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Relativistic three-body effects in black hole coalescence

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(Received 27 September 2005; published 18 October 2006)

Three-body interactions are expected to be common in globular clusters and in galactic cores hosting supermassive black holes. We consider an equal-mass binary black hole system in the presence of a third black hole. Using numerically generated binary black hole initial data sets, and first and second-order post-Newtonian (1PN and 2PN) techniques, we find that the presence of the third black hole has non-negligible relativistic effects on the location of the binary's innermost stable circular orbit (ISCO), and that these effects arise at 2PN order. For a stellar-mass black hole binary in orbit about a supermassive black hole, the massive black hole has stabilizing effects on the orbiting binary, leading to an increase in merger time and a decrease of the terminal orbital frequency, and an amplification of the gravitational radiation emitted from the binary system by up to 6%.

DOI: 10.1103/PhysRevD.74.087503 PACS numbers: 04.25.Dm, 04.25.Nx, 04.30.Db, 04.70.Bw

Gravitational radiation, as predicted by general relativity, plays an important observable role in the relativistic dynamics of astrophysical systems. Collisions of compact objects, such as neutron stars and black holes, produce characteristic gravitational wave signals that are observable up to very high redshifts. Binary black hole systems are the prime scientific target source of gravitational waves for both the current generation of ground-based detectors, such as LIGO [1], and for the next generation of space-based detectors, such as LISA [2].

Theoretically accurate models of black hole coalescence based on the theory of general relativity are expected to provide crucial information for the interpretation of these gravitational wave observations. So far, all numerical models have focused on isolated binary black hole systems. However, in a realistic scenario it is reasonable to expect that some of these black hole binaries will, at some point in their lifetime, interact gravitationally with a third compact object. The presence of the third body may influence the evolution and gravitational radiation emission of the black hole binary during the inspiral and merger phase. This may produce some important observational effects on the gravitational waveforms.

Relativistic three-body interactions are expected to play an important role in astrophysical scenarios including (1) multibody interactions in high-density cores of globular clusters [3,4], (2) stellar-mass binary black hole systems interacting with a central supermassive black hole, and (3) hierarchical triples of massive black holes that might be formed in the nuclei of galaxies undergoing sequential mergers [5,6]. In the globular cluster scenario, a possible mechanism to produce hierarchical triple systems is through binary-binary interactions [4,7–9]. At least 20%–50% of binary-binary encounters may actually result in a stable hierarchical three-body system [4]. Estimates of the lifetime of a triple system are made in [8], where it is noted that a three-body system can survive several hundred times longer than the orbital period of one of the original binaries [5]. Further astrophysically motivated studies of three-body systems have been considered in Refs. [10–12].

In this paper we consider the effect of the presence of a third black hole on the location of the innermost stable circular orbit (ISCO) of a binary nonspinning black hole system. We use numerical and post-Newtonian techniques to study two three-body scenarios: (1) a test case, in which the third black hole has comparable mass to each black hole in the binary, and for simplicity is stationary, and (2) the more astrophysically realistic case of a stellar-mass black hole binary in orbit about a supermassive black hole. Scenario (2) could be treated using only post-Newtonian methods, but in scenario (1) we find that calculations at 2PN accuracy give results consistent with the fully general-relativistic numerical approach.

Method: We consider two black holes separated by a distance d, and assumed to be in quasicircular orbit. A third black hole is located on the axis of rotation of the binary, a distance l from the origin. Each black hole in the binary has equal and opposite momentum P, and the total angular momentum of the binary is $J = Pd$. In the first set of results the third black hole is instantaneously stationary; we later relax this assumption.

Each black hole in the binary has mass m. When referring to numerical calculations, m corresponds to the bare mass of each black hole, as defined later; in analytic post-Newtonian calculations m is the Newtonian mass of the body. The third body has mass m_3. The total mass of the three black holes is $M = 2m + m_3$. In numerical calculations, the total Arnowitt-Deser-Misner (ADM) mass of the spacetime (which will differ from M), is denoted by M_{ADM}.

We model this configuration using two approaches. The first is to consider numerically generated initial data sets, and apply the techniques described in [13,14] to locate quasicircular orbits and the ISCO of the binary. The second approach is to use post-Newtonian calculations.

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Quasicircular orbits and the ISCO have been identified for binary systems (see, for example, [15,16]), and we extend this procedure to a three-body system.

