Distributed formation tracking control of multiple car-like robots

Chunyu Chen
University of Texas-Pan American

DISTRIBUTED FORMATION TRACKING

CONTROL OF MULTIPLE

CAR-LIKE ROBOTS

A Thesis

by

Chunyu Chen

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COMMITTEE MEMBERS

Dr. Wenjie Dong
Chair of Committee

Dr. Jun Peng
Committee Member

Dr. Xiang Lian
Committee Member

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ABSTRACT

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In this thesis, distributed formation tracking control of multiple car-like robots is studied. Each vehicle can communicate and send or receive states information to or from a portion of other vehicles. The communication topology is characterized by a graph. Each vehicle is considered as a vertex in the graph and each communication link is considered as an edge in the graph. The unicycles are modeled firstly by both kinematic systems. Distributed controllers for vehicle kinematics are designed with the aid of graph theory. Two control algorithms are designed based on the chained-form system and its transformation respectively. Both algorithms achieve exponential convergence to the desired reference states. Then vehicle dynamics is considered and dynamic controllers are designed with the aid of two types of kinematic-based controllers proposed in the first section. Finally, a special case of switching graph is addressed considering the probability of vehicle disability and links breakage.

DEDICATION

I would like to dedicate this thesis to my families who always have faith in me in respect to every decision I make and encourage me by giving me unconditional support. They possess upright ethics and are best exemplarities who teach me basic principles of being a conscientious and optimistic human.

As an international student, it is especially difficult to adapt to the new environment when I first came to America. But thanks to all my local friends and my teachers, I became accustomed to campus life very soon. I would like to especially give my heartfelt thanks to my advisor Dr. Dong who is always generous and helpful whenever I met some problems throughout my graduate studies. The completion of this thesis would not be possible without the guidance of you and I am so grateful to all the support you give me.

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CHAPTER I

INTRODUCTION

Multi-agent systems are composed of multiple interacting intelligent agents and can solve complex problems which cannot be achieved by monolithic system. Multi-agent systems have been widely applied into industrial utilization, different applications of multi-agent systems are addressed in [1]-[14] including formation control [1]-[4], rendezvous [5]-[7] and flocking [8]-[10], distributed sensor networks [11][12] and cooperative control of unmanned air vehicles [13][14].

Consensus problem for multi-agent systems has been intensively studied in recent years with the aid of new techniques from distributed computing [15]-[18] and graph theory [19]-[21]. Consensus for net-worked agents means they reach an agreement in respect with a certain quantity of interests. In real-life operations, agents in net-worked systems are always expected to be operated synchronically and preserve common quantity of states thus fulfill the tasks cooperatively.

General forms for networks of dynamic agents are studied in [22]-[32]. Since different dynamic systems can be modeled by combination of first-order or multi-order systems, it is necessary to study control algorithms for those simplified subsystems.

Distributed control of multiple Single-integrator systems is considered in [22]-[28], multi-agent systems with higher order are studied in [29]-[32]. In [26], delayed-state-derivative feedback control method is proposed for sample-data consensus of first-order multi-agentsystems. In [27], sampled-data based consensus problem of first-order multi-agent systems with quantized communication is studied and control methods are proposed. In [28], novel distributed adaptive consensus controllers are designed for multiple first-order nonlinear systems with unknown parameters and external disturbances. In [29], cooperative tracking control of higher-order nonlinear systems with a dynamic leader is studied for a weighted communication graph with fixed topologies. In [30], distributed control methods for asymptotic consensus of first-order and second-order linear networks are addressed with communication time-delays. In [31], double integrator dynamics with switching topologies are addressed for consensus with a reference nonlinear model. In [32], a second-order consensus protocol is introduced and then applied to achieve altitude alignment among a team of micro air vehicles. In [33], distributed coordination problem for multiple Lagrangian systems is studied with parametric uncertainties. In [34], synchronizing networks of nonidentical, nonlinear dynamical Lagrangian systems is addressed for connected graph with constant unknown time delays, adaptive controllers are designed to achieve global full-sate synchronizations. In [35], distributed finite-time containment control for multiple Lagrangian systems is addressed and a model-independent control law is proposed using both the one-hop and two-hop neighbors' information. In [36], leaderless consensus algorithms for Euler-Lagrangian systems are analyzed, special scenarios of actuator saturation and unavailability of measurements of generalized coordinate derivatives are considered and corresponding control algorithms are proposed.

In [37]-[41], distributed Control methods for consensus problem are addressed under different assumptions of the agent systems and communication topologies including time-delays of communication links, communication switching and node/link failures. In [37], the authors propose consensus algorithms for multi-agent networked systems, directed graph is considered for communication and disturbances of time-delays and node/link failures are considered for verifying robustness of proposed control laws. In [38], both linear and nonlinear consensus protocols are proposed for distributed control of multi-agent consensus, filtering effects of communication channels are considered and maximum tolerance of time-delays is calculated. In [39], both fixed and switching communication topologies are studied for directed and undirected graph, algebraic connectivity is utilized for studying the convergence velocity of multi-agent systems, communication networks with directed information flow is studied for performance of the control algorithms. In [40], the agents' inputs are supposed to be subjected to a constantly albeit possibly unknown time delay, it is proved the time delay has restrictions on communication topologies thus influences the consensus conditions. In [41], the multi-agent systems are considered time-discrete with directed fixed communication topologies. The proposed control laws display dependence of the consensus condition on the agent's unstable poles, non-minimum phase zeros and their relative degree.

Robotic vehicle systems offer advantages of performing complex engineering tasks due to their robustness and precision with the aid of external accessories such as infrared detector, hand gripper and wireless sensor. Mobile robots can complete more complex tasks including rescue and navigation and have been applied into military and industrial and security environments. Vehicles can be operated in isolated fashion of which control information is solely vehicles' own states. However, control of multiple vehicles enables improving existing single-vehicle application by developing new capabilities. Compared with tasks performed by solo automobile, advantages of multivehicle systems include increasing reliability, efficiency and spatially distributed operation. For multivehicle system control techniques, one of the most important problems is coordination of motions of individual vehicles, which can either be realized by controlling each vehicle respectively or utilizing distributed control methods with the aid of communication between vehicles. One common problem of distributed coordination of multivehicle system is consensus seeking. Each vehicle agent in the multivehicle system needs to reach an agreement for certain quantities of interest, the common interest might be convergence of vehicles' states to predefined values. In this case, the vehicles either converge to a static point known as rendezvous problem or track the common trajectory or reference system. In reality, vehicles are usually equipped with sensors which can detect information from neighbors thus realize the communication between different vehicles with certain spatial distance. The communication between multivehicle systems can be described by communication graph, every vehicle system is treated as a vertex in the communication graph and there exist communication links between two vehicles if one vehicle can receive information from another vehicle. The links can be either directed or undirected based on the characteristics of communication type. Distributed control of multivehicle systems for consensus seeking only utilize information from neighbor vehicles instead of accessing the consensus states directly for self-control since not all the systems can communicate with the reference system. It has been proved when the communication topology satisfies specific requirements, all the vehicle systems will converge to consensus states by applying distributed control methods. High expense of communication equipments with wide-bandwidth communication channels are withdrawn and system redundancy is reduced. Even if during the operating process some links or vehicle nodes are disabled, it does not influence the overall systems' performance and robustness is improved.

Before further capabilities of multivehicle systems can be developed, the most important problem is to solve the multivehicle coordination and design distributed coordination strategies. One common problem of multivehicle systems coordination is formation control. Consensus algorithms for multi-agent systems have been utilized for designing distributed formation control in [42]-[47]. Leader-follower formation control for trajectory tracking is addressed in [42], Lyapunov-based techniques are developed for distributed tracking control with one vehicle acting as a leader while others acting as followers tracking the path in certain formation pattern. In [43], distributed formation tracking controllers are designed for nonholonomic vehicles by combing consensus-based controllers with cascaded systems. In [44], the authors translate multivehicle formation control problem into a leader-following consensus problem and design distributed control methods to achieve constant velocities for all vehicle agents and constant spacing between vehicles with three types of communication topologies. In [45], backstepping techniques are utilized for distributed controller design with multivehicle system formation tracking problem, constant communication delays are also considered for the controller design. In [46], state feedback control laws for multivehicle systems formation tracking are proposed with the aid of graph theory and Lyapunov theory. In [47], asymptotical control of multiple wheeled robots is designed with a leader-follower communication topologies.

In these papers, the control laws are designed for intermediate parameters of translational and rotational velocities. However, in practical application, it is the external torques generated by vehicle engines that control motions of a real vehicle. Then dynamic vehicle model is considered for distributed formation tracking control of multivehicle systems. In [48], cooperative control of higher-order multivehicle systems with dynamic uncertainties is addressed with a local cooperative controller and a vehicle-level controller. In [49], neural network is applied to estimate the uncertainties and external disturbances of the dynamic systems. In [50], adaptive cooperative control laws are proposed with the aid of the passivity property of system dynamics with uncertainties. In [51], robust adaptive neural network (NN) control of multiple unmanned ground vehicles is addressed with a virtual leader-follower format. Neural network is introduced to solve nonlinearities and uncertainties of dynamic systems.

In this thesis, distributed formation tracking control of multivehicle systems is studied. The net-worked vehicle agents are supposed to be information transmittable and receivable for specific neighbors. The reference trajectory signals are supposed to be a virtual leader with the same system structure as the follower vehicle agents and the communication graph is a leader-follower topology, and control algorithms for both vehicles' kinematics and dynamics are proposed. For vehicles' kinematic systems, the unicycle models have three generalized states with two Cartesian coordinates in respect to x-axis, y-axis and the steering angle with respect to x-axis. The control inputs for kinematics are translational and rotational velocities. The formation tracking problem is redefined as consensus with the reference trajectory with the same known kinematics which is self-regulated, thus the control goals can be ultimately concluded into designing control laws to stabilize the multiple error systems obtained by subtracting the reference trajectory from each follower's kinematic system. In this thesis, a new distributed control method with the aid of cascaded system theory and graph theory is proposed for kinematic systems. Firstly variable transformations are implemented to transform kinematics into multiple chained-form systems, then the multiple chained-form systems are written in a cascaded structure. The graph theory is utilized to characterize the communication topologies for multiple vehicles and estimate the reference trajectory since it is assumed only a portion of vehicles can receive control information from the reference trajectory directly and all the other followers can only receive information from their neighbors. It is proved the unavailable reference signal can still be estimated with the states information of neighboring vehicles when the communication graph satisfies certain conditions. Then with the aid of exponential stability theorem from the cascaded systems, the multiple transformed chained-form systems are stabilized by states feedback control with the system's own states information and the estimated transformed reference trajectory (the reference trajectory is also transformed into chained form). The stability of the chained-form error systems is proved to guarantee the consensus of vehicles' kinematics with the reference trajectory. Moreover, this thesis considers the vehicles' dynamics considering the impracticality of control design through velocities in real-life operations. The control velocities designed in kinematics are considered intermediate variables and states of vehicles' dynamics while the control torques in vehicles' dynamics are the real control inputs. With the aid of backstepping methods, distributed formation tracking control laws are considered for vehicle dynamics with the aid of kinematics-based controllers. Both cases of dynamics with and without parametrical uncertainties are considered since at some circumstances the physical quantities of vehicles may not be known, sliding mode control is utilized to estimate the uncertainties. This thesis also addresses the switching communication topologies considering in real-life operations the graph is unfixed due to disconnection and creation of communication links and vehicle disabilities. To confirm the effectiveness of proposed control algorithms, simulations are done for four identical unicycles with the aid of SIMULINK of MATLAB.

CHAPTER II

PRELIMINARY RESULTS

In this chapter, main analytical tools for distributed control of multivehicle system consensus are discussed. Graph theory is utilized for characterizing communication topology of multivehicle system, Laplacian matrix is introduced to analyze the communication graph mathematically. In order to define consensus for multivehicle system, stability is introduced to characterize consensus of the overall systems.

2.1 Introduction of Communication Graph

A graph is usually defined as a group of vertices and the edges connecting these vertices. The set of vertices are defined as $V_n = \{v_1, v_2, ..., v_n\}$, where n is the number of vertices. E_n is the set of edges and satisfies $E_n \subseteq V_n \times V_n$ as not all the vertices are connecting with each other. Graph G_n is a pair of sets (V_n, E_n). A graph can be either undirected or directed based on characteristic of communication links, namely elements in E_n. If the links are bidirectional then the communication graph is called undirected graph otherwise directed graph. For directed graph, E_n is called arrow sets where communication links $e_{ij} = (v_i, v_j)$ are directional, v_i is called the tail of arrow and v_j is called head of arrow, v_j can receive information from v_i. Take the directed communication topology in Figure 2.1 as an example.

Figure 2.1 is the communication topology.

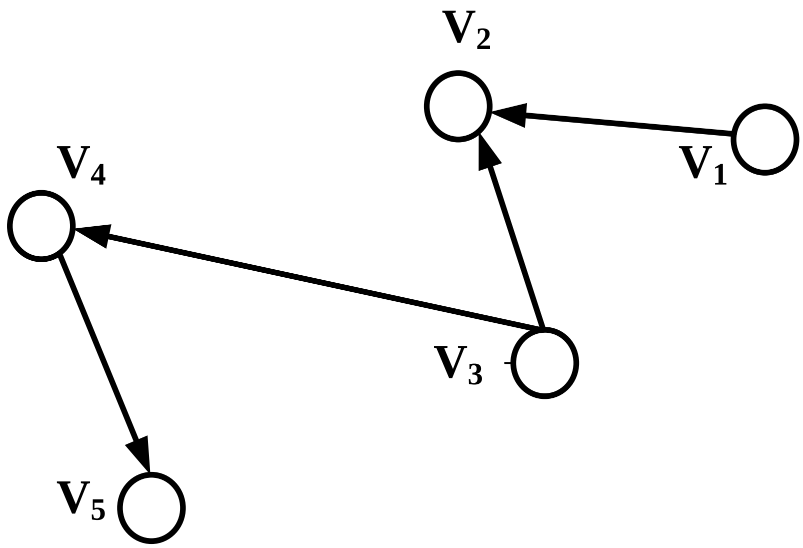

Figure 2.1 Communication topology

G_5={V_5,E_5} where V_5={v_1, v_2,v_3, v_4,v_5 }, E_5={{e_{12} },{e_{32} },{e_{34} },{e_{45} }}.

2.2 Fundamental Properties of Graph

From Section 2.1, it is learned the communication between multivehicle systems can be described by directed or undirected graph. Characteristics of graph determine whether consensus states can be achieved. In this section, fundamental conception of graph is introduced including connectivity and tree structure. In latter control design these properties will be utilized to achieve consensus states.

Definition 2.1 A **path** is a sequence of edges connecting series of vertices.

Definition 2.2 Sub**path** G_1 of G_2 is defined as a graph which satisfies $V(G_1) \subseteq V(G_2)$ and $E(G_1) \subseteq E(G_2)$.

Definition 2.3 A graph is **connected** if for each pair of vertices (v_i, v_j), there exists a path from v_i to v_j, for directed graph, connectivity means both paths from v_i to v_j and v_j to v_i exist.

Definition 2.4 A **tree** is an undirected connected graph, for directed graph, the **oriented** or **directed tree** is a tree if directions of edges are ignored.

Definition 2.5 A **spanning tree** is a subgraph containing all the vertices of the connected graph.

Definition 2.6 For the vertex v_i in the vertex set of a digraph $G_d = (V_n, E_n)$, the number of its tail end-points is called the outdegree of v_i denoted by $deg^+(v_i)$. the number of its head -points is called the indegree of v_i denoted by $deg^-(v_i)$.

Definition 2.7 For the vertex v_i in the vertex set of Graph $G_n = (V_n, E_n)$, the **neighbor** of v_i is a vertex which forms an edge in the edge set with v_i, for a digraph. The **out-neighbor** of v_i is the vertex which send information to v_i. On the contrary, the **in-neighbor** of v_i is the vertex which receive information from v_i.

2.3 Laplacian Matrix

From Section 2.1, it is learned interacting multivehicle systems with specific communication topology can be characterized by a graph with each vehicle the vertex of the graph and intercommunication the link of edge sets. Mathematical formula is utilized for consequent control algorithm design with the aid of Laplacian Matrix. For communication graph of n vehicles $G_n = (V_n, E_n)$, adjacency matrix $A = [a_{ij}]_{nxn}$ is defined as $a_{ij} = 1$, if v_j can receive information from v_i, otherwise $a_{ij} = 0$. Since the vehicle can acquire its own information without communicating with other vehicles, a_{ii} is supposed to be zero. In reality, due to the unevenness of communication intensity between different links, additional weight factor is added to describe the communication links $A_w = [w_{ij}a_{ij}]_{nxn}$. It is proved the weights only influence the convergence time for overall systems while having no effects on the consensus performance.

Consider $D = diag(\sum_{j=1}^{n} a_{1j}, \sum_{j=1}^{n} a_{2j} \dots, \sum_{j=1}^{n} a_{nj})$, then Laplacian matrix L is calculated by $L = D–A$. For the directed graph in Figure 2.1, its associated Laplacian matrix is

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

Let X be a $n \times n$ matrix with entries x_{ij}, *l*et $R_i = \sum_{j \neq i} |x_{ij}|$ be the sum of absolute values of non-diagonal entries in the i^{th} row, ***Greshgrin disc*** $D(x_{ii}, R_i)$ is defined as the closed disc centered at x_{ii} with the radius of R_i.

Theorem 2.1 Every eigenvalue of X lies in at least one of the Greshogrin discs $D(x_{ii}, R_i)$.

Theorem 2.2 Let $G_n = (V_n, E_n)$ be the graph associated with n vertices with Laplacian matrix L, then all the eigenvalues of $-L$ are located in the left half of the complex plane.

Proof By Theorem 2.1, it can be proved all the eigenvalues of L are located within the union of the following discs

$$\cup D_i = \left\{ |\lambda_i - l_{ii}| \leq \sum_{j \neq i} |l_{ij}| \right\}$$

Notice for the Laplacian matrix, it follows that $l_{ii} = \sum_{j \neq i} |l_{ij}| = deg^+(v_i)$, then the union of discs can be expressed as

$$\cup D_i = \{ |\lambda_i - deg^+(v_i)| \leq deg^+(v_i) \}$$

Moreover, all the eigenvalues of *L* are contained in the largest disc of the union

$$|\lambda_i - deg^+_{max}(v_i)| \leq deg^+_{max}(v_i)$$

Clearly, all the eigenvalues of -*L* are contained in the mirror image of the largest disc

$$|\lambda_i + deg^+_{max}(v_i)| \leq deg^+_{max}(v_i)$$

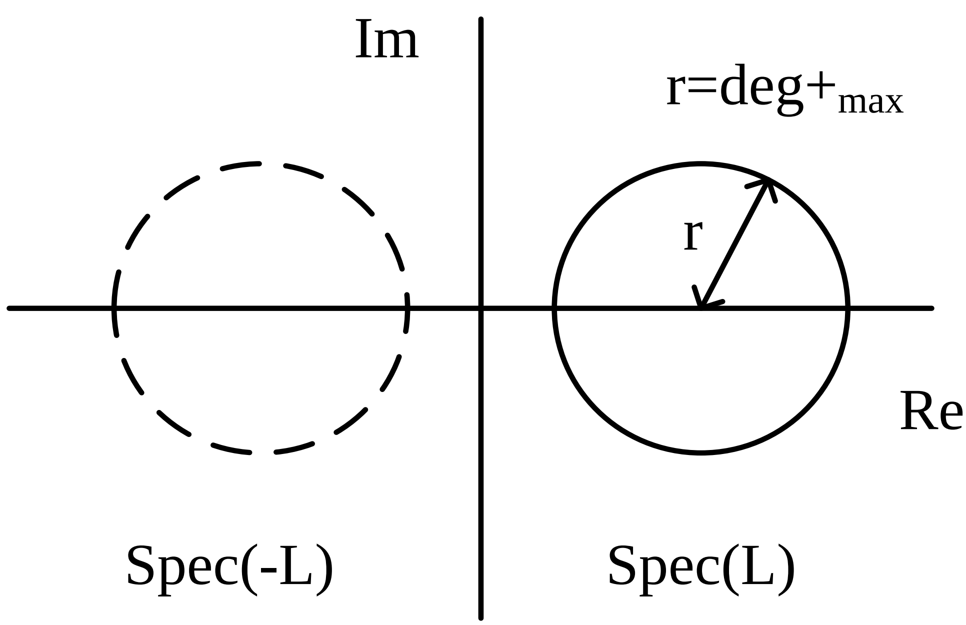

Figure 2.2 Greshgrin disc of the Laplacian matrix

Since the row sum of $-L$ is zero and it is known there exists at least one eigenvalue λ_1 which is equal to zero with the rest eigenvalues $\lambda_n \leq \lambda_{n-1} \leq \cdots \leq \lambda_2 \leq 0$.

Theorem 2.3 Let $G_n = (V_n, E_n)$ be the graph associated with n vertices with Laplacian matrix L, eigenvalues of $-L$ satisfy $\lambda_n \leq \lambda_{n-1} \leq \cdots \leq \lambda_2 < \lambda_1 = 0$ if G_n is connected.

Proof Since G_n is connected then rank of $-L$ $Rank(-L) = n - 1$ and $-L$ has a simple zero eigenvalue, by Theorem 2.2, $\lambda_n \leq \lambda_{n-1} \leq \cdots \leq \lambda_2 < \lambda_1 = 0$.

Theorem 2.4 Let $G_n=(V_n, E_n)$ be the graph associated with n vertices with Laplacian matrix L, eigenvalues of $-L$ satisfy $\lambda_n \leq \lambda_{n-1} \leq \cdots \leq \lambda_2 < \lambda_1 = 0$ if G_n has a spanning tree.

2.4 Fundamental Properties of Nonlinear Dynamic System

Consider the nonlinear dynamic system defined by the following n-dimensional first-order vector differential equation

$$\dot{x} = f(t, x, u) \tag{2.1}$$

where $x = [x_1, x_2, \dots, x_n]^T$, $u = [u_1, u_2, \dots, u_n]^T$ and

$$f(t, x, u) = [f(t, x_1, u_1), f(t, x_2, u_2), \dots f(t, x_n, u_n)]$$

The equation above is called the state equation and x is the ***state*** while u is the input. Since the function f depends explicitly on t, the nonlinear system is called non-autonomous or time-varying.

A point $\mathrm{x} = \tilde{\mathrm{x}}$ in the state space is said to be an ***equilibrium point*** of (2.1) if it has the property that any state starting from $\tilde{\mathrm{x}}$ will remain at $\tilde{\mathrm{x}}$ in certain time. For non-autonomous system (2.1) the equilibrium points are roots of

$$f(t,x) = 0 \tag{2.2}$$

If all the solutions of (2.2) starting out from $\tilde{x}$ will ultimately stay near $\tilde{x}$ in certain time, $\tilde{x}$ is said to be ***Lyapunov stable***. If all solutions starting out from $\tilde{x}$ will converge to $\tilde{x}$ then $\tilde{x}$ is said to be ***asymptotically stable***. If in addition to being asymptotically stable, the convergence velocity is at a exponential decay rate then $\tilde{x}$ is said to be ***exponentially stable***.

$\tilde{x}$ is said to be globally asymptotically stable (GUS) or globally exponentially stable (GES) if the solutions can start out from any point in the state space in addition to the requirement for asymptotical stableness and exponential stableness.

Consider a function $\alpha: [0, a) \rightarrow [0, \infty)$, if α is strictly increasing and $\alpha(0) = 0$, then α is said to be a class κ function. More strongly, if $\alpha(a) = \infty$ as $a \rightarrow \infty$ then α is said to be a class κ_∞ function.

Consider a function $\beta: [0, a) \times [0, \infty) \rightarrow [0, \infty)$, if for each fixed s, $\beta(r, s)$ belongs to class κ with respect to r, for each fixed r, $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \rightarrow 0$ as $s \rightarrow \infty$, then $\beta(r, s)$ is said to be a class κι function.

Assume there exists a class κι function β such that for any initial state $x(t_0)$

$$||\mathrm{x(t)}|| \leq \beta\left(||x(t_0)||, t - t_0\right)$$

for $\forall t \geq t_0$ then the origin of (2.1) is globally uniformly asymptotically stable.

The origin is said to be globally exponentially stable if there exist positive constants k and λ such that

$$||x(t)|| \leq k||x(t_0)||e^{-\lambda(t-t_0)}$$

for any initial state $x(t_0)$, $\forall t \geq t_0$

CHAPTER III

CONSENSUS ON MULTIPLE LINEAR SYSTEMS

From the kinematic vehicle system in Chapter 4, it is learned that vehicle kinematic system is characterized by a first-order system. In this chapter, distributed control laws for first-order system consensus is discussed. Consensus states are represented by reference signals. Consensus signals are divided into time-invariant and time-varying reference signals. In this paper, the latter scenario of time-varying trajectory convergence is studied. Firstly control laws in [37][38] for time-invariant reference states and in [22] for time-varying reference states are introduced, then a novel method that will be utilized in Chapter 4 for distributed multivehicle system formation tracking control is proposed. Compared with control algorithms in [22], the proposed first-order system consensus law removes utilization of derivatives of states information such as coordinates and angles, which cannot be easily acquired by sensors. Then consensus algorithms for multiple double-integrator systems are designed based on the novel first-order algorithm.

3.1 Consensus of Multiple First-order Systems

A first-order system is defined by

$$\dot{\delta}_i = u_i \tag{3.1}$$

where δ_i is the state of the the first- order system i, u_i is the control input.

