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Contextualizing the Mathematical Knowledge for Teaching Framework for teachers of Emergent Bilinguals

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We are amid a rapid demographic shift, with Emergent Bilinguals (EBs) being the fastest growing K-12 student population. This has created an ambitious goal for teacher education programs as they must prepare mathematics pre-service teachers (PSTs) to assess the needs of EBs. As a response, this study conducted a qualitative analysis of 16 PSTs to propose a contextualized version of the Mathematical Knowledge for Teaching (MKT) framework as an emerging knowledge base of teaching that can be used to further guide the planning and enactment of teacher education programs in the mathematics education of EBs, specifically.

KEYWORDS

mathematics teacher education, English learner students, Mathematical Knowledge for Teaching (MKT), pre-service teachers, emergent bilingual learners

Introduction

As we traverse our current technology-driven era, it has become clear that our nation's prosperity will increasingly depend on advances and growth in the science, technology, engineering, and mathematics (STEM) fields (English, 2017). Simultaneously, we are amid a rapid demographic shift in which a race, cultural, and linguistic diversification within our population is inevitable (Bauman and Murray, 2017), with Emergent Bilingual (EB) students, historically labeled as "Limited English Proficient" or "English as a Second Language" students, being the fastest growing K-12 student population (Jiménez-Castellanos and García, 2017). This has created a two-fold goal for many teacher education programs as they not only have to familiarize pre-service teachers (PSTs) with newer curricula built upon stronger mathematical foundations, but they must also prepare PSTs on how to make the curricula accessible to EBs (Bohon et al., 2017). Despite the urgency of these two matters, however, countless international reports have repeatedly highlighted the substantial achievement gaps experienced by many of our K-12 students in mathematics (Woessmann, 2016; DeSilver, 2017; Wu et al., 2020), with EBs significantly underperforming in standardized mathematics measures when compared to their non-EB peers (Polat et al., 2016; Powell et al., 2020).

Such disparities in mathematics achievement have prompted mathematics education researchers to explore and identify the knowledge PSTs need to develop when serving the mathematical needs of EBs, specifically (e.g., Domínguez, 2011; Moschkovich, 2013, 2018; Turner et al., 2013; Krause et al., 2020). Teacher preparation programs, for example, reported that PSTs respond to multicultural opportunities that lead them to grow more accustomed to diversity (Schellen and King, 2014). However, the multicultural education efforts found in many existing teacher preparation programs vary significantly in effectiveness (Cherng and Davis, 2019; Chang and Cochran-Smith, 2022), and therefore making it difficult to prepare teachers to face the demand of educating EBs. This has resulted on placing teachers that have neither knowledge nor understanding of the "cultures of the children in their classes" (Lyon, 2009, p. 52) and ultimately producing teachers with limited experiences that do not understand the students they teach.

This study aims to further respond to such urgency by further enhancing our understandings of the processes of learning to teach mathematics to EBs. Mainly, we propose a contextualized version of Ball et al. (2008) Mathematical Knowledge for Teaching (MKT), an already commonly used framework within teacher preparation programs that delineates the knowledge base of teaching mathematics (Depaepe et al., 2013; Carrillo-Yañez et al., 2018; Torbeyns et al., 2020), as a means to further guide the planning and enactment of teacher education programs in the mathematics education of EBs, specifically. To do so, we first consulted the literature on the mathematics education of EBs. This allowed us to create a contextualized version of the MKT and its subdomains which was further refined through a qualitative study in which the beliefs of 16 PSTs were explored.

Literature overview

Teachers' Mathematical Knowledge for Teaching (MKT)

One of the most influential conceptualizations of what teachers need to know to carry out their mathematics teaching responsibilities stems from Shulman's (1986) work on teachers' knowledge base. His criticism toward the lack of attention given to research on subject matter teaching led to his introduction of seven categories he considered indispensable aspects of teachers' knowledge base, particularly Pedagogical Content Knowledge (PCK), or the knowledge that is specific to teaching a particular content area. However, despite the wide acceptance of Shulman's conceptualization of PCK, several scholars have noted issues with it, including the embedded cognitivist perspective within PCK that might fail to take into consideration a more dynamic and context-dependent view of mathematical learning (e.g., Bednarz and Proulx, 2009; Hodgen, 2011; Smestad, 2015).

In an effort to respond to these criticisms, several researchers have expanded on Shulman's PCK framework (e.g., Hill et al., 2004; Ball et al., 2008; Hill et al., 2008; Schoenfeld and Kilpatrick, 2008; Baumert and Kunter, 2013; Carrillo-Yañez et al., 2018). Perhaps the most widely adopted re-conceptualization of Shulman's PCK was developed by Ball et al. (2008) in their MKT framework, a highly influential theoretical framework within the field of mathematics education and teacher preparation (Depaepe et al., 2013; Carrillo-Yañez et al., 2018; Torbeyns et al., 2020). Having a more robust theoretical and empirical foundation (Ball et al., 2008), MKT differs from Shulman's (1986) conceptualization of teachers' knowledge base in that it integrates several of Shulman's contributions, including PCK, with additional subdomains that take an overall more contextualized view of mathematics teaching, including key knowledge on both teachers *and* their students.

According to Ball et al. (2008), PCK can be expanded into three subdomains: (1) Knowledge of Content and Students (KCS), or the combination of content and student knowledge such that it allows for teachers to anticipate, predict, or interpret students' mathematical ideas; (2) Knowledge of Content and Teaching (KCT), or the combination of content and teaching knowledge such as the identification and sequencing of mathematical tasks; and (3) Knowledge of Content and Curriculum (KCC), or the knowledge about instructional materials and programs. Additionally, teachers must also possess Subject Matter Knowledge (SMK), which is also expanded into three subsequent subdomains: (4) Common Content Knowledge (CCK), or the mathematical knowledge and skills for which mathematics as a discipline stands upon; (5) Specialized Content Knowledge (SCK), or the *unpacking* of mathematics required only by mathematics teachers but not by other professions; and (6) Horizon Content Knowledge (HCK), or the consideration of the subsequent mathematical knowledge that students will learn in future courses.

