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A GARCH-MIDAS approach to modelling stock returns

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Abstract

Measuring stock market volatility and its determinants is critical for stock market participants, as volatility spillover effects affect corporate performance. This study adopted a novel approach to analysing and implementing GARCH-MIDAS modelling methods. The classical GARCH as a benchmark and the univariate GARCH-MIDAS framework are the GARCH family models whose forecasting outcomes are examined. The outcome of GARCH-MIDAS analyses suggests that inflation, interest rate, exchange rate, and oil price are significant determinants of the volatility of the Johannesburg Stock Market All Share Index. While for Nigeria, the volatility reacts significantly to the exchange rate and oil price. Furthermore, inflation, exchange rate, interest rate, and oil price significantly influence Ghanaian equity volatility, especially for the long-term volatility component. The significant shock of the oil price and exchange rate to volatility is present in all three markets using the generalized autoregressive conditional heteroscedastic-mixed data sampling (GARCH-MIDAS) framework. The GARCH-MIDAS, with a powerful fusion of the GARCH model's volatility-capturing capabilities and the MIDAS approach's ability to handle mixed-frequency data, predicts the volatility for all variables better than the traditional GARCH framework. Incorporating these two techniques provides an innovative and comprehensive approach to modelling stock returns, making it an extremely useful tool for researchers, financial analysts, and investors.

Keywords: GARCH-MIDAS, Johannesburg stock market, all share index, Nigeria stock exchange market, Ghana stock exchange

1. Introduction

For deepening inclusive development that generates wealth and reduces poverty incidence in Africa, positive economic policy outcomes and stock market performance are crucial (Sarpong and Nketiah-Amponsah, 2022). Due to the significant implications for investors, policymakers, and market participants, stock return analysis and forecasting have always been at the forefront of financial research. The financial landscape is ever-changing, so it is important to have accurate and reliable models that capture dynamic stock returns (Rumaly, 2023). Traditional methods for modelling financial time series data volatility, such as the generalized autoregressive conditional heteroscedasticity model (GARCH), have been extensively used. It is often challenging, however, for these models to effectively incorporate short-term and long-term data. Financial researchers have recently attempted to overcome

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conventional limitations by developing more comprehensive and sophisticated models. The generalized autoregressive conditional heteroscedastic-mixed data sampling (GARCH-MIDAS) approach is one such innovative framework that has attracted considerable interest in recent years. With this new model, the strengths of GARCH are seamlessly combined with the flexibility of mixed data sampling (MIDAS), resulting in more accurate and robust stock return predictions. Many researchers, scholars, practitioners, and various stakeholders agree that the financial market is the foundation of a sound financial system and an essential driver of economic growth in both advanced and developing economies (Jiang *et al.*, 2021; Abaidoo and Agyapong, 2022). Since the inception of the Ghana Stock Exchange in 1990 as part of economic reforms, successive governments have implemented extensive restructuring to attract more participation and promote the progress of the capital market and its allied institutions in the country (Owusu-Antwi G, 2009). These reforms included the establishment of regulatory and supervisory frameworks; the introduction of automated trading; market liberalization; and fair diversification of products and services.

Even though these interventions led to some improvement in the stock exchange, the performance of Ghanaian stocks has been dismal compared to their peers in other regions of the world. The total market capitalization witnessed a steep fall from GH 57.1 billion in 2015 to GH 56.8 billion in 2019 and equity trading has not seen any significant improvement over the last ten years as trading volumes fell from about GH 331 million in 2010 to about GH 324 million in 2019. Capitalization of the stock market as a percentage of GDP has witnessed a sharp decline from about 31.7% in 2015 to about 16.3% in 2019 (Ghana Stock Exchange, 2019). Similarly, Nigeria has one of the largest economies in Sub-Saharan Africa, but capital market participation is low in comparison to the country's economy and population. In October 2018, the Nigeria All Share Index decreased from 32,383 to 32,763 points. Market capitalization dropped from 11.961 trillion Naira to 11.822 trillion Naira in September the same year (Nigeria Stock Exchange, 2018). These markets have been characterized by low level of liquidity and capitalization, often centered on a few listed stocks. As the performance of businesses is linked to macroeconomic conditions, the situation is exacerbated by the perennially volatile macroeconomic environment in Sub-Saharan Africa (Acheampong *et al.*, 2021). The highly volatile and unpredictable macroeconomic conditions have hampered the returns and growth of businesses in the subregion (Yusuf, 2022; Abaidoo and Agyapong, 2023).

Based on methodological considerations, over the years, a multiplicity of research has been conducted in advanced and developing nations to understand the period-varying and conditional stock price volatility and their underlying causes using various statistical tools. Traditional techniques, such as vector autoregressive (VAR) family type, and generalised autoregressive conditional heteroscedastic (GARCH) family type, were widely used to study the stylized behavior of stock returns (see for instance, (Ali *et al.*, 2020; Nasir *et al.*, 2021)). Though these models have been useful in finance and economics, they are bedevilled by data frequency incompatibility as a number of macroeconomic indicators are reported on a monthly, quarterly, and annual basis while stock prices are measured on a daily or hourly basis. To address the issue of frequency disparity, researchers often convert variables sampled at different (uneven) frequencies to uniform (match) frequencies by means of extrapolation or averaging and aggregating. Consequently, conversion of variables of diverse frequencies to avoid frequency mismatch would lead to loss of vital information in the data, which adversely affects forecast performance (Nobre and Neves, 2019). Similarly, this phenomenon is usually accompanied by estimation bias (Dieppe *et al.*, 2021). Additionally, these models are not able to segregate volatility into short- and long-run components and also fail to capture time-varying conditional variance. In this regard, many studies on volatility dynamics have been conducted using the novel generalised autoregressive conditional heteroscedastic-mixed data sampling (GARCH-MIDAS) methodology because the component model allows for a concise representation of complex dependence (Abebe *et al.*, 2022; Aghabazaz *et al.*, 2022). Furthermore, volatility component models have recently attracted considerable curiosity not only because of their potential to express complex dynamic data with a parsimonious model structure but also due to their ability to deal effectively with non-stationarity or structural breaks in equity price swings (Wang and Ghysels, 2015).

Additionally, substantial works related to GARCH-MIDAS have been conducted elsewhere in the world other than Africa and their recognition is important to this study. Yu *et al.* (2021) conducted a study using sample data from 1 January 2002 to 31 October 2018 to examine the effects of global economic policy uncertainty on stock volatility for nine emerging economies including Turkey, Brazil, India, China, South Africa, Indonesia, Mexico, Russia, and South Korea. The monthly global economic policy uncertainty is regressed on daily stock observations for each country. A GARCH-MIDAS model was used in the analysis and fluctuation test to examine the model's time-varying forecast performance. In all countries examined, the study found that GEPU significantly influences stock market volatility. Wang *et al.* (2020) extended the GARCH-MIDAS model to account for the asymmetry and extreme effects on the long and short-term volatility components. In their analysis, they used daily price data from the S&P 500 index from 1993 to 2016. The forecast was generated using fifteen extended GARCH-MIDAS models. They discovered significant evidence that bad news has a greater influence than good news, and negative and positive extreme shocks have significant and differential impacts on stock volatility. They also found that negative extreme shocks are more likely to affect stock volatility than positive extreme shocks, and that both the asymmetry effect and the extreme volatility effect can affect stock volatility in both the long and short term. Furthermore, they conducted MCS tests using five error function metrics to assess the model's prediction performance, and they discovered that models containing both the asymmetry and extreme volatility effects performed best out-of-sample, whereas the standard model performed worst, implying that a model comprising both the asymmetry and extreme volatility effects can forecast stock volatility.