In [39], a first-order consensus protocol is proposed as

$$u_i = -\sum_{j=1}^{n} a_{ij}(\delta_i - \delta_j) \tag{3.2}$$

where a_{ii} = 0 and $a_{ij} \geq 0$ if information flows from vehicle v_j to vehicle v_i and 0 otherwise, by applying u_i in (3.2), system (3.1) can be written as

$$\dot{\delta} = -L\delta \tag{3.3}$$

where $\delta = [\delta_1, \delta_2, \ldots\delta_n]^T$, $L = [l_{ij}]$ where $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij} \forall i = j$.

Theorem 3.1 (3.2) guarantees δ=[δ_1, δ_2, ..., δ_n]^T converge to the same value known as group decision value $\delta^*(t) = \sum_{i=1}^{n} \alpha_i \delta_i(0)$ where α=[α_1, α_2, ..., α_n] is a nonnegative left vector of L associated with eigenvalue 0 with the property that $\alpha_i \geq 0$, $i=1,2,\ldots,n$. and the sum of $\alpha_i \sum_{i=1}^{n} \alpha_i = 1$ if graph G is connected.

From Theorem 3.1 it can be learned all the agents will converge to the consensus state, the consensus value is actually decided by the weight of each agent. It is proved that if the communication link is bidirectional and the undirected graph is connected, all the agents share the same weight and the consensus value is the average of the agents' initial states. If the graph is a digraph, each agent's weight in the final consensus value is different, the weight depends on the information accessibility to other agents, which means if the agent can have its information sent to more follower agents, the higher weight it will have in the consensus state.

In the vehicle tracking problem, a predefined signal is regarded as a reference trajectory, this signal can also be modeled as a vehicle system known as the virtual leader, the virtual leader don't receive information from any other agent and it has the highest weight in the decision value, assume the reference signal is denoted by v_{n+1}, then $\alpha_{n+1} = 1$ and $\alpha_i = 0$ for $1 \leq i \leq n$, the control law in (3.2) is rewritten as

$$u_i = -\sum_{j=1}^{n} a_{ij}(\delta_i - \delta_j) - a_{i,n+1}(\delta_i - \delta_{n+1}) \tag{3.4}$$

where $a_{i,n+1} > 0$. if v_i can receive information from v_{n+1}, otherwise $a_{i,n+1} = 0$.

Theorem 3.2 For a directed graph G, if $\dot{\delta}_{n+1} = 0$ and G has a spanning tree with v_{n+1} as its root, (3.4) guarantee δ=[$\delta_1, \delta_2, \ldots, \delta_n]^T$ converge to the same value known as group decision value δ^*(t) = δ_{n+1}.

Theorem 3.2 holds only if the reference state is constant signal. However, the tracking problem usually involves time-varying reference signal. If the algorithms in (3.4) are applied to time-varying signal case, $\dot{\delta}^r$ will be introduced onto the right hand of (3.4)

$$u_i = \dot{\delta}_{n+1} - \sum_{j=1}^{n} a_{ij}(\delta_i - \delta_j) - a_{i,n+1}(\delta_i - \delta_{n+1}) \tag{3.5}$$

It is known that only a portion of agents can receive information from v_{n+1}, which means (3.5) holds only if all the agents can receive information from v_{n+1}. Then the modified control laws for time-varying reference signal are proposed in [22].

Theorem 3.3 Let G ={V, E} be the communication topology associated with n+1 agents (the n+1'th agent is the time-varying reference signal), algorithm (3.6) solve the consensus problem with a time-varying reference signal if and only if there exists a spanning tree with the virtual agent as its root [22].

$$u_i = \frac{1}{\sum_{j=1}^{n} a_{ij} + a_{i,n+1}} \sum_{j=1}^{n} a_{ij}[\dot{\delta}_j - \gamma_i(\delta_i - \delta_j)] + \frac{a_{i,n+1}}{\sum_{j=1}^{n} a_{ij} + a_{i,n+1}} \sum_{j=1}^{n} [\dot{\delta}_{n+1} - \gamma_i(\delta_i - \delta_{n+1})]$$

(3.6)

3.2 First-order Systems Consensus with Time-varying Reference Signals

In this section, a different control law for first-order system consensus with time-varying reference signals is studied. The proposed control laws will be used in designing the distributed control algorithms for the vehicle kinematic and dynamic systems in Chapter 4 and Chapter 5. In this proposed first-order control algorithm, derivative term $\dot{\delta}_j$ and $\dot{\delta}_{n+1}$ are removed since in practical applications sensors may only sense physical states of coordinates and angles but not translational and rotational velocities.

The desired trajectory is assumed to be time-varying signal, the main problem for trajectory tracking is to find a term to replace $\dot{\delta}_{n+1} = u_{n+1}$, which cannot be acquired directly for each slave system, consider system (3.4), define the tracking errors $\tilde{\delta}_{\mathrm{i}} = \delta_{\mathrm{i}} - \delta_{n+1}$, then (3.4) is transformed into

$$\dot{\tilde{\delta}}_i = \sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_i - \tilde{\delta}_j\right) - a_{i,n+1}\tilde{\delta}_i - \dot{\delta}_{1,n+1} \qquad (3.7)$$

Replace $\dot{\delta}_{1,n+1}$ in (3.7) with $\rho\mathrm{sign}(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_i - \tilde{\delta}_j\right) - a_{i,n+1}\tilde{\delta}_i)$, it follows that

$$u_i = -\sum_{j=1}^{n} a_{ij}(\delta_i - \delta_j) - a_{i,n+1}(\delta_i - \delta_{n+1}) - psign(\sum_{j=1}^{n} a_{ij}\left(\delta_i - \delta_j\right) + a_{i,n+1}(\delta_i - \delta_{n+1}))$$

(3.8)

Theorem 3.4 Let G ={V, E} be the communication topology associated with n+1 agents (agent v_{n+1} is the time-varying reference signal), algorithm (3.8) guarantee that $\delta_{\mathrm{i}} - \delta_{n+1}$ globally exponentially stable if there exists a spanning tree in the communication graph with v_{n+1} as the root of the spanning tree.

Proof Substitute u_i in (3.8) into (3.1), define the tracking error $\tilde{\delta}_{\mathrm{i}} = \delta_{\mathrm{i}} - \delta_{n+1}$ it follows that

$$\dot{\tilde{\delta}}_i = \sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_i - \tilde{\delta}_j\right) - a_{i,n+1}\tilde{\delta}_i - \rho sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_i - \tilde{\delta}_j\right) - a_{i,n+1}\tilde{\delta}_i\right) - \dot{\delta}_{n+1}$$

With the aid of Laplacian matrix in Chapter 2 the equation above can be written in

$$\dot{\tilde{\delta}} = -(L+B)\tilde{\delta} - \rho sign\left((L+B)\tilde{\delta}\right) - \dot{\delta}_{n+1}\mathbf{1} \tag{3.9}$$

where $\delta = \left[\delta_{1,} \dots, \delta_{n,}\right]$, L is the Laplacian matrix associated with G, $B = diag(a_{1,n+1}, \dots, a_{n,n+1})$. It can be proved of $L + B$ is positive symmetric matrix with eigenvalues in the right half of the complex plane.

Choose the Lyapunov function $V = \frac{1}{2}\tilde{\delta}^2$, differentiate V along (3.9), it follows that

$$\dot{V} = -\tilde{\delta}^T(L+B)\tilde{\delta} - \tilde{\delta}^T \rho sign\left((L+B)\tilde{\delta}\right) - \tilde{\delta}^T\dot{\delta}_{n+1}\mathbf{1}$$

$$= -\tilde{\delta}^T(L+B)\tilde{\delta} - \tilde{\delta}^T \rho sign\left((L+B)\tilde{\delta}\right)$$

$$-\left((L+B)\tilde{\delta}\right)^{\mathrm{T}}(L+B)^{-1}\dot{\delta}_{n+1}\mathbf{1}$$

Let ϵ be the minimum eigenvalue of $(L+B)^{-1}$, then

$$\dot{V} \leq -\tilde{\delta}^T(L+B)\tilde{\delta} - \epsilon\rho\left((L+B)\tilde{\delta}\right)^T sign\left((L+B)\tilde{\delta}\right)$$

$$-\left((L+B)\tilde{\delta}\right)^{\mathrm{T}}(L+B)^{-1}\dot{\delta}_{n+1}\mathbf{1}$$

If ρ satisfies

$$\rho \geq \frac{||(L+B)^{-1}||\,|\dot{\delta}_{n+1}|}{\epsilon}$$

then $\dot{V} \leq -\tilde{\delta}^T(L+B)\tilde{\delta}$, let $\sigma_{\min}$ be the smallest eigenvalue of $L+B$, it follows that

$$\dot{V} \leq -\tilde{\delta}^T(L+B)\tilde{\delta} \leq -\sigma_{\min}\tilde{\delta}^2 = \frac{\sigma_{min}}{0.5} \times \frac{1}{2}\tilde{\delta}^2$$

$$\dot{V} \leq 2\sigma_{min}V$$

Then $\tilde{\delta}$ exponentially globally converge to zero, which means $\lim_{t\to\infty} \delta_i - \delta_{n+1} = 0$.

Compared with the first-order system control methods in [22], the distributed control law in (3.8) has low redundancy, more states transference means wider bandwidth of communication channels and higher requirements on communication facilities, in addition, (3.8) removes the usage of inaccessible quantity of states and has broader potential applications in real-life operations.

Example 3.1 Consider a directed spanning tree topology in Figure 3.1. Assume the virtual leader is the time-varying reference signal which in this case is the sinusoid signal *sin(t)*. Suppose only agent v_1 could get access with the leader directly. Simulation results in Figure 3.2 and 3.3 show that all the follower agents converge to the reference signal as $t \to \infty$.

Figure 3.1 is the information exchange graph.

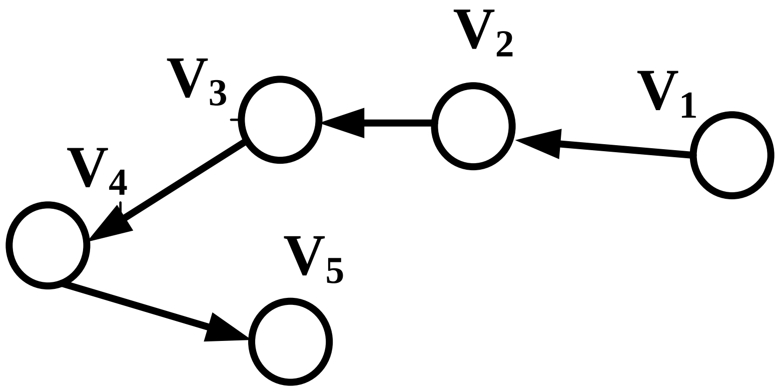

Figure 3.1 Information exchange graph

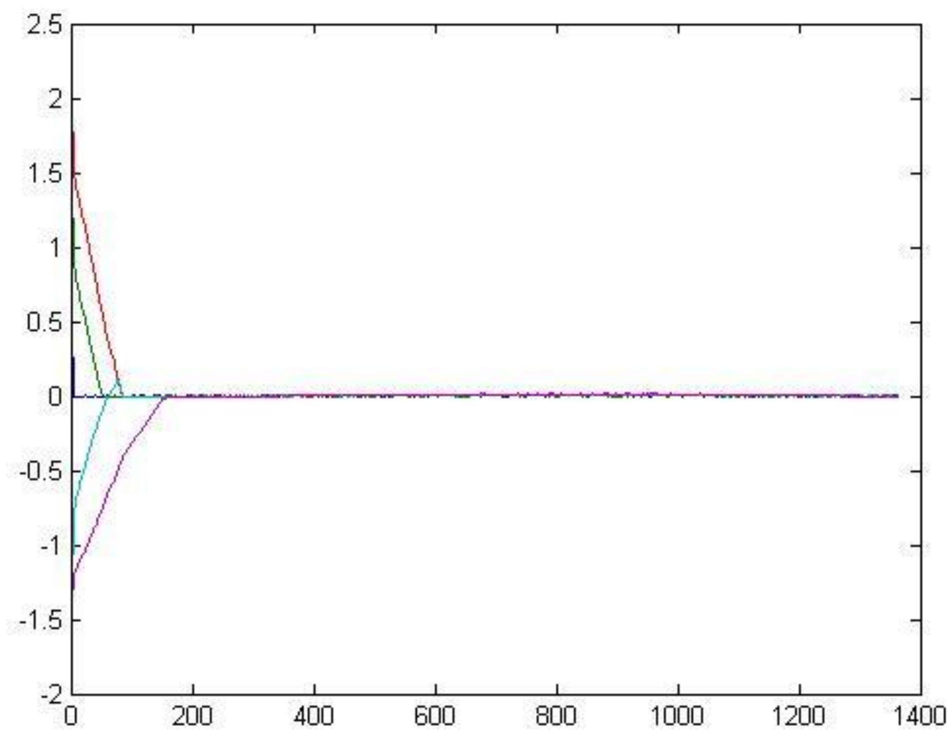

Figure 3.2 $\delta_j - \delta_0$ for $1 \leq j \leq 5$ (I)

Figure 3.3 is the trajectory of δ_j for $1 \le j \le 5$.

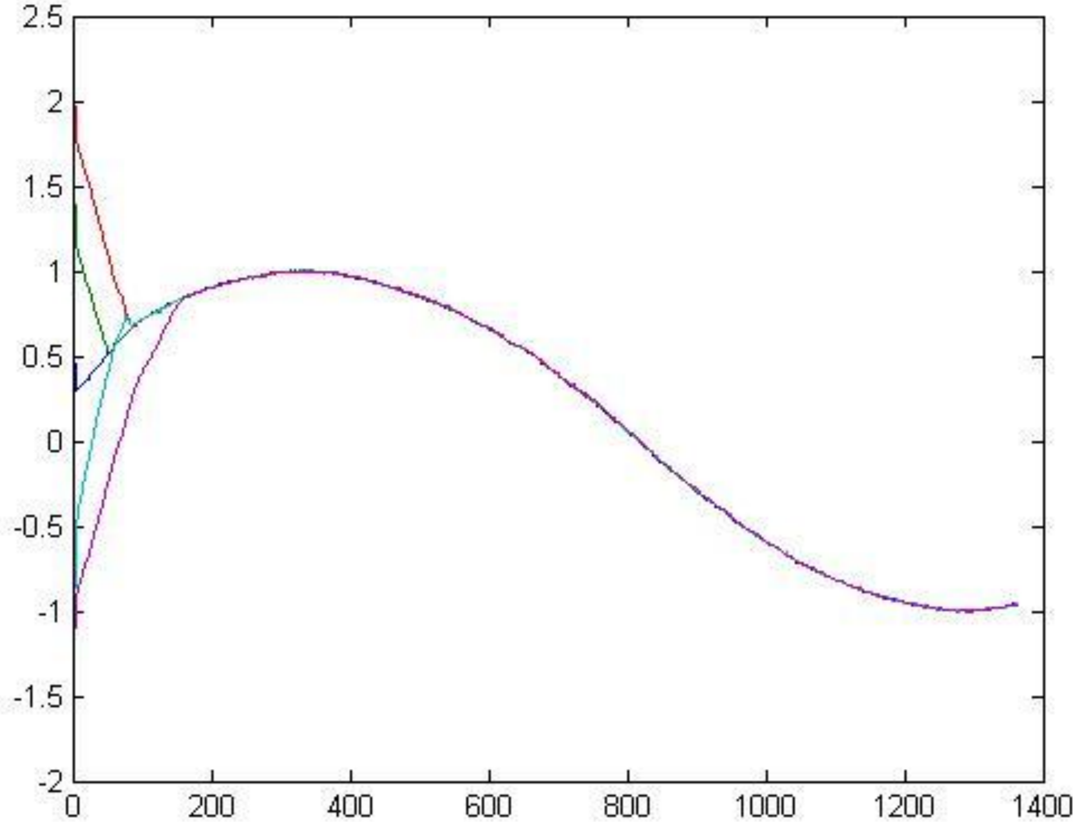

Figure 3.3 Trajectory of δ_j for $1 \le j \le 5$ (I)

3.3 Second-order Systems Consensus with Time-varying Reference Signals

Consider multiple double-integrator systems

$$\dot{\sigma}_j = \delta_j \tag{3.10}$$

$$\dot{\delta}_j = u_j \tag{3.11}$$

for $1 \le j \le n$, where δ_j and σ_j are the states of system j, u_j is the control input.

Assume the reference trajectory is also characterized by the second order system with the same structure of (3.10) and (3.11)

$$\dot{\sigma}_0 = \delta_0 \tag{3.12}$$

$$\dot{\delta}_0 = u_0 \tag{3.13}$$

In order to track the reference signal in (3.12)-(3.13), the multiple second-order error systems between each individual agent and the reference trajectory are utilized.

Define $\tilde{\sigma}_{\mathrm{j}} = \sigma_j - \sigma_0$, $\tilde{\delta}_{\mathrm{j}} = \delta_j - \delta_0$, the multiple second-order error systems are defined by

$$\dot{\tilde{\sigma}}_j = \delta_j - \delta_0 \tag{3.14}$$

$$\dot{\tilde{\delta}}_j = u_j - u_0 \tag{3.15}$$

Notice δ_0 and u_0 are supposed not to be available to all the follower double-integrator systems. Estimation of these two reference states are implemented based on the communication topology of networked multi-agent systems. The novel first-order consensus algorithm proposed in previous section is utilized to help design the double integrator systems.

Define $\chi_j = k_1\sigma_j + \delta_j$, then (3.10) is transformed into

$$\dot{\sigma}_j = -k_1\sigma_j + \chi_j \tag{3.16}$$

Similarly, (3.12) is transformed into

$$\dot{\sigma}_0 = -k_1\sigma_0 + \chi_0 \tag{3.17}$$

Differentiate $\chi_j = k_1\sigma_j + \delta_j$, it follows that

$$\dot{\chi}_j = k_1\dot{\sigma}_j + \dot{\delta}_j$$

$$= k_1\delta_j + u_j \tag{3.18}$$

Lemma 3.1 If $\chi_j - \chi_0$ is bounded and converges to zero, then σ_j exponentially converges to σ_0.

Proof Subtract (3.16) by (3.17) it follows that

$$\dot{\sigma}_j - \dot{\sigma}_0 = -k_1(\sigma_j - \sigma_0) + (\chi_j - \chi_0) \tag{3.19}$$

Since $\dot{\sigma}_j - \dot{\sigma}_0 = -k_1(\sigma_j - \sigma_0)$ is exponentially stable and $\chi_j - \chi_0$ is asymptotically stable, then $\sigma_j - \sigma_0$ is bounded and converges to zero exponentially.

Lemma 3.2 If $\chi_j - \chi_0$ is bounded and converges to zero, then δ_j converges to δ_0.

Proof

$$\chi_j - \chi_0 = k_1\sigma_j + \delta_j - k_1\sigma_0 - \delta_0$$

$$= k_1(\sigma_j - \sigma_0) + (\delta_j - \delta_0)$$

By Lemma 3., $\sigma_j - \sigma_0$ is bounded and exponentially converges to zero, then it can be proved $\delta_j - \delta_0$ exponentially converges to zero.

Notice that the convergence of χ_j to χ_0 ensures

$$\lim_{t\to\infty}(\sigma_j - \sigma_0) = 0$$

$$\lim_{t\to\infty}(\delta_j - \delta_0) = 0$$

Reference signal tracking of multiple double-integrator systems is transformed into designing distributed control laws for χ_j such that

$$\lim_{t\to\infty}(\chi_j - \chi_0) = 0$$

Theorem 3.5 Let $G = \{V, E\}$ be the communication topology associated with n+1 agents (agent v_{n+1} is the time-varying reference signal), algorithm (3.20) guarantee that $\sigma_j - \sigma_0$ and $\delta_j - \delta_0$ are globally exponentially stable if there exists a spanning tree in the communication graph with v_{n+1} as the root of the spanning tree.

$$u_j = -\sum_{i=1}^{n} a_{ji}(\chi_i - \chi_i) - a_{j,n+1}(\chi_j - \chi_0) - psign(\sum_{j=1}^{n} a_{ij}(\chi_j - \chi_i) + a_{j,n+1}(\chi_j - \chi_0) - k_1\delta_j \tag{3.20}$$

where $\chi_0 = k_1\sigma_0 + \delta_0$. $a_{j,n+1} = 1$ if v_j can receive information from the virtual leader directly, otherwise $a_{j,n+1} = 0$.

Proof Substitute u_j in (3.20) into (3.18), define the tracking error $\tilde{\chi}_j = \chi_j - \chi_{1,n+1}$ it follows that

$$\dot{\tilde{\chi}}_j = \sum_{i=1}^{n} a_{ji}(\tilde{\chi}_j - \tilde{\chi}_i) - a_{j,n+1}\tilde{\chi}_j - \rho sign\left(\sum_{i=1}^{n} a_{ji}(\tilde{\chi}_j - \tilde{\chi}_i) - a_{j,n+1}\chi_j\right) - \dot{\chi}_0$$

With the aid of Laplacian matrix in Chapter 2 the equation above can be written in

$$\dot{\tilde{\chi}} = -(L+B)\tilde{\chi} - \rho sign\big((L+B)\tilde{\chi}\big) - \dot{\chi}_0\mathbf{1} \tag{3.21}$$

where $\chi = [\chi_{1,} \dots, \chi_{n,}]$, L is the Laplacian matrix associated with G, $B = diag(a_{1,n+1}, \dots, a_{n,n+1})$. It can be proved of $L + B$ is positive symmetric matrix with eigenvalues in the right half of the complex plane.

Choose the Lyapunov function $V_x = \frac{1}{2}\tilde{\chi}^2$, differentiate V_x along (3.21), it follows that

$$\dot{V}_{\mathrm{x}} = -\tilde{\chi}^T(L+B)\tilde{\chi} - \tilde{\chi}^T \rho sign\big((L+B)\tilde{\chi}\big) - \tilde{\chi}^T \dot{\chi}_0 \mathbf{1}$$

$$= -\tilde{\chi}^T(L+B)\tilde{\chi} - \tilde{\chi}^T \rho sign\big((L+B)\tilde{\chi}\big)$$

$$-\big((L+B)\tilde{\chi}\big)^{\mathrm{T}}(L+B)^{-1}\dot{\chi}_0\mathbf{1}$$

Let ϵ be the minimum eigenvalue of $(L+B)^{-1}$, then

$$\dot{V}_{\mathrm{x}} \le -\tilde{\chi}^T(L+B)\tilde{\chi} - \epsilon\rho\big((L+B)\tilde{\chi}\big)^T sign\big((L+B)\tilde{\chi}\big)$$

$$-\big((L+B)\tilde{\chi}\big)^{\mathrm{T}}(L+B)^{-1}\dot{\chi}_0\mathbf{1}$$

If ρ satisfies

$$\rho \ge \frac{\big||(L+B)^{-1}|\big||\dot{\chi}_0|}{\epsilon}$$

then $\dot{V} \le -\tilde{\chi}^T(L+B)\tilde{\chi}$, let σ_{min} be the smallest eigenvalue of $L+B$, it follows that

$$\dot{V}_{\mathrm{x}} \le -\tilde{\chi}^T(L+B)\tilde{\chi} \le -\sigma_{min}\tilde{\chi}^2 = \frac{\sigma_{min}}{0.5} \times \frac{1}{2}\tilde{\chi}^2$$

$$\dot{V}_{\mathrm{x}} \le 2\sigma_{min}V_x$$

Then it can be proved $\chi_j - \chi_0$ exponentially converge to zero. By Lemma 3.1 σ_j exponentially converges to σ_0. By Lemma 3.2 δ_j exponentially converges to δ_0.

Example 3.2 Consider the communication topology for five double-integrator systems in Figure 3.4. Assume the virtual leader is the time-varying reference signal which in this case is the sinusoid signal $(\sigma_0, \delta_0) = (cos(t), -sin(t))$. Suppose only agent v_1 could get access to the leader directly. Simulation results in figure 3.5 and 3.6 show that $\lim_{t\to\infty}(\delta_j - \delta_0) = 0$. Figure 3.7 and Figure 3.8 show that $\lim_{t\to\infty}(\sigma_j - \sigma_0) = 0$.

Figure 3.4 represents the communication graph.