We agree with Ball et al. (2008) that the knowledge for teaching mathematics should stem from the actual teaching practice and, therefore, with the overall contextualized nature inherent to mathematics teaching. Indeed, we echo these and other similar sentiments from other researchers that in order for the MKT framework to be meaningful in the identification of the knowledge required for teachers to teach mathematics efficiently, it must be contextualized in a way that it considers the actual students' needs (e.g., Smestad, 2015; Carrillo-Yañez et al., 2018). Carrillo-Yañez et al. (2018), for example, adapted the MKT into the Mathematics Teacher's Specialized Knowledge (MTSK) model where each of the sub-domains comprising MTSK emerge from the mathematics itself, including the teaching and learning attributes which are specific to it, as opposed to more general qualities that all teachers of mathematics must possess. Similarly, Jackson et al. (2020) adapted the MKT into the Mathematical Knowledge for Teaching Future Teachers (MKT-FT) with the goal of providing more guidance toward teacher preparation programs on how to better prepare its programs' instructors in their preparation of mathematics PSTs.

In an effort to provide even more robust guidance in the current preparation of future mathematics teachers of EBs, specifically, we also attempted to contextualize the MKT framework by first consulting the relevant literature on the education of mathematics toward EBs. Although this was not an exhaustive literature review, this allowed us to identify key emerging themes that were later refined via our study's qualitative approach. It is imperative to clarify that by doing so, our intention is not to imply that Ball et al.' (2008) MKT model is faulty. On the contrary, and similar to Carrillo-Yañez et al. (2018) and Jackson et al. (2020), we build on the MKT's malleable nature to provide a contextualized MKT with additional considerations that, we argue, must *also* be taken into account when teaching mathematics to EBs.

Emergent Bilinguals: What's in a name?

Before addressing the needs of EBs in mathematics, we first made the decision to adopt the term EB as an attempt to acknowledge the complex and dynamic nature of language acquisition among K-12 students who come from homes in which the home language is not English and who therefore present various degrees of linguistic competencies in K-12 school settings (García, 2009). The term "emergent," for example, suggests that bilingualism is a process, rather than a fixed state, and recognizes that language proficiency can continue to develop and evolve over time. Similarly, the term "bilingual" emphasizes that these individuals are fluent in two or more languages, rather than suggesting that one language is dominant over the other. The term EB also avoids the negative connotations of terms such as "Limited English Proficient" or "English Language Learner," which can imply a generalization about these individuals having a deficit in their language skills. This allowed us to better frame our positionality toward the EB student population as being possessors of valuable linguistic and cultural resources that undoubtedly intersect with their school mathematics experiences and that warrant further research, including this study.

Subject Matter Knowledge (SMK) within the EBs' context

Within SMK lies CCK, or the common mathematical knowledge and skills used in settings outside of teaching. Although Ball et al. (2008) do not mean for the term *common* to mean knowledge possessed by everyone, they do leave the construct open for interpretation by stating that this type of knowledge is "used in a wide variety of settings…not unique to teaching" (p. 399). It is these interpretations, however, that we argue are of extreme importance when considering the mathematical knowledge that we may consider as *common* and that might inadvertently have inequitable educational repercussions. For example, among teachers and non-teachers, there exists a prominent idea that mathematics is a universal language (Mosqueda, 2010; Barrow, 2014), and although one can argue that mathematical concepts, patterns, and relationships are common across the globe, the language (and symbols) in which we choose to represent these, and therefore prioritize, is undeniably not common. Indeed, failing to acknowledge our EBs' identities and funds of knowledge (including linguistic practices), as imperative aspects of their mathematics explorations can contribute to episodes of severe identity crises shown to negatively impact many aspects of their lives (Cummins, 1981; Valenzuela, 1999; García, 2009).

According to Ball et al. (2008), teachers of mathematics should also possess knowledge that is unique to their teaching profession, such as SCK. This includes being able to identify mathematical patterns in students' written work, construct and implement mathematical representations effectively, and distinguish between different types of tasks within similar mathematical concepts, to name a few examples. A contextualization of SCK that includes the EBs' emphasis, however, must also consider the high linguistic demand that is embedded within mathematics (Mosqueda, 2010; Barrow, 2014; Waller and Flood, 2016). By failing to do so, problems can arise when we as educators make instructional decisions and accommodations based what we might perceive as content difficulties for EBs when in fact these might be linguistic challenges. Hence, a contextualized SCK intersects what we know about teaching mathematics with an additional awareness of the linguistic complexities present within the subject matter.

Lastly, teachers must also possess HCK, or a mathematical peripheral view such that it allows them to orient their instruction of the discipline through well informed judgments about what is mathematically important and eliciting (Jakobsen et al., 2012). HCK can also include knowledge about historical developments in mathematics such that it can orient and culturally frame the teachers' instructional practice (Ball and Bass, 2009). In other words, teachers of mathematics should know the mathematics that students have experienced, are experiencing, and will experience across their educational trajectories. We also know that mathematics is culturally, and therefore also linguistically, contextualized (e.g., Mosqueda, 2010; Abrams et al., 2013; Barrow, 2014; Rubel and McCloskey, 2021). Therefore, in the case of EBs, a contextualization of HCK must also include knowledge about the various types of linguistic supports, if any, that EBs have engaged with or will engage with throughout their schooling, and how mathematics is being presented in such models. Having a better understanding of EBs' linguistic supports, both in their past and future, could allow mathematics teachers to make well-informed instructional decisions based on more accurate judgments of their students' mathematical performance.