We first outline the numerical initial-data approach. Black hole initial data consist of solutions to the four initial-value equations that result from the (3 + 1) decomposition of Einstein's equations. These equations are the Hamiltonian and momentum constraints for the spatial metric γ_{ij} and extrinsic curvature K_{ij} on one time slice [17]. One way to solve the constraint equations is through the conformal transverse-traceless decomposition [17]. In this decomposition there exist analytic solutions of the momentum constraint, the Bowen-York solutions, that can describe any number of boosted black holes. In particular, we can write down a solution for two boosted black holes, plus a single stationary black hole (which is a trivial solution). The Hamiltonian constraint remains to be solved numerically, and we do this using the puncture approach of Brandt and Brügmann [18].

Given the freedom in specifying the orbital parameters of the initial data sets for black hole spacetimes, we would like to identify which of those sets represents two black holes in quasicircular orbit. To this end we implement the effective-potential method of Cook [13] in which minima are located in sequences of total ADM energy, $E_{\rm ADM}$, versus coordinate separation, d, for constant values of the orbital angular momentum of the binary, J, and individual black hole masses m and m_3. In this work the ADM mass calculated at each puncture is used to denote the bare mass of each black hole [19]. This procedure is carried out for many sequences until the lowest value of J that produces a sequence with a minimum is located. We denote this last minimum as the innermost stable circular orbit (ISCO) of the binary. Once the ISCO is located, its orbital angular frequency, Ω, is calculated via

$$\Omega = \frac{\partial E_{\rm ADM}}{\partial J}\bigg|_{m_i}. \tag{3}$$

In this approach we make a number of assumptions and approximations. We assume that the effective-potential method reliably yields parameters consistent with quasicircular orbits, and that the initial data we use are a reasonable approximation to the "correct" initial data for two black holes in orbit. In this numerical approach we ignore entirely any motion of the third black hole: we consider only the effect of a stationary third black hole on the instantaneous binding energy of the binary system, and make the loose assumption that its effect on the locations of quasicircular orbits will be indicative of the effect of a third black hole passing near the binary. Our purpose is to determine the qualitative effect of the presence of a third body on the ISCO of a binary.

We now turn to the procedure to find quasicircular orbits and the ISCO in post-Newtonian theory. The ISCO for two-body systems has been calculated using post-Newtonian methods by a number of authors [15,16]. The approach can be generalized to three bodies and compared with our numerical results. The post-Newtonian approach has the advantage that it is straightforward to generalize to cases where the binary is in circular orbit about the third body.

In the two-body case, the post-Newtonian and numerical (Bowen-York data) results differ, but in this work we are interested in the differential effect of the presence of a third body, and whether that effect is comparable when calculated using numerical or post-Newtonian methods.

We locate the post-Newtonian ISCO using the procedure outlined in [16]. We start with the 2PN Hamiltonian for a three-body system, which can be found in [20–22]. The full three-body Hamiltonian [22] is a function of the masses, positions, and momenta of the three particles, $H(m_a, \mathbf{x}_a, \mathbf{p}_a)$; a total of 21 parameters. In the simple configuration described above, this number can be reduced to six: $m_1 = m_2 = m$ (an equal-mass binary), $\mathbf{x}_1 = (d/2, 0, 0) = -\mathbf{x}_2$, $\mathbf{x}_3 = (0, 0, l)$, $\mathbf{p}_1 = (0, P, 0) = -\mathbf{p}_2$, and $\mathbf{p}_3 = (P_3, 0, 0)$. In the first case we consider the third body is stationary and $P_3 = 0$. The orbital angular momentum of the binary is $\mathbf{J} = Pd\hat{\mathbf{z}}$. We have been careful to choose $\mathbf{p}_3 = P_3\hat{\mathbf{x}}$ (when it is nonzero), so that the angular momentum due to the third body is perpendicular to that due to the binary; this is essential for an unambiguous definition of the binary's angular velocity in terms of (3).

To identify circular orbits, one first locates minima order by order in the post-Newtonian expansion of the Hamiltonian. This procedure was performed semianalytically with *Mathematica*, to yield the separation d of a circular orbit to 2PN accuracy in terms of the total binary angular momentum, J. This value of d was inserted into post-Newtonian Hamiltonian to get the energy, $E_{\rm circ}$, of circular orbits to 2PN accuracy and to calculate the angular velocity of the orbit via (3), with $E_{\rm ADM}$ now replaced by $E_{\rm circ}$. This procedure was performed for different values of J, to produce a plot of $E_{\rm circ}$ versus Ω; the minimum in this plot corresponds to the ISCO. See [16] for an example of this algorithm in the case of a binary system. See also [23] for a review of the use of the post-Newtonian techniques to study the stability of neutron stars in a binary system.

Note that these methods tell us nothing about the stability of the "orbits" that have been identified, and indeed in general we do not expect a general-relativistic three-body system to be any more stable than its Newtonian counterpart. However, we can still identify parameters that meet the requirements of quasicircular orbits and suggest configurations that we reasonably expect to be stable. In particular, a stellar-mass black hole binary in orbit about a third supermassive black hole, which is the astrophysical scenario we ultimately wish to study.