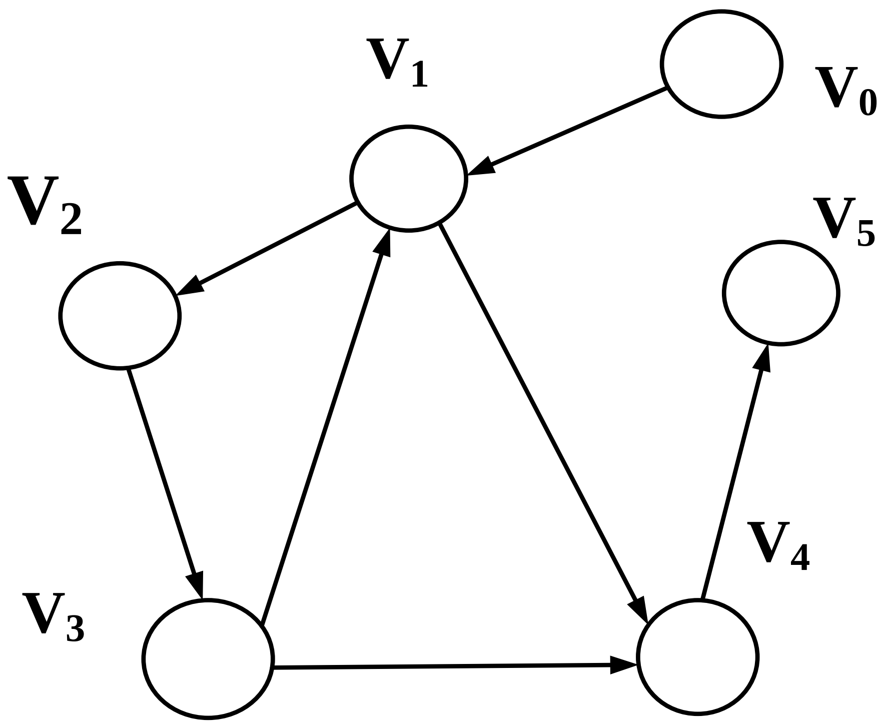

Figure 3.4 Communication topology for multiple double-integrator systems

Figure 3.5 is the convergence result of σ_j for $1 \leq j \leq 5$

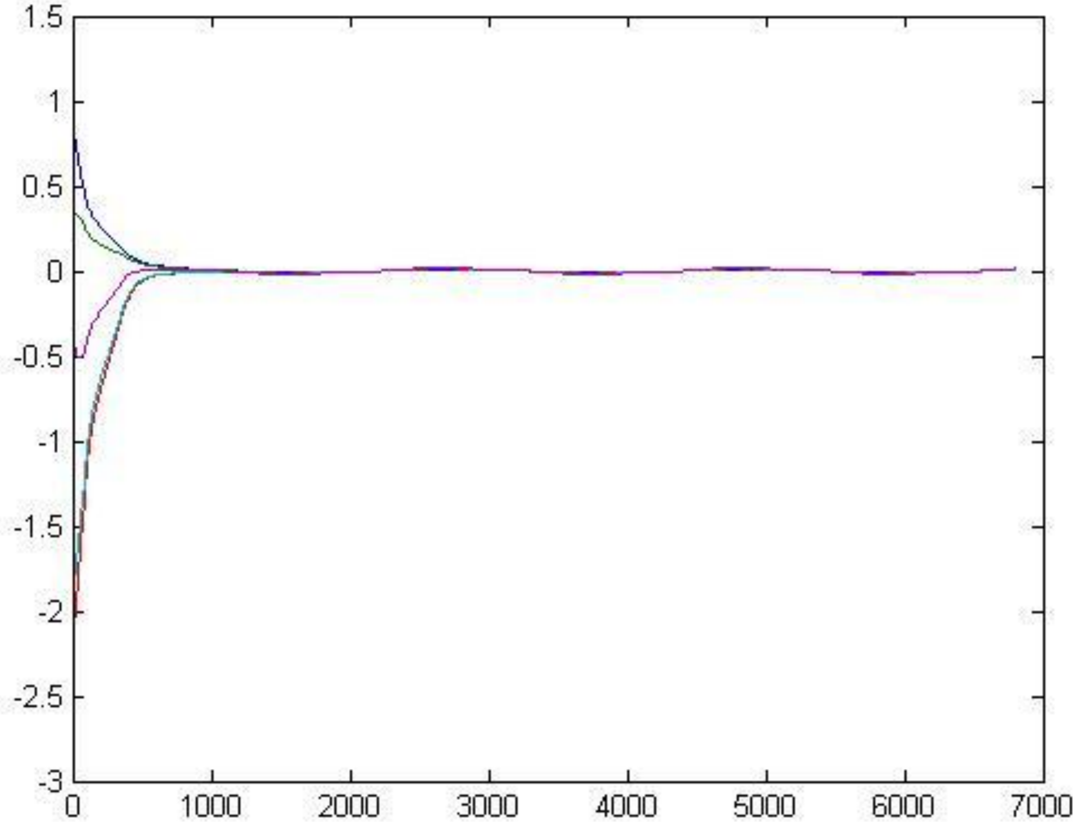

Figure 3.5 $\sigma_j - \sigma_0$ for $1 \leq j \leq 5$

Figure 3.6 is the trajectory of σ_j for $1 \leq j \leq 5$

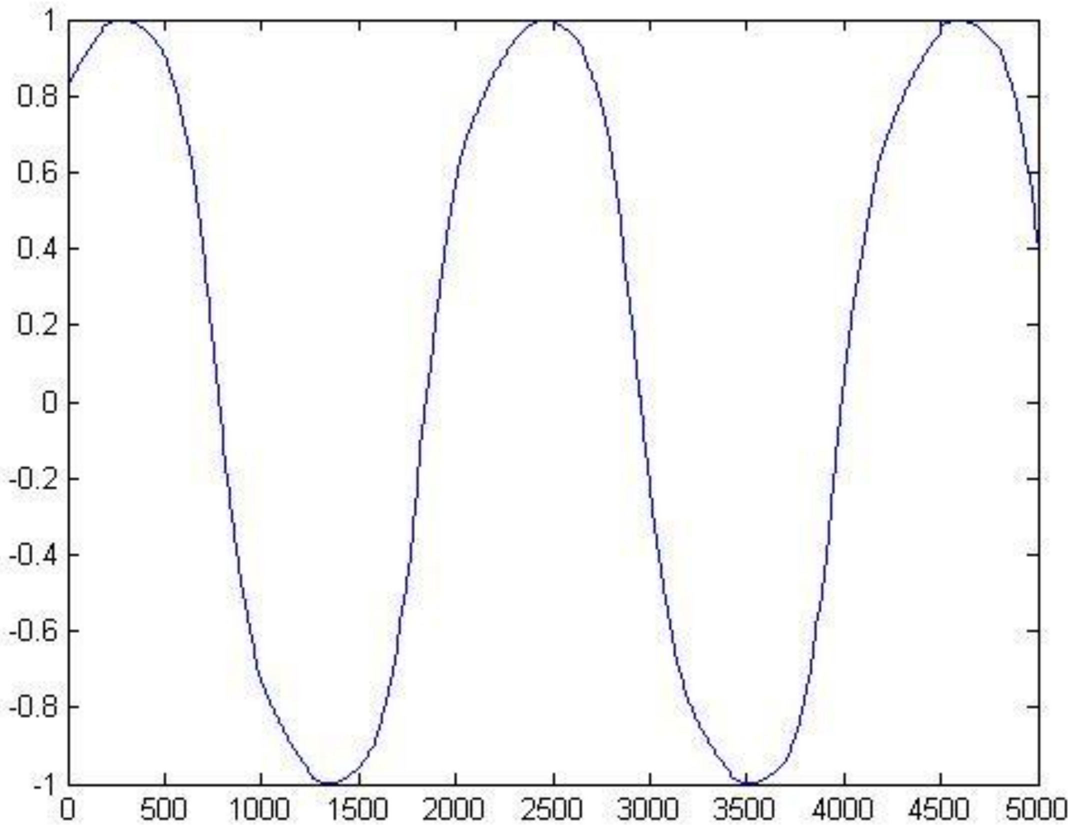

Figure 3.6 Trajectory of σ_j for $1 \leq j \leq 5$

Figure 3.7 is the convergence result of δ_j for $1 \leq j \leq 5$

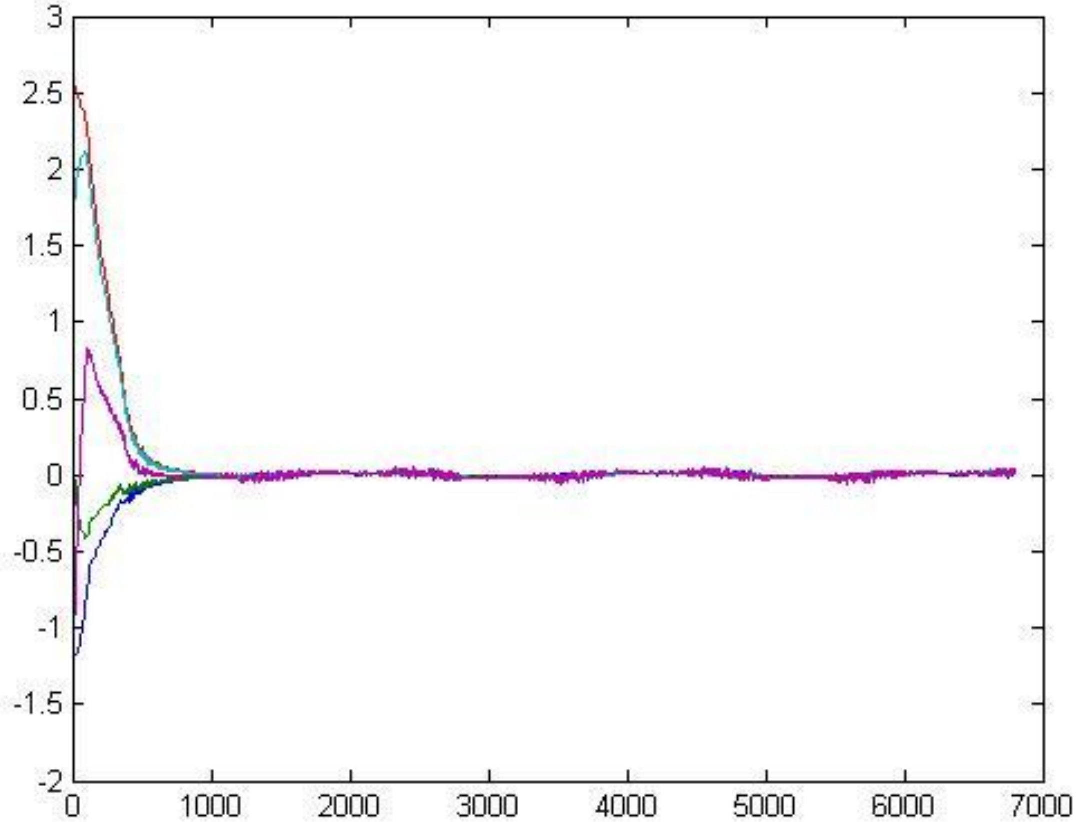

Figure 3.7 $\delta_j - \delta_0$ for $1 \leq j \leq 5$ (II)

Figure 3.8 is the trajectory of δ_j for $1 \leq j \leq 5$

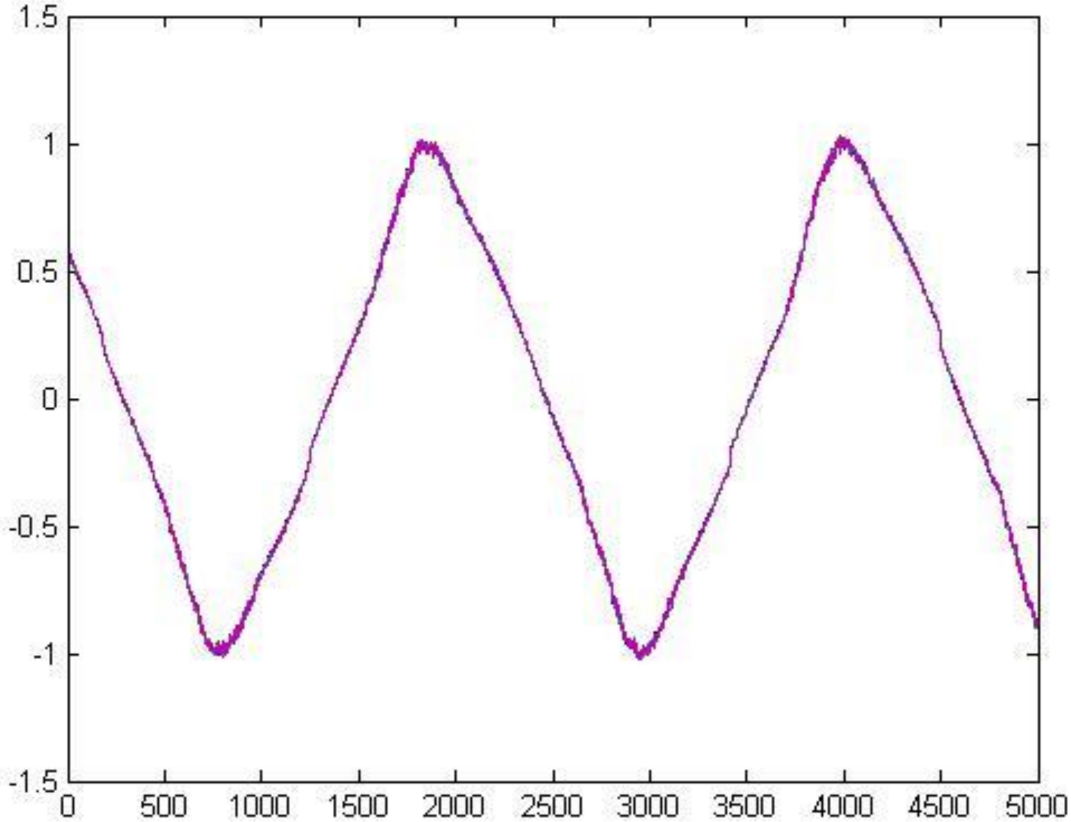

Figure 3.8 Trajectory of δ_j for $1 \leq j \leq 5$ (II)

3.4 Summary

In this chapter, distributed control for consensus problem on first-order multi-agent systems is addressed with the aid of graph theory and Lyapunov theory. Control methods are introduced for exponential stability of multi-error systems in [22][38][39]. A novel consensus algorithm for time-varying reference signals tracking is proposed and has potential practical applications in real-life operations. This method will be utilized to design distributed formation tracking laws of multiple car-like robotic systems in Chapter IV and V

CHAPTER IV

COOPERATIVE FORMATION TRACKING CONTROL OF MULTIPLE KINEMATIC VEHICLES

In this chapter, firstly the kinematic model of the vehicle is derived and fundamental properties of the corresponding system are analyzed, then distributed formation tracking control of multiple kinematic systems is studied with the aid of graph theory and theories from cascaded systems and chained-form systems.

There are many types of mobile vehicles of which car-like vehicle is widely used. The simplest mobile robot structure is a single chassis installed with two wheels, which can be simplified as a unicycle under the consumption both wheels have identical configuration and motion behaviors. Then the two wheels can be treated as collapsing into the middle point of the chassis. Four-wheel car-like robot is more similar as the real vehicle. Some are four-wheel drive with all four wheels receiving torque from the engine, the others are either front-wheel drive or rear-wheel drive which means either the front wheels or the rear wheels are driven by the engine. The wheels are considered to be employed with nonholonomic constraints under the assumption that there is no slippage at the wheel. The nonholonomic constraint is not integrable and can be characterized by the relation between the velocities (both translational and angular) and the steering angle of the vehicle.

$$\dot{x}\,sin(\theta) - \dot{y}\,cos(\theta) = 0$$

where (x, y) is coordinate of the middle point of the axis and θ is the steering angle with respect to x-axis， $\dot{x}$ and $\dot{y}$ are the velocities with respect to x-axis and y-axis.

4.1 Kinematic Modeling of Vehicle

Consider the vehicle as a unicycle rolling on a plane, as is shown in Figure 4.1. The configuration of a unicycle could be described with a vector of three generalized coordinates, namely the orthogonal coordinates (x, y) of the point contacting with the plane in a fixed frame and θ measuring the orientation of the wheel from the contacting point with respect to x-axis, which is shown in Figure 4.2. The vector of generalized coordinates is defined by $q = [x, y, \theta]$, the generalized velocities$\dot{q} = [\dot{x}, \dot{y}, \dot{\theta}]$ is under nonholonomic constraints in the presence of rolling without slipping conditions, which can be characterized by a form of Pfaffian constraint of a set of linearly independent constraints linear in velocity.

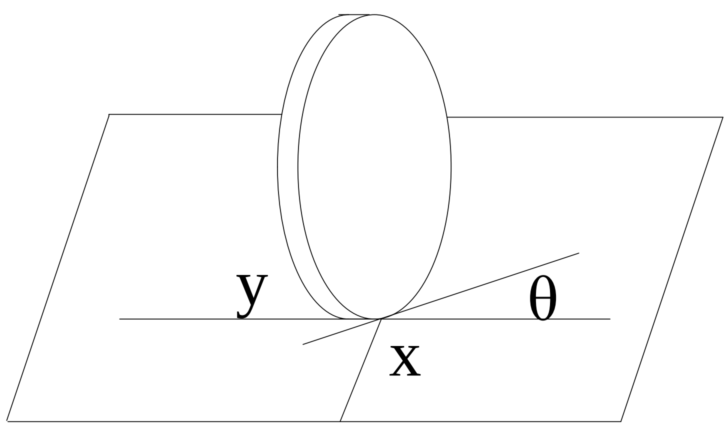

Figure 4.1 A unicycle

Figure 4.2 is the simplified model of unicycle.

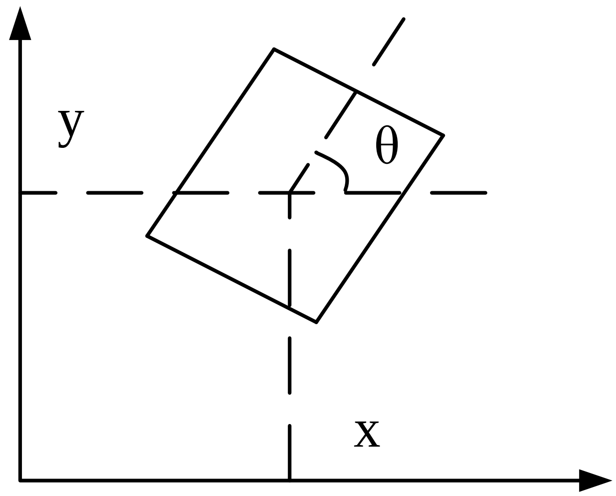

Figure 4.2 Simplified model of unicycle

$$[sin(\theta) \quad -cos(\theta) \quad 0]\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = 0$$

The generalized velocities can be written as

$$\dot{q} = \begin{bmatrix}\cos(\theta)\\ sin(\theta)\\ 0\end{bmatrix} v + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix} w \tag{4.1}$$

where *w* and *v* are the unicycle's rotational and translational velocities.

Consider a car-like vehicle model having the same kinematic of a real car. A general front-wheel steer, rear-wheel drive vehicle is commonly utilized in practical application, which means the external torque is implemented onto the real wheels. The front wheels are driven wheels and can be steered while the rear wheel orientation is fixed. The generalized coordination of four-wheel car-like vehicle can be defined by vector $q = [x, y, \theta, \varphi]$, where x, y are the cartesian coordinates of the rear wheel, θ is the orientation of the car body with respect to x-axis, φ is the steering angle, as is shown in Figure 4.3.

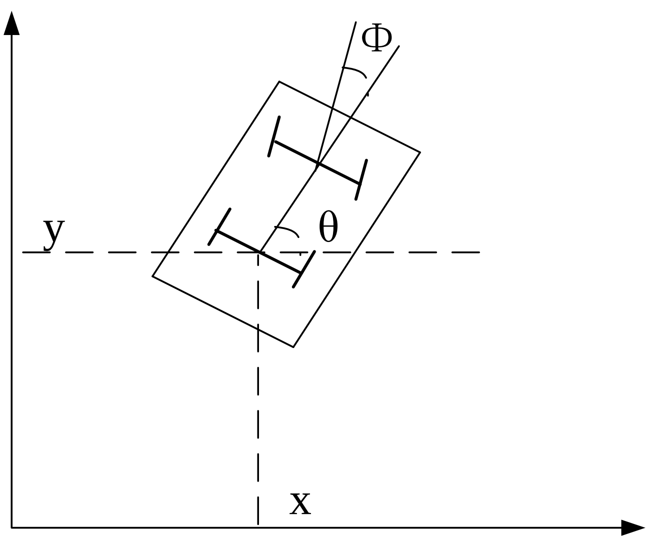

Figure 4.3 Simplified model of four-wheeled car

Similar as the unicycle, the four-wheel vehicle model is also subject to nonholonomic constraints on both its real and front wheels with the form for each wheel,

$$\dot{x}_f \sin(\theta + \varphi) - \dot{y}_f \, cos(\theta + \varphi) = 0$$

$$\dot{x}_r \sin(\theta) - \dot{y}_r \cos(\theta) = 0$$

where x_f and y_f denote the cartesian coordinates of the middle point of the axis connecting the two front wheels， x_r and y_r denote the cartesian coordinates of the middle point of the axis connecting the two rear wheels. Assume the distance between the middle point of front axis and rear axis is *l* and the vehicle has a rigid body, it follows that

$$x_f = x_r + lcos(\theta)$$

$$y_f = y_r + lsin(\theta)$$

Then the Pfaffian constraint matrix *A* can be denoted by

$$A = \begin{bmatrix} sin(\theta+\varphi) & -cos(\theta+\varphi) & -lcos\varphi & 0 \\ sin(\theta) & -cos(\theta) & 0 & 0 \end{bmatrix}$$

The kinematic model can be written in

$$\dot{q} = \begin{bmatrix} cos(\theta) \\ sin(\theta) \\ tan(\varphi)/l \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} w \tag{4.2}$$

Notice in. (4.2), the steering angle cannot achieve $\pm\frac{1}{2}\pi$ which implies the front wheel cannot be perpendicular to the longitudinal axis of the car body.

Throughout this paper, the unicycle model in Figure 4.2 is used, when the two axises in the four-wheel car model in Figure 4.3 is considered overlapped and regarded as a single axis. Then four-wheel car-like models can be simplified into unicycles.

In Section 4.2, state controllability of kinematic system (4.1) at a point and a path is addressed with the aid of controllability theory of nonlinear system. Controllability of the vehicle's kinematics determines whether feedback control could be utilized for stability of the nonlinear kinematic system. Trajectory tracking systems (4.4) is proved to be controllable and can be stabilized by states feedback control thus the solo vehicle can track and converge to the reference states.

4.2 Controllability Analysis

Kinematic model in (4.1) is denoted by

$$\dot{q} = g_1 v + g_2 w \tag{4.3}$$

where

$$g_1 = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{bmatrix}, g_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(4.3) is a nonlinear and driftless system, notice the number of inputs is less than that of the generalized coordinates, then (4.3) is an underactuated system.

Controllability at a point: Consider q_e as an equilibrium point with zero input, in order to check whether system (4.3) is linearized at q_e,

$$\dot{\tilde{q}} = g_1(q_e)v + g_2(q_e)w = G(q_e)V$$

where $\tilde{q} = q - q_e$, the linearized system has rank of two, which implies it is locally controllable. In order to test the controllability of nonlinear system (4.3), Lie Algebra rank condition is introduced. Define the Lie bracket as

$$[X \quad Y](q) = \frac{\partial Y}{\partial q} X - \frac{\partial X}{\partial q} Y$$

The controllability problem boils down to checking whether

$$rank\left[g_1, g_2, [g_1, g_2], \left[g_1, [g_1, g_2]\right], \dots\right] = 3$$

$[g_1, g_2] = [\sin(\theta), -\cos(\theta), 0]$, it can be verified

$$rank\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ \sin(\theta) & 0 & -\cos(\theta) \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} \sin(\theta)\cos(\theta) & 0 & -cos^2(\theta) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3$$

then (4.3) is globally controllable. However, the system cannot be stabilized by a smooth feedback control law, which implies the stability of (4.3) requires either giving up the continuity condition or using time-varying control laws.

Controllability about a trajectory: The desired reference trajectory is denoted by $q_d(t) = [x_d(t), y_d(t), \theta_d(t)]$, define $\tilde{q}(t) = q(t) - q_d(t)$, $\tilde{V}(t) = V(t) - V_d(t)$,

$$\tilde{q}(t) = f\big(q(t), V(t), q_d(t), V_d(t)\big)$$

$$= G\big(q(t)\big)V(t) - G(q_d(t))V_d(t)$$

$$\dot{\tilde{q}}(\mathrm{t}) = \frac{\partial f\big(q_d(t), V_d(t)\big)}{\partial q(t)}\big(q(t) - q_d(t)\big) + \frac{\partial f\big(q_d(t), V_d(t)\big)}{\partial V(t)}\big(V(t) - V_d(t)\big) \tag{4.4}$$

Then it follows that the linearization of (4.3) is

$$\dot{\tilde{q}} = \begin{bmatrix} 0 & 0 & -sin\big(\theta_d(t)\big)\,v_d(t) \\ 0 & 0 & cos\big(\theta_d(t)\big)\,v_d(t) \\ 0 & 0 & 0 \end{bmatrix}\tilde{q} + \begin{bmatrix} cos\big(\theta_d(t)\big) & 0 \\ sin\big(\theta_d(t)\big) & 0 \\ 0 & 1 \end{bmatrix}\tilde{V} \tag{4.5}$$

Notice when the trajectory is designed to be time-invariant signal with $v_d(t) = v_{d0}$ and $\theta_d(t) = \theta_{d0}$, the controllability condition matrix is

$$[B \quad AB \quad AB^2] = \begin{bmatrix} cos(\theta_{d0}) & 0 & 0 & -sin(\theta_{d0})v_{d0} & 0 & 0 \\ sin(\theta_{d0}) & 0 & 0 & cos(\theta_{d0})v_{d0} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} = 3$$

which implies the linearized system (4.4) is locally controllable with a linear reference trajectory. By linear feedback, eigenvalues of (4.5) can be placed in the left half of complex plane, then the equilibrium point $\tilde{q}(t) = q(t) - q_d(t)$ is locally asymptotically stable. When the reference trajectory is nonlinear with inconstant velocities, Gramian's singularity determines the controllability and state-transition matrix should satisfy specific conditions to make the system locally stable. Due to the nonlinearity of kinematic system, if the nonlinear system can be transformed into linear form with state transformation, linear feedback control laws can be designed to guarantee stability of transformed system thus stability of original nonlinear system.