This study therefore aims to investigate and document appropriate macroeconomic variables that explain volatility in stock returns in three African countries (i.e., South Africa, Nigeria and Ghana) using GARCH-MIDAS modeling techniques while providing an in-depth exploration of the GARCH-MIDAS approach to modelling stock returns, highlighting its theoretical underpinnings and practical implications. Through empirical analyses and comparisons with other traditional models, we aim to demonstrate the superiority of GARCH-MIDAS model in forecasting accuracy and risk assessment. Specifically, this research seeks to assess the relationship between inflation, money supply (MS), exchange rate (EXR), interest rate and oil price on stock returns in South Africa, Nigeria and Ghana using GARCH-MIDAS model. The study will further examine the impact of these macro economic variables on the components' volatility in South Africa, Nigeria and Ghana. Additionally, it will examine the performance of competing GARCH family models and test the forecasting potential of the models by examining hold-over performance.

2. Data and methods

Secondary data of the daily stock price of the stock market Indexes were gathered from three African stock exchanges (South Africa, Nigeria, Ghana) as presented in Table 1. Choosing these markets was based on their significance to the growth of the region as well as the fact that they are the largest economies in the region. Data on macroeconomic variables, comprising exchange rate (EXR) per US dollar, money supply (MS), black consumer price index (CPI), interest rates (IR), and oil prices (OP), are based on information from the central banks of the respective countries, the statistical service, and

Table 1: Summary of variables used in the study and their sources

Country	Variables	No of obs	Data source
South Africa	Stock returns	3,077	JSE
	CPI	144	IFS database
	EXR	144	IFS database
	IR	144	SARB
	OP	144	SARB
	MS	144	SARB
Nigeria	Stock returns	3,078	NSE
	CPI	144	CBN
	EXR	144	CBN
	IR	144	CBN
	OP	144	CBN
	MS	144	CBN
Ghana	Stock returns	2,948	GSE
	CPI	144	GSS
	EXR	144	IFS database
	IR	144	BOG
	OP	144	BOG
	MS	144	BOG

the International Finance Statistics database. The sample period runs from January 2010 to December 2021. In-sample periods range from January 2010 to December 2019 while the hold-out periods run from January 2020 to December 2021. Regarding South Africa, financial times stock exchange (FTSE) / Johannesburg stock exchange (JSE) all share index, which is a component of about 90% of companies listed on Johannesburg Exchange, was used. Nigerian stock exchange (NGX) All Share Index represents weighted performance of Nigerian companies listed on NGX, and GSE All Share Index represents companies listed on GSE. The rationale for choosing that time frame depends on the availability of credible data for all selected countries in order to avoid gaps and disparities in the analysis and to ensure that the results represent the contemporary patterns in the markets.

2.1. Augmented Dickey-Fuller (ADF) stationarity test

The ADF test is conducted at each level by differencing to determine the existence of unit root in the variables. The unit root test is performed on the response variable (stock returns) and explanatory variables: inflation (INF), currency exchange rate (EXR), interest rate (IR), money supply (MS), and crude oil price (OP). black The test's null hypothesis is that the series has a unit root, indicating that it is non-stationary. Δy_t is given in (2.1) by;

$$\Delta y_t = U + \alpha_t + \sum_{j=1}^{p} \delta \Delta y_{t-j} + U_t + \phi y_{t-1}, \tag{2.1}$$

where $U_t \sim \mathrm{IID}\, N(0, \sigma^2)$, U is the constant trend, α_t is the parameter of time trend, δ is the unit root and Δy_t denotes the difference between the variable y_t and its own lag.

2.2. Heteroscedasticity test

A homoscedasticity test is important to make sure that the regression can predict the dependent variable consistently across all independent variables. Homoscedasticity is necessary to guarantee efficient estimators of our parameters. Moreover, a violation of homoscedasticity has implications for the validity of t and F statistics for inference.

2.3. Normality test

The Jarque-Bera (JB) test was the adopted measure to test normality in the stock returns dataset. Based on the property of normal distribution, only the first two moments of a distribution completely describes the distribution completely, that is, its mean and variance. In statistical distributions, it is standard to measure the third and fourth moments by skewness and kurtosis.
The JB test, S_k and K are defined in (2.2), (2.3) and (2.4) respectively by;

$$JB = \frac{n}{6}\left[S_k^2 + \frac{1}{4}(K-3)^2\right] \tag{2.2}$$

$$S_k = \frac{\hat{\mu}_3}{\hat{\sigma}^3} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^3}{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2\right]^{\frac{3}{2}}} \tag{2.3}$$

$$K = \frac{\hat{\mu}_4}{\hat{\sigma}^4} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^4}{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2\right]^{2}}, \tag{2.4}$$

where n stands for the number of observations, S_k denotes the sample skewness, and K represents the sample kurtosis. $\hat{\mu}_3$and $\hat{\mu}_4$ are the third and fourth central moment sample estimates, respectively.

2.4. The Lagrange multiplier (LM) test for ARCH disturbances

The GARCH family of models must contain ARCH effect in the series, which is a basic requirement. ARCH-LM test is employed in this study to test the existence of ARCH effect in ARMA residuals (mean equation). Depending on the difference between the real and expected values, also known as residuals of the mean equation, the square residuals are used to determine conditional heteroscedasticity. Engle (1982) proposed the LM test, which comprises regressing the squared residuals on past lag squared residuals values represented in (2.5) as

$$\varepsilon_{i,t}^2 = \psi_0 + \psi_1\varepsilon_{i,t-1}^2 + \psi_2\varepsilon_{i,t-2}^2 + \cdots + \psi_q\varepsilon_{i,t-q}^2 + \omega_{i,t}, \tag{2.5}$$

where $\varepsilon_{i,t}^2$ is the squared residuals from the mean equation, $\omega_{i,t}$ denotes the white noise process.
The null and alternative hypotheses are stated as:
Null Hypothesis (H_0): There are no ARCH effects in the residuals of the time series model.

$$H_0 : \psi_1 = \psi_2 = \cdots = \psi_q = 0.$$

Alternative Hypothesis (H_1): There are ARCH effects in the residuals of the time series model. At least one of the coefficients of the lagged squared residuals is different from zero.

$$H_1 : \text{At least one } \psi_i \neq 0 \quad \text{for } i = 1, 2, \ldots, q.$$

The LM test statistic can be conveniently obtained from the coefficient of determination R^2 of the regression in (2.5). More precisely, the ARCH-LM statistic, represented by LM(q) is given by

$\mathrm{LM}(q) = \mathrm{TR}^2$, where T is the number of observations we have in the series. In the absence of residual autocorrelation, it has an asymptotic χ^2_q distribution if the null hypothesis of no conditional heteroscedasticity holds (Engle, 1982). Large values of the test statistic indicate that H_0 is false and, hence, there may be ARCH effects in the residuals. In that case, it may be useful to consider fitting an ARCH or ARCH-type model to the residuals. An F version of the statistic with potentially better small sample properties may also be considered (Harvey, 1990; Hendry and Doornik, 1997). It has the form in (2.6) as:

$$F_{LM} = \frac{R^2}{1-R^2}.\frac{T-p-q-1}{q} \sim F(q, T-p-q-1). \tag{2.6}$$

2.5. Ljung-box test

The Ljung-Box test evaluates squared residuals based on their immediate preceding values.
The modified test statistic for finite samples is given by (2.7) as:

$$Q = T(T+2)\sum_{i=1}^{q}\frac{\hat{\rho}_i^2}{T-i} \sim \chi^2(q), \tag{2.7}$$

where $\hat{\rho}_i^2$ is the estimator of the autocorrelation function and T is the sample size. The parameter Q is distributed as chi-square with q degrees of freedom under the null hypothesis of no autocorrelation of residuals. The null hypothesis of no autocorrelation of residuals is rejected if the calculated value of the test statistic is greater than the critical value of the distribution with q degrees of freedom.