Pedagogical Content Knowledge (PCK) within the EBs' context

Within PCK lies KCS, or the idea that teachers of mathematics must have a robust understanding of how students think about, know, and learn mathematics (Ball et al., 2008). For instance, teachers must know *about* mathematics in a way that enables them to anticipate what students may find confusing, identify topics their students might find motivating, and interpret the mathematics students' spoken and written words (Jankvist et al., 2015; Nolan et al., 2015), among other things. Therefore, in order to have a more accurate understanding of the interactions between mathematics and EBs, a contextualization of KCS must also include a shift in orientation that focuses on what they can contribute to mathematics rather than focusing on the challenges that may come with being multilingual. Indeed, many scholars have already begun to point out critical areas required for the assurance of a more equitable mathematics education, such as shying away from deficit-oriented views about EBs and their linguistic and academic abilities and on how to incorporate these into their mathematics classrooms (e.g., Domínguez, 2011; Moschkovich, 2013, 2018; Turner et al., 2013; Krause et al., 2020; Esquierdo, 2021). We echo these words by arguing that the prioritizing of what EBs *bring* to the classroom rather than focusing on what they *lack* serves to a more just educational experience for EBs.

Research also suggests that teachers of mathematics must possess KCT via evidence-based instructional practices that foster students' learning. This means adopting the best sequencing of mathematical tasks to introduce a particular topic, making informed decisions on which students' examples to use to launch a classroom-wide discussion, and selecting instructional strategies judiciously such as to strengthen students' mathematical explorations (Ball et al., 2008; Nolan et al., 2015), to name a few examples. A contextualized KCT must also include evidence-based mathematics instructional practices specific for a more culturally and linguistically diverse student population. For instance, Moschkovich (2013) advocates mathematics instruction for EB students that includes a focus on students' mathematical reasoning and practices rather than on accuracy in using technical language and vocabulary. Similarly, Chval and Chávez (2012) encourage mathematics teachers to attain a multi-modal view of mathematical discussions and explorations, including the implementation of language development practices such as teachers writing down important ideas, concepts, terms, and representations on the board to which students can then refer (Stigler et al., 1996) and student writing and co-revisions within their mathematical assignments (Chval and Khisty, 2009).

Lastly, teachers must also possess KCC, or a "particular grasp of the materials and programs that serve as 'tools of the trade' for teachers" (p. 8), including the idea of lateral (within the same grade level) and horizontal (across upper and lower grade levels) curriculum knowledge (Jankvist et al., 2015). Teachers of mathematics who primarily serve the needs of EBs, on the other hand, must also be critical about such mathematics curricula that they enact in their classrooms (e.g., Smith et al., 2016; Kelly, 2018). Harper et al. (2021) call our attention toward an ideological shift in which teachers of mathematics no longer adopt a neutral political stance but rather disrupt the *whiteness* that is embedded within mathematics curricula and that has historically marginalized disadvantaged student populations. It is through empowerment that comes with being mathematically literate in which teachers *and* students are given a platform to voice these injustices and disturb cycles of discriminatory actions (Gutstein, 2007; Gutiérrez, 2013).

Methodology

Exploring teachers' beliefs about the epistemologies of mathematics has the potential to identify key aspects of mathematical pedagogies that would otherwise remain invisible (Lovin et al., 2012). Therefore, to further refine the contextualization of the MKT, we explored the beliefs regarding the mathematics education of EBs shared by PSTs enrolled in a teacher preparation program where all PSTs take several courses on multicultural education and are also required to certify as either an English as a Second Language (ESL) or Bilingual teacher. To accomplish this, we frame PSTs' *beliefs* as a flexible yet robust collection of "what teachers know, believe, and think" in relationship to teaching and learning (Borg, 2003, p. 81). This process is reported below.

Setting and participants

Participants were 16 PSTs, 15 of them identifying as women and one as a man, enrolled in a teacher-preparation program at a university located in the southwest area of the United States. Six PSTs self-identified as White, seven as Latinx/Hispanic, and three as Asian. All 16 PSTs were in their penultimate semester of their teacher-preparation program enrolled in Elementary Mathematics Methods, a course designed to expose PSTs to research-based recommendations for the learning and instruction of K-5 mathematics. Eight of the PSTs were in the same cohort pursuing a degree in Early Childhood to Sixth Grade Generalist Certification with the additional requirement to be certified in teaching as ESL teachers. The remaining eight PSTs were enrolled in a separate cohort pursuing the same bachelor's degree but with a focus on teaching bilingually (English and Spanish). Regardless of the degree, both cohorts had already taken the same amount of coursework that focused specifically on the education of bilingual, bicultural, and biliterate children in the United States. Furthermore, due to the high degree of Latinxs/Hispanics in the surrounding communities, their teacher-training placements had all been, thus far, in classrooms that had at least 50% of EBs present.

Data collection

All 16 PSTs participated in a semi-structured interview (Roulston, 2010) designed to explore in more detail the PSTs' beliefs toward the mathematics education of EBs (see Appendix for interview protocol). Similar to De Araujo (2017), these semi-structured interviews were supplemented with a curriculum interview where they were presented with a 6th grade mathematics handout named "Make the Dream Team" (Kunetz and Webb, 2012, p. 7), an activity that prompted students to compare fractions, decimals, and percentages through a hypothetical basketball scenario. Such handout was purposely selected as it offered additional opportunities for PSTs to comment on the appropriateness, or not, of its illustrations, tables, and overall theme for its use with EBs. All interviews were conducted and recorded by the first author, and these were then transcribed for further analysis. Combined, both the semi-structured interview and the curriculum interview lasted approximately one to one and a half hours per PST. Additionally, in order to use familiar terminology with the PSTs, all the data collection materials refer to EBs as either English Language (EL) Learners, English learners, or ESL learners.