Since solo kinematic system is proved controllable about a trajectory and state feedback control laws can be designed for consensus, it is accessible to achieve multivehicle systems Since solo kinematic system is proved controllable about a trajectory and state feedback control laws can be designed for consensus with the desired trajectory. It is accessible to achieve multivehicle systems consensus by states feedback control using not only the vehicle's own states information but also its neighbor vehicles' states information.

4.3 Formation Tracking with Chained-form Systems

The canonical form of kinematic model makes it easier to analyze the vehicle system in the chained form system. Chained system is a qausi-linear system, the first two equation of chained system is the standard linear system and can be stabilized by linear feedback control laws if not considering the states of following equations. And in [52][53], different control laws have been proposed for stabilization of all states of chained form system. Chained-form nonlinear system is firstly introduced and then combined with multivehicle systems for designing distributed formation tracking methods.

Consider the unicycle model in (4.1), with the following states transform

$$q_1 = \theta$$

$$q_2 = x\cos(\theta) + y\sin(\theta)$$

$$q_3 = x\sin(\theta) - y\cos(\theta)$$

$$v = q_3 u_1 + u_2$$

$$w = u_1$$

(4.1) can be written as

$$\begin{aligned} \dot{q}_1 &= u_1 \\ \dot{q}_2 &= u_2 \\ \dot{q}_3 &= q_2 u_1 \end{aligned} \tag{4.6}$$

Actually, the transform equations could always be found to change the two-inputs *n*-states driftless systems to the chained-from system

$$\dot{q}_1 = u_1$$

$$\dot{q}_2 = u_1$$

$$\dot{q}_n = q_{n-1} u_1$$

The virtual agent vehicle, namely the desired trajectory, is also transformed into the chain-like systems

$$\dot{q}_{10} = u_{10}$$

$$\dot{q}_{20} = u_{10}$$

$$\dot{q}_{30} = q_{2r}u_{10} \tag{4.7}$$

Problem statement The generalized coordinates of vehicle v_j is denoted by [x_{1j}, y_{1j}, θ_{1j}] The control problem is to design control laws and make the follower vehicle track the virtual leader [x_0, y_0, θ_0] in a fixed pattern [p_{jx}, p_{jy}]. The control problem is defined as designing control laws for v_j and w_j such using its own state information and the knowledge from its neighbors such that

$$\lim_{t\to\infty} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} p_{ix} - p_{jx} \\ p_{iy} - p_{jy} \end{bmatrix} \tag{4.8}$$

$$\lim_{t\to\infty} (\theta_j - \theta_0) = 0 \tag{4.9}$$

$$\lim_{t\to\infty} \left[\sum_{i=1}^{m} \frac{x_j}{m} - x_0 \right] = 0 \tag{4.10}$$

$$\lim_{t\to\infty} \left[\sum_{i=1}^{m} \frac{y_j}{m} - x_0 \right] = 0 \tag{4.11}$$

for $1 \le i \le j \le m$, in order to introduce the desired pattern into the transformed states, the state transformation is redefined by

$$q_{1j} = \theta_j$$

$$q_{2j} = (x_j - p_{jx})cos\theta_j + (y_j - p_{jx})sin\theta_j$$

$$q_{3j} = (x_j - p_{jx})sin\theta_j - (y_j - p_{jx})cos\theta_j$$

$$v_j = q_{3j}u_{1j} + u_{2j}$$

$$w_j = u_{1j}$$

Then the chained-form system for vehicle v_j is

$$\dot{q}_{1j} = u_{1j} \tag{4.12}$$

$$\dot{q}_{2j} = u_{2j} \tag{4.13}$$

$$\dot{q}_{3j} = q_{2j}u_{1j} \tag{4.14}$$

Lemma 4.1 if

$$\lim_{t\to\infty}(q_{1j} - q_{10}) = 0 \tag{4.15}$$

$$\lim_{t\to\infty}(q_{2j} - q_{20}) = 0 \tag{4.16}$$

$$\lim_{t\to\infty}(q_{3j} - q_{30}) = 0 \tag{4.17}$$

For 1≤j ≤*m*, then (4.8)-(4.11) hold.

Proof It is straightforward to prove that (4.9) holds by (4.15), by the transformation equations

$$\begin{bmatrix} x_j - p_{jx} \\ y_j - p_{jx} \end{bmatrix} = \begin{bmatrix} cos\theta_j & sin\theta_j \\ sin\theta_j & -sin\theta_j \end{bmatrix} \begin{bmatrix} q_{2j} \\ q_{3j} \end{bmatrix}$$

Therefore, by (4.16) and (4.17)

$$\lim_{t\to\infty} \begin{bmatrix} x_j - p_{jx} \\ y_j - p_{jx} \end{bmatrix} = \lim_{t\to\infty} \begin{bmatrix} cos\theta_j & sin\theta_j \\ sin\theta_j & -sin\theta_j \end{bmatrix} \begin{bmatrix} q_{2j} \\ q_{3j} \end{bmatrix}$$

$$= \begin{bmatrix} cos\theta_0 & sin\theta_0 \\ sin\theta_0 & -sin\theta_0 \end{bmatrix} \begin{bmatrix} q_{20} \\ q_{30} \end{bmatrix} = \begin{bmatrix} x_0 - p_{0x} \\ y_0 - p_{0y} \end{bmatrix}$$

$[p_{0x}, p_{0y}]$ = $[0, 0]$ is considered the center of the desired pattern, namely the position of the virtual leader, which means $\lim_{t\to\infty}[x_j - p_{jx} - x_0, y_j - p_{jy} - y_0] = [0,0]$.

By Lemma 4.1, the control problem can be defined as designing u_{1j} and u_{2j} with aids of its own state information and its neighbors' information such that

$$\lim_{t\to\infty}(q_{*j} - q_{*0}) = 0 \tag{4.18}$$

where $q_{*j} = [q_{1j}, q_{2j}, q_{3j}]$ and $q_{*0} = [q_{10}, q_{20}, q_{30}]$.

Before designing the control laws for u_{1j} and u_{2j}, several theorems which will be utilized in designing distributed control methods are proposed, consider the following system

$$\dot{x}_1 = f_1(t, x_1) + g(t, x_1, x_2)x_2$$

$$\dot{x}_2 = f_2(t, x_2) \tag{4.19}$$

where $x_1 \epsilon \mathbb{R}^n$, $x_2 \epsilon \mathbb{R}^n$, $f_1(t, x_1)$ is continuously differentiable in (t, x_1) and $g(t, x_1, x_2)$, $f_2(t, x_1)$ are continuous and locally Lipschitz in x_1 and (x_1, x_2), it could be seen that the state equation of x_1 depends on state x_2 thus (4.19) is a cascaded system.

Theorem 4.1 The cascaded system (4.19) is globally uniformly exponentially stable if the following three assumptions hold [54].

Assumption 1 $\dot{x}_1 = f_1(t, x_1)$ is globally uniformly exponentially stable and there exists a continuously differentiable function $V(t, x_1)$:$\mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}$ that satisfies

$$k_1||x_1||^a \leq V(t, x_1) \leq k_2||x_1||^a$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_1} f_1(t, x_1) \le -k_3 ||x_1||^a$$

where k_1,k_2 and k_3 and a are positive constants.

Assumption 2 The interconnection function $g(t, x_1, x_2)$ satisfies for all $t \ge t_0$

$$\left||g(t, x_1, x_2)|\right| \le \theta_1(\left||x_2|\right|) + \theta_2(\left||x_2|\right|)x_1$$

where θ_1 and θ_2 are continuous functions.

Assumption 3 $\dot{x}_2 = f_2(t, x_1)$ is globally uniformly exponentially stable and for all

$t_0 \ge 0$, it follows that

$$\int_{t_0}^{\infty} ||x_2(t_0, t, x_2(t_0))|| \le k(||x_2(t_0)||)$$

where k is a class κ function.

Theorem 4.1 has been proved in [54] and [55], the proof is limited here.

Lemma 4.2 The linear equation is uniformly exponentially stable if there exists a symmetric, continuously differentiable matrix function $Q(t)$ and such that

$$c_1 I \le Q(t) \le c_2 I$$

$$Q(t) f_1(t) + f_1^T(t) Q(t) + \dot{Q}(t) \le vI$$

where c_1, c_2 and v are finite positive constants.

Theorem 4.2 for the system

$$\dot{x} = (f_1(t) + f_2(t))x \tag{4.20}$$

If $\dot{x} = f_1(t)x$ is globally exponentially stable and $f_2(t)$ converges to zero exponentially, then the system (4.20) is globally exponentially stable under the assumption that both $f_1(t)$and $f_2(t)$are bounded.

Proof Consider the linear time-varying system $\dot{x} = f_1(t)x$, since it is globally exponentially stable, let $Q(t)$ be a continuous, bounded positive symmetric matrix,

$$Q(t) = \int_t^\infty \varphi^T(\tau,t)\varphi(\tau,t)d\tau$$

And $Q(t)$ satisfies the following conditions

$$c_1 I \leq Q(t) \leq c_2 I$$

$$Q(t)f_1(t) + f_1^T(t)Q(t) + \dot{Q}(t) = -I$$

where c_1 and c_1 are finite positive constants, $\varphi(\tau,t)$ is the state transmission matrix of $\dot{x} = f_1(t)x$, it follows that

$$Q(t)(f_1(t) + f_2(t)) + \left(f_1^T(t) + f_2^T(t)\right)Q(t) + \dot{Q}(t) = -I + Q(t)f_2(t) + f_2^T(t)Q(t)$$

Since $Q(t)$ and $f_2(t)$ are bounded, $f_2(t)$ is exponentially stable, then

$$Q(t)f_2(t) + {f_2}^T(t)Q(t) \leq I$$

for $\mathrm{t} \geq \mathrm{t}_0$, it follows that

$$Q(t)(f_1(t) + f_2(t)) + \left(f_1^T(t) + f_2^T(t)\right)Q(t) + \dot{Q}(t) = vI$$

By Lemma 4.2 (4.20) is globally exponentially stable.

Consider the linear time-varying system

$$\dot{x} = \begin{bmatrix} 0 & \dots & \dots & \dots & 0 \\ \varphi(t) & 0 & \dots & \dots & 0 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \varphi(t) & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} u \qquad (4.21)$$

where $\varphi(t)$ is a bounded continuously differentiable Lipschiz signal which satisfies $|\varphi(t)| \leq M$.

Theorem 4.3 System (4.21) is globally exponentially stable if the control inputs

$$u = -k_i \varphi(t)^{mod(i,2)} x_i$$

where k_i are coefficients of the following polynomial

$$\lambda^{\mathrm{n}} + k_1 \lambda^{\mathrm{n}-1} + \cdots + k_n$$

such that this polynomial is Hurwitz.

4.4 Distributed State Feedback Tracking Control of Kinematic Cascaded Systems

In order to track the desired trajectory, the tracking error $q_{*je} = q_{*j} - q_{*0}$ is defined as

$$\dot{q}_{1je} = u_{1j} - u_{1r}$$

$$\dot{q}_{2je} = u_{2j} - u_{2r}$$

$$\dot{q}_{3je} = q_{2je} u_{1r} + q_{2j}(u_{1j} - u_{1r}) \qquad (4.22)$$

Since the stability of system (4.22) implies the $\lim_{t\to\infty} q_{*je} = q_{*j} - q_{*0} = 0$, then the control problem is to design u_{1j} and u_{2j} with the knowledge of q_{*j}, q_{*0} and u_{*r} such that (4.22) is globally exponentially stable.

Consider u_{*j} in the following form

$$u_{1j} = u_{1j}(t, q_{*j}, q_{*0}, u_{*r})$$

$$u_{2j} = u_{2j}(t, q_{*j}, q_{*0}, u_{*r})$$

which means the control laws utilize the state information and are thus state-feedback methods. Notice in (4.22) u_{1j} can be easily designed to stabilize q_{1je}, which means $\dot{x}_2 = f_2(t, x_1)$ is globally exponentially stable, $\dot{x}_1 = f_1(t, x_2)$ is

$$\begin{bmatrix} \dot{q}_{2je} \\ \dot{q}_{3je} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ u_{1r} & 0 \end{bmatrix} \begin{bmatrix} q_{2je} \\ q_{3je} \end{bmatrix}$$

Then the chained from cascaded system is

$$\dot{x}_1 = f_1(t, x_1) + g(t, x_1, x_2)x_2$$

$$\dot{x}_2 = f_2(t, x_2) \tag{4.23}$$

where $x_1 = [q_{2je}, q_{3je}]^T$, $x_2 = q_{1je}, f_1(t, x_1) = \begin{bmatrix} 0 & 0 \\ u_{1r} & 0 \end{bmatrix} \begin{bmatrix} q_{2je} \\ q_{3je} \end{bmatrix}$, $g(t, x_1, x_2)x_2 = q_{2j}(u_{1j} - u_{1r})$ and $f_2(t, x_2) = u_{1j} - u_{1r}$.

Theorem 4.4 the control laws

$$u_{1j} = u_{1r} - k_1 q_{1je}$$

$$u_{2j} = u_{2r} - k_2 q_{2je} - k_3 u_{1r} q_{3je} \qquad (4.24)$$

guarantee (4.23) are globally exponentially stable, provided that $k_1 > 0$, k_2 and k_3 are such that the polynomial

$$\lambda^2 + k_2\lambda + k_3$$

is Hurwitz.

Proof The closed-loop dynamics of (4.22) can be expressed by $f_1(t, x_1) = (A(t) - BK(t))x_1$, where $A(t) = \begin{bmatrix} 0 & 0 \\ u_{1r} & 0 \end{bmatrix}$, $B = [1,0]^T$ and $K(t) = [k_2, k_3 u_{1r}], f_2(t, x_2) = -k_1 x_2$, $g(t, x_1, x_2) = -k_1[0, q_{2j}]^T$. Since the reference signal u_{1r} is persistently excited signal which is bounded and continuously differentiable, by Theorem 4.3 it is proved $\dot{x}_1 = f_1(t, x_1)$ is globally exponentially stable. For the Assumption 2 in Theorem 4.1, it follows that

$$\begin{aligned} ||g(t, x_1, x_2)|| &= ||-k_1[0, q_{2r}]^T - k_1[0, q_{2e}]^T|| \\ &\leq k_1 C + k_1||[0, q_{2e}]^T|| \end{aligned}$$

Then Assumption 2 holds. For the third assumption, since the closed-loop dynamics of $f_2(t, x_2)$ is $\dot{x}_2 = -k_1 x_2$ which is globally exponentially stable, then Assumption 3 holds.

Notice in the control laws in (4.24), both u_{1j} and u_{2j} utilize the information of reference states u_{*r} directly. From the communication topology it is learned that not all the vehicles could receive information directly from the virtual leader, which means other methods should be found to replace u_{*r} while the new term should still converge to u_{*r}. Then the distributed control algorithms on first-order system with time-varying reference signals in Theorem 3.2 is utilized for achieving estimation of the u_{*r}. Moreover, in order to express the communication between vehicles, the constant positive figure k_1 in (4.24) is replaced with Laplacian Matrix from graph theory. Consider the communication topology for *m* vehicles as graph *G*, the adjacency matrix *A* and the corresponding Laplacian matrix *L* are defined as in Chapter 2. Regard the reference signals as a virtual vehicle agent and suppose there exists a spanning tree in the communication graph and the virtual agent is the root of the spanning tree, it can be proved *Le* (containing the virtual leader) is positive symmetric thus has the same effects as k_1.

Theorem 4.5 For *m* systems in (4.12), if there exists a spanning tree with the virtual leader v_{m+1} as the root of the spanning tree, then the distributed control laws

$$u_{1j} = \eta_{1j} \tag{4.25}$$

where

$$\eta_{1j} = -\sum_{i\in N_j} a_{ji}\left(q_{1j} - q_{1i}\right) - a_{j,m+1}\left(q_{1j} - q_{1,m+1}\right) + \delta_{1j} \tag{4.26}$$

$$\dot{\delta}_{1j} = -\sum_{i\in N_j} a_{ji}\left(\delta_{1j} - \delta_{1i}\right) - a_{j,m+1}\left(\delta_{1j} - \delta_{1,m+1}\right) - \rho sign[\sum_{i\in N_j} a_{ji}\left(\delta_{1j} - \delta_{1i}\right) - a_{j,m+1}\left(\delta_{1j} - \delta_{1,m+1}\right)] \tag{4.27}$$

guarantee that q_{1j} globally exponentially converge to $q_{1,m+1}$, for $1 \leq j \leq m$

Proof Define the tracking error $\tilde{\delta}_{1j} = \delta_{1j} - \delta_{1,m+1}$ it follows that

$$\dot{\tilde{\delta}}_{1j} = \sum_{j=1}^{m} a_{ij}\left(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}\right) - a_{i,n+1}\tilde{\delta}_{1j} - \rho sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}\right) - a_{j,m+1}\tilde{\delta}_{1j}\right) - \dot{\delta}_{1,m+1}$$

By theorem 2.4 it can be proved $\delta_{1\mathrm{j}} - \delta_{1,m+1}$ globally exponentially converge to zero. Substitute u_{1j} in (4.25) into (4.12) it follows that

$$\dot{q}_{1j} = -\sum_{i\in N_j} a_{ji}\left(q_{1j} - q_{1i}\right) - a_{j,m+1}\left(q_{1j} - q_{1,m+1}\right) + \delta_{1j}$$

Define $\tilde{q}_{1j} = q_{1j} - q_{1,m+1}$ then the equation above can be written in

$$\dot{\tilde{q}}_{1j} = -\sum_{i\in N_j} a_{ji}\left(\tilde{q}_{1j} - \tilde{q}_{1i}\right) - a_{j,m+1}\tilde{q}_{1j} + \delta_{1j} - \dot{q}_{1,m+1} \qquad (4.28)$$

With the aid of the Laplacian Matrix in graph theory, (4.28) can be written in

$$\dot{\tilde{q}}_{1*} = -(L + B)\tilde{q}_{1*} + \tilde{\delta}_{1*} \qquad (4.29)$$

(4.29) is a linear system (since $L + B$ is a positive symmetric time-invariant matrix) subjected to globally exponentially stable disturbance$\tilde{\delta}_{1*}$. By Theorem 4.2, $\tilde{q}_{1*}$ is globally exponentially stable thus q_{1j} globally exponentially converge to $q_{1,m+1}$.

Theorem 4.6 For *m* systems in (4.13)-(4.14), if there exists a spanning tree with the virtual leader v_{m+1} as the root of the spanning tree, then the distributed control laws

$$u_{2j} = \eta_{2j} \qquad (4.30)$$

where

$$\eta_{2j} = -k_2 q_{2j} - k_3 u_{1j} q_{3j} + \delta_{2j} \tag{4.31}$$

$$\dot{\delta}_{2j} = -\sum_{i \in N_j} a_{ji}\left(\delta_{2j} - \delta_{2i}\right) - a_{j,m+1}\left(\delta_{2j} - \delta_{2,m+1}\right) - \beta sign[\sum_{i \in N_j} a_{ji}\left(\delta_{2j} - \delta_{2i}\right) - a_{j,m+1}\left(\delta_{2j} - \delta_{2,m+1}\right)] \tag{4.32}$$

guarantee that q_{2j} globally exponentially converge to $q_{2,m+1}$ and q_{3j} globally exponentially converge to $q_{3,m+1}$, for $1 \le j \le m$

where

$$\delta_{2,m+1} = u_{2,m+1} + k_2 q_{2,m+1} + k_3 u_{1,m+1} q_{3,m+1}$$

k_2, k_3 are chosen such roots of polynomial $\lambda^2 + k_2 \lambda + k_3$ are in the left-half of complex plane.

Proof. Substitute u_{2j} in (4.30) into (4.13) it follows that

$$\dot{q}_{2j} = -k_2 q_{2j} - k_3 u_{1j} q_{3j} + \delta_{2j} \tag{4.33}$$

Define

$$s_{1j} = \sum_{i \in N_j} a_{ji}\left(q_{2j} - q_{2i}\right) + a_{j,m+1}\left(q_{2j} - q_{2,m+1}\right)$$

$$s_{2j} = \sum_{i \in N_j} a_{ji}\left(q_{3j} - q_{3i}\right) + a_{j,m+1}\left(q_{3j} - q_{3,m+1}\right)$$

Differentiate s_{1j}, s_{2j} along (4.31) and (4.14) it follows that

$$\dot{s}_{1j} = \sum_{i\in N_j} a_{ji}\left(\left(-k_2 q_{2j} - k_3 u_{1j} q_{3j} + \delta_{2j}\right) - \left(-k_2 q_{2i} - k_3 u_{1i} q_{3i} + \delta_{2i}\right)\right)$$

$$+a_{j,m+1}\left(\left(-k_2 q_{2j} - k_3 u_{1j} q_{3j} + \delta_{2j}\right) - \left(-k_2 q_{2,m+1} - k_3 u_{1,m+1} q_{3,m+1} + \delta_{2,m+1}\right)\right)$$

Then one has

$$\dot{s}_{1j} = -k_2\left(\sum_{i\in N_j} a_{ji}\left(q_{2j} - q_{2i}\right) + a_{j,m+1}\left(q_{2j} - q_{2,m+1}\right)\right)$$

$$-k_3 u_{1,m+1}\left(\sum_{i\in N_j} a_{ji}\left(q_{3j} - q_{3i}\right) + a_{j,m+1}\left(q_{3j} - q_{3,m+1}\right)\right)$$

$$+\sum_{i\in N_j} a_{ji}\left(\delta_{2j} - \delta_{2i}\right) + a_{j,m+1}\left(\delta_{2j} - \delta_{2,m+1}\right)$$

$$-k_3\left(\sum_{i\in N_j} a_{ji}\left((u_{1j} - u_{1,m+1})q_{3j} - (u_{1j} - u_{1,m+1})q_{3i}\right) + (u_{1j} - u_{1,m+1})a_{j,m+1} q_{3j}\right)$$

By mathematical manipulation it follows that

$$\dot{s}_{1j} = -k_2 s_{1j} - k_3 u_{1,m+1} s_{2j} + \sum_{i\in N_j} a_{ji}\left(\delta_{2j} - \delta_{2i}\right) + a_{j,m+1}\left(\delta_{2j} - \delta_{2,m+1}\right)$$

$$-k_3\left(\sum_{i\in N_j} a_{ji}\left((u_{1j} - u_{1,m+1})q_{3j} - (u_{1i} - u_{1,m+1})q_{3i}\right) + (u_{1j} - u_{1,m+1})a_{j,m+1} q_{3j}\right) \tag{4.34}$$

Similarly

$$\dot{s}_{2j} = \sum_{i \in N_j} a_{ji} \left(u_{1j} q_{2j} - u_{1i} q_{2i} \right) + a_{j,m+1} \left(u_{1j} q_{2j} - u_{1,m+1} q_{2,m+1} \right)$$

$$\dot{s}_{2j} = u_{1,m+1} (\sum_{i \in N_j} a_{ji} \left(q_{2j} - q_{2i} \right) + a_{j,m+1} \left(q_{2j} - q_{2,m+1} \right))$$

$$+ \sum_{i \in N_j} a_{ji} \left((u_{1j} - u_{1,m+1}) q_{2j} - (u_{1i} - u_{1,m+1}) q_{2i} \right) + (u_{1j} - u_{1,m+1}) a_{j,m+1} q_{2j}$$

By mathematical manipulation it follows that

$$\dot{s}_{2j} = u_{1,m+1} s_{1j} + \sum_{i \in N_j} a_{ji} \left((u_{1j} - u_{1,m+1}) q_{2j} - (u_{1i} - u_{1,m+1}) q_{2i} \right) + (u_{1j} - u_{1,m+1}) a_{j,m+1} q_{2j} \tag{4.35}$$

Let $\tilde{\delta}_{2\mathrm{j}} = \delta_{2j} - \delta_{2,m+1}$, $\tilde{u}_{1j} = u_{1j} - u_{1,m+1}$, (4.34) and (4.35) can be written as

$$\dot{s}_{1j} = -k_2 s_{1j} - k_3 u_{1,m+1} s_{2j} + \sum_{i \in N_j} a_{ji} \left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i} \right) + a_{j,m+1} \tilde{\delta}_{2j}$$

$$-k_3 \left(\sum_{i \in N_j} a_{ji} \left(\tilde{u}_{1j} q_{3j} - \tilde{u}_{1j} q_{3i} \right) + \tilde{u}_{1j} a_{j,m+1} q_{3j} \right) \tag{4.36}$$

$$\dot{s}_{2j} = u_{1,m+1} s_{1j} + \sum_{i \in N_j} a_{ji} \left(\tilde{u}_{1j} q_{2j} - \tilde{u}_{1j} q_{2i} \right) + \tilde{u}_{1j} a_{j,m+1} q_{2j} \tag{4.37}$$

(4.27), (4.28) and (4.32) are written in

$$\dot{\tilde{\delta}}_{1j} = -\sum_{i\in N_j} a_{ij}\left(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}\right) - a_{i,m+1}\tilde{\delta}_{1j} - \dot{\delta}_{1,m+1} - \rho sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}\right) - a_{j,m+1}\tilde{\delta}_{1j}\right) \tag{4.38}$$

$$\dot{\tilde{q}}_{1j} = -\sum_{i\in N_j} a_{ji}\left(\tilde{q}_{1j} - \tilde{q}_{1i}\right) - a_{j,m+1}\tilde{q}_{1j} + \tilde{\delta}_{1j} \tag{4.39}$$

$$\dot{\tilde{\delta}}_{2j} = -\sum_{i\in N_j} a_{ij}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) - a_{i,m+1}\tilde{\delta}_{2j} - \dot{\delta}_{2,m+1} - \beta sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) - a_{j,m+1}\tilde{\delta}_{2j}\right) \tag{4.40}$$

Define

$$x_1 = [[s_{11}, s_{21}], [s_{12}, s_{22}], \dots, [s_{1m}, s_{2m}]]$$

$$x_2 = [\left[\tilde{\delta}_{21}, \tilde{q}_{11}, \tilde{\delta}_{11}\right], \left[\tilde{\delta}_{22}, \tilde{q}_{12}, \tilde{\delta}_{12}\right], \dots, \left[\tilde{\delta}_{2m}, \tilde{q}_{1m}, \tilde{\delta}_{1m}\right]]$$

Then it follows that

$$f_{1j} = \begin{bmatrix} -k_2 & -k_3 u_{1,m+1} \\ u_{1,m+1} & 0 \end{bmatrix} \begin{bmatrix} s_{1j} \\ s_{2j} \end{bmatrix}$$

$$f_1(t, x_1) = [f_{11}, \dots, f_{1m}]$$

$$f_{2j} = \begin{bmatrix} -\sum_{i\in N_j} a_{ij}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) - a_{i,m+1}\tilde{\delta}_{2j} \\ -\beta sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) - a_{j,m+1}\tilde{\delta}_{2j}\right) - \dot{\delta}_{2,m+1} \\ -\sum_{i\in N_j} a_{ji}\left(\tilde{q}_{1j} - \tilde{q}_{1i}\right) - a_{j,m+1}\tilde{q}_{1j} + \tilde{\delta}_{1j} \\ -\sum_{i\in N_j} a_{ij}\left(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}\right) - a_{i,m+1}\tilde{\delta}_{1j} \\ -\rho sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}\right) - a_{j,m+1}\tilde{\delta}_{1j}\right) - \dot{\delta}_{1,m+1} \end{bmatrix}$$

$$f_2(t, x_2) = [f_{21}, f_{22}, \dots, f_{2m}]$$

$$g_j = \begin{bmatrix} \sum_{i\in N_j} a_{ji}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) + a_{j,m+1}\tilde{\delta}_{2j} \\ -k_3\left(\sum_{i\in N_j} a_{ji}\left(\tilde{u}_{1j}q_{3j} - \tilde{u}_{1j}q_{3i}\right) + \tilde{u}_{1j}a_{j,m+1}q_{3j}\right) \\ \sum_{i\in N_j} a_{ji}\left(\tilde{u}_{1j}q_{2j} - \tilde{u}_{1j}q_{2i}\right) + \tilde{u}_{1j}a_{j,m+1}q_{2j} \end{bmatrix}$$

$$g(t, x_1, x_2)x_2 = [g_1, g_2, \dots, g_m]$$

By Theorem 2.4 it can be proved $\tilde{\delta}_{2j} = \delta_{2j} - \delta_{2,m+1}$ globally exponentially converge to zero, which implies

$$-\sum_{i\in N_j} a_{ij}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) - a_{i,m+1}\tilde{\delta}_{2j} - \dot{\delta}_{2,m+1} - \beta sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) - a_{j,m+1}\tilde{\delta}_{2j}\right)$$

globally exponentially converge to zero.