2.6. Model evaluation measures

1. Akaike information criteria (AIC): Model selection criteria of this type were the first to be widely accepted. By extending the maximum likelihood principle to the AIC, once the structure of the model is understood, its parameters can be estimated using the maximum likelihood principle. AIC computed in (2.8) by

$$\mathrm{AIC} = -2\ln(\mathrm{Likelihood}) + 2K. \tag{2.8}$$

2. Bayesian information criteria: The BIC is a criterion developed under the Bayesian paradigm for selecting among a set of finite models. It selects the best model from a candidate model space for more inferences. The likelihood function is used to determine whether a model is good. It evaluates various models with varying asymptotic features. BIC computed in (2.9) by

$$\mathrm{BIC} = -2\ln(\mathrm{Likelihood}) + 2K\ln(T), \tag{2.9}$$

where T denote the number of observations and K represent the number of parameters to be estimated. The ideal lags for GARCH-MIDAS and GARCH models are arrived at using AIC and BIC information criteria.

2.7. GARCH model

Using the ARMA-GARCH model, the mean equation is a linear ARMA (p, q) model that is univariate and comprises an autoregressive and moving average component. The combination of ARMA and

GARCH model has been widely used to model both fluctuating stationary and non-stationary effects in financial and economic time series. Thus, the general ARMA (p, q) is defined in (2.10) as:

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}, \tag{2.10}$$

where y_t is the value of the time series at time t, ϕ_0 is the model intercept, ε_t is the white noise process at time t, ϕ_i are the parameters of the autoregressive part of the model (for $i = 1, 2, \ldots, p$), θ_j are the parameters of the moving average part of the model (for $j = 1, 2, \ldots, q$), p and q are the orders of the autoregressive and moving average respectively.
The general form of the GARCH (p, q) model consists of the mean and variance equations. The mean equation of GARCH (p, q) is written in (2.11) as:

$$y_t = \mu + \varepsilon_t, \tag{2.11}$$

where $\varepsilon_t = \sigma_t z_t$ and $z_t \sim N(0, 1)$.
Likewise, the variance equation of GARCH (p, q) is written in (2.12) as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \tag{2.12}$$

where σ_t^2 is the conditional variance of ε_t at time t, α_0 is the constant term, α_i are the coefficients of the lagged squared error terms (for $i = 1, 2, \ldots, p$), and β_j are the coefficients of the lagged conditional variances (for $j = 1, 2, \ldots, q$). To ensure non-negativity and to get the value of the conditional variance to be always non-negative, the following conditions have to be met:

$$\alpha_0 > 0, \;\; \alpha_i \geq 0, \;\; \forall i \;; \;\; \beta_j \geq 0, \;\; \forall j \,.$$

2.7.1. GARCH-MIDAS

In this study, we employed a multiplicative two class volatility component GARCH-MIDAS model introduced by (Engle *et al.*, 2013). The GARCH-MIDAS technique models return as the product of a unit variance GARCH model, which denotes volatility swings in a short-term perspective, and an explanatory variable based on components, which illustrate macroeconomic changes over time. The model combines short-run GARCH (1, 1) high frequency volatility with low-frequency macroeconomic variables taken care of by the MIDAS regression introduced by Ghysels *et al.* (2007). The GARCH-MIDAS incorporates lagged values of stock returns at different frequencies, allowing for a more comprehensive and nuanced representation of the underlying volatility dynamics (Yao and Li, 2023). This feature enables the model to better capture short-term and long-term dependencies in the data, making it particularly well-suited for predicting stock returns time series with irregularly spaced observations (Salisu *et al.*, 2020). The daily Stock return is calculated in (2.13) as

$$r_{i,t} = \ln P_{i,t} - \ln P_{i,t-1}. \tag{2.13}$$

The daily return is assumed to follow the following processes given in (2.14):

$$r_{i,t} = \mu + \sqrt{T_t g_{i,t}}\varepsilon_{i,t}, \quad \forall\, i = 1, 2, \ldots, N_t, \tag{2.14}$$

where $\varepsilon_{i,t} \sim N(0,1)$, $r_{i,t}$ represents the return of stock i at time t. It is the dependent variable in the model, μ is the constant mean stock return. In many financial models, it is assumed that the returns have a constant mean, μ, over time, T_t represents the long-term component of the volatility at time t. In the GARCH MIDAS framework, T_t is typically modelled as a slowly changing process influenced by macroeconomic or other long-term factors, $g_{i,t}$ represents the short-term (GARCH) component of the volatility for stock i at time t. The GARCH component captures the more rapidly changing aspects of volatility, typically influenced by recent past returns,$\varepsilon_{i,t}$ is the error term for stock i at time t. It is usually assumed to be independently and identically distributed (i.i.d.) with a standard normal distribution, N_t denotes the number of trading days month t. This is specified for each stock i within the set of all stocks considered at time t.
From (2.14), squaring both sides yields (2.15) and (2.16) as:

$$\left(r_{i,t-\mu}\right)^2 = \left(\sqrt{T_t g_{i,t}}\varepsilon_{i,t}\right)^2. \tag{2.15}$$

$$\left(r_{i,t-\mu}\right)^2 = T_t g_{i,t}\varepsilon_{i,t}^2. \tag{2.16}$$

Dividing both sides by T_t yields (2.17) as:

$$g_{i,t}\varepsilon_{i,t}^2 = \frac{(r_{i,t} - \mu)^2}{T_t}. \tag{2.17}$$

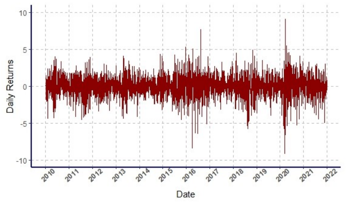
The short-term component of the volatility, $g_{i,t}$ in the GARCH-MIDAS framework is typically modeled using a GARCH(1,1) process given in (2.18) by:

$$g_{i,t} = (1 - \alpha - \beta) + \alpha\frac{(r_{i-1,t} - \mu)^2}{T_t} + \beta g_{i-1,t}, \tag{2.18}$$

where, $(1 - \alpha - \beta) = \omega$ and while β and α are GARCH and ARCH parameters respectively such that$\beta \geq 0$, $\alpha > 0$, $\alpha + \beta \leq 1$.

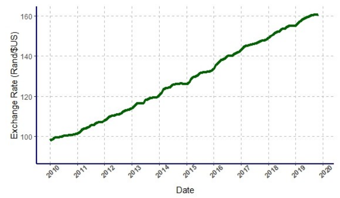
$$\ln T_t = m + \theta\sum_{k=1}^{K}\varphi_k(\omega_1, \omega_2)X_{t-k} \tag{2.19}$$

with $T_t = \exp(m + \theta\sum_{k=1}^{K}\varphi_k(\omega_1, \omega_2)X_{t-k})$ as found in (2.19). $\ln T_t$ represents the logarithm of the time-varying component of the volatility process at time t, m is a constant term that represents the intercept or baseline level of the logarithmic volatility component, θ quantifies the effect of the low-frequency factors on long-term volatility, $\sum_{k=1}^{K}$ is the summation runs over K lagged values of the explanatory variables, K is the maximum lag considered in the model, $\varphi_k(\omega_1, \omega_2)$ represents the weighting function applied to the lagged values of the explanatory variables. The weights depend on parameters ω_1 and ω_2, which are often used to control the shape and decay of the weights. Common choices for $\varphi_k(\omega_1, \omega_2)$ include the Beta polynomial or exponential decay functions, X_{t-k} are the lagged values of