Data analysis

Using MAXQDA, a qualitative data analysis software, the first author transcribed and initially analyzed all the PSTs' interviews using a line-by-line approach (Charmaz, 2006). Open coding followed by axial and selective coding were then used by the first author to synthesize the data into merging and coherent themes (Charmaz, 2006). The first author also adopted a constant comparative method that allowed the further development of themes from the raw qualitative data through a constant comparing of instances within such data (Glaser and Strauss, 1967; Fram, 2013). Simultaneously, memo-writing was implemented throughout the entirety of the analytic process to help theorize the emerging themes.

To establish a rigorous and trustworthy research study, the first author conducted member checks with participants to corroborate on the interpretations that were given to their interview statements. In addition, and throughout the process, the first and second author participated in peer-debriefing activities with two other colleagues in which they read transcripts and provided feedback on data analyses to ensure credibility and validity of the study. This led to a total of four rounds of coding from which consensus and, ultimately, theoretical saturation was achieved (Thornberg and Charmaz, 2014). In all, this process allowed the research team to identify emerging themes within the data, incorporate these into the initially adapted MKT, and ultimately refine the revised MKT framework.

Results

The analysis of the interviews led to the identification of six underlying subthemes pertaining to the PSTs' beliefs about the mathematics education of EBs (see Table 1). In this section, we present findings that illustrate the subthemes that emerged and their relationship within the contextualized MKT framework. It is important to note that these subthemes, and the entire contextualization of the MKT for that matter, are not to be taken as

TABLE 1 Frequencies of themes in interview data mapped onto MKT.

MKT (Ball et al., 2008)	Subthemes found in the interview data and that were mapped onto the MKT	Count (n)	Share within domain (%)	Share of total (%)
Subject matter knowledge	CCK: the non-universality of mathematics	19	0.35	0.13
	SCK: the linguistic embeddedness in mathematics	24	0.44	0.17
	HK: the urgency of an English-only mathematics	12	0.22	0.08
	Total	55	1.00	0.38
Pedagogical content knowledge	KCS: the challenges and advantages of EBs	21	0.24	0.15
	KCT: research-based instructional practices for EBs	57	0.66	0.40
	KCC: curriculum meta-awareness	9	0.10	0.06
	Total	87	1.00	0.61

mutually exclusive from each other but rather as a continuation in an effort to address the need of a stronger mathematics experience for EBs via multiple angles.

Subject matter knowledge

Content Knowledge (CCK): The non-universality of mathematics

Thinking of mathematics as a universal subject implies that mathematical concepts and procedures are the same everywhere, and that they can be learned and applied in the same way regardless of language or cultural background. However, this assumption can be problematic when considering the education of linguistically diverse students, such as EBs, as it fails to consider the important role that language and culture play in mathematical learning (Moschkovich, 2013, 2018; Setati and Barwell, 2013). For that reason, it is imperative that teachers of mathematics have an awareness of the non-universality of mathematics, especially when working closely with EBs.

Indeed, the PSTs in this study seem to have realized that although mathematical concepts can be thought of as universal, the actual representation of mathematics varies significantly. This awareness was most salient in the interview data when discussing the idea of whether mathematics is a universal language or not. Most PSTs answered by making a distinction between mathematical *concepts* and mathematical *register*, that is, the language used to talk about mathematics (Pimm, 1987). The degree to which how important they felt this was when educating EBs, however, remained unclear.

Out of the 16 PSTs, 13 agreed that although one can think of mathematical concepts as universal, such as the four main operations that are emphasized throughout the K-5 curriculum, the language in which we decide to teach these concepts, and other similar ones, is not. For instance, when discussing the content embedded within the mathematics classroom activity, Maggie noted:

> I think that the math concepts are universal, you know? Like adding and subtracting and such. But they're not translated in every language. Like word problems are going to be in different languages, obviously. Also, numbers and things like that all look totally different and there are different ways to do those problem, but the concepts themselves I think are universal between languages.

More interestingly, within the 13 PSTS, nine demonstrated awareness of diverse written numerical systems from across different cultures and added that, although concepts can be taught as universal, the language we employ, including written number systems, is not. For example, when probed about responses about different number systems, PSTs like Emma demonstrated familiarity with numeric systems that differ to those familiar to herself:

> So, I'm thinking in Mandarin where the symbols are different. A three does not look like a three in Mandarin. The word for three does not sound like three. So, learning and manipulating numbers is probably the same. It's just done in a different way. I don't agree that it's a universal language, but I think math is universal.

Only three PSTs agreed with the idea that mathematics is a universal language. However, after carefully examining their responses, it could have been interpreted that they also were referring to the universality of mathematics regarding its concepts and not the mathematical register. For instance, Aubrey claimed that because mathematics is, in essence, numbers and the manipulation of such, this could be understood across diverse languages as these concepts seem to be present everywhere:

> Yeah. Mathematics is a universal language, you know? To me, math is only numbers, right? Math is mostly adding and multiplying numbers, and it can be understood by everyone regardless of your language background. And so your numbers can make sense regardless of whether you speak that language or not.