By theorem 4.5 $\tilde{q}_{1j} = q_{1j} - q_{1,m+1}$ and $\tilde{\delta}_{1j} = \delta_{1j} - \delta_{1,m+1}$ globally exponentially converge to zero, which implies

$$-\sum_{i\in N_j} a_{ij}(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}) - a_{i,m+1}\tilde{\delta}_{1j} - \dot{\delta}_{1,m+1} - \rho sign\left(\sum_{j=1}^{n} a_{ij}(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}) - a_{j,m+1}\tilde{\delta}_{1j}\right)$$

globally exponentially converge to zero, and

$$-\sum_{i\in N_j} a_{ji}\left(\tilde{q}_{1j} - \tilde{q}_{1i}\right) - a_{j,m+1}\tilde{q}_{1j} + \tilde{\delta}_{1j}$$

globally exponentially converge to zero. From the definition of f_{2j}, then system $\dot{x}_2 = f_2(t, x_2)$ is exponentially stable,the third assumption in Theorem 4.1 holds. Since f_{1j} have the same form as (4.24), by theorem 4.3 $\dot{x}_1 = f_1(t, x_1)$ is exponentially stable, then the first assumption in Theorem 4.1 holds. From the definition of g_j, it can be proved the second assumption in Theorem 4.1 holds, which means $x_1 = [[s_{11}, s_{21}], [s_{12}, s_{22}], \dots, [s_{1m}, s_{2m}]]$ is globally exponentially stable. From the definition of s_{1j} and s_{2j} one has

$$\begin{aligned} s_{1j} &= \sum_{i\in N_j} a_{ji}\left(q_{2j} - q_{2i}\right) + a_{j,m+1}\left(q_{2j} - q_{2,m+1}\right) \\ &= \sum_{i\in N_j} a_{ji}\left(\tilde{q}_{2j} - \tilde{q}_{2i}\right) + a_{j,m+1}\tilde{q}_{2j} \\ &= (L+B)\tilde{q}_{2j} \end{aligned}$$

Since $(L + B)$ is positive symmetric matrix then $q_{2j} - q_{2,m+1} = \tilde{q}_{2j} = (L + B)^{-1}s_{1j}$ globally exponentially converge to zero,

$$s_{2j} = \sum_{i\in N_j} a_{ji}\left(q_{3j} - q_{2i}\right) + a_{j,m+1}\left(q_{3j} - q_{3,m+1}\right)$$

$$= \sum_{i \in N_j} a_{ji} \left(\tilde{q}_{3j} - \tilde{q}_{3i} \right) + a_{j,m+1} \tilde{q}_{3j}$$

$$= (L + B) \tilde{q}_{3j}$$

Similarly, $q_{3j} - q_{3,m+1} = \tilde{q}_{3j} = (L + B)^{-1} s_{2j}$ globally exponentially converge to zero.

Theorem 4.7 For m systems in (4.12)-(4.14), if there exists a spanning tree with the virtual leader v_{m+1} as the root of the spanning tree, then the distributed control law (4.25) and (4.30) guarantee that (4.8)-(4.11) hold.

Proof By Theorem 4.5, control laws in (4.25) ensure q_{1j} globally exponentially converge to $q_{1,m+1}$. By Theorem 4.6, control laws in (4.30) ensure q_{2j} globally exponentially converge to $q_{2,m+1}$ and q_{3j} globally exponentially converge to $q_{3,m+1}$. By Lemma 4.1, (4.8)-(4.11) hold.

4.5 Simulation I

To show the effectiveness of proposed control algorithms in Section 4.4 for vehicles kinematics, simulation is done for five identical unicycles. Robot models are as shown in Figure 4.1. The desired pattern and communication graph are shown in Figure 4.4 and Figure 4.5.

In Figure 4.4, it is assumed there are four identical car-like mobile robots. The desired formation of the five robots is assumed to be a regular pentagon with each side exactly identical. The sum of the internal angles is 540 degree, all sides have equal length and each interior angle is 108 degree. The diagnosis in this case is 0.5.

Figure 4.5 represents the communication graph of the five robots, v_0 is assumed to be the virtual leader, from the communication topology it is learned that only v_1 can receive information directly from the desired trajectory, other follower robots communicate with the leader indirectly. It can be also proved a spanning tree exist in the communication graph in Figure 4.5 with the virtual leader as the root of the spanning tree.

Figure 4.4 is the desired formation.

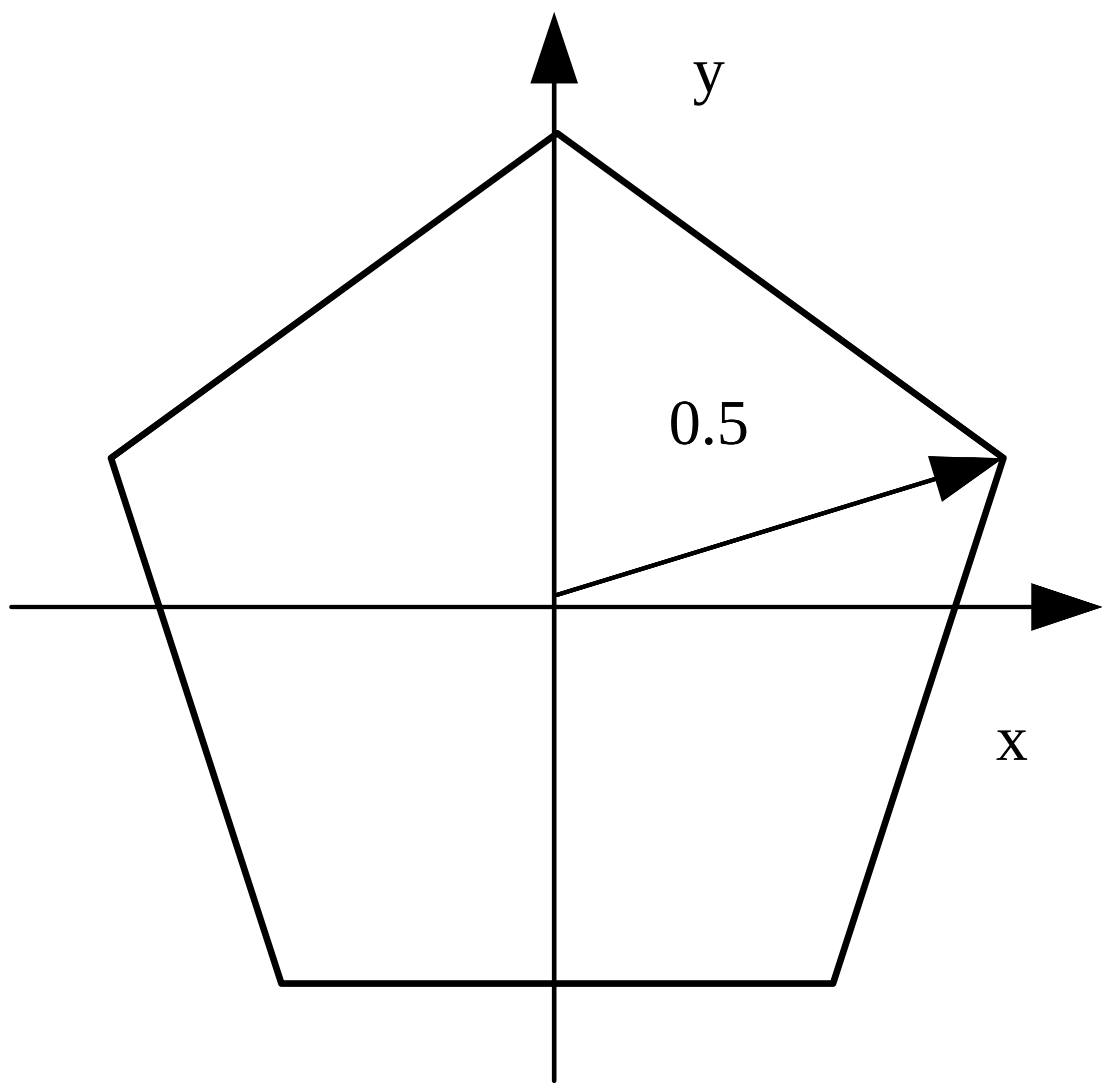

Figure 4.4 Desired formation of five vehicles I

Figure 4.5 is the communication graph.

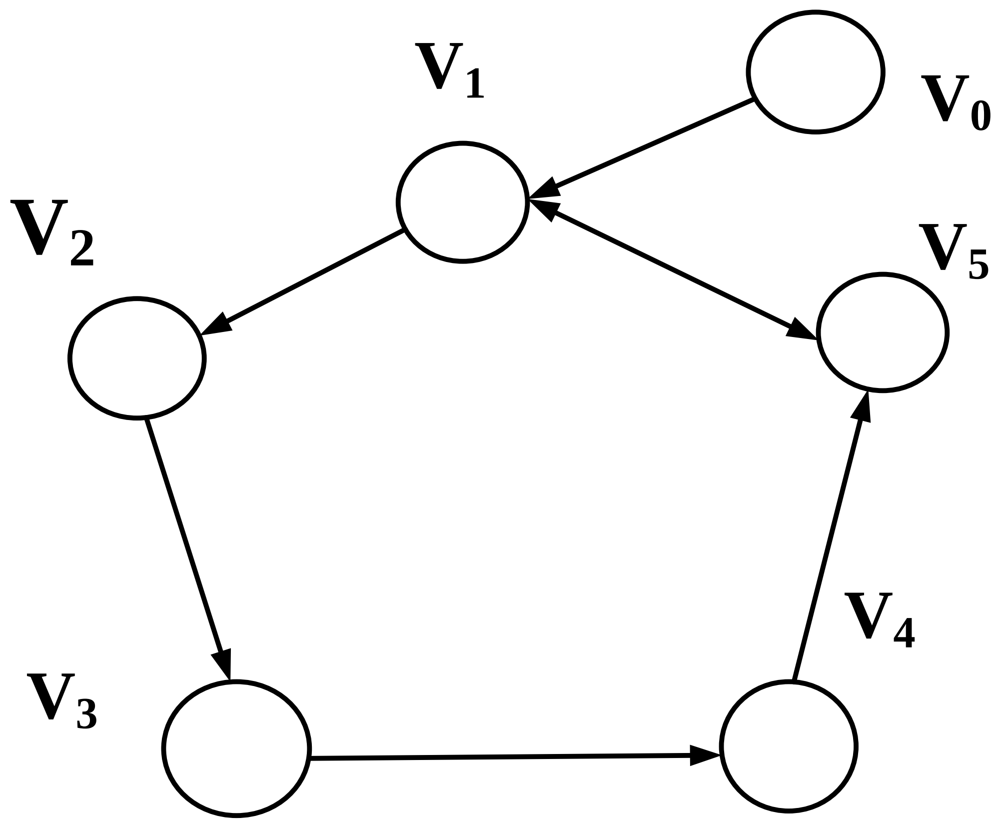

Figure 4.5 Information exchange topology I

For the five vehicles' kinematic systems, Figure 4.6 represents the centroid *of* x_j ($1\leq j \leq5$) and the desired trajectory x_0. Figure 4.7 represents the centroid of y_j ($1\leq j \leq5$) and the desired trajectory y_0. Figure 4.8 represents $(\theta_0 - \theta_j)$ (1≤ j ≤5). Figure 4.9 represents the formation tracking of five follower robots.

Figure 4.6 is the convergence results of x_j.

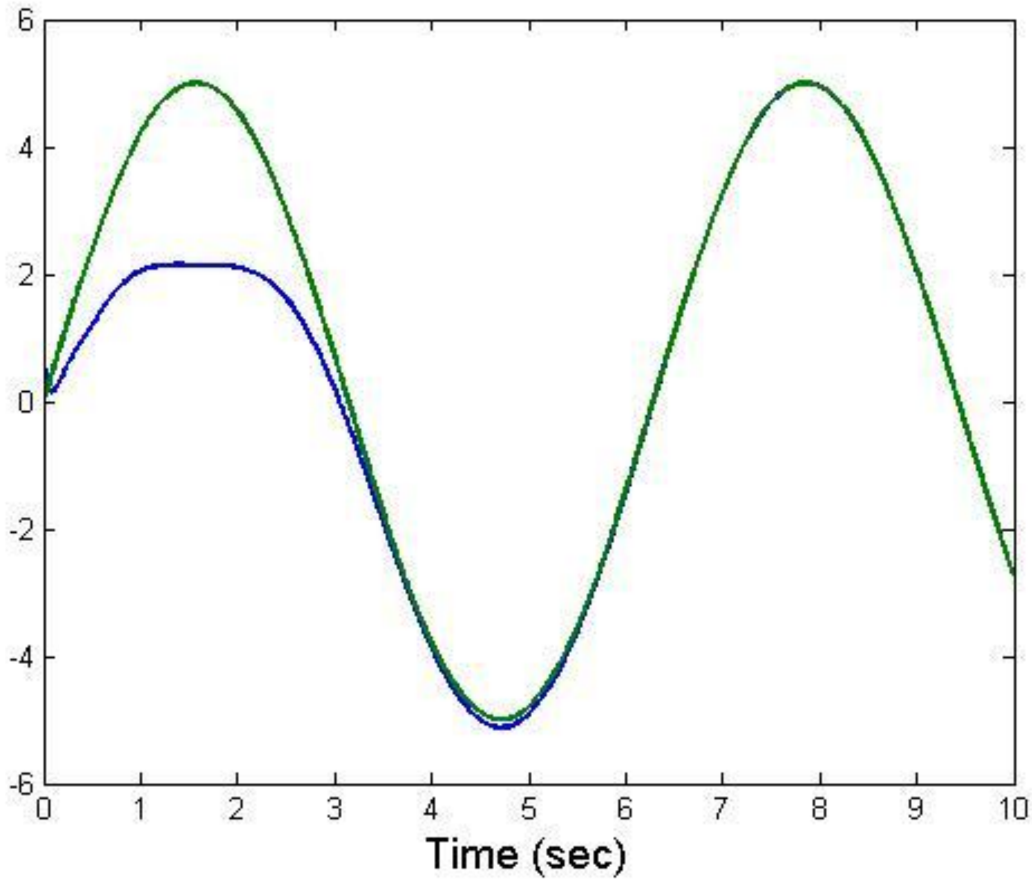

Figure 4.6 The centroid of x_j ($1\leq j \leq 5$) and the desired trajectory x_0

Figure 4.7 is the convergence result of y_j.

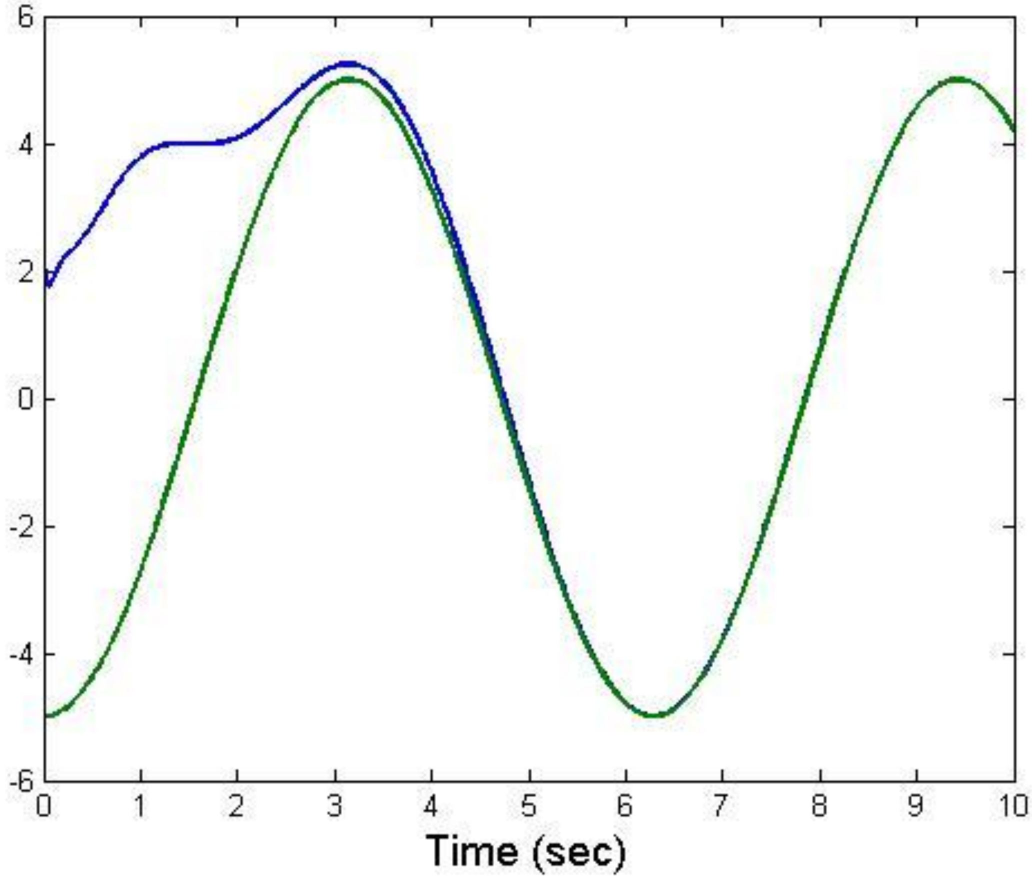

Figure 4.7 The centroid of y_j ($1\leq j \leq 5$) and the desired trajectory y_0

Figure 4.8 is the convergence result of θ_j.

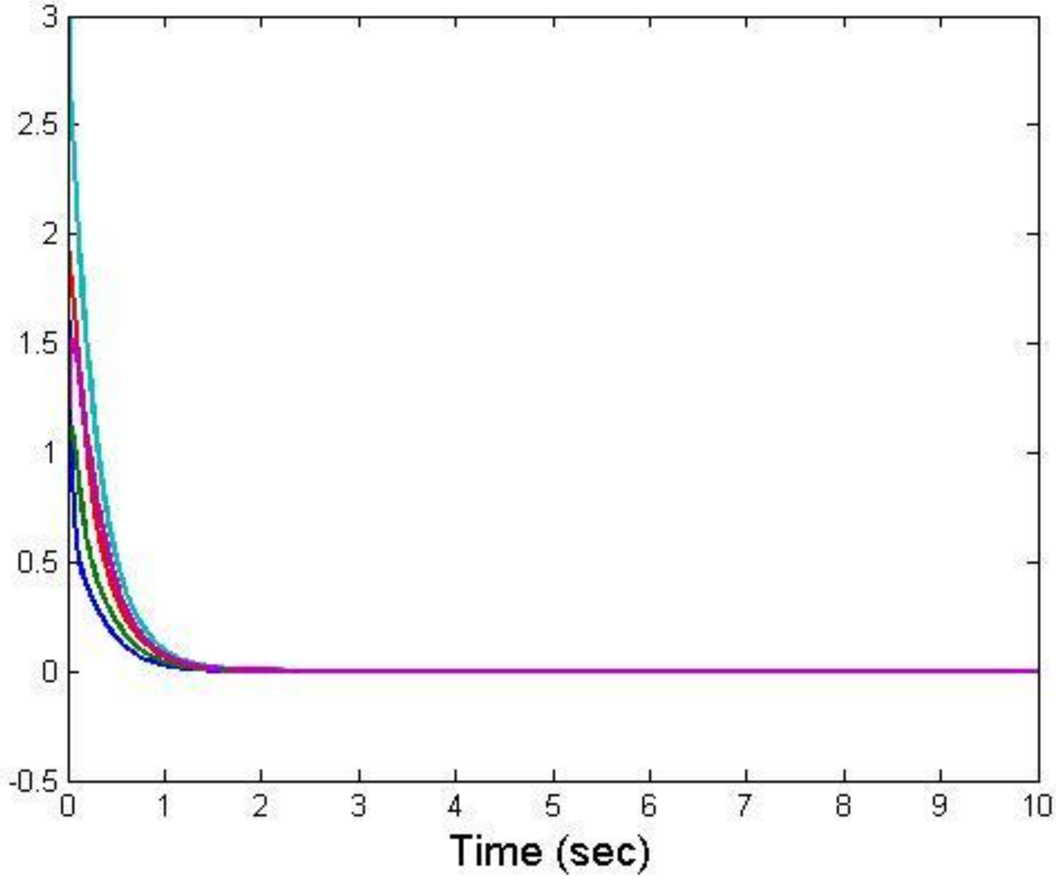

Figure 4.8 $\theta_j - \theta_0$ ($1 \leq j \leq 5$)

Figure 4.9 is the formation tracking of multi-vehicle system.