(a) Time series plot of daily stock returns

(b) Time series plot of monthly exchange rate



(c) Time series plot of monthly consumer price index

(d) Time series plot of monthly treasury bill rate

(e) Time series plot of monthly money supply

(f) Time series plot of oil price

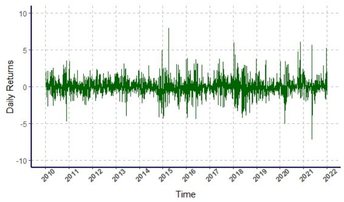
Figure 1: *Daily plot of stock returns and monthly economic variables for South Africa.*

the explanatory variable X at time $t - k$. The variable X are the relevant economic micro-economic variables that affects the long-term component of volatility.
In this study, we adopted the Beta polynomial weighting scheme to filter the macroeconomic variables specified in (2.20) by:

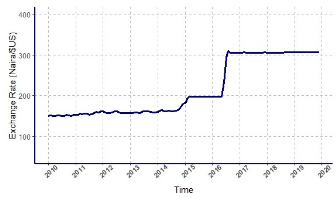
$$\varphi_k(\omega_1, \omega_2) = \frac{\left(\frac{k}{K}\right)^{\omega_1 - 1}\left(1 - \frac{k}{K}\right)^{\omega_2 - 1}}{\sum_{j=1}^{K}\left(\frac{j}{K}\right)^{\omega_1 - 1}\left(1 - \frac{j}{K}\right)^{\omega_2 - 1}}. \tag{2.20}$$

A flexible or unrestricted Beta smoothing scheme can be used to estimate ω_1, ω_2 in (2.20). These are weights to be computed in the Beta polynomial function as a weighted factor of low-frequency economic series. k denotes the number of lag periods. In these situations, K represents the maximum lag order of the series under consideration and defines the length of the time window over which the lagged values are considered. MIDAS regressions and their input variables are independently smoothed by the Beta polynomial smoothing. $\varphi_k(\omega_1, \omega_2)$are the Beta polynomial function which is the weight corresponding to the macroeconomic variables which is contained in the lag period of k. This study employed a restricted version of the Beta polynomial scheme by fixing ω_1 to 1 and ω_2 is estimated as ω to determine the decaying pattern. A high value of ω indicates rapid decay, whereas a low value indicates slow decay. Therefore, the two two-parameter polynomials now become a single parameter beta polynomial and are defined in (2.21) as:

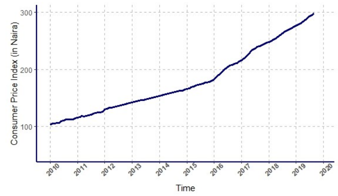
$$\varphi_k(\omega_2) = \frac{\left(1 - \frac{k}{K}\right)^{\omega_2 - 1}}{\sum_{j=1}^{K}\left(1 - \frac{j}{K}\right)^{\omega_2 - 1}}. \tag{2.21}$$



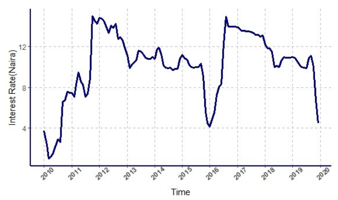
(a) Time series plot of daily stock returns



(b) Time series plot of monthly exchange rate

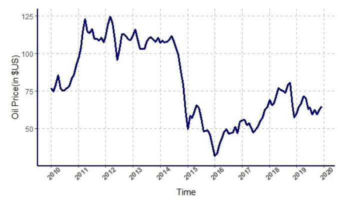


(c) Time series plot of monthly consumer price index



(d) Time series plot of monthly treasury bill rate

(e) Time series plot of monthly money supply



(f) Time series plot of oil price

Figure 2: *Daily plot of stock returns and monthly economic variables for Nigeria.*

Based on a variance ratio analysis, it can be determined what influence each macroeconomic variable has on total conditional volatility (Engle *et al.*, 2013). To determine whether large or small swings in the stock market are driven by macroeconomic variance, it is important to assess the importance of the variance ratio of each of the exogeneous variables. As a result, (2.22) is used to calculate the variance ratio.

$$\mathrm{V(R)} = \frac{\mathrm{Var}[\ln(T_t^X)]}{\mathrm{Var}\left[\ln(T_t^X \cdot g_{i,t}^X)\right]}, \tag{2.22}$$

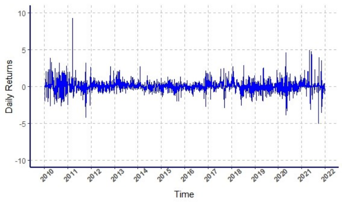
where $V(R)$ represents the variance ratio.

2.8. GARCH-MIDAS model estimation process

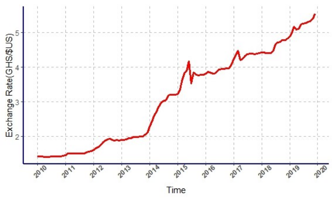
In the GARCH-MIDAS model, the parameters are estimated using the maximum likelihood estimation method. An estimation of log-likelihood seeks to determine the parameter value that is most similar to the value predicted by the actual data. The log-likelihood function is then estimated by estimating the parameters that maximize it. Given that $r_{i,t} = \mu + \sqrt{T_t g_{i,t}}\varepsilon_{it} \ \forall i = 1, 2, \ldots, N_t$ which specifies the parameters involved in the GARCH-MIDAS equation to be estimated, the log-likelihood function is represented by $\mathcal{L} = \ln L(\mu, \alpha, \beta, \theta, \omega)$ for restricted version with the log-likelihood function for the estimation. From (2.14),

Assuming the errors $\varepsilon_{i,t}$ are normally distributed, the conditional density is given by (2.23):

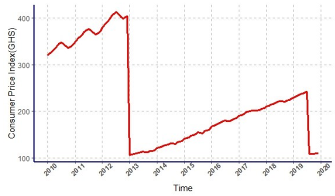
$$f(r_{i,t} \mid \mathcal{F}_{t-1}) = \frac{1}{\sqrt{2\pi T_t g_{i,t}}} \exp\left(-\frac{(r_{i,t} - \mu)^2}{2T_t g_{i,t}}\right). \tag{2.23}$$



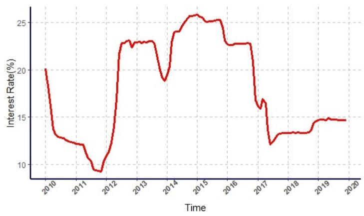
(a) Time series plot of daily stock returns



(b) Time series plot of monthly exchange rate



(c) Time series plot of monthly consumer price index



(d) Time series plot of monthly interest rate

(e) Time series plot of monthly money supply

Figure 3: *Daily plot of stock returns and monthly economic variables for Ghana.*

The log-likelihood function for a sample of T observations is given in (2.24) by:

$$\mathcal{L} = \sum_{t=1}^{T} \sum_{i=1}^{N_t} \log f(r_{i,t} \mid \mathcal{F}_{t-1}). \tag{2.24}$$

Substituting the density function yields (2.25):

$$\mathcal{L} = -\frac{1}{2} \sum_{t=1}^{T} \sum_{i=1}^{N_t} \left[\log(2\pi) + \log(T_t g_{i,t}) + \frac{(r_{i,t} - \mu)^2}{T_t g_{i,t}} \right]. \tag{2.25}$$

The parameters $\mu, \alpha, \beta, m, \theta, \omega$ are estimated by maximizing the log-likelihood function using some numerical optimization techniques.

3. Model evaluation

In order to assess the predictive power of the different models, we employed the following loss functions in evaluating the model's forecasting accuracy.