Specialized Content Knowledge (SCK): The linguistic embeddedness in mathematics

Understanding what it means to know and do mathematics critically impacts how all teachers of mathematics will teach the subject. Recognizing the high linguistic demand embedded within mathematics, however, is especially important for PSTs working with EBs given how challenging it can be for EBs (Mosqueda, 2010; Barrow, 2014; Waller and Flood, 2016), particularly when asking EBs to understand English-written word problems, to explain their mathematical reasoning, and to show their work in written form. In that regard, all 16 PSTs demonstrated a high awareness of the intersection between language in mathematics and EBs, particularly among word problems. After reading the mathematics handout given in the curriculum interview, for example, Emma exclaimed the following:

> I mean it's [*language*] a big role in that you have to understand the math language in all word problems, and you have to be able to almost decode them in a different way, a unique way of understanding information and processing it. I'm a native English speaker, so like it was pretty easy for me that way because I understood everything. But I think of my EL [*a.k.a. EB*] students, and how I cannot make the same assumption for them, right? Like they might not be familiar with all the wording in problems like these [*e.g., Free Throws, Field Goals, Three-Pointers; all basketball-specific terms*].

A more profound analysis of PSTs' responses revealed that most PSTs shared a deeper and more technical awareness of the various types of words that EBs could encounter in mathematics and how these might be obstacles to their full participation in mathematical explorations. Beck et al. (2002), for example, place vocabulary words into three major categories: "Tier 1 which consists of basic or common words, Tier 2 which involves words that are used across the curriculum and multiple meaning words, and Tier 3 which is content specific vocabulary" (p. 24). All 16 PSTs, including Leticia, seemed cognizant that the vocabulary embedded within mathematics lies among different categories, or tiers, including words specific to mathematics such as those from Tier 3:

> I'm thinking that there's so many terms in math that you don't use it regular life. Uh, like if I told you to find a 'reciprocal', like I would never use that in a sentence outside of a classroom setting in math. And so I feel that language plays a really big role, especially when teaching ESL students.

This also included an awareness from the PSTs of vocabulary words from Tiers 1 and 2, despite how, historically, such tiers are not emphasized when working with the English development of EBs of mathematics (Donley and Reppen, 2001). Examples, as illustrated by Aubrey, included, but were not limited to, prepositions, cognates, and nouns:

> I made a number line, with zero being in the middle, and everything over this way [*left-hand side*] was negative. And then everything over here [*right-hand side*] was positive. But there're so many words that we could use to describe these sides in relation to zero: under, less than, above, greater than…if you don't understand the difference between these, that's hard. I think that's where language and math can be even more difficult because this kind of language is just difficult for those not familiar with English.

Horizon Content Knowledge (HCK): The urgency of an English-only mathematics

When teachers are aware of the language acquisition models their students are a part of, they can tailor their instruction to meet their students' linguistic and academic needs (DelliCarpini and Alonso, 2014; DelliCarpini, 2021). They can also help them understand their students' cultural and linguistic backgrounds which can promote a more inclusive and culturally responsive classroom environment (García and Kleifgen, 2018). For that reason, possessing HCK within the EB context means that PSTs must also be knowledgeable on language acquisition theories and models, and how these interplay with their students' long-term language *and* mathematics development.

Throughout this analysis, however, results revealed that only five PSTs demonstrated beliefs regarding such matters, namely the perceived urgency of having to transition into an English-only instructional environment even if counterintuitive to their EBs' linguistic development. For example, when asked about why she believed that she needed to focus on rapid English acquisition during her K-5 instruction, including mathematics, Joanna responded by saying:

> I mean, I have to do it because bilingual programs don't go up to middle school and high school, and therefore math is going to be taught in English only, and I need to prepare kids to be able to do well. Like it's [*math*] not a universal language, and thus I need to prepare them to do it in English, especially on those district assessments.

In this case, Joanna not only expressed concern over the linguistic preparation of her future students but acknowledges that students may not receive the linguistic accommodations they could be receiving in a K-5 bilingual program. Furthermore, Joanna perceives this lack of accommodations could have an impact on her students' mathematical achievement, particularly in standardized exams.

Another belief shared among the five PSTs regarding the urgency of many to transition to an English-only teaching environment was the perceived collective push to English-only as a means for job attainment and security in the United States, particularly jobs in the STEM fields. As expressed by Alicia:

> I mean we push math and science so much from the very beginning. Like 'Oh this is so important for you to get jobs and if you want to get a job here [*U.S.A.*], you need to speak English.' I do think that that they're basing it on a true argument.

However, there seemed to be a strongly shared discomfort toward this idea. As shared by Susana:

> The people that create these [*STEM*] programs think of math as very formulaic. They're wanting everyone to become doctors and engineers and there is a push for STEM and so of course they're going to want to teach math in English. They probably think that's the only way they [*EBs*] could have a chance. What people don't understand is that if you teach math in two languages, you can offer a richer and fuller context of what math is.

Pedagogical Content Knowledge

Knowledge of Content and Students (KCS): The challenges and advantages of EBs

Two subthemes emerged related to teachers' knowledge of content and students, namely the challenges and advantages that emerge when mathematics and EBs come into contact. One challenge shared by 14

PSTs involved knowing the linguistic diversity that exists within the EBs' label and how that affects their students' mathematical explorations. For example, EBs may possess differing linguistic skills, including a high degree of bilingualism in both English and another language(s), that go beyond the prescribed idea that EBs possess limited English proficiency. To this, Sophia noted the following:

> It's important to recognize that there are EL [*a.k.a. EB*] students that mostly speak Spanish and others that speak both English and Spanish at around the same level. So language is very important in math. It's important to translate terms or give them an anchor chart in both languages [*in a mathematics class*] even if it's not something that you need to do so that you can help them a little bit.

Sophia's observations point out that EBs vary in linguistic abilities, including varying degrees of fluency in the students' first language. Sophia also describes how accommodations in a mathematics class should not only include those in the students' target language but also in their native language.