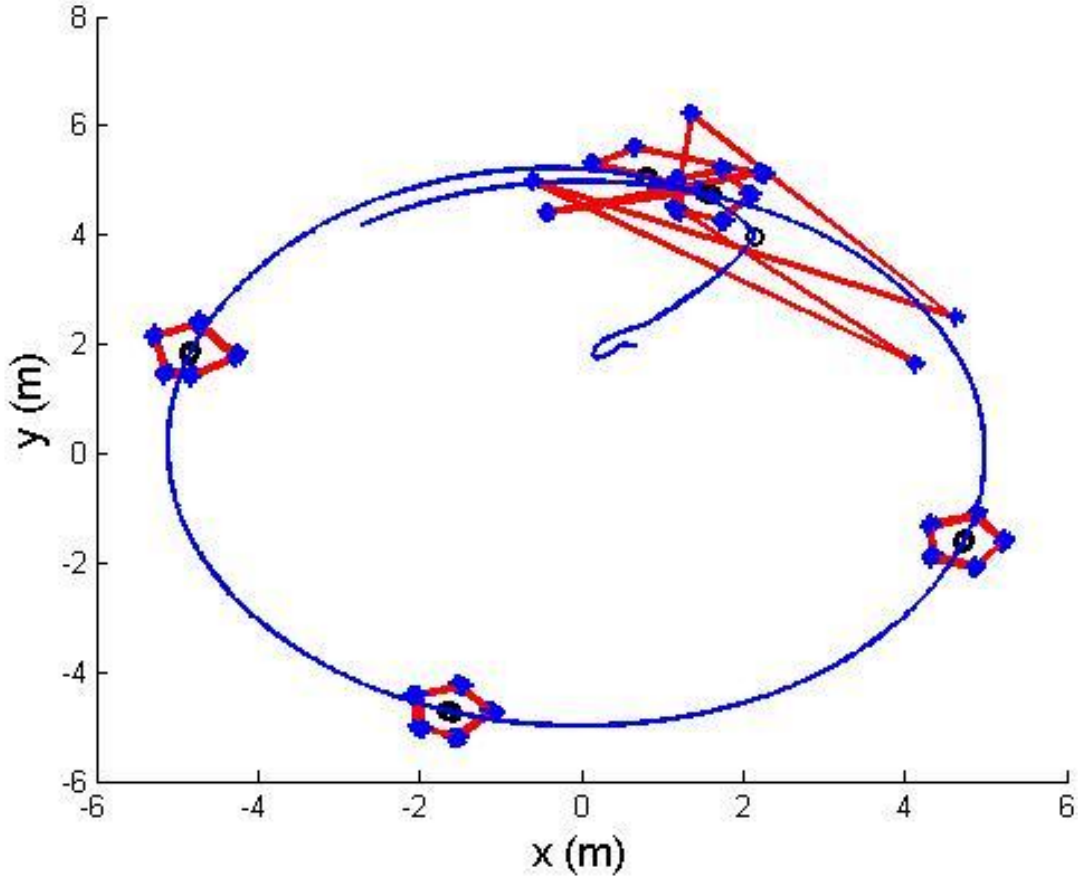

Figure 4.9 Formation tracking of five follower robots I

4.6 Novel Distributed Controllers of Vehicle Kinematics

In Section 4.4, distributed formation tracking control algorithms are proposed based on transformed chained-form structures. The chained-form systems are rewritten in a cascaded form in (4.19). Exponential stability theorem is utilized to stabilize the chained-from systems thus the original vehicle kinematics converges to the reference trajectory exponentially.

In this section, modified kinematic controllers are designed. Firstly, the following lemma is proposed.

Lemma 4.3 For the system

$$\dot{\delta} = -\varphi_1(t)^2\delta + \varphi_2(t) \tag{4.41}$$

where $\varphi_1(t)$ is a PE signal, if $\varphi_2(t)$ is bounded and converges to zero, then δ converges to zero.

Notice in the chained-form systems (4.12)-(4.14). Since (4.12) and (4.13) are first-order linear systems, it is easy to stabilize them by linear feedback control laws. However, (4.14) has a nonlinear structure, stability of (4.12) and (4.13) cannot ensure the stability of (4.14). Further variable transformations require to be implemented in order to stabilize q_{3j}.

Consider the following variable transformations,

$$z_{*j} = q_{*j} - \mu_{*j} \tag{4.42}$$

where $q_{*j} = [q_{1j}, q_{2j}, q_{3j}]$ are defined in (4.12)-(4.13).

$\mu_{*j} = [\mu_{1j}, \mu_{2j}, \mu_{3j}]$, where $\mu_{1j} = \mu_{3j} = 0$, $\mu_{2j} = -k_u u_{1j} z_{3j}$, k_u is a positive constant.

By (4.39), the chained-form system in (4.12)-(4.13) is transformed into

$$\dot{z}_{1j} = u_{1j} \tag{4.43}$$

$$\dot{z}_{2j} = u_{2j} + k_u(\dot{u}_{1j} z_{3j} + \dot{z}_{3j} u_{1j}) \tag{4.44}$$

$$\dot{z}_{3j} = -k_u u_{1j}^2 z_{3j} + u_{1j} z_{2j} \tag{4.45}$$

Similarly, the chained-form reference trajectory is also transformed into

$$\dot{z}_{10} = u_{10} \tag{4.46}$$

$$\dot{z}_{20} = u_{20} + k_u(\dot{u}_{10}z_{30} + \dot{z}_{30}u_{10}) \tag{4.47}$$

$$\dot{z}_{30} = -k_u u_{10}^2 z_{30} + u_{10}z_{20} \tag{4.48}$$

Compared with the chained-form systems in (4.12)-(4.13), (4.43) has the same structure as (4.12). Since the nonlinear term in (4.44) can be removed by state feedback, (4.44) can be stabilized by the control methods of (4.13). (4.45) has the same structure as (4.41) in Lemma 4.3. If (4.43) and (4.44) are stabilized to (4.46) and (4.47), when u_{1j} is PE signal, (4.42) can be stabilized to (4.45).

Before the distributed control algorithms for (4.43) to (4.45) are designed, the equivalence between consensus of systems in (4.43)-(4.45) and control goals in (4.8)-(4.11) is proved.

Lemma 4.4 For the transformed systems in (4.43) - (4.45), if

$$\lim_{t\to\infty}(z_{1j} - z_{10}) = 0 \tag{4.49}$$

$$\lim_{t\to\infty}(z_{2j} - z_{20}) = 0 \tag{4.50}$$

$$\lim_{t\to\infty}(z_{3j} - z_{30}) = 0 \tag{4.51}$$

$$\lim_{t\to\infty}(u_{1j} - u_{10}) = 0 \tag{4.52}$$

then (4.8)-(4.11) hold.

Proof

$$\lim_{t\to\infty}(q_{1j} - q_{10}) = \lim_{t\to\infty}(z_{1j} + \mu_{1j} - z_{10} - \mu_{10})$$

$$\lim_{t\to\infty}(q_{1j}-q_{10})=\lim_{t\to\infty}(z_{1j}-z_{10})$$

$$\lim_{t\to\infty}(q_{3j}-q_{30})=\lim_{t\to\infty}(z_{3j}+\mu_{3j}-z_{30}-\mu_{30})$$

$$\lim_{t\to\infty}(q_{3j}-q_{30})=\lim_{t\to\infty}(z_{3j}-z_{30})$$

$$\lim_{t\to\infty}(q_{2j}-q_{20})=\lim_{t\to\infty}(z_{2j}+\mu_{2j}-z_{20}-\mu_{20})$$

$$\lim_{t\to\infty}(q_{2j}-q_{20})=\lim_{t\to\infty}(z_{2j}-k_u u_{1j}z_{3j}-z_{20}+k_u u_{10}z_{30})$$

Notice when $\lim_{t\to\infty}(\mathrm{z}_{3\mathrm{j}}-\mathrm{z}_{30})=0$, $\lim_{t\to\infty}(\mathrm{u}_{1\mathrm{j}}-\mathrm{u}_{10})=0$,

$$\lim_{t\to\infty}(k_u u_{1j}z_{3j}-k_u u_{10}z_{30})=0$$

which means

$$\lim_{t\to\infty}(q_{2j}-q_{20})=\lim_{t\to\infty}(z_{2j}-z_{20})$$

Then it follows that

$$\lim_{t\to\infty}\begin{bmatrix}z_{1j}-z_{10}\\z_{2j}-z_{20}\\z_{3j}-z_{30}\end{bmatrix}=\lim_{t\to\infty}\begin{bmatrix}q_{1j}-q_{10}\\q_{2j}-q_{20}\\q_{3j}-q_{30}\end{bmatrix}$$

By Lemma 4.1, (4.8)-(4.11) hold.

From Lemma 4.4, it is learned that the control goal is to design $u_{*j}=[u_{1j},u_{2j}]$ for the transformed systems in (4.43)-(4.45) such that

$$\lim_{t\to\infty}(z_{1j}-z_{10})=0$$

$$\lim_{t\to\infty}(z_{2j} - z_{20}) = 0$$

$$\lim_{t\to\infty}(z_{3j} - z_{30}) = 0$$

From Lemma 4.3, it is learned if $\varphi_2(t)$ is bounded and converges to zero, $\varphi_1(t)$ is a persistently excited signal, then δ is asymptotically stable. Notice (4.42) has the same structure as (4.38). If $u_{1j}z_{2j}$ is bounded and converges to zero, u_{1j} is a persistently excited signal, then z_{3j} can be stabilized.

Assumption 4 $\frac{d^i u_{10}}{dt^i}$ $(0 \leq \text{i} \leq 2)$ is bounded and

$$\int_t^{t+T} u_{10}^2\,(\tau)d\tau > \alpha$$

for some $\alpha > 0$ and $T > 0$.

Assumption 4 implies u_{10} is a persistently excited signal.

Lemma 4.5 For the *m* systems in (4.40)-(4.42), under Assumption 4, if

$$\lim_{t\to\infty}(z_{2j} - z_{20}) = 0$$

$$\lim_{t\to\infty}(u_{1j} - u_{10}) = 0$$

then

$$\lim_{t\to\infty}(z_{3j} - z_{30}) = 0.$$

Proof Subtract (4.42) by (4.45), it follows that

$$\dot{z}_{3j} - \dot{z}_{30} = -(k_u u_{1j}^2 z_{3j} - k_u u_{10}^2 z_{30}) + u_{1j} z_{2j} - u_{10} z_{20}$$

$$\dot{z}_{3j} - \dot{z}_{30} = -k_u u_{1j}^2 (z_{3j} - z_{30}) - k_u (u_{1j}^2 - u_{10}^2) z_{30} + u_{1j} z_{2j} - u_{10} z_{20}$$

Denote $\tilde{z}_{3j} = z_{3j} - z_{30}$, one has

$$\dot{\tilde{z}}_{3j} = -k_u u_{1j}^2 \tilde{z}_{3j} - k_u (u_{1j}^2 - u_{10}^2) z_{30} + u_{1j} z_{2j} - u_{10} z_{20}$$

Since $\lim_{t\to\infty}(z_{2j} - z_{20}) = 0$, $\lim_{t\to\infty}(u_{1j} - u_{10}) = 0$ and z_{30} is bounded, then

$$-k_u (u_{1j}^2 - u_{10}^2) z_{30} + u_{1j} z_{2j} - u_{10} z_{20}$$

is bounded and converge to zero. u_{1j} is a PE signal, then by Lemma 4.3, $\tilde{z}_{3j}$ is asymptotically stable.

Since the consensus with z_{20} for z_{2j} can ensure that z_{3j} converges to z_{30}, then by designing control laws for z_{2j} and z_{1j}, the control goals in (4.49)-(4.52) can be achieved.

Theorem 4.8 For the m systems in (4.43), if there exists a spanning tree with the virtual leader v_{m+1} as the root of the spanning tree, then the distributed control laws

$$u_{1j} = \eta_{1j} \tag{4.53}$$

where

$$\eta_{1j} = -\alpha_1 z_{1j} + \zeta_{1j} \tag{4.54}$$

$$\dot{\zeta}_{1j} = -\sum_{i\in N_j} a_{ji}\left(\zeta_{1j} - \zeta_{1i}\right) - a_{j,m+1}\left(\zeta_{1j} - \zeta_{10}\right)$$

$$- \rho_1 sign\left[\sum_{i\in N_j} a_{ji}\left(\zeta_{1j} - \zeta_{1i}\right) - a_{j,m+1}\left(\zeta_{1j} - \zeta_{10}\right)\right] \tag{4.55}$$

guarantee that

$$\lim_{t\to\infty}(z_{1j} - z_{10}) = 0$$

$$\lim_{t\to\infty}(u_{1j} - u_{10}) = 0$$

for $1 \le j \le m$, where $a_{j,m+1} = 1$ if the virtual leader is available to vehicle v_j, otherwise $a_{j,m+1} = 0$. α_1 is a positive constant, $\zeta_{10} = u_{10} + \alpha_1 z_{10}$.

Proof (4.55) is rewritten by

$$\dot{\zeta}_{1*} = -L\zeta_{1*} - B(\zeta_{1*} - \zeta_{10}\mathbf{1}) - \rho_1 sign(L\zeta_{1*} + B(\zeta_{1*} - \zeta_{10}\mathbf{1}))$$

where $\zeta_{1*} = [\zeta_{11}, \zeta_{12}, \dots, \zeta_{1m}]$, $B = diag(a_{1,m+1}, a_{2,m+1}, \dots, a_{m,m+1})$.

Let $\tilde{\zeta}_{1j} = \zeta_{1j} - \zeta_{10}$, it follows that

$$\dot{\tilde{\zeta}}_{1*} = -(L+B)\tilde{\zeta}_{1*} - \rho_1 sign\left((L+B)\tilde{\zeta}_{1*}\right) - \dot{\zeta}_{10}$$

By Theorem 3.4, it is known when ρ_1 satisfies

$$\rho_1 \ge \frac{\left\|(L+B)^{-1}\right\| |\dot{\zeta}_{10}|}{\epsilon}$$

where ϵ is the minimum eigenvalue of $(L+B)^{-1}$, then ζ_{1j} exponentially converges to ζ_{10} for $1 \le j \le m$.

Choose a nonnegative symmetric function $V_z = \sum_{j=1}^{m} \frac{1}{2} \tilde{z}_{1j}^2$. Differentiate V_z along (4.54), it follows that

$$\dot{V}_z = -\alpha_1 \tilde{z}_{1*}^2 + \tilde{z}_{1*} \tilde{\zeta}_{1*}$$

$$\dot{V}_z \le -\alpha_1 \tilde{z}_{1*}^2 + ||\tilde{z}_{1*}||\,||\tilde{\zeta}_{1*}||$$

$$= -2\alpha_1 V_z + ||\tilde{\zeta}_{1*}||\sqrt{2V_z}$$

Choose $V_2 = \sqrt{V_z}$, one has

$$\dot{V}_2 \le -\alpha_1 V_2 + \frac{1}{\sqrt{2}} ||\tilde{\zeta}_{1*}||$$

Then V_2 is exponentially stable, which implies z_{1j} exponentially converges to z_{10} for $1 \le j \le m$.

By (4.50), it is known

$$\lim_{t\to\infty} (u_{1j} - u_{10}) = \lim_{t\to\infty} (-\alpha_1 z_{1j} + \zeta_{1j} - u_{10})$$

Since ζ_{1j} exponentially converges to ζ_{10}, then

$$\lim_{t\to\infty} (-\alpha_1 z_{1j} + \zeta_{1j} - u_{10}) = \lim_{t\to\infty} (-\alpha_1 z_{1j} + u_{10} + \alpha_1 z_{10} - u_{10})$$

$$= -\lim_{t\to\infty} (\alpha_1 (z_{1j} - z_{10}))$$

Since $\lim_{t\to\infty} (z_{1j} - z_{10}) = 0$.It can be proved that $\lim_{t\to\infty} (u_{1j} - u_{10}) = 0$.

Theorem 4.9 For the m systems in (4.44)-(4.45), if there exists a spanning tree with the virtual leader v_{m+1} as the root of the spanning tree, then the distributed control laws

$$u_{2j} = \eta_{2j} \tag{4.56}$$

where

$$\eta_{2j} = -\sum_{i\in N_j} a_{ji}\left(z_{2j} - z_{2i}\right) - a_{j,m+1}\left(z_{2j} - z_{20}\right)$$
$$- \rho_2 sign\left[\sum_{i\in N_j} a_{ji}\left(z_{2j} - z_{2i}\right) - a_{j,m+1}\left(z_{2j} - z_{20}\right)\right] - k_u\left(\dot{u}_{1j}z_{3j} + \dot{z}_{3j}u_{1j}\right) \tag{4.57}$$

guarantee that

$$\lim_{t\to\infty}(z_{2j} - z_{20}) = 0$$

for $1 \le j \le m$, where $a_{j,m+1} = 1$ if the virtual leader is available to vehicle v_j, otherwise $a_{j,m+1} = 0$.

Proof Substitute u_{2j} in (4.56) into (4.44) one has

$$\dot{z}_{2*} = -Lz_{2*} - B(z_{2*} - z_{20}\mathbf{1}) - \rho_2 sign(Lz_{2*} + B(z_{2*} - z_{20}\mathbf{1}))$$

where $z_{2*} = [z_{21}, z_{22}, \dots, z_{2m}]$, $B = diag(a_{1,m+1}, a_{2,m+1}, \dots, a_{m,m+1})$.

Let $\tilde{z}_{2j} = z_{2j} - z_{20}$, it follows that

$$\dot{\tilde{z}}_{2*} = -(L + B)\tilde{z}_{2*} - \rho_2 sign\big((L + B)\tilde{z}_{2*}\big) - \dot{z}_{20}$$

By Theorem 3.4, it is known when ρ_2 satisfies

$$\rho_2 \geq \frac{\left|\left|(L+B)^{-1}\right|\right||\dot{z}_{20}|}{\epsilon}$$

where ϵ is the minimum eigenvalue of $(L+B)^{-1}$, then z_{2j} exponentially converges to z_{20} for $1 \leq j \leq m$.

Theorem 4.10 For the m systems in (4.43)-(4.45), if there exists a spanning tree with the virtual leader v_{m+1} as the root of the spanning tree, then the distributed control laws in (4.53) and (4.56) guarantee that the control goals in (4.8)-(4.11) hold.

Proof By Theorem 4.8 $\lim_{t\to\infty}(z_{1j} - z_{10}) = 0$, $\lim_{t\to\infty}(u_{1j} - u_{10}) = 0$. By Theorem 4.9, $\lim_{t\to\infty}(z_{2j} - z_{20}) = 0$. By Lemma 4.5, $\lim_{t\to\infty}(z_{3j} - z_{30}) = 0$. By Lemma 4.4, the control goals in (4.8)-(4.11) hold.

4.7 Simulation II

To show the effectiveness of proposed control algorithms in Section 4.6 for vehicles kinematics, simulation is done for four identical unicycles. The desired pattern and communication graph are shown in Figure 4.10 and Figure 4.11. The reference trajectory is

$$(x_0, y_0, \theta_0) = (2sin(0.5t), -2cos(0.5t), 0.5t)$$

In Figure 4.10, it is assumed there are five identical car-like mobile robots. The desired formation of the five robots is assumed to be a regular pentagon with each side exactly identical. The sum of the internal angles is 540 degree, all sides have equal length and each interior angle is 108 degree. The diagnosis in this case is 0.5.

Figure 4.11 represents the communication graph of the five robots, v_0 is assumed to be the virtual leader, from the communication topology it is learned that only v_1 can receive information directly from the desired trajectory, other follower robots communicate with the leader indirectly. Unlike the communication topology in Figure 4.5, in Figure 4.11, the communication pattern between follower vehicles is changed but it can be also proved a spanning tree exist in the communication graph in Figure 4.11 with the virtual leader as the root of the spanning tree.

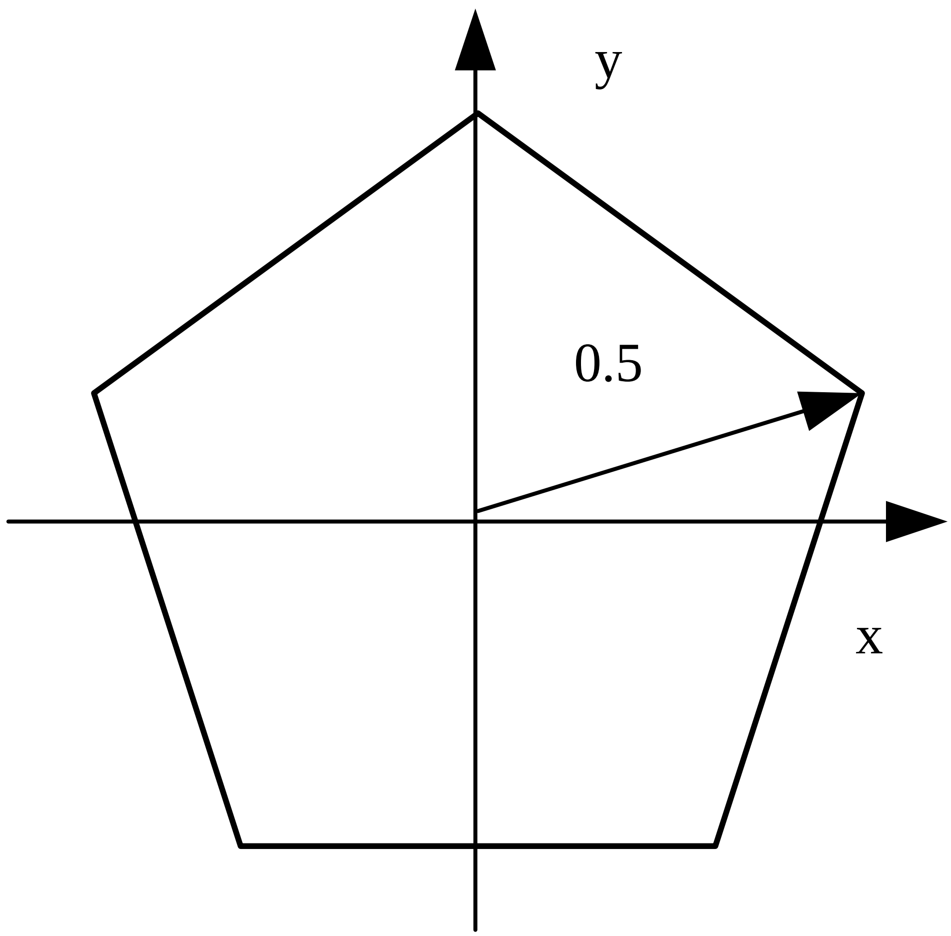

Figure 4.10 Desired formation of five vehicles II

Figure 4.11 is the communication graph.

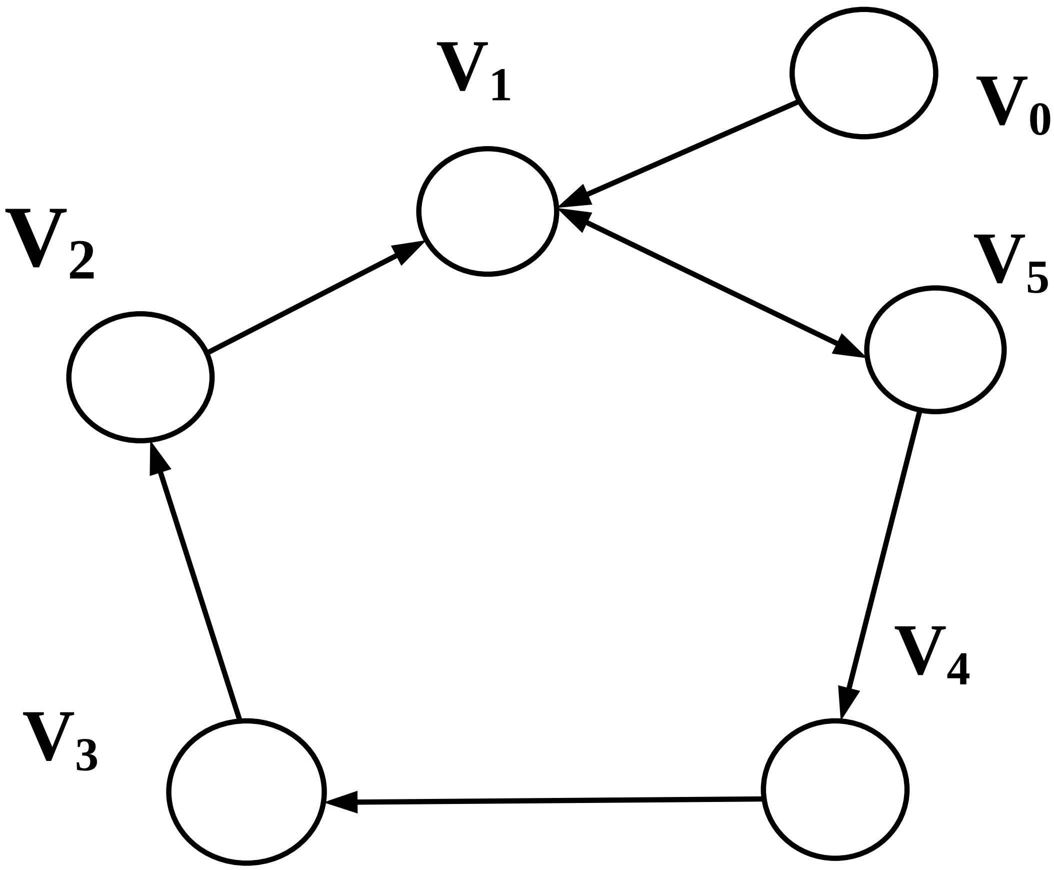

Figure 4.11 Information exchange topology II

For the five vehicles' kinematic systems, Figure 4.12 represents the centroid of x_j (1≤ j ≤5) and the desired trajectory x_0. Figure 4.13 represents the centroid of y_j (1≤ j ≤5) and the desired trajectory y_0. Figure 4.14 represents $(\theta_0 - \theta_j)$ (1≤ j ≤5). Figure 4.15 represents the formation tracking of five follower robots.

Figure 4.12 is the convergence result of x_i.