3.1. Mean square error (MSE)

In this statistical loss function, large deviations between predictions and actual values are given more weight. Outliers can adversely affect the MSE estimate, so if the prediction is substantially different from the observed value, the MSE given by (3.1) will be high.

$$\text{MSE} = \frac{1}{N} \sum_{t=1}^{N} \left(\sigma^2_{t+1} - \hat{\sigma}^2_{t+1} \right)^2, \tag{3.1}$$

Table 2: Summary statistics for stock return & economic variables of each country

Variables	Obs	Mean	Sd	Median	Min	Max	Skew	Kurtosis
South Africa								
Returns	2,574	0.00	1.42	0.03	−8.37	7.73	−0.22	1.64
INF	120	11.18	2.84	11.52	6.74	16.37	−0.13	−1.43
EXR	120	128.48	19.50	126.32	98.17	161.15	0.09	−1.31
IR	120	6.31	0.86	6.21	4.92	7.61	−0.08	−1.54
OP	120	80.01	25.73	75.80	31.93	124.62	0.09	−1.44
MS	120	2,787,167	551,837	2,708,945	1,924,798	3,806,876	0.18	−1.22
Nigeria	**Obs**	**Mean**	**Sd**	**Median**	**Min**	**Max**	**Skew**	**Kurtosis**
Returns	2,581	0.01	1.01	0	−4.66	7.98	0.28	4.88
INF	120	183.07	59.25	165.09	103.13	307.47	0.54	−0.99
EXR	120	215.78	68.05	181.06	150.08	309.73	0.5	−1.64
IR	120	10.21	3.25	10.79	1.04	15	−0.94	0.52
OP	120	80.01	25.73	75.8	31.93	124.62	0.09	−1.44
MS	120	18,560,673	5,375,874	18,458,138	10,446,374	29,137,800	0.2	−1.17
Ghana	**Obs**	**Mean**	**Sd**	**Median**	**Min**	**Max**	**Skew**	**Kurtosis**
Returns	2,461	−0.02	3.90	0.005	−30.5	21.49	−46.20	61.75
INF	120	209.93	99.16	179.35	106.5	412.4	0.80	−0.82
EXR	120	3.57	1.55	3.84	1.42	5.95	−0.05	−1.44
IR	120	17.16	5.10	14.70	9.25	25.83	0.41	−1.41
OP	120	76.13	25.95	71.67	26.63	124.62	0.21	−1.2
MS	120	40,282.10	27,410.45	33,204.02	7,753.02	105,997.55	0.72	−0.59

Note: The return is the Natural Log of first difference of daily stock index and monthly economic variables for the three Africa countries from January, 2010 to December, 2019.

where N stands for the number of observations in the out-of-sample data, σ^2_{t+1} is realized variance which is the proxy for actual volatility and $\hat{\sigma}^2_{t+1}$ is the forecast from the model.

3.2. Mean absolute error (MAE)

The MAE given by(3.2) is a measure of the average magnitude of the difference between the actual loss and the forecast loss. To calculate the error, the average of the absolute variation between the actual value and the forecast value is taken. The robustness of this measure comes from the fact that it is not heavily influenced by outliers.

$$\text{MAE} = \frac{1}{N}\sum_{t=1}^{N}\left|\sigma^2_{t+1} - \hat{\sigma}^2_{t+1}\right|, \tag{3.2}$$

where σ_{t+1} represent actual value and $\hat{\sigma}_{t+1}$ represents the forecast from the model.

3.3. Heteroscedasticity-adjusted MSE (HMSE)

The HMSE represented in (3.3) is a non-linear statistical loss function. It serves as heteroscedasticity-adjusted version of mean square error.

$$\text{HMSE} = \frac{1}{N}\sum_{t=1}^{N}\left(1 - \frac{\sigma^2_{t+1}}{\hat{\sigma}^2_{t+1}}\right)^2. \tag{3.3}$$

3.4. Heteroscedasticity-adjusted MAE

(HMAE) HMAE is the non-linear loss measure computed by (3.4) as:

$$\text{HMAE} = \frac{1}{N}\sum_{t=1}^{N}\left|1 - \frac{\sigma_{t+1}^2}{\hat{\sigma}_{t+1}^2}\right|. \tag{3.4}$$

4. Results and discussions

For South Africa (in Figure 1), the treasury bill rate, which serves as a substitute for interest rates, displayed downward movement between 2011 and 2014. In Nigeria (Figure 2), the interest rate displayed an irregular pattern, which is the same as the case with Ghana's interest rate. Money supply shows an upward trend for all the countries, indicating an increase in money supply for the period. The exchange rate for South Africa and Ghana (in Figure 3) demonstrates upward trend, indicating that their currencies are depreciating against the US dollar. On the other hand, the Nigerian Naira appears to be more stable between 2017 to 2019. Consumer price index, which is used as approximation for inflation, shows an upward and downward pattern for all countries.

4.1. Descriptive statistics

Table 2 presents summary statistics of raw data for macroeconomic indicators and the natural log for daily stock returns for the period spanning from 1/1/2010 to 12/31/2019. From the tables, it can be observed that the South African stock index has been more volatile (with a standard deviation of 1.42) compared to the Nigerian stock index, which recorded a standard deviation of 1.01. This indicates that the Nigerian equity market is more stable than the South African stock market. This is because the latter has a standard deviation value above the SA returns. When compared to counterpart markets, the standard deviation for the GSE-All Share Index is high, indicating that the Ghanaian stock market experienced greater volatility, and the average return is negative and insignificant. This is not surprising since the mean of log returns is always close to zero. The South Africa and Nigeria exchange index returns have positive and insignificant mean values. The average monthly inflation for Nigeria is 183.07 and for South Africa is 11.8. The inflation cannot be compared between the two countries because the study used the consumer price index as an equivalent for inflation, which is computed in their respective local currencies. The expected monthly exchange rate for Nigerian Naira per US dollar is N215.78 and the average exchange rate for the South African Rand against the US dollar is R128.48. Furthermore, South Africa's interest rate is low (6.3%), compared to a higher average of 10.21% in Nigeria and 17% in Ghana. For South Africa and Ghana, there is a negative skewness of stock returns.

From Table 3, we observe that in Ghana, there is a negative relationship between the exchange rate and inflation. They move in the opposite direction; whereas money supply and inflation have a positive relationship, which shows an increase in money supply leads to a corresponding increase in inflation and vice versa. This implies that as the money supply grows, inflation increase as well. Exchange and inflation both move in the same direction, so inflation rises when the value of the cedi declines. Money supply and exchange rate have a strong and positive relationship. The results of fitting an appropriate ARMA model is given in Table 5.

4.2. ARCH-LM Test

A heteroscedastic test was conducted on the selected model to determine whether ARCH effects exist. By using the ARCH-LM test, the heteroscedasticity test is conducted on 12 lags of residuals of the chosen ARMA and GARCH models for all countries. The observe p-values of the model indicate

Table 3: Correlation between the macroeconomic variables of each country

South Africa	INF	EXR	IR	MS	OP
INF	1				
EXR	0.916	1			
IR	0.913	0.997	1		
MS	0.649	0.682	0.679	1	
OP	−0.788	−0.647	−0.653	−0.781	1
Nigeria	INF	EXR	IR	MS	OP
INF	1				
EXR	0.934	1			
IR	0.263	0.33	1		
MS	0.979	0.908	0.267	1	
OP	−0.591	−0.655	0.119	−0.638	1
Ghana	INF	EXR	IR	MS	OP
INF	1				
EXR	−0.62	1			
IR	−0.34	−0.18	1		
MS	0.54	0.95	−0.35	1	
OP	0.46	0.72	−0.01	−0.57	1

Table 4: Lagrange-multiplier test and Box-Ljung test results of each country

ARCH-LM			Box-Ljung test	
Series	T-statistic	P-value	Q	P-value
South Africa				
ARMA (1,0)	150	0.0000	0.0145	0.904
GARCH (1,2)	239.77	2.2e-16	2.6322	0.1054
Nigeria				
ARMA (1,0)	148	0.0000	0.0145	0.9093
GARCH (1,2)	239.9	2.2e-16	2.2899	0.1308
Ghana				
ARMA (2,2)	507	0.0000	0.05266	0.8185
GARCH (1,3)	613.66	2.2e-16	18.362	1.83e-05

that it is the most appropriate fit because the residuals of the fitted models show ARCH effects. For all market indices, the ARCH-LM test rejects the null hypothesis of no ARCH effects, indicating that there are no serial correlations. Based on the results, the series exhibits volatility clustering, therefore it is suitable for implementation using GARCH family models. Moreover, In order to assess the possibility of serial correlation in the residuals of appropriate ARMA (p,q) and GARCH (p,q) models, the Box-Ljung test was performed. The results of the test on ARMA (p,q) and GARCH (p,q) rejected the null hypothesis of no ARCH effects for all countries.