Similarly, 11 out of the same 14 PSTs shared the importance of understanding students' language comprehension level, as it may vary significantly and critically impact students' development of mathematical knowledge. As expressed by Jessie:

> I'm always evaluating them [*EBs*]. They mostly speak Spanish, and I know they're working on their English. And so for me, I'm always asking myself, 'Do they not understand the math concept or what I am actually saying? Do they understand the language?' To me, it all boils down to them not knowing the concept versus them not knowing what I'm actually saying to them, and this can complicate teaching.

In the previous example, Jessie describes a common dilemma that occurs in mathematics classes with EBs and acknowledges how it can appear that students are not comprehending the mathematical concept at hand when in fact students might not be able to understand the actual language of instruction.

The second major subtheme that emerged involved the existing advantages of EBs in mathematics classes. Most PSTs, including Nancy, strongly believed that EBs brought with them multiple perspectives critical in the exploration and discussion of mathematical concepts beneficial to all students:

> I think collaboration is especially good for [*mathematics*] classes where there are a lot of English Learners because then they can share their resources with other ELs. Because English Learners, they obviously have different language resources and experiences in their heads, and so they could then help each other solve the math problems I give them.

As further elaborated by Aubrey, many PSTs saw the exchange of ideas among EBs beneficial to all learners, regardless of linguistic status, as they believed these were sound instructional practices to foster in mathematics classes:

> I worked in [*mathematics*] classrooms that have mostly English Language Learners, but I will continue to take those same strategies with me, regardless. To me, whenever you accommodate for your learners who need more support, you're helping everyone. You're also assisting the kids who maybe wouldn't struggle with language but see math in a different way.

Knowledge of Content and Teaching (KCT): Research-based instructional practices for EBs

Possibly the greatest indictor of PSTs' beliefs about the mathematics education of EBs emerged in their familiarity with research-based recommendations for EB's instructional practices. Overwhelmingly, all 16 of the PSTs demonstrated a vast knowledge of practices that consider the academic needs of EBs while still maintaining the same mathematical rigor. Indeed, there was a total of 68 instances in which PSTs mentioned research-based instructional practices specific to the EB's population. Among these, providing EBs with different representations of the same mathematical concepts were a salient practice referenced the most. As exemplified by John, these representations included the utilization of pictures, gestures, graphs, regalia, anchor charts, and other similar learning aides designed to support the linguistic comprehension of all students, particularly EBs:

> I would say that the best math teaching for ELs is multi-modal. It's using little videos or hands-on materials or pictures. Trying different aspects to reach my [*multilingual*] students as I'm teaching math. They need to touch and see the math. Only like that do I know they are understanding it even if it's in English.

The least referenced instructional practice, on the other hand, was the incorporation of EBs' home language(s) and culture into their mathematical explorations. Only six PSTs shared a belief that mathematics should be culturally relevant to EBs if we expect them to develop a more robust understanding of mathematics, as well as to develop a genuine interest in mathematics. As exemplified by Isabel:

> I want them [*EBs*] to succeed as well, you know? But how can I do that if I am talking about things they don't know about? No. I need to include their knowledge, their cultural wealth into every math lesson. How can they learn if they don't even see themselves in math?

In this case, Isabel acknowledges that students, particularly EBs, possess pre-existing knowledge relevant to their mathematical explorations, and she also highlights how difficult it can be for students to identify with mathematics when the subject itself is presented in an unfamiliar fashion devoid of any cultural markers. She continues by expressing how this later shared how this view does not imply a *watering down* of the curriculum but rather she sees it as an empowering and equitable tool that enables EBs' inclusion into the mathematical discussion and ultimately their education:

> I don't think that's watering it [*curriculum*] down. I want to make sure my EL [*a.k.a. EB*] students are included too. And if I have to change things to do so, I'll do it. I want them to see like 'Hey guys, like I'm going to go beyond what I see in the curriculum, because y'all deserve to know this.' And this applies to all subjects, you know? Not just math, but all of their education. It's about being fair.

Knowledge of Content and Curriculum (KCC): Curriculum meta-awareness

Knowledge about mathematics and its relation to curricula and EBs was not an emphasized theme by the PSTs in comparison with the other. Nonetheless, two sub-themes did emerge and highlighted some degree of awareness about how mathematics and traditional curricula interact among EBs, namely the advantages bilingualism could bring to a mathematics curriculum and the power dynamics that seem to be embedded in such curricula.

For example, six PSTs shared a belief that mathematics curricula, and the entire education system for that matter, is in English because that 'is the dominant language in the United States' and thus provides access to the job market. Out of these six, however, only three PSTs demonstrated a more critical stance when it came to discussing mathematics curricula, particularly when it came to the mathematics education of EBs. Having experienced a bilingual (English-Spanish) mathematics curriculum during one of her classroom observations, for example, Jessie pointed out how such curricula could potentially provide an advantage for Spanish-dominant speaking EBs in developing a more robust mathematical understanding. In her observation of a lesson dealing with large two-digit numbers, Jesse claimed that:

> I saw them [*EBs*] discussing larger numbers. There's *dieciseis* [sixteen] which literarily translates to six and ten. Students were then like, 'Oh, so there's always that 10 and then a number, right?' To me, it was less confusing for the students because they're learning not just how to say these numbers but how they are constructed using the power of ten. It's embedded within the language [*Spanish*] too.

Through her observation, Jessie hints toward the potential advantage of having a mathematics curriculum that facilitates students' comprehension through the incorporation of students' linguistic practices. In this case, Spanish was used not only as a source to facilitate comprehension among EBs, but the nature of how numbers are expressed in Spanish also made the base-ten system more explicit as compared to English.