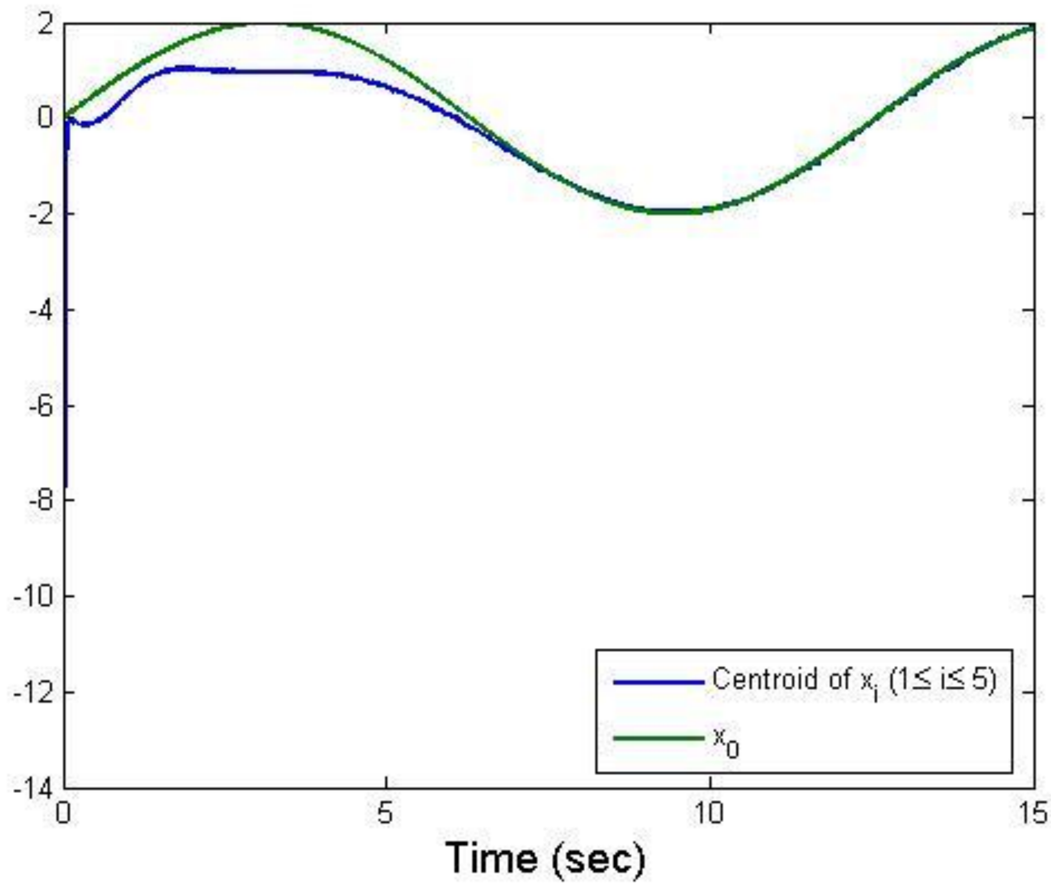

Figure 4.12 The centroid of x_i (1≤ i ≤5) and the desired trajectory x_0

Figure 4.13 is the convergence result of y_i.

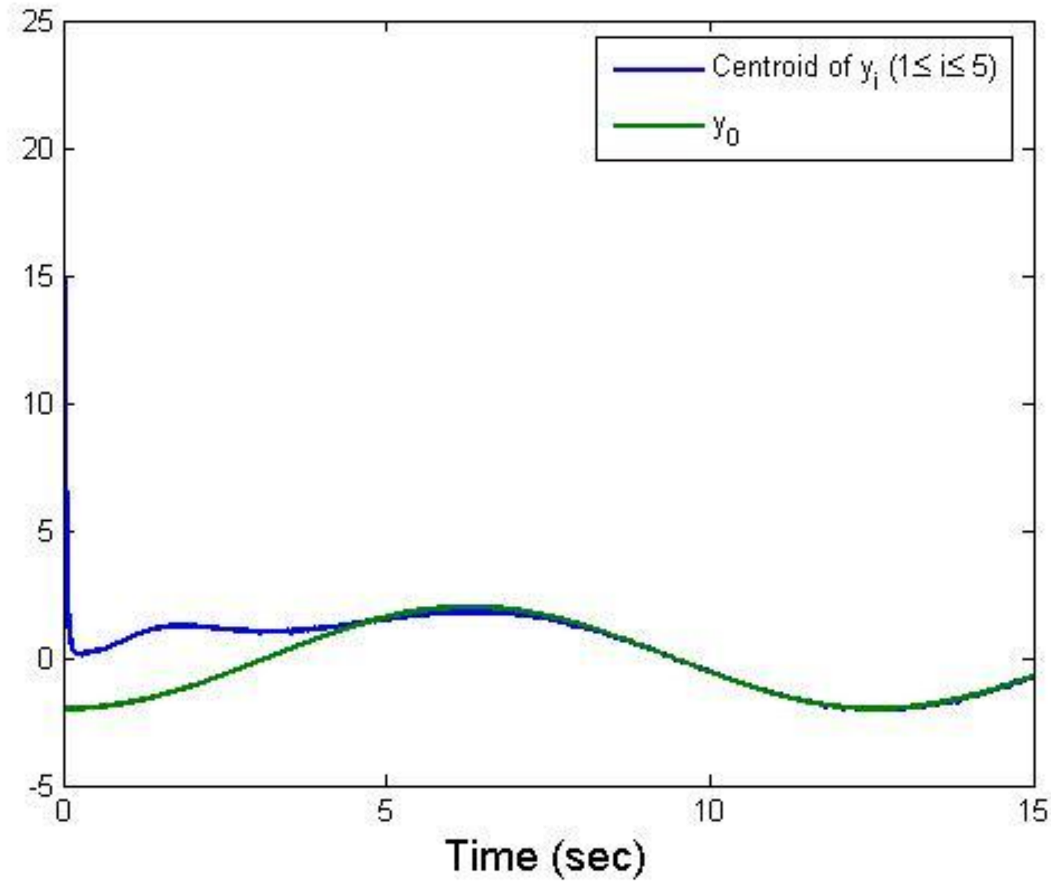

Figure 4.13 The centroid of y_i (1≤ i ≤5) and the desired trajectory y_0

Figure 4.14 is the convergence result of θ_i.

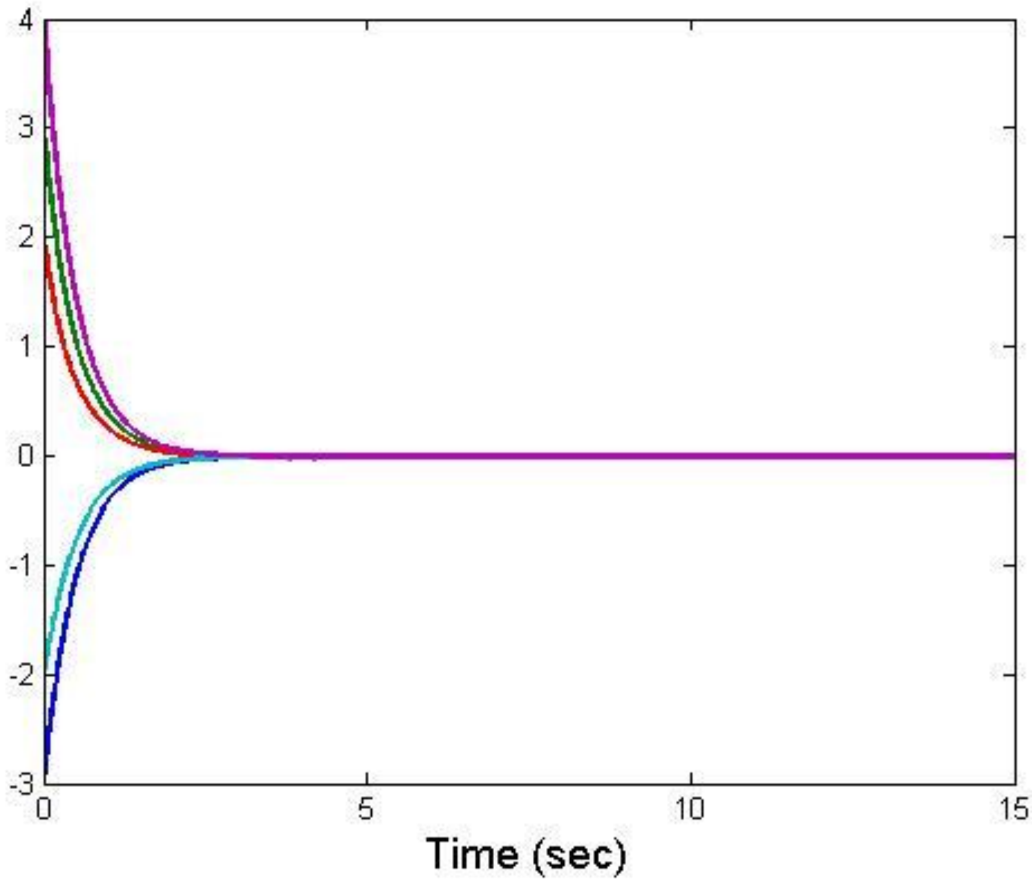

Figure 4.14 $\theta_i - \theta_0$ ($1 \leq i \leq 5$)

Figure 4.15 is the formation tracking of multi-vehicle system.

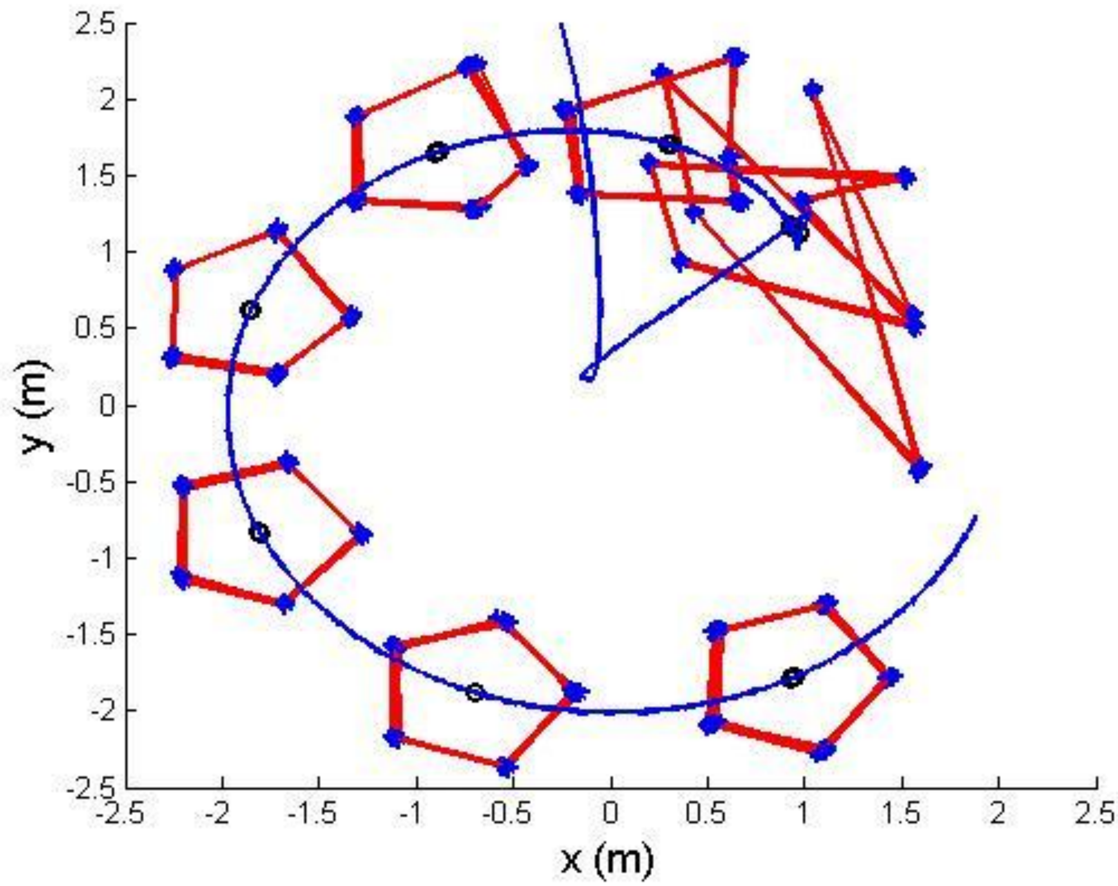

Figure 4.15 Formation tracking of five follower robots II

4.8 Summary

In this chapter, distributed formation tracking control of multiple wheeled robots is discussed. The reference trajectory is considered as sinusoidal signals which are persistently excited. Cooperative formation tracking is achieved with the aid of cascaded system stability theorem and graph theory. Variable transformation is utilized to transfer the original nonlinear kinematics into chained-form system. Stability of chained-form system is proved to guarantee the stability of original kinematics.

Thus, consensus of the multiple chained-form systems guarantees the formation tracking of multiple robots. In Section 4.4, the chained-form systems are rewritten in cascaded form such that the exponential stability theory of cascade systems could be utilized to stabilize the transformed systems. In Section 4.6, variable transformations are further implemented onto the chained-form systems such that formation tracking is realized with the aid of the novel distributed consensus algorithm on multiple first-order systems and Lemma 4.3. Both algorithms solve the multivehicle formation tracking problem. Compared with distributed kinematic controllers in (4.50)-(4.52), (4.25),(4.26) and (4.29),(4.30) are much more concise since the control laws are designed only for the chained-form systems while the controllers in Section 4.6 are implemented on the transformed chained-form systems. Moreover, Derivative terms are removed since in real-life operations these data information cannot be acquired by the sensors.

CHAPTER V

COOPERATIVE FORMATION TRACKING CONTROL OF MULTIPLE DYNAMIC VEHICLES

In Chapter IV, kinematic model of car-like robot is studied and control algorithms for kinematic system are proposed. However, in practical application the torques generated by vehicle engines actually control the motions of vehicles, then it is necessary to consider the dynamic models of vehicles with respect to control torques. Vehicle dynamics is derived with the aid of Euler-Lagrangian equations and distributed formation tracking control methods are designed for multiple vehicles of dynamic systems. Two cases of dynamics of without parametric uncertainties and with parametric uncertainties are both addressed considering in real-life operations the physical quantities might not be available precisely. The dynamic controllers are based on kinematic controllers in Section 4.4, Section 4.6 and backstepping methods. Considering that time-switching communication topology occurs in real-life operation due to vehicle disability and communication link breakage, a special case of time-varying communication graph is addressed and distributed consensus control laws are proposed under certain assumptions of the switching graph.

5.1 Dynamics of Vehicle

Consider unicycle vehicle model in Figure 4.1, for vehicle v_j, the kinetic energy is

$$K_j = \frac{1}{2} m_j \left(\dot{x}_j{}^2 + \dot{y}_j{}^2 \right) + \frac{1}{2} I_j \dot{\theta}_j{}^2$$

where $\dot{x}_j$ and $\dot{y}_j$ are translational velocities with respect to x-axis and y-axis respectively, $\dot{\theta}_j$ is rotational velocity, m is mass of the vehicle and I_j is moment of inertia. Potential energyP_j is considered zero as the plane on which the vehicles drive is regarded as zero potential energy surface, then the Lagrangian equation for the vehicle dynamics is

$$L_j = k_j - P_j = \frac{1}{2} m_j \left(\dot{x}_j{}^2 + \dot{y}_j{}^2 \right) + \frac{1}{2} I_j \dot{\theta}_j{}^2$$

By definition of Euler-Lagrange equation it follows that

$$\frac{d}{dt} \frac{\partial L_j}{\partial \dot{x}_j} - \frac{\partial L_j}{\partial x_j} = f_{jx}$$

$$\frac{d}{dt} \frac{\partial L_j}{\partial \dot{y}_j} - \frac{\partial L_j}{\partial y_j} = f_{jy}$$

$$\frac{d}{dt} \frac{\partial L_j}{\partial \dot{\theta}_j} - \frac{\partial L_j}{\partial \theta_j} = f_{j\theta}$$

$$m_j \ddot{x}_j = \frac{\tau_{1j}}{R_{1j}} cos\, \theta_j + \frac{\tau_{2j}}{R_{2j}} cos\, \theta_j + \lambda_j sin\theta_j$$

$$m_j \ddot{y}_j = \frac{\tau_1}{R_1} sin\, \theta_j + \frac{\tau_{2j}}{R_{2j}} sin\, \theta_j - \lambda_j cos\theta_j$$

$$I_j \ddot{\theta}_j = \frac{\tau_{1j}}{R_{1j}} - \frac{\tau_{2j}}{R_{2j}} \qquad (5.1)$$

where $\tau_{*j} = [\tau_{1j}, \tau_{2j}]$ are drive torques implemented on two wheels respectively. R_{1j} and R_{2j} are radius of two wheels, λ_j is nonholonomic constraint force.

Define $q_{*j} = [x_j, y_j, \theta_j]$ can be expressed as

$$M_j(q_{*j})\ddot{q}_{*j} + C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j} + G_j(q_{*j}) = B_j(q_{*j})\tau_{*j} + {J_j}^T(q_{*j})\lambda_j \qquad (5.2)$$

where $M_j(q_{*j}) = \begin{bmatrix} m_j & 0 & 0 \\ 0 & m_j & 0 \\ 0 & 0 & I_j \end{bmatrix}$, C_j represents Centripetal and Coriolis force, G_j is gravitational force, both C_j and G_j are three-order null matrix,

$$B_j(q_{*j}) = \begin{bmatrix} \frac{cos\,\theta_j}{R_{1j}} & \frac{cos\,\theta_j}{R_{2j}} \\ \frac{sin\,\theta_j}{R_{1j}} & \frac{sin\,\theta_j}{R_{2j}} \\ \frac{1}{R_{1j}} & \frac{1}{R_{2j}} \end{bmatrix}$$

$$I_j = [sin\theta_j, -\cos\theta_j\,, 0]^T$$

The nonholonomic constraints are defined as

$$\left[sin\theta_j, -\cos\theta_j\,, 0\right]q_{*j} = 0$$

Notice on the right hand of (5.2) the nonholonomic constraints $[sin\theta_j, -\cos\theta_j\,, 0]^T\lambda_j$ has nothing to do with the control torques, the disturbance term should be removed for control design of τ_{*j}.

Define the velocity vector $v_{*j} = [v_1, v_2]$ with v_1 its translational velocity and v_2 its rotational velocity, it follows that

$$\dot{q}_{*j} = g(q_{*j})v_{*j} = \begin{bmatrix} cos\theta & 0 \\ sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{5.3}$$

Notice that

$$J_j{}^T * g(q_{*j}) = [sin\theta_j, -cos\,\theta_j\,, 0] * \begin{bmatrix} cos\theta_j & 0 \\ sin\theta_j & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Then the nonholonomic constraints $J_j{}^T(q_{*j})\lambda_j$ can be removed by substituting q_{*j} in (5.2) with v_{*j} in (5.3), it follows that

$$M_j(q_{*j})g(q_{*j})\dot{v}_{*j} + M_j(q_{*j})\dot{g}(q_{*j})v_{*j} + C_j(q_{*j}, \dot{q}_{*j})g(q_{*j})v_{*j} + G_j(q_{*j}) = B_j(q_{*j})\tau \tag{5.4}$$

By multiplying $g^T(q_{*j})$ on both sides of (5.4) one has

$$\bar{M}_j(q_{*j})\dot{v}_{*j} + \bar{C}_j(q_{*j}, \dot{q}_{*j})v_{*j} + \bar{G}_j(q_{*j}) = \bar{B}_j(q_{*j})\tau \tag{5.5}$$

where

$$\bar{M}_j(q_{*j}) = g^T(q_{*j})M_j(q_{*j})g(q_{*j})$$

$$\bar{C}_j(q_{*j}, \dot{q}_{*j}) = g^T(q_{*j})M_j(q_{*j})\dot{g}(q_{*j}) + g^T(q_{*j})C_j(q_{*j}, \dot{q}_{*j})g(q_{*j}),$$

$$\bar{G}_j(q_{*j}) = g^T(q_{*j})G(q_{*j}), \bar{\mathrm{B}}_{\mathrm{j}}(q_{*j}) = g^T(q_{*j})B_j(q_{*j}).$$

In Chapter 4, distributed control laws are proposed for the control inputs of the chained-form system. Notice that the control inputs in (4.12) are not velocities of the vehicle kinematics v_{*j}. In order to associate the transformed dynamic system (5.5) with kinematic system (4.12), the chained-form transform equations are considered, define $u_{*j} = [u_1, u_2]^{\mathrm{T}}$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \Psi * u_{*j} = \begin{bmatrix} q_3 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{5.6}$$

Substitute v_{*j} in (5.5) with u_{*j} by the transformation in (5.6) and multiply Ψ^{T} on both sides of (5.5) it follows that

$$\widetilde{M}_j(q_{*j})\dot{u}_{*j} + \tilde{C}_j(q_{*j}, \dot{q}_{*j})u_{*j} + \tilde{G}_j(q_{*j}) = \tilde{B}_j(q_{*j})\tau \tag{5.7}$$

where

$$\widetilde{M}_j(q_{*j}) = \Psi^{\mathrm{T}} \overline{M}_j(q_{*j})\Psi$$

$$\tilde{C}_j(q_{*j}, \dot{q}_{*j}) = \Psi^{\mathrm{T}} \overline{M}_j(q_{*j})\dot{\Psi} + \Psi^{\mathrm{T}} \bar{C}_j(q_{*j}, \dot{q}_{*j})\Psi$$

$$\tilde{G}_j(q_{*j}) = \Psi^{\mathrm{T}} \bar{G}_j(q_{*j}),\ \tilde{B}_j(q_{*j}) = \Psi^{\mathrm{T}} \bar{B}_j(q_{*j}).$$

The transformed dynamic system (5.7) has the following two properties

(1) $\dot{\tilde{M}}_{\mathrm{j}} - 2\tilde{C}_j$ is skew-symmetric

(2) $\widetilde{M}_j\left(q_{*j}\right)\dot{u}_{*j} + \tilde{C}_j\left(q_{*j}, \dot{q}_{*j}\right)u_{*j} + \tilde{G}_j\left(q_{*j}\right) = Y_j(\dot{u}_{*j}, u_{*j}, \dot{q}_{*j}, q_{*j})a_j$

where $\widetilde{M}_j\left(q_{*j}\right)$ is the a 3×3 positive-definite symmetric matrix, $\tilde{C}_j\left(q_{*j}, \dot{q}_{*j}\right)$ represents centripetal and Coriolis force and $\tilde{G}_j\left(q_{*j}\right)$ is the gravitational force. a_j is the inertia vector which is composed of the mass and moment of inertia of the vehicle, Y_j is a known function of $\dot{u}_{*j}, u_{*j}, \dot{q}_{*j}, q_{*j}$.

In Chapter 4, distributed control of multiple vehicles with kinematic systems is studied with the aid of control algorithms of chained-form systems and graph theory. Since the u_{*j} in (4.25) and (4.30) are the intermediate states of vehicle dynamics rather than the real control inputs, distributed control of dynamic systems is based on the kinematic control methods with the aid of backstepping methods.

5.2 Distributed Control of Dynamic Systems without Parametric Uncertainties

Define $\hat{u}_{*j} = u_{*j} - \eta_{*j}$, where $\eta_{*j} = [\eta_{1j}, \eta_{2j}]$ are defined in (4.26) and (4.31).

Theorem 5.1 For m systems in (5.7), if there exists a spanning tree with the virtual leader v_{m+1} as the root of the spanning tree, then the distributed control laws

$$\tau_j = \tilde{B}_j^{-1}(\widetilde{M}_j\dot{\eta}_{*j} + \tilde{C}_j\eta_{*j} + \tilde{G}_j - k\hat{u}_{*j}) \tag{5.8}$$

Guarantee that (4.8)-(4.11) holds, where k is a positive constant, η_{*j} are defined in (4.26) and (4.31).

Proof Choose Lyapunov function $V_u = \frac{1}{2}\hat{u}_{*j}^T\widetilde{M}_j\hat{u}_{*j}$, differentiate it along (5.7) one has

$$\dot{V}_u = \frac{1}{2}\hat{u}_{*j}^T\dot{\widetilde{M}}_j\hat{u}_{*j} + \hat{u}_{*j}^T\widetilde{M}_j\dot{\hat{u}}_{*j} \tag{5.9}$$

Substitute τ_j in (5.8) into (5.7) it follows that

$$\widetilde{M}_j\dot{u}_{*j} + \tilde{C}_j u_{*j} + \tilde{G}_j = \widetilde{M}_j\dot{\eta}_{*j} + \tilde{C}_j\eta_{*j} + \tilde{G}_j - k\hat{u}_{*j}$$

$$\widetilde{M}_j(\dot{u}_{*j} - \dot{\eta}_{*j}) + \tilde{C}_j(u_{*j} - \eta_{*j}) = -k\hat{u}_{*j}$$

$$\tilde{M}_j \dot{\hat{u}}_{*j} = -\tilde{C}_j \hat{u}_{*j} - k\hat{u}_{*j} \tag{5.10}$$

Substitute $\tilde{M}_j \dot{\hat{u}}_{*j}$ in (5.10) into (5.9) one has

$$\dot{V}_u = \frac{1}{2}\hat{u}_{*j}^T \dot{\tilde{M}}_j \hat{u}_{*j} + \hat{u}_{*j}^T * (-\tilde{C}_j \hat{u}_{*j} - k\hat{u}_{*j})$$

$$= \frac{1}{2}\hat{u}_{*j}^T (\dot{\tilde{M}}_j - 2\tilde{C}_j)\hat{u}_{*j} - \hat{u}_{*j}^T k \hat{u}_{*j}$$

Since $\dot{\tilde{M}}_{\mathrm{j}} - 2\tilde{C}_j$ is skew-symmetric, then it follows that $\dot{V}_u = -\hat{u}_{*j}^T k \hat{u}_{*j}$. Let λ_m be the maximal eigenvalue of $\tilde{M}_j$ it can be learned that $\dot{V}_u < -\frac{2k}{\lambda_m} V_u$, then $\hat{u}_{*j}$ is exponentially stable which means $u_{*j} - \eta_{*j}$ exponentially converge to zero.