The results from 5 indicate that ARMA (1,0), ARMA (1,0) and ARMA (2,2) models are best suited for South Africa, Nigeria and Ghana, respectively. For each country, this is determined by ARMAs (p, q) fitted to returns depending on a minimum AIC and BIC. According to this, the lag return of one period affects volatility on the South African and Nigerian stock markets when using the ARMA model. Whereas there are consequences of a lag return of two periods for the volatility of the Ghanaian stock exchange. Hence, Ghana's stock exchange has a longer memory than those in South Africa and Nigeria.

4.3. Fitting ARMA-GARCH model

The study experimented with every potential pairing of the ARMA and GARCH approach in order to find the best suitable one. Based on the results of fitting ARMA (p,q) - GARCH (p,q), the best

Table 5: Results of fitting appropriate ARMA model of each country

South Africa			**Nigeria**			**Ghana**		
ARMA(p, q)	AIC	BIC	ARMA(p, q)	AIC	BIC	ARMA(p,q)	AIC	BIC
(0,1)	9108.36	9125.92	**(1,0)***	**7222.58**	**7240.15**	**(2,2)***	**5166.11**	**5200.95**
(1,0)*	**9108.36**	**9125.92**	(2,0)	7224.28	7247.7	(2,1)	5168.82	5197.85
(2,0)	9110.36	9133.77	(1,1)	7224.28	7247.7	(3,1)	5168.87	5203.71
(0,2)	9110.36	9133.77	(0,2)	7224.86	7248.28	(3,3)	5168.94	5215.39
(1,1)	9110.36	9133.77	(2,1)	7226.28	7255.56	(3,0)	5242.98	5272.01
(1,2)	9112.33	9141.6	(1,2)	7226.28	7255.56	(0,3)	5249.79	5278.82
(2,1)	9112.35	9141.62	(2,2)	7228.2	7263.34	(2,0)	5258.60	5281.83
(0, 4)	9113.74	9148.86	(0,1)	7231.66	7249.22	(0,2)	5261.22	5284.45
(4,0)	9113.84	9148.96	(2,3)	7228.36	7269.35	(1,0)	5275.37	5292.78
(3,1)	9113.86	9148.98	(3,1)	7228.15	7263.29	(0,1)	5275.37	5292.79

*Indicates the best fitted ARMA (p,q) model based on minimum AIC and BIC model selection criteria.

Table 6: Result of fitting appropriate ARMA-GARCH model of each country

South Africa			**Nigeria**			**Ghana**		
ARMA(1,0)-GARCH(p,q)	AIC	BIC	ARMA(1,0)-GARCH(p,q)	AIC	BIC	ARMA(2,2)-GARCH(p,q)	AIC	BIC
(1,1)*	**2.4713**	**2.499**	(1,1)	2.6094	2.6207	(1,1)	1.8164	1.8424
(1,2)	2.6022	2.6294	**(1,2)***	**2.6028**	**2.616**	(1,2)	1.8162	1.8446
(1,3)	2.6009	2.6304	(1,3)	2.6029	2.6187	**(1,3)***	**1.8082**	**1.8390**
(2,1)	2.6068	2.634	(2,1)	2.6101	2.6238	(2,1)	1.8109	1.8392
(3,1)	2.6076	2.6371	(3,1)	2.6109	2.6267	(3,1)	1.8117	1.8424
(2,2)	2.6030	2.6325	(2,2)	2.6036	2.6195	(2,2)	1.8176	1.8412

Note: *indicates appropriate fitted ARMA (p,q)-GARCH (p,q) model based on minimum AIC and BIC model criteria.

fitted models for each country were determined to be the ARMA (1,0) - GARCH (1,1), ARMA (1,0) - GARCH (1,2), and ARMA (2,2) - GARCH (1,3) models for South Africa, Nigeria, and Ghana, respectively, taking into consideration the goodness of fit test. The GARCH model for Ghana with higher-order terms indicates that the Ghanaian stock exchange has higher memory, and it is necessary to include more lags to explain how past information influences current market behavior. As shown in Table 6, these models displayed the least information criteria among all the fitted models.

4.4. ARMA-GARCH parameter estimate results

we present the volatility of Nigeria Stock Market, Johannesburg and Ghanaian Stock Market for the in-sample period. The fitted GARCH (p, q) parameters are estimated for each country. As the competing models, GARCH coefficients are estimated without the inclusion of exogenous variables. Table 7 contain the estimate of conditional variance and mean equation. It has been shown that almost all parameters calculated using the best fitting ARMA-GARCH models are significant for all countries at the 1% levels with the exception of the mean which statistical insignificant. The intercept from the variance equation is significant at the 1% level. There is statistical significance at the 1% level for the mean equation terms AR(1), AR(2), MA(1) and MA(2) in all three markets. Additionally, statistical significance is observed for the coefficients β_1 that assess volatility persistence. Combining ARCH and GARCH results in a sum roughly equal to one and close to unity for all three countries. This implies that volatility clustering is common across all the stock exchanges studied and expected to be triggered by past conditional variance for South Africa since the ARCH coefficient is smaller than the GARCH coefficient (0.1539 < 0.7791). On the contrary the ARCH parameter estimate is greater than the GARCH (1) parameter estimates for Nigeria and Ghana. This shows the weight of the past period shocks is associated with the variance of the current period residual.

Table 7: Parameter estimates for fitted ARMA-GARCH model of each country

South Africa				
Coefficients	Estimates	Std Error	T-value	P-value
μ	−0.0129	0.0199	−0.64814	0.5169
ϕ_1	0.1939	0.0227	8.5471	0.0000
ω	0.0685	0.0144	4.7533	0.0000
α	0.1539	0.0214	7.1834	0.0000
β	0.7791	0.0315	24.7595	0.0000
Nigeria				
Coefficients	Estimates	Std Error	T-value	P-value
μ	−0.00879	0.0199	−0.4412	0.6591
ϕ_1	0.1961	0.0228	8.589	0.0000
ω	0.0883	0.0166	5.323	0.0000
α	0.2055	0.0242	8.4771	0.0000
β_1	0.1774	0.0772	2.2964	0.0217
β_2	0.5285	0.0766	6.9009	0.0000
Ghana				
Coefficients	Estimates	Std Error	T-value	P-value
μ	−0.00599	0.0242	−0.2477	0.8043
ϕ_1	1.6145	0.0144	112.253	0.0000
ϕ_2	−0.6387	0.0143	−44.712	0.0000
θ_1	−1.5676	0.000033	−47606.98	0.0000
θ_2	0.6215	0.000126	4933.13	0.0000
ω	0.0274	0.0059	4.648	0.0000
α	0.2465	0.0312	7.913	0.0000
β_1	0.1854	0.0811	2.286	0.0223
β_2	0.1723	0.0516	3.337	0.0008
β_3	0.3668	0.0551	6.657	0.0000

4.5. GARCH-MIDAS results

The estimated coefficients of the GARCH-MIDAS model are presented in Table 8. The result was classified for each country and how each macroeconomic variables affect volatility in each country. The lags periods included in each model is based on the fast-decaying pattern of pictorial or visual presentation of various Beta polynomial scheme. Based on the probabilistic analysis for all possible choice, 36 months lag periods of K are included in the MIDAS filter for all macroeconomic variables for South Africa. In Nigeria 36 months lag periods of K was included in the MIDAS filter with exception of inflation which achieves 24 months of K in the MIDAS weighting scheme. Ghana's data exhibit slight deviation from her peers, as 48 months lag periods of K fit inflation, 24 months lag period of K for interest rate and 36 months lag periods of K fit exchange rate, oil price and money supply. These results were arrived at taking into consideration minimum Bayesian Information Criteria and maximum log-likelihood values (Table 8). The magnitude impact of various macroeconomic variables is measured as a percentage change effect of these variables on the volatility using $e^{\theta\frac{\psi(\omega)}{k}-1}$ (Engle *et al.*, 2013), where θ represents the directional effect of macroeconomic series, $\psi(\omega)$ denotes the weight assigned to the lags and k signifies the number of lags used for the parameter estimation. Usually, the smaller the weight, the slower the wind-off pattern, while the higher the weight, the faster the wind-off pattern. The short-run parameters in the model (α and β) sum up to 1.

