}$$

Since

$$\int_0^t e^{-\lambda s}\,ds = \frac{1}{\lambda}(1-e^{-\lambda t}),$$

the sum in (**??**) can be evaluated and we arrive at,

$$(|\alpha\rangle\langle\beta|)_t = \exp\big(\frac{1}{2}(e^{-\lambda t}-1)(|\alpha|^2-2\alpha\beta^*+|\beta|^2)\big)\cdot|\alpha e^{-\lambda t/2}\rangle\langle\beta e^{-\lambda t/2}| = \langle\alpha|\beta\rangle^{1-e^{\lambda t}}|\alpha e^{-\lambda t/2}\rangle\langle\beta e^{-\lambda t/2}|. \tag{2.12}$$

Suppose the initial density operator $\rho(0)$ is given by

$$\rho(0) = N\big(|\alpha\rangle\langle\alpha| + |\beta\rangle\langle\beta| + |\alpha\rangle\langle\beta| + |\beta\rangle\langle\alpha|\big). \tag{2.13}$$

Using (**??**) and since $R_m(t)$ is linear, the density operator at a later time t is

$$\rho(t) = N \sum_{\tau,\tau'=\alpha}^{\beta} \langle \tau | \tau' \rangle^{1-e^{-\lambda t}} |\tau e^{-\lambda t/2}\rangle \langle \tau' e^{-\lambda t/2}|. \tag{2.14}$$

The off-diagonal elements in the density matrix in a coherent state basis are all dephased by the factor $\langle \alpha | \beta \rangle^{1-e^{\lambda t}}$. This predicts the off-diagonal elements are the more rapidly dephased when the separation is greater between the initial states. To obtain a physical explaination for this kind of rapid dephasing, consider the state of the coupling of system plus environment before and after one quantum passes from the system to the environment.

Considering a general, arbitrary density operator which has been expressed in terms of an off-diagonal coherent state representation as

$$\rho(0) = \int F(\alpha, \alpha') \frac{|\alpha\rangle\langle\alpha'|}{\langle\alpha'|\alpha\rangle} \, d\nu(\alpha, \alpha'), \tag{2.15}$$

where ν is the integration measure, the expression for this density operator at time $t > 0$ is given by

$$\rho(t) = \int P(\alpha, \alpha') \frac{|\alpha e^{-\lambda t/2}\rangle\langle\alpha' e^{-\lambda t/2}|}{\langle\alpha' e^{-\lambda t/2}|\alpha \, e^{-\lambda t/2}\rangle} \, d\nu(\alpha, \alpha'). \tag{2.16}$$

Squeezed states and number states are quantum states which are not possible to express in terms of a temporal distribution using the diagonal P representation and requires an off-diagonal representation instead.

If an interference experiment were to be carried out in the presence of damping such that

$$\rho(t) = \sum_{i,j=1}^{2} |\alpha_i(t)\rangle\langle\alpha_j(t)|, \qquad |\alpha_i(t)\rangle = |\alpha_i e^{-\lambda t/2} e^{i\omega t}\rangle, \tag{2.17}$$

it would be found that when the initial excitations are of equal amplitude and opposite phase so $\alpha_1 = -\alpha_2 = \alpha$ with α real and $|x\rangle$ a coordinate basis element,

$$\langle x|\rho(t)|x\rangle = I_+ + I_- + 2e^{-|\alpha|^2(1-e^{-\lambda t})} I_+ I_- \cos\theta(t),$$

$$I_\pm = (\frac{\omega}{\pi\hbar})^{1/4} \exp\big(-(\sqrt{\frac{\omega}{\hbar}}\, x \pm \sqrt{2}\alpha e^{-\lambda t/2}\cos(\omega t))^2\big), \quad \theta(t) = 2\sqrt{\frac{2\omega}{\hbar}} \sin(\omega t) \cdot \alpha e^{-\lambda t/2}\, x. \tag{2.18}$$

The oscillation is damped out due to the interaction with the environment at a rate of $\lambda/2$. From the coefficient of the interference term in (**??**), the interference term is damped out at a rate of $e^{-|\alpha|^2(1-e^{-\lambda t})}$. So the greater the initial separation of the two wave packets, the more rapidly the coherence between them is damped. The conclusion is that a quantum state prepared in a macroscopic superposition is very rapidly reduced to a mixed state.