Results: In the first configuration, the masses are $m = 1$, $m_3 = 0.5$, and the third body is stationary ($P_3 = 0$). The results of the numerical and post-Newtonian approaches are compared in Fig. 1, which shows the percentage change

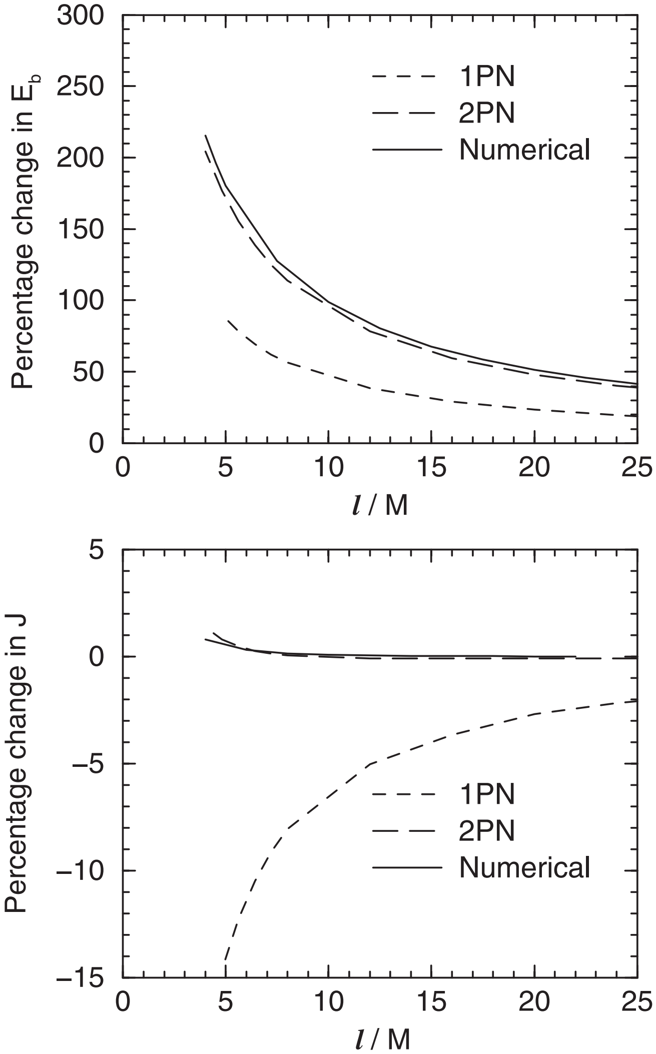

FIG. 1. The percentage change in the binding energy of the three-body system, E_b, and the total angular momentum of the binary, J, at the binary's ISCO, as a function of the coordinate distance of a third body from the center of the binary. $m = 1$; $m_3 = 0.5$, and the third body is stationary.

in the binding energy of the system, E_b, and the total angular momentum of the binary, J, at the binary's ISCO, as a function of the coordinate distance of a third body from the center of the binary is shown. The percentage differences are with respect to the corresponding two-body ISCO for numerical, 1PN, and 2PN calculations. Note that the numerical, 1PN, and 2PN results are in different gauges, and the coordinate distance of the third black hole, l/M, may correspond to (slightly) different physical distances in each of the three gauges. We therefore do not make a quantitative comparison of these results. Note also that we are considering the binding energy of the entire three-body system, not only the binary. However, we can make the qualitative observations that (1) the binding energy of the system increases dramatically as the third black hole is placed closer to the binary, (2) there is better agreement between 2PN and numerical data than between 1PN and numerical data, and (3) most significantly, we need at least 2PN accuracy to see the qualitatively correct effect on the binary's angular momentum, J. For this reason we will use 2PN data in subsequent calculations.

Having seen that the presence of a third body has a significant effect on the binary's ISCO, we now consider an astrophysically more realistic situation: the third body is far larger than each of the bodies in the binary (by a factor of 10^5), and the binary is in orbit about the third body. It is not practical to study this scenario numerically; in particular, we cannot achieve suitable accuracy for such extreme mass ratios with our finite-difference code. However, the previous results show that 2PN calculations are adequate.