4.5.1. Discussion on inflation

The short run parameters(α and β) are statistically significant at 5%, indicating that the short-run term components are highly volatile when inflation is included in the estimation of GM parameters

Table 8: Parameter estimate for GARCH-MIDAS model of each country

SA	μ	α	β	m	θ	ω_2	VR	LL	BIC
EXR	0.00859	0.03585***	0.9574***	0.10676	5.52692*	1.00*	50.647	−3115.8	6283.59
	(0.03147)	(0.00857)	(0.01108)	(0.33803)	(3.02656)	(0.5752)			
INF	0.0103**	0.0354**	0.958**	−3.2854	6.9165	1.2267*	49.46	−3119.3	6276.62
	(0.0307)	(0.0074)	(0.0091)	(1.3518)	(2.4774)	(0.1633)			
IR	0.0086	0.0378***	0.9531***	0.4928**	8.3078**	1.0808***	39.5	−3119.4	6283.69
	(0.0309)	(0.009)	(0.0117)	(0.1743)	(4.1317)	(0.2768)			
OP	0.0075	0.0402***	0.9497***	0.4748***	−0.2403*	1.5181***	36.89	−3120.5	6285.93
	(0.0312)	(0.0101)	(0.0134)	(0.1748)	(0.1669)	(0.3306)			
MS	0.0084	0.0369***	0.9554***	0.3513	61.044	2.4503***	0.22	−3122.1	6289.16
	(0.0316)	(0.0094)	(0.0137)	(0.5906)	(0.8811)	(0.3378)			
NIG	μ	α	β	**m**	θ	ω_2	**VR**	**LL**	**BIC**
INF	−0.033*	0.166**	0.78***	0.06	0.102	1.014	1.34	−2445	4935.04
	(0.217)	(0.0582)	(0.0836)	(0.4369)	(0.0034)	(1.7474)			
EXR	−0.0395*	0.1562***	0.8017***	0.3148	−0.0300*	253.56***	10.75	−2438.8	4922.54
	(0.0234)	(0.044)	(0.0573)	(0.3103)	(0.0174)	(36.23)			
IR	−0.0354*	0.1661***	0.779***	0.2349	0.4406	1.4409***	2.94	−2444.5	4933.94
	(0.0219)	(0.0582)	(0.0837)	(0.2562)	(0.5059)	(0.3367)			
OP	−0.0418**	0.1741***	0.7626***	0.1419	0.0641**	253.86***	20.77	−2428.8	4902.61
	(0.0212)	(0.0549)	(0.0833)	(0.2305)	(0.0221)	(74.4028)			
MS	−0.00334	0.1832	0.7519	0.8639	−152.12	1.00	7.03	−2442	4928.87
	(0.0742)	(0.3336)	(0.7908)	(1.1453)	(452.23)	(9.7467)			
GH	μ	α	β	**m**	θ	ω_2	**VR**	**LL**	**BIC**
INF	0.004513	0.1539*	0.8306***	0.4390	0.2304*	1.00	67.68	−1339	2722
	(0.0334)	(0.0903)	(0.1004)	(0.786)	(0.8116)	(6.0352)			
EXR	0.0348*	0.1591***	0.8281***	−0.0102	1.4278*	114.32	3.59	−1529	3102
	(0.023)	(0.0514)	(0.0629)	(0.7744)	(1.045)	(255.8)			
IR	0.02969*	0.1968*	0.7805***	0.00728*	-1.7141*	1.0001***	31.8	−1527	3099
	(0.0207)	(0.1088)	(0.1292)	(0.8215)	(1.0594)	(0.3236)			
OP	0.0155***	0.5159**	0.7485**	−1.0298	36.33**	1.7717	24.78	−1313	2672
	(0.0124)	(0.0392)	(0.0704)	(0.1526)	(15.678)	(0.3758)			
MS	0.0315*	0.1396*	0.8455***	5.711	−71.59	1.0574***	42.8	−1519	3082
	(0.0203)	(0.104)	(0.1224)	(5.079)	(0.2614)	(57.29)			

Note: ***(significant at 1%), **(significant at 5%) and *(significant at 10%). Values in the bracket represent Bollerslev-Wooldridge standard errors. Where the value with the asterisks is the estimated coefficient of short-run (α) and long-run (β) components of GARCH-MIDAS model, $V(R)$ represents the variance ratio and LL is the log-likelihood function.

for South Africa. This shows a pattern of volatility clustering, where periods with high volatility extend into subsequent periods and periods with low volatility extend into succeeding periods. The coefficient θ is positive and significant for South Africa, showing a direct relationship, which means that increasing inflation is associated with greater stock volatility. Additionally, inflation contributes 49.46% to the total conditional variance, as computed by the variance ratio. Thus, a growth in inflation of 1% for the previous period would increase volatility by 26.59% for the next period. This result corresponds with the result of similar work of Engle *et al.* (2013) that inflation is positively related with the long-term component of the market return. For Nigeria, the MIDAS slope parameter θ is positive but not significant. As a result, inflation and stock market volatility are not directly related. An insignificant slope parameter implies that inflation has an insignificant effect on the long-term volatility of the Nigerian equity market. This outcome is congruent with that obtained in 2017 by Tarza Sokpo *et al.* (2017) who similarly discovered that there was no substantial relationship between inflation and Nigerian stock returns. For Ghana, there is a 67.68% contribution to volatility from inflation. It is anticipated that inflation will contribute a significant amount to total volatility. This is attributed to the fact that the country experienced high inflation from 2012 to 2017.

4.5.2. Discussion on exchange rate

Conditional volatility in South Africa is 50.65% explained by the exchange rate. Compared to other countries, the Johannesburg Stock Exchange is highly impacted by the volatility of the exchange rate. θ is positive and significant at the 10% level, which suggests the depreciation of the South African Rand against the US dollar accelerates the volatility of the stock market. Thus, the incremental effect of the exchange rate on the long-term component of the GM model shows that 1% depreciation of the South Rand against the US dollar in the past period resulted in 16.61% surge in the long-term volatility component of Johannesburg stock returns.The situation is a bit different in Nigeria. The coefficient θ is negative and significant at the 10% level. This revealed that the exchange rate can reduce volatility, implying that depreciation of the Nigerian Naira against the US dollar results in a deceleration of long-term component volatility; hence, 1% depreciation of the Naira against the US dollar reduces the volatility of the long-term component by 19.05%. By indicating a negative slope coefficient, the depreciating Nigerian naira is causing stocks to be less attractive to investors, therefore decelerating volatility. Currency depreciation could be caused by high import patronage, as is common in Sub-Saharan African countries. For Ghana, the exchange rate contributes so little to Ghana's stock market volatility since the nation has experienced instability in its local currency over the years. It has contributed to high uncertainty and higher prices, but it can also be said that the local currency enjoyed relative stability in 2010 and 2011. This may have accounted for the exchange rate's insignificant contribution to market volatility.