Similarly, the second sub-theme that emerged was related to how these same three PSTs interpreted English-only mathematics curriculum as a means of power being enforced when implemented with linguistically diverse student populations. For instance, Isabel shared how she believes the language she uses in her mathematics lessons could be sending indirect messages to her EBs due to the power that she possesses as their teacher:

> I think the language we use with our lessons is really important. I mean, if I only speak English and I know I have students that don't, well I am just enforcing my authority as the teacher and also ignoring their needs. It gets me thinking what message am I sending, you know? Like the way I speak to a student and if he or she is understanding a concept. I am just imposing my own power over them as the teacher.

The Mathematical Knowledge for Teaching EBs: an emerging contextualization

The initial overview of the literature highlighted several key considerations involving the mathematics education of EBs, focusing on the importance that is developing a critical stance toward the *how* and *why* we should consider the specific needs for such student population. Additionally, interviewing PSTs embedded within a EBs-focused teacher preparation program unveiled further considerations, as well as complexities, that are present when educating EBs. Combined, this provides us with a contextualized MKT that can further guide teacher education programs in their design and implementation of relevant preparation coursework in teaching mathematics to EBs (see Table 2).

Discussion

Collectively, the beliefs shared by the PSTs in this study, as well as recommendations from the literature, serve to contextualize the MKT framework into one that is focused explicitly on EBs. Results also highlight areas that we must continue to emphasize in teacher education programs. First, we, as teacher educators, must foster among PSTs a flexible interpretation of what is considered common mathematical knowledge. This by no means disregards the importance of axioms and theorems in mathematics but rather acknowledges the possible repercussions that dogmatic ideologies about what mathematics is could have on those whose mathematical knowledge and experience might be perceived as foreign or unfamiliar. Aspects of this view were reflected in the data as many PSTs demonstrated an awareness of how the idea of mathematics as a universal language could be problematic when working with the linguistically diverse population such as EBs. Rather than assuming all children, regardless of cultural or linguistic backgrounds, ought to be familiarized with Westernized conceptions about mathematics, including symbol systems and content-specific terminology, they recognized the diversity in mathematical algorithms and representations and how critical this awareness is in the mathematics education of such student population.

Both, Mosqueda (2010) and Barrow (2014) argue against conceptions of mathematics as a universal language for similar reasons, claiming that ideologies like these, in conjunction with deficit orientations, often contribute to the inadequate attention given to the linguistic needs of EBs. On the contrary, rather than assuming mathematics should be understood everywhere regardless of language, we argue that we consider mathematical concepts as universal. This view of mathematics could lead to a more successful cultural mediator when trying to bridge the diverse conceptions that come with different student groups by tapping into known mathematical principles and foundations rather than the language used to communicate such ideas (Waller and Flood, 2016).

Second, in order to make better-informed instructional decisions, we must help teachers of mathematics to acknowledge the linguistic difficulties that EBs share in mathematics classrooms. When selecting appropriate word problems, for instance, it is critical that mathematics teachers are aware of the linguistic complexity that can be present and not let this get conflated with mathematical competency among their EBs. Indeed, Martiniello (2008) found that the linguistic complexity found in some 4th grade mathematics problems play a more significant role with EBs that non-EBs. Through a very thorough linguistic analysis of 4th grade word, Martiniello (2008) identified

TABLE 2 A contextualized MKT for the teaching of EBs.

MKT (Ball et al., 2008)	According to Ball et al. (2008), teachers of mathematics must be able to...	Additionally, to address the needs of EB students, teachers of mathematics must also...
Subject matter knowledge		
Common content knowledge	Know of common mathematical characteristics and properties, use proper mathematical terms and notations, and identify inaccurate mathematical statements.	Remain critical as to what is accepted as common in mathematics, especially when considering the diversity that exists among numeration systems, numerals, and the mathematics register.
Subject content knowledge	Identify patterns in students' contributions, present mathematical ideas effectively, and link students' contributions to underlying ideas.	Acknowledge the high degree of linguistic demands embedded in mathematics that are not common for non-dominant English speakers. This includes basic vocabulary, words that are used across the curriculum and with multiple meanings, and content-specific terminology.
Horizon content knowledge	Be familiar with students' past and future mathematics curricula, as well as historical mathematical landmarks to culturally frame instruction.	Know how mathematics is presented in common language-support models (e.g., ESL submersion or pull-out programs, Dual-Language programs, etc.) such that they can make judicious pedagogical decisions within an educational system that historically aligns with English-only linguistic policies.
Pedagogical content knowledge		
Knowledge of content and students	Select tasks that students might find interesting, anticipate what students are likely to think or do, and be familiar with common conceptions and misconceptions.	Adopt an asset perspective toward the mathematical capabilities of EB students, especially the academic richness within their funds of knowledge and linguistic repertoire. This also requires a deep understanding about the challenges *and* advantages that EBs bring with them to mathematics, such as the varying degree of multilinguism and cultural experiences that is present within the EB-label.
Knowledge of content and teaching	Know how to design and sequence instruction, identify appropriate tasks and examples for instructional purposes, and evaluate advantages and disadvantages of using specific representations.	Know of a variety of research-based mathematical instructional practices and recommendations specifically for EBs. This includes providing them with multiple mathematical representations and manipulatives, and with ample opportunities for collaborative and culturally relevant tasks that are also rich in language.
Knowledge of content and curriculum	Know the materials and programs that serve as the curriculum, including those across grade levels.	Remain critical about the messages of disenfranchisement that could unintentionally be sent to EBs by any given curricula, including the role of languages other than English and of bilingual education in the United States.

word problems with a higher degree of linguistic complexity that seemed to favor non-EBs over their EB peers in terms of content comprehension. Besides unfamiliarity with vocabulary terms, both content and non-content specific, it seemed that many EBs demonstrated difficulties grasping word problems with multiple clauses, long noun phrases, and those with a limited syntactic transparency. Instead of providing EBs with language-free mathematical problems and activities, both Martiniello (2008) and Abedi (2006) advocate for a simplification of unnecessarily complex language in mathematics while still maintaining the same rigor.