Figure 2 shows the percentage change in the binding energy of the binary due to the presence of a third body with a mass 10^5 times larger than that of each body in the

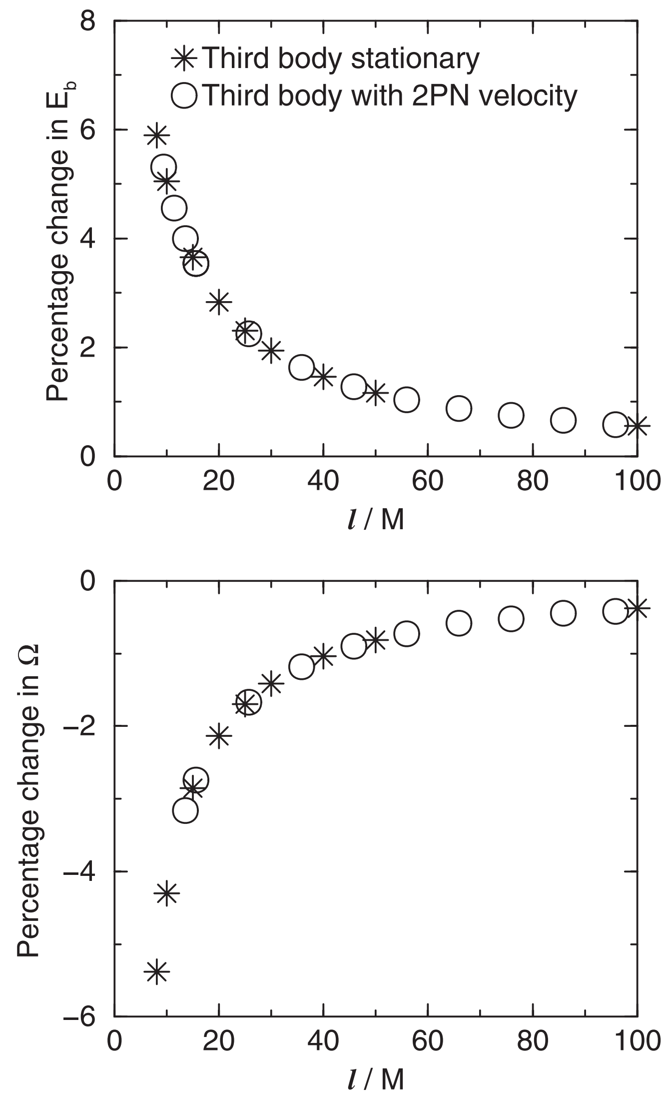

FIG. 2. The percentage change in the binding energy, E_b, and orbital frequency, Ω, of the binary at the ISCO, as a function of the distance of a third body from the center of the binary. The masses are $m_1 = m_2 = 1$; $m_3 = 10^5$.

binary. This configuration models a stellar-mass binary orbiting a supermassive black hole in a galactic core. All results were calculated to 2PN accuracy. The effect of the location of the third body was calculated when the third body was stationary (starred points), and when it had momentum P_3 (circles) consistent with the 2PN circular orbit of a two-body system of masses $2m$ and m_3. We can see that the effect is practically the same in both cases. This is expected since the speed of the third body $v_3 = p_3/m_3$ is very small and enters as a second PN order effect. Note that we are now considering the binding energy of the binary only (using the two-body 2PN Hamiltonian with the binary parameters found in the orbit search procedure), not that of the entire system, as we did in Fig. 1. The effect of the third body is much smaller, but is still appreciable, around 6% when the third body is close to its own ISCO with the binary.

To examine the generality of this effect, we placed the third body in different orientations relative to the binary: in the plane of the binary, at 45 degrees to that plane, and we changed the direction of the third body's momentum (keeping $\mathbf{J}_3 \cdot \mathbf{J} = 0$). The results were unaffected to within the accuracy of the method.

In addition to being an interesting purely relativistic effect, the displacement of the ISCO due to the presence of the third body has observational consequences on the emission of gravitational waves. A larger negative value of the binding energy at the ISCO can be associated with a larger loss of energy radiated by the system during the inspiral phase (comparable to that during the plunge and ring-down phase [24,25].) We thus expect (1) an increase in the terminal amplitude of the inspiral gravitational waveform; (2) an increase in the duration of the preplunge phase; and (3) since the orbital frequency of the ISCO decreases due to the presence of the third body (see Fig. 2), the corresponding waveform will display a lower preplunge frequency. Similarly, the rotation parameter of the remnant black hole will be smaller, due to a decrease in $J_{\rm ISCO}$. This will reduce the value of the least damped quasinormal frequency of the final Kerr black hole.

We wish to thank Luc Blanchet, Marc Freitag, David Merritt, and Cole Miller for helpful discussions, and for carefully reading this manuscript. The authors gratefully acknowledge the support of the NASA Center for Gravitational Wave Astronomy (CGWA) at The University of Texas at Brownsville (No. NAG5-13396), and NSF grants No. PHY-0140326 and No. PHY-0354867. M. H. was also supported in part by the SFB/Transregio 7 on Gravitational Wave Astronomy of the German Science Foundation. Numerical results were obtained on the CGWA *Funes* cluster.

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