In the envariance formulation of quantum mechanics Zurek has shown that the environment has an action on the quantum system that can often be thought of as a repeated measurement of a pointer observable Ω of the system. The pointer observable has to commute with the system-environment interaction Hamiltonian H_{SE},

$$[\Omega, H_{SE}] = 0. \tag{2.19}$$

This interaction serves to create or single out a preferred pointer basis in the Hilbert space of the quantum system.

As introduced already, a pointer basis is made up of the eigenspaces of the pointer observable Ω. When operator Ω commutes as well with the Hamiltonian as in (**??**), it is referred to as a nondemolition observable of the measurement performed by the environment.

In the case under consideration here, the measuring apparatus may be thought of as playing the role of the environment which measures the complex amplitude of the oscillator through the coupling defined by (**??**). The initial state of the oscillator has undergone a preparation to yield a superposition of two eigenstates of operator a by the first stage of the measurement. The interaction of the system with the environment reduces this coherent superposition to a mixture rather quickly. On a short time scale, the reduction of the wavepacket is done through the system interaction with the environment measuring apparatus.

3 Damping of Phases

In the following model, an interaction with the environment is considered and is implemented by means of the number operator of the oscillator. If Γ represents a coupling operator, the Hamiltonian of the model is given by

$$H = \hbar \omega \, a^\dagger \, a + a^\dagger \, a \, \Gamma. \tag{3.1}$$

This kind of coupling does not lead to energy damping, however there will occur phase damping. In this model the environment may be thought of as making a measurement of the number quanta in the system. The number operator $a^\dagger a$ has to be an exact pointer variable of Hamiltonian (**??**) since $[a^\dagger a, H] = 0$. It is also a quantum nondemolition observable. For this system, the number state vectors $|n\rangle$ form the pointer basis of the system, eigenvectors of the number operator.

The master equation for the reduced density operator of the system in the interaction picture for a finite temperature bath is

$$\frac{\partial \rho}{\partial t} = \frac{\lambda}{2} (2a^\dagger a \rho \, a^\dagger a - \rho a^\dagger a a^\dagger a - a^\dagger a \, a^\dagger a \, \rho), \tag{3.2}$$

where $\lambda = \lambda' / k_B T$ and λ' is the damping constant.

Expand $\rho(t)$ in terms of the eigenbasis of the operator N as follows

$$\rho(t) = \sum_{m,n} \rho_{mn}(t) |n\rangle\langle m|, \tag{3.3}$$

in (**??**) and use the facts that $a^\dagger a|n\rangle = n\,|n\rangle$ and $\langle m| a^\dagger a = (a^\dagger a|m\rangle)^\dagger = m\langle m|$. The right side of (**??**) is

$$\begin{aligned} 2a^\dagger a \rho \, a^\dagger a - \rho \, a^\dagger a a^\dagger a - a^\dagger a a^\dagger a \, \rho &= \sum_{m,n} (2mn - m^2 - n^2) \, \rho_{mn}(t) |n\rangle\langle m| \\ &= - \sum_{m,n} (n-m)^2 \rho_{mn}(t) \, |n\rangle\langle m|. \end{aligned} \tag{3.4}$$

This has shown that the ρ_{mn} satisfy the differential equation,

$$\frac{\partial \rho_{mn}(t)}{\partial t} = -\frac{\lambda}{2} (n-m)^2 \, \rho_{mn}(t). \tag{3.5}$$

This equation can be solved in closed form, and it has the solution

$$\rho_{mn}(t) = e^{-\lambda (n-m)^2 t/2} \, \rho_{mn}(0). \tag{3.6}$$

This result has the implication that the coherence between a superposition of two different number states is damped by the exponential factor on the right of (**??**).

Consider a system where such a decay of coherence between a superposition of number states could be observed. Any coherent state may be expressed as a superposition of number states by means of the linear combination

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \tag{3.7}$$

The density operator for an initial coherent state of the form (**??**) evolves into the form,

$$\rho(t) = e^{-|\alpha|^2} \sum_{mn} \frac{\alpha^n (\alpha^*)^m}{\sqrt{m!}\,\sqrt{n!}}\, e^{-\lambda(m-n)^2 t/2}\, |n\rangle\langle m|. \tag{3.8}$$