Lastly, it is important for our teachers to note that issues like these become of greater importance as mathematical classrooms are adopting more discussion-based practices aligned with recent reform efforts in mathematics education. To this matter, research has shown that EBs are less likely to participate in English-dominated discussion-based instructional practices if they are not given appropriate scaffolding (e.g., Planas and Gorgorió, 2004; Banes et al., 2018). A multi-modal approach to student participation, on the other hand, shows promise in overcoming language-based barriers, such as code-switching and gesturing (e.g., Moschkovich, 2013). As such, scaffolding our teachers in possessing a peripheral view of not only mathematics but also the classroom linguistic policies in EBs' previous, current, and future academic trajectories can further inform then on the types of instructional practices that they implement in their classrooms.

Conclusion

Findings made it possible for the identification of contextualized concrete steps we can take to continue fostering equitable mathematical practices toward EBs. This requires PSTs to develop skills such as in highlighting the linguistic demands embedded in mathematics, the various types of language support programs offered in classes of mathematics, and overall research-based instructional practices. Simultaneously, PSTs must develop an ideology toward mathematics that is malleable enough, yet still true in its mathematical nature, such that it creates equitable learning opportunities for students whose knowledge and contributions might deviate from what is considered *traditional* or *normative* mathematical practices.

Such findings, however, come with some limitations. First, it is worth mentioning that a sample of 16 PSTs is not enough to assess, not to mention summarize, the high degree of complexity involved in the equitable mathematics education of EBs. Another limitation is on how the researcher's lens unavoidable influences qualitative efforts, including data collection, analyses, and synthesis. Lastly, the PSTs selected for this study were still in their teacher preparation program, and therefore their responses may not reflect those of in-service mathematics teachers with years of experience working with EBs.

For those reasons, we recommend that further research expands upon our contextualized MKT by exploring the beliefs of more experienced in-service teachers of mathematics who primarily serve EBs. Not only would this serve to further cement the contextualization of the MKT as a valuable framework for PSTs and their respective teacher preparation programs, but it could also uncover other key factors in the education of EBs and mathematics not previously highlighted. As a means of comparison, we would also argue for studies that can compare how a contextualized MKT framework might differ when exploring the experiences of in-service teachers with less familiarity in working with EBs and mathematics. This could reveal novel areas of improvement in the preparation of teachers of mathematics in more equity-oriented manners. Lastly, the impact of teachers adopting such contextualized MKT framework should be explored in students' achievement measures. The degree of the effect this framework could have on EBs' actual achievement in mathematics as measured by common state-mandated benchmarks could shed light on issues of teacher implementation of instructional practices and how these could be mitigated to produce more robust results in student achievement.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material. Further inquiries can be directed to the corresponding author.

Ethics statement

The studies involving human participants were reviewed and approved by the Institutional Review Board for Human Subjects Research. The patients/participants provided their written informed consent to participate in this study.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Appendix

Semi-structured interview and curriculum interview protocol

Hi and thank you for agreeing to participate in this interview. The purpose of this will be to learn more about your beliefs regarding mathematics and ELs in the mathematics classroom. Please feel free to skip any of these questions if you experience any discomfort. You may also end your participation in this interview at any time.

Part 1. Semi-structured interview

1. Mathematics at a personal level
   a. Tell me about your experience learning math in school.
      i. What did you like the most about learning math? Why?
      ii. What did you like the least about learning math? Why?
      iii. What kinds of activities did you do in math class?
      iv. What role do you think language played in your learning of math?
      v. Is language in mathematics important? Why or why not?
      vi. If yes, how important is it?
   b. In your opinion, what does it mean to know and do mathematics?
   c. What is your take on the idea that "Mathematics is a universal language"?

2. Mathematics within a teacher/school context
   a. Tell me a story about a time when you had to teach a math lesson.
   b. What is the easiest part about teaching math?
   c. What is the hardest part about teaching math?
   d. What activities did you use, if any?
   e. What was the role of language in your teaching/students' learning?
   f. What does good mathematics teaching look like (in elementary school)?
   g. Is this the same for both EL and non-EL classes?

3. What does student-learning look like in a mathematics classroom?
   a. How do EL students participate in mathematical discussions?
   b. How long would it take for a new EL student to be successful in math class?
   c. How long would it take them to be able to participate in mathematical discussions?
   d. What kind of accommodations, if any, would you have to make for EL students?

4. Why do you think dual language programs dictate mathematics be taught only in English? Is this a helpful practice? Why or why not?
5. Is mathematics an ideal subject for EL students who are beginning to acquire the English language? Why or why not?

Part 2. Curriculum interview

Please take several minutes to go over the following mathematics activity. Feel free to make as many annotations as you want on it. Afterwards, I will ask you a couple of questions. When done, I will collect the document, ask a few more questions, and that will finalize our interview.

[Show handout "Make the Dream Team" (Kunetz and Webb, 2012, p. 7)]

Now that you have looked at it, imagine you are presenting this handout to your mathematics class.

1. What is your overall impression of this handout?
2. What do you think is the mathematical concept of focus in here?
3. Is there anything else you might want to add / modify in this activity to make it better for your instruction?
4. How would you use this activity to teach a lesson in a typical mathematics class?
5. Would you need to make any further modifications to the handout if you were using it with EL students?