4.5.3. Discussion on interest rate

For South Africa, it has been determined that interest rates contribute 39.50% of volatility of stock market in South Africa. The coefficient θ, which indicates the directional effect of a variable in the model on the volatility of the long-run component is positive and significant at 10%. This suggests that interest rate increases can trigger high volatility of the long-run component. In terms of the magnitude of the directional effect, a 1% rise in interest rate in the previous month would cause 28.34% acceleration in volatility on the Johannesburg stock exchange in the following month. For Ghana, according to the observation, interest drove conditional volatility by 31.8%. This is expected due to the high interest rates witnessed in the country over the years, which make the cost of running a business high. Interestingly, the most relevant parameter θ is negative and significant at 10%. In this case, the slope value being negative means the high interest rate (T-bill rate) leads to low volatility.

4.5.4. Discussion on oil price

According to the empirical results for South Africa, observations on the proportion of the contribution of oil price to conditional volatility have shown that oil prices account for 36.89% of the variation on Johannesburg Stock Exchange. In Nigeria, the oil price drives inflation by 20%, as calculated by the variance ratio. This is anticipated because the oil price dictates the pace of economic activities in the country. In the case of Ghana, the oil price has a positive and 5% significant outcome for the MIDAS slope coefficient θ. The marginal effect of the positive coefficient reported a 1% hike in oil price at $t-1$ would raise long-term component volatility by 40% at time $t+1$.

Table 9: Results of out-of-sample forecast loss function for GARCH family models of each country

LOSS FUNCTIONS	GARCH	GM-INF	GM-EXR	GM-IR	GM-MS	GM-OP
South Africa						
MSE	174.96	170.16	169.97	168.87	168.85	168.85
MAE	4.32	3.94	3.96	3.98	3.99	3.98
HMSE	168.33	146.86	144.34	104.5	105.5	104.37
HMAE	3.9	3.54	3.44	2.97	2.95	2.96
Nigeria						
MSE	632.02	629.35	630.2	629.35	629.18	630.09
MAE	3.56	3.26	3.05	2.88	3.3	3.07
HMSE	635.18	314.55	519.38	932.9	287.67	490.01
HMAE	3.03	2.24	2.77	2.71	2.31	3.5
Ghana						
MSE	19.57	9.13	9.14	9.15	9.13	9.45
MAE	3.95	1.32	1.41	1.44	1.35	0.97
HMSE	85.17	11.05	8.47	7.82	1.13	9.99
HMAE	2.92	1.47	1.37	1.35	0.97	1.42

Note: GM; GARCH-MIDAS

4.5.5. Discussion on money supply

For South Africa, it was observed that the short-term coefficients (ARCH and GARCH terms) sum up to 0.9922 and are significant at 1%, demonstrating evidence of high volatility in the short-term component of FTSE/ALL index stock returns. It is interesting to note that the money supply makes an insignificant contribution of 0.22% to the total conditional volatility. The coefficient θ of the long-term component is positive but not significant. It is imperative to note that the Nigerian instance presents a negative value that is insignificant. For both the ARCH term and the GARCH term, there are no significant parameters. The MIDAS slope coefficient θ is negative but insignificant. All the estimated parameters show money supply do not have a significant impact on both short-run and long-run component volatility of the stock market. However, money supply accounts of 7.02% to total conditional volatility based on the variance ratio. For Ghana, the unconditional mean is positive and significant at 10% the level. The short-term parameters α and β are significant at the 10% and 1% levels, respectively. The weight is 1.05 and significant at the 1% level showing it considered the most recent observations in the estimation. It also revealed that the quantity of money in circulation contribute 42.8% to the total conditional variance in the stock market.

4.6. Out of sample evaluation for GARCH and GARCH-MIDAS family

The out-of-sample assessment of a model is one of the important areas in predictive modelling since stakeholders are concerned about the ability of the model to accurately forecast future market volatility. This section explores the evaluation of the univariate GARCH as the benchmark and the bivariate GARCH-MIDAS model. It presupposes that incorporating macroeconomic conditions will have any significant impact on the model's performance. Table 9 reports the comparative performance of six different statistical loss functions. The assessment measures the extent of variation between forecasted and actual volatility (proxied as realized variance). Since volatility is hard to observe in empirical setting, we used realised volatility as a substitute for observed volatility. The lower value indicates a lower variation between the actual value and forecasted volatility. Thus, the model that produced the lower statistical loss function has better predictive performance. The result of the evaluation shows the model of GARCH-MIDAS produces improve predictive performance compared to the traditional GARCH model. Models with exogenous variables result in lower loss functions than the benchmark GARCH model. All the loss functions used provide a smaller minimum error for GARCH-MIDAS than the GARCH. The GARCH-MIDAS better predicts the realized volatility for all variables than the traditional GARCH framework. The results of this study are in agreement with those obtained by Conrad and Loch (2015) and Engle *et al.* (2013).

5. Conclusions and recommendations

There is some evidence that the macroeconomic environment has some impact on business performance in Sub-Saharan Africa particularly Ghana, Nigeria, and South Africa (Benson *et al.*, 2022; Nyeadi, 2023). GARCH family models were used to perform the analysis, with varying results due to the models varying behavior patterns. According to the results, the GARCH-MIDAS models tend to outperform the simple GARCH model. The crude oil price is found to have a significant impact on stock returns in all three countries, and its impact is stronger than that of other macroeconomic variables. Although macroeconomic variables like money supply, interest rates, and inflation play a key role in economic conditions in Africa, crude oil prices tend to dominate the conversation. The continent's economies are characterized by resource dependence (Usman and Landry, 2021), so oil export revenue substantially influences government spending, infrastructure development, and investment opportunities (Adegboye and Akinyele, 2022). A plausible reason would be the high dependence of African countries on crude oil importation, despite the fact that most of these countries are oil producing economies. Consequently, oil prices influence stock returns more than other macroeconomic factors. The outcome of this research provides some insightful perspective on the extent to which macroeconomic policies exert favorable or adverse impacts on activity in the equity market. Sub-Saharan Africa policymakers need to focus on the real sectors of the economy in order to regain macroeconomic stability, as oil price, especially international price of crude oil, and foreign exchange rate are the main factors destabilizing these economies that have a source of shocks to the business environment. To save their economies destabilizing shocks emanating from surging crude oil price, investors should also invest in cheaper and alternative energy sources or improve their ability to refine their own crude. As oil-exporting nations grapple with global oil market dynamics, understanding the complex interplay between oil prices, macroeconomic variables, and investor sentiment becomes paramount for sound investment strategies and effective economic policies. Stabilizing their currencies requires economic and structural reforms as well as reducing reliance on foreign goods. A vigorous policy of controlling the surging demand for foreign products should be pursued across Sub-Saharan Africa. The findings of this research is evident that the macroeconomic environment exerts a discernible influence on business performance within Sub-Saharan Africa, with particular focus on Ghana, Nigeria, and South Africa. Notably, crude oil prices emerged as a pivotal factor, exerting a more pronounced impact on stock returns than other macroeconomic variables. This phenomenon underscores the resource-dependent nature of the continent's economies and the preeminent role of oil export revenues in shaping governmental fiscal capabilities and investment landscapes. For investors, the findings suggest a critical need to factor in oil price volatility when constructing portfolios in these markets. Policymakers are advised to derive actionable strategies that mitigate the destabilizing effects of oil price fluctuations. This study has shed more light on the true potential of GARCH-MIDAS and its implications for understanding and predicting stock returns in modern financial markets. Future research could pivot towards exploring the integration of additional macroeconomic indicators that may influence stock market dynamics, such as technological innovation or geopolitical factors

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The authors report there are no competing interests to declare.

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