An experiment which studied the decay of coherence between different number states would perform measurements of the k-th moment of the amplitude. To give briefly an idea as to how these moments can be computed, we start with

$$\langle \alpha^k(t)\rangle = \mathrm{Tr}\,[\rho(t)\, a^k] = \sum_s \langle s|\rho(t)\, a^k|s\rangle = \sum_s \langle s|\rho(t)|s-k\rangle (s-(s-k+1))^{1/2}$$

$$= \sum_s \langle s|e^{-|\alpha|^2} \sum_{m,n} \frac{\alpha^n(\alpha^*)^m}{\sqrt{n!}\sqrt{m!}} e^{-\lambda(m-n)^2 t/2} |n\rangle\langle m|s-k\rangle (\frac{s!}{(s-k)!})^{1/2}$$

$$= e^{-|\alpha|^2} \sum_n \frac{\alpha^n(\alpha^*)^{n-k}}{\sqrt{n!}\sqrt{(n-k)!}} e^{-\lambda k^2 t/2} (\frac{n!}{(n-k)!})^{1/2} = e^{-|\alpha|^2} \sum_n \frac{\alpha^n(\alpha^*)^{n-k}}{(n-k)!}\, e^{-\lambda k^2 t/2}. \tag{3.9}$$

In a similar way, we find that

$$\langle a^{\dagger\, k}(t)\rangle = \mathrm{Tr}[\rho(t)\, a^{\dagger\, k}] = \sum_s \langle s|\rho(t) a^{\dagger\, k}|s\rangle = \sum_s \langle s|\rho(t)|s+k\rangle\, (\frac{(s+k)!}{s!})^{1/2}$$

$$= e^{-|\alpha|^2} \sum_n \frac{\alpha^n(\alpha^*)^{n-k}}{\sqrt{n!}\sqrt{(n+k)!}} e^{-\lambda k^2 t/2} (\frac{(n+k)!}{n!})^{1/2} = e^{-|\alpha|^2} \sum_n \frac{\alpha^n(\alpha^*)^{n+k}}{n!}\, e^{-\lambda k^2 t/2}. \tag{3.10}$$

It can be concluded from this analysis that if there exists a pointer observable which is also a constant of the motion, the system has to quickly approach a state exactly diagonal in the pointer basis.

The quadrature phase amplitude ζ, where $a = \zeta + i\eta$, of a harmonic oscillator may be coupled to the environment for example. This is a quantum nondemolition kind of coupling. It could lead to developments in gravity wave detectors. The interaction Hamiltonian is then given by

$$H = \zeta\, \Gamma. \tag{3.11}$$

The matrix elements of ρ in the pointer basis $|\zeta\rangle$ decay according to the relation

$$\langle \zeta'|\rho(t)|\zeta\rangle = \mathcal{N}\, e^{-(\zeta-\zeta')^2 t/2} \langle \zeta'|\rho(0)|\zeta\rangle. \tag{3.12}$$

The system of N spin-1/2 particles or N two-level atoms such that the total angular momentum is $J = \sum_{i=1}^{N} \sigma_i$ should also be noted. If the system is coupled to the reservoir by means of the operator J_z, with Hamiltonian is given by

$$H = \hbar\omega J_z + J_z\Gamma. \tag{3.13}$$

the off-diagonal matrix elements in the pointer basis $|m\rangle$, that is the eigenstates of J_z should decay as,

$$\langle m|\rho(t)|n\rangle = \mathcal{N} e^{-\lambda(m-n)^2 t/2} \langle m|\rho(0)|n\rangle. \tag{3.14}$$

4 Summary

The implications of this kind of investigation apply not only to measurement theory, but to basic questions such as the interpretation of quantum mechanics. For example, as the temperature of the bath decreases, the effective coupling of the system with environment increases. Hence a zero-temperature limit is inapplicable in the weak coupling regime. The zero temperature limit should be viewed as a mathematical model in that domain. Zurek, who worked a lot on the role of envariance in quantum mechanics, has pointed out that the effect of the environment on a quantum system can be thought of as a repeated measurement of the pointer states of the system. Environmental interaction sifts out a preferred pointer basis in the Hilbert space of the quantum system. This basis is made up of eigenspaces of a pointer observable Ω, and the stable case is that in which the Ω commutes with the Hamiltonian of the system. Although the equations referred to here are common, it is hoped this kind of treatment will give a push to obtaining experimental information that can further contribute to the study of envariance. Some work has been done on master equations that emerge out of more elaborate systems, and one such example will be introduced now **[14-16]**. Gravitational decoherence refers to the effect of gravity on the decoherence of quantum systems. A massive scalar field which interacts with a gravitational field as the environment. Proceeding in a very similar way as in decoherence studies by means of open quantum systems, gravitational degrees of freedom can be traced over and a master equation for the quantum matter degrees of freedom can be found. So a master equation which describes a moving particle interacting with a weak gravitational field has been developed based on standard theories for quantum matter and classical gravity.

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