The distributed controllers in (4.25) and (4.30) are rewritten as

$$u_{1j} = -\sum_{i \in N_j} a_{ji}\left(q_{1j} - q_{1i}\right) - a_{j,m+1}\left(q_{1j} - q_{1,m+1}\right) + \delta_{1j} + \hat{u}_{1j} \tag{5.11}$$

$$\dot{\delta}_{1\mathrm{j}} = -\sum_{i \in N_j} a_{ji}\left(\delta_{1j} - \delta_{1i}\right) - a_{j,m+1}\left(\delta_{1j} - \delta_{1,m+1}\right) - \rho sign[\sum_{i \in N_j} a_{ji}\left(\delta_{1j} - \delta_{1i}\right) - a_{j,m+1}\left(\delta_{1j} - \delta_{1,m+1}\right)] \tag{5.12}$$

$$u_{2j} = -k_2 q_{2j} - k_3 u_{1j} q_{3j} + \delta_{2j} + \hat{u}_{2j} \tag{5.13}$$

$$\dot{\delta}_{2j} = -\sum_{i \in N_j} a_{ji}\left(\delta_{2j} - \delta_{2i}\right) - a_{j,m+1}\left(\delta_{2j} - \delta_{2,m+1}\right) - \beta sign[\sum_{i \in N_j} a_{ji}\left(\delta_{2j} - \delta_{2i}\right) - a_{j,m+1}\left(\delta_{2j} - \delta_{2,m+1}\right)] \tag{5.14}$$

s_{1j} and s_{2j} are defined as

$$s_{1j} = \sum_{i \in N_j} a_{ji} \left(q_{2j} - q_{2i}\right) + a_{j,m+1}\left(q_{2j} - q_{2,m+1}\right)$$

$$s_{2j} = \sum_{i \in N_j} a_{ji} \left(q_{3j} - q_{3i}\right) + a_{j,m+1}\left(q_{3j} - q_{3,m+1}\right)$$

The derivative s_{1j} is

$$\dot{s}_{1j} = -k_2 s_{1j} - k_3 u_{1,m+1} s_{2j} + \sum_{i \in N_j} a_{ji} \left(\delta_{2j} - \delta_{2i}\right) + a_{j,m+1}\left(\delta_{2j} - \delta_{2,m+1}\right)$$

$$+ \sum_{i \in N_j} a_{ji} \left(\hat{u}_{2j} - \hat{u}_{2i}\right) + a_{j,m+1}\left(\hat{u}_{2j}\right)$$

$$-k_3 \left(\sum_{i \in N_j} a_{ji} \left((u_{1j} - u_{1,m+1}) q_{3j} - (u_{1i} - u_{1,m+1}) q_{3i}\right) + (u_{1j} - u_{1,m+1}) a_{j,m+1} q_{3j} \right)$$

By mathematical manipulation it follows that

$$\dot{s}_{1j} = -k_2 s_{1j} - k_3 u_{1,m+1} s_{2j} + \sum_{i \in N_j} a_{ji} \left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) + a_{j,m+1} \tilde{\delta}_{2j} + \sum_{i \in N_j} a_{ji} \left(\hat{u}_{2j} - \hat{u}_{2i}\right)$$

$$+ a_{j,m+1}\left(\hat{u}_{2j}\right) - k_3 \left(\sum_{i \in N_j} a_{ji} \left(\tilde{u}_{1j} q_{3j} - \tilde{u}_{1i} q_{3i}\right) + \tilde{u}_{1j} a_{j,m+1} q_{3j} \right)$$

(5.15)

Similarly one has

$$\dot{s}_{2j} = u_{1,m+1}s_{1j} + \sum_{i\in N_j} a_{ji}\left(\tilde{u}_{1j}q_{2j} - \tilde{u}_{1j}q_{2i}\right) + \tilde{u}_{1j}a_{j,m+1}q_{2j} \quad (5.16)$$

$\dot{\tilde{\delta}}_{1j}$, $\dot{\tilde{q}}_{1j}$ and $\dot{\tilde{\delta}}_{2j}$ are written as

$$\dot{\tilde{\delta}}_{1j} = -\sum_{i\in N_j} a_{ij}\left(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}\right) - a_{i,m+1}\tilde{\delta}_{1j} - \dot{\delta}_{1,m+1} - \rho sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}\right) - a_{j,m+1}\tilde{\delta}_{1j}\right)$$

$$(5.17)$$

$$\dot{\tilde{q}}_{1j} = -\sum_{i\in N_j} a_{ji}\left(\tilde{q}_{1j} - \tilde{q}_{1i}\right) - a_{j,m+1}\tilde{q}_{1j} + \tilde{\delta}_{1j} + \hat{u}_{1j} \quad (5.18)$$

$$\dot{\tilde{\delta}}_{2j} = -\sum_{i\in N_j} a_{ij}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) - a_{i,m+1}\tilde{\delta}_{2j} - \dot{\delta}_{2,m+1} - \beta sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) - a_{j,m+1}\tilde{\delta}_{2j}\right)$$

$$(5.19)$$

Define

$$x_1 = [[s_{11}, s_{21}], [s_{12}, s_{22}], \dots, [s_{1m}, s_{2m}]$$

$$x_2 = [[\tilde{\delta}_{21}, \tilde{q}_{11}, \tilde{\delta}_{11}], [\tilde{\delta}_{22}, \tilde{q}_{12}, \tilde{\delta}_{12}], \dots, [\tilde{\delta}_{2m}, \tilde{q}_{1m}, \tilde{\delta}_{1m}]]$$

Then it follows that

$$f_{1j} = \begin{bmatrix} -k_2 & -k_3u_{1,m+1} \\ u_{1,m+1} & 0 \end{bmatrix} \begin{bmatrix} s_{1j} \\ s_{2j} \end{bmatrix}$$

$$f_1(t, x_1) = [f_{11}, \dots, f_{1m}]$$

$$f_{2j} = \begin{bmatrix} -\sum_{i\in N_j} a_{ij}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) - a_{i,m+1}\tilde{\delta}_{2j} \\ -\beta sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) - a_{j,m+1}\tilde{\delta}_{2j}\right) - \dot{\delta}_{2,m+1} \\ -\sum_{i\in N_j} a_{ji}\left(\tilde{q}_{1j} - \tilde{q}_{1i}\right) - a_{j,m+1}\tilde{q}_{1j} + \tilde{\delta}_{1j} + \hat{u}_{1j} \\ -\sum_{i\in N_j} a_{ij}\left(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}\right) - a_{j,m+1}\tilde{\delta}_{1j} \\ -\rho sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}\right) - a_{j,m+1}\tilde{\delta}_{1j}\right) - \dot{\delta}_{1,m+1} \end{bmatrix}$$

$$f_2(t, x_2) = [f_{21}, f_{22}, \ldots, f_{2m}]$$

$$g_j = \begin{bmatrix} \sum_{i\in N_j} a_{ji}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) + a_{j,m+1}\tilde{\delta}_{2j} \\ \sum_{i\in N_j} a_{ji}\left(\hat{u}_{2j} - \hat{u}_{2i}\right) + a_{j,m+1}\left(\hat{u}_{2j}\right) \\ -k_3\left(\sum_{i\in N_j} a_{ji}\left(\tilde{u}_{1j}q_{3j} - \tilde{u}_{1j}q_{3i}\right) + \tilde{u}_{1j}a_{j,m+1}q_{3j}\right) \\ \sum_{i\in N_j} a_{ji}\left(\tilde{u}_{1j}q_{2j} - \tilde{u}_{1j}q_{2i}\right) + \tilde{u}_{1j}a_{j,m+1}q_{2j} \end{bmatrix}$$

$$g(t, x_1, x_2)x_2 = [g_1, g_2, \ldots, g_m]$$

By Theorem 2.4 it can be proved $\tilde{\delta}_{2j} = \delta_{2j} - \delta_{2,m+1}$ globally exponentially converge to zero, which implies

$$-\sum_{i\in N_j} a_{ij}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) - a_{i,m+1}\tilde{\delta}_{2j} - \dot{\delta}_{2,m+1} - \beta sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}\right) - a_{j,m+1}\tilde{\delta}_{2j}\right)$$

globally exponentially converge to zero.

By Theorem 4.5 $\tilde{q}_{1j} = q_{1j} - q_{1,m+1}$ and $\tilde{\delta}_{1j} = \delta_{1j} - \delta_{1,m+1}$ globally exponentially converge to zero, which implies

$$-\sum_{i\in N_j} a_{ij}\left(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}\right) - a_{i,m+1}\tilde{\delta}_{1j} - \dot{\delta}_{1,m+1} - \rho sign\left(\sum_{j=1}^{n} a_{ij}\left(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}\right) - a_{j,m+1}\tilde{\delta}_{1j}\right)$$

globally exponentially converge to zero, and

$$-\sum_{i\in N_j} a_{ji}\left(\tilde{q}_{1j} - \tilde{q}_{1i}\right) - a_{j,m+1}\tilde{q}_{1j} + \tilde{\delta}_{1j}$$

globally exponentially converge to zero. From the definition of f_{2j}, then system $\dot{x}_2 = f_2(t, x_2)$ is exponentially stable,the third assumption in Theorem 4.1 holds. f_{1j} have the same form as (4.24), by Theorem 4.3 $\dot{x}_1 = f_1(t, x_1)$ is exponentially stable, then the first assumption in Theorem 4.1 holds. From the definition of g_j, it can be proved the second assumption in Theorem 4.1 holds, which means $x_1 = [[s_{11}, s_{21}], [s_{12}, s_{22}], \dots, [s_{1m}, s_{2m}]]$ is globally exponentially stable, from the definition of s_{1j} and s_{2j},

$$s_{1j} = \sum_{i\in N_j} a_{ji}\left(q_{2j} - q_{2i}\right) + a_{j,m+1}\left(q_{2j} - q_{2,m+1}\right)$$

$$= \sum_{i\in N_j} a_{ji}\left(\tilde{q}_{2j} - \tilde{q}_{2i}\right) + a_{j,m+1}\tilde{q}_{2j}$$

$$= (L + B)\tilde{q}_{2j}$$

Since $(L + B)$ is positive symmetric matrix then $q_{2j} - q_{2,m+1} = \tilde{q}_{2j} = (L + B)^{-1}s_{1j}$ globally exponentially converge to zero,

$$s_{2j} = \sum_{i\in N_j} a_{ji}\left(q_{3j} - q_{2i}\right) + a_{j,m+1}\left(q_{3j} - q_{3,m+1}\right)$$

$$= \sum_{i \in N_j} a_{ji} \left(\tilde{q}_{3j} - \tilde{q}_{3i} \right) + a_{j,m+1} \tilde{q}_{3j}$$

$$= (L + B) \tilde{q}_{3j}$$

Similarly, $q_{3j} - q_{3,m+1} = \tilde{q}_{3j} = (L + B)^{-1} s_{2j}$ globally exponentially converge to zero.

5.3 Distributed Control of Dynamic Systems with Parametric Uncertainties

In Section 5.2, distributed control laws are proposed for dynamic systems without uncertainties. However, in practice inertia parameters of the mass and moment of inertia are unknown and require to be estimated.

In this section, distributed controllers for vehicle dynamics with parametric uncertainties are designed based on both the kinematic controllers in Section 4.4 and Section 4.6. The parametrical uncertainties are estimated by robust control methods and adaptive control methods respectively.

Define $\hat{u}_{*j} = u_{*j} - \eta_{*j}$, where $\eta_{*j} = [\eta_{1j}, \eta_{2j}]$ are defined in (4.26) and (4.31).

Theorem 5.2 For m systems in (5.7), if there exists a spanning tree with the virtual leader v_{m+1} as the root of the spanning tree, then the distributed control laws

$$\tau_j = \tilde{B}_j^{-1} (\hat{M}_j \dot{\eta}_{*j} + \hat{C}_j \eta_{*j} + \hat{G}_j - k \hat{u}_{*j} - Y_j \beta sign(Y_j^T \hat{u}_{*j})) \qquad (5.20)$$

guarantee that (4.8)-(4.11) holds, where k satisfies $|\hat{a} - \tilde{a}| < \beta$, $\hat{a}$ and $\tilde{a}$ are the estimated and actual inertia parameters vector. η_{*j} are defined in (4.26) and (4.31).

Proof Denote the inertia parameter errors by $\breve{a} = \hat{a} - \tilde{a}$, choose the Lyapunov function $V_u = \frac{1}{2}\hat{u}_{*j}^T \tilde{M}_j \hat{u}_{*j}$, differentiate it along (5.7) one has

$$\dot{V}_u = \frac{1}{2}\hat{u}_{*j}^T \dot{\tilde{M}}_j \hat{u}_{*j} + \hat{u}_{*j}^T \tilde{M}_j \dot{\hat{u}}_{*j} \tag{5.21}$$

Substitute τ_j in (5.8) into (5.7) it follows that

$$\tilde{M}_j \dot{u}_{*j} + \tilde{C}_j u_{*j} + \tilde{G}_j = \hat{M}_j \dot{\eta}_{*j} + \hat{C}_j \eta_{*j} + \hat{G}_j - k\hat{u}_{*j} - Y_j \beta sign(Y_j^T \hat{u}_{*j})$$

$$\tilde{M}_j \dot{u}_{*j} + \tilde{C}_j u_{*j} + \tilde{G}_j$$

$$= \left(\breve{M}_j + \tilde{M}_j\right)\dot{\eta}_{*j} + \left(\breve{C}_j + \tilde{C}_j\right)\eta_{*j} + (\breve{G}_j + \tilde{G}_j) - k\hat{u}_{*j} - Y_j \beta sign(Y_j^T \hat{u}_{*j})$$

$$\tilde{M}_j\left(\dot{u}_{*j} - \dot{\eta}_{*j}\right) + \tilde{C}_j\left(u_{*j} - \eta_{*j}\right) = -k\hat{u}_{*j} + Y_j \breve{a} - Y_j \beta sign(Y_j^T \hat{u}_{*j})$$

$$\tilde{M}_j \dot{\hat{u}}_{*j} = -\tilde{C}_j \hat{u}_{*j} - k\hat{u}_{*j} + Y_j \breve{a} - Y_j \beta sign(Y_j^T \hat{u}_{*j}) \tag{5.22}$$

Substitute $\tilde{M}_j \dot{\hat{u}}_{*j}$ in (5.22) into (5.21) one has

$$\dot{V}_u = \frac{1}{2}\hat{u}_{*j}^T \dot{\tilde{M}}_j \hat{u}_{*j} + \hat{u}_{*j}^T * (-\tilde{C}_j \hat{u}_{*j} - k\hat{u}_{*j} + Y_j \breve{a} - Y_j \beta sign(Y_j^T \hat{u}_{*j}))$$

$$= \frac{1}{2}\hat{u}_{*j}^T (\dot{\tilde{M}}_j - 2\tilde{C}_j)\hat{u}_{*j} - \hat{u}_{*j}^T k\hat{u}_{*j} + \hat{u}_{*j}^T Y_j \breve{a} - \hat{u}_{*j}^T Y_j \beta sign(Y_j^T \hat{u}_{*j})$$

Since $\dot{\tilde{M}}_j - 2\tilde{C}_j$ is skew-symmetric, then $\dot{V}_u < -\hat{u}_{*j}^T k\hat{u}_{*j}$, let λ_m be the maximal eigenvalue of $\tilde{M}_j$ it can be learned that $\dot{V}_u < -\hat{u}_{*j}^T k\hat{u}_{*j} < -\frac{2k}{\lambda_m}V_u$, then $\hat{u}_{*j}$ is exponentially stable which means $u_{*j} - \eta_{*j}$ exponentially converge to zero. The rest proof is similar as Theorem 5.1 and not repeated here.

In Theorem 5.2, distributed controllers for vehicle dynamics with parametrical uncertainties are designed based on the kinematic controllers in (4.25) and (4.30). Parametrical uncertainties are estimated by sliding mode control. Next the distributed controller for vehicle dynamics with parametrical uncertainties designed based on the controllers in (4.53) and (4.56).

Define $\hat{u}_{*j} = u_{*j} - \eta_{*j}$, where $\eta_{*j} = [\eta_{1j}, \eta_{2j}]$ are defined in (4.54) and (4.57).

Theorem 5.3 For m systems in (5.7), if there exists a spanning tree with the virtual leader v_{m+1} as the root of the spanning tree, then the distributed control laws

$$\tau_j = \tilde{B}_j^{-1}(-k\hat{u}_{*j} - Y_j(\dot{u}_{*j}, u_{*j}, \dot{q}_{*j}, q_{*j})\hat{a}_j) \tag{5.23}$$

and the update laws for $\hat{a}_j$

$$\dot{\hat{a}}_j = -\Gamma_j Y_j^T(\dot{u}_{*j}, u_{*j}, \dot{q}_{*j}, q_{*j})\hat{u}_{*j} \tag{5.24}$$

guarantee that (4.8)-(4.11) holds, where $\hat{a}_j$ is the estimated inertial parameter vector and $\hat{a}_j$ is bounded. η_{*j} are defined in (4.54)-(4.57).

Proof Substitute τ_j in (5.23) into (5.7) it follows that

$$\tilde{M}_j\dot{u}_{*j} + \tilde{C}_j u_{*j} + \tilde{G}_j = -k\hat{u}_{*j} - Y_j(\dot{u}_{*j}, u_{*j}, \dot{q}_{*j}, q_{*j})\hat{a}_j$$

$$\tilde{M}_j\dot{u}_{*j} + \tilde{C}_j u_{*j} + \tilde{G}_j = \hat{M}_j\dot{\eta}_{*j} + \hat{C}_j\eta_{*j} + \hat{G}_j - k\hat{u}_{*j}$$

$$\tilde{M}_j\dot{u}_{*j} + \tilde{C}_j u_{*j} + \tilde{G}_j = \left(\breve{M}_j + \tilde{M}_j\right)\dot{\eta}_{*j} + \left(\check{C}_j + \tilde{C}_j\right)\eta_{*j} + (\check{G}_j + \tilde{G}_j) - k\hat{u}_{*j}$$

$$\tilde{M}_j\left(\dot{u}_{*j} - \dot{\eta}_{*j}\right) + \tilde{C}_j\left(u_{*j} - \eta_{*j}\right) = -k\hat{u}_{*j} + Y_j\breve{a}_j$$

$$\widetilde{M}_j\dot{\hat{u}}_{*j} = -\tilde{C}_j\hat{u}_{*j} - k\hat{u}_{*j} + Y_j\breve{a}_j \tag{5.25}$$

where $\breve{a}_j = \hat{a}_j - a_j$ is the estimation error of inertial parameter vector.

Choose the Lyapunov function $V_v = \hat{u}_{*j}^T\widetilde{M}_j\hat{u}_{*j} + \breve{a}_j^T\Gamma_j^{-1}\breve{a}_j$, differentiate it along (5.7) one has

$$\dot{V}_v = \hat{u}_{*j}^T\dot{\widetilde{M}}_j\hat{u}_{*j} + 2\hat{u}_{*j}^T\widetilde{M}_j\dot{\hat{u}}_{*j} + 2\breve{a}_j^T\Gamma_j^{-1}\breve{a}_j \tag{5.26}$$

Substitute $\widetilde{M}_j\dot{\hat{u}}_{*j}$ in (5.25) into (5.26), it follows that

$$\dot{V}_v = \hat{u}_{*j}^T\dot{\widetilde{M}}_j\hat{u}_{*j} + 2\hat{u}_{*j}^T(-\tilde{C}_j\hat{u}_{*j} - k\hat{u}_{*j} + Y_j\breve{a}_j) + 2\breve{a}_j^T\Gamma_j^{-1}\breve{a}_j$$

With the aid of properties of vehicle dynamics one has

$$\dot{V}_v = -2\hat{u}_{*j}^Tk\hat{u}_{*j} + 2\hat{u}_{*j}^TY_j\breve{a}_j + 2\breve{a}_j^T\Gamma_j^{-1}\breve{a}_j \tag{5.27}$$

Substitute $\dot{\hat{a}}_j$ in (5.24) into (5.27), it follows that

$$\dot{V}_v = -2\hat{u}_{*j}^Tk\hat{u}_{*j}$$

Therefore V_v is bounded, which means $\hat{u}_{*j}$ and $\breve{a}_j$ are bounded. From (5.25) it is known that $\dot{\hat{u}}_{*j}$ is bounded. By Barbalat's Lemma, it can be proved that $\lim_{t\to\infty}\hat{u}_{*j} = 0$.

Since the real inputs are $u_{*j} = \hat{u}_{*j} + \eta_{*j}$, where $\eta_{*j} = [\eta_{1j}, \eta_{2j}]$ are defined in (4.54) and (4.57). Then the distributed controllers in (4.53) and (4.56) are rewritten as

$$u_{1j} = -\alpha_1 z_{1j} + \zeta_{1j} + \hat{u}_{1j} \tag{5.28}$$

$$u_{2j} = -\sum_{i \in N_j} a_{ji} \left(z_{2j} - z_{2i} \right) - a_{j,m+1}\left(z_{2j} - z_{20} \right)$$

$$- \rho_2 sign \left[\sum_{i \in N_j} a_{ji} \left(z_{2j} - z_{2i} \right) - a_{j,m+1}\left(z_{2j} - z_{20} \right) \right] - k_u\left(\dot{u}_{1j} z_{3j} + \dot{z}_{3j} u_{1j} \right)$$

$$+ \hat{u}_{2j} \tag{5.29}$$

It follows that

$$\dot{\tilde{z}}_{1j} = -\alpha_1 \tilde{z}_{1j} + \tilde{\zeta}_{1j} + \hat{u}_{1j}$$

Choose a nonnegative symmetric function $V_z = \sum_{j=1}^{m} \frac{1}{2} \tilde{z}_{1j}^2$. Differentiate V_z along (4.50), it follows that

$$\dot{V}_z = -\alpha_1 \tilde{z}_{1*}^2 + \tilde{z}_{1*} \tilde{\zeta}_{1*} + \tilde{z}_{1*} \hat{u}_{1j}$$

$$\dot{V}_z \le -\alpha_1 \tilde{z}_{1*}^2 + ||\tilde{z}_{1*}|| \ \left|\left|\tilde{\zeta}_{1*}\right|\right| + ||\tilde{z}_{1*}|| \ |\left|\hat{u}_{1j}\right||$$

$$= -2\alpha_1 V_z + K\sqrt{2V_z}$$

where $K = \left|\left|\tilde{\zeta}_{1*}\right|\right| + |\left|\hat{u}_{1j}\right||$ Choose $V_2 = \sqrt{V_z}$, one has

$$\dot{V}_2 \le -\alpha_1 V_2 + \frac{1}{\sqrt{2}} K$$

Then V_2 is exponentially stable, which implies z_{1j} exponentially converges to z_{10} for $1 \leq j \leq m$. Thus $\lim_{t\to\infty}(u_{1j} - u_{10}) = 0$.

$$\dot{z}_{2*} = -Lz_{2*} - B(z_{2*} - z_{20}\mathbf{1}) - \rho_2 sign\big(Lz_{2*} + B(z_{2*} - z_{20}\mathbf{1})\big) + \hat{u}_{2j}$$

$$\dot{\tilde{z}}_{2*} = -(L + B)\tilde{z}_{2*} - \rho_2 sign\big((L + B)\tilde{z}_{2*}\big) - \dot{z}_{20} + \hat{u}_{2j}$$

By Theorem 3.4, it is known when ρ_2 satisfies

$$\rho_2 \geq \frac{||(L + B)^{-1}||(|\dot{z}_{20}| + \left||\hat{u}_{2j}|\right|)}{\epsilon}$$

where ϵ is the minimum eigenvalue of $(L + B)^{-1}$, then z_{2j} exponentially converges to z_{20} for $1 \leq j \leq m$. By Lemma 4.4, (4.8)-(4.11) hold.

5.4 Distributed Controller with Time-varying Communication Topology

In previous sections, the communication graph is considered time-invariant. In real-life operations the communication topology is sometimes switching resulting from links creation, disconnection and nodes disability. It has been proved through an infinite sequence of non-overlapping, uniformly bounded time interval, if the union of the graphs cross each interval has a directed spanning tree, then multi-agent systems can still reach consensus with the aid of SIA matrix theories. Now consider the dynamic systems (5.7) with parametric uncertainties.

Theorem 5.4 For m systems in (5.7), if there exists a spanning tree with the virtual leader v_{m+1} as the root of the spanning tree at any finite time interval, then the distributed control laws

$$\tau_j = \tilde{B}_j^{-1}(\widehat{M}_j\dot{\eta}_{*j} + \hat{C}_j\eta_{*j} + \hat{G}_j - k\hat{u}_{*j} - Y_j\beta sign(Y_j^T\hat{u}_{*j}))$$

guarantee that (4.8)-(4.11) holds, where k satisfies $|\hat{\alpha} - \tilde{\alpha}| < \beta$, $\hat{\alpha}$ and $\tilde{\alpha}$ are the estimated and actual inertia parameters vector.

5.5 Simulation III

To show the effectiveness of proposed control algorithms for vehicles dynamics, simulation is done for four identical robot models as shown in Figure 4.1. For the dynamic controllers in without parametrical uncertainties, the desired pattern and communication graph are shown in Figure 5.1 and Figure 5.2. Figure 5.3 to Figure 5.6 represent the convergence results from dynamic systems without parametric uncertainties. For the dynamic controllers with parametrical uncertainties in Theorem 5.2, the desired pattern and communication graph are shown in Figure 5.7 and Figure 5.8. Figure 5.9 to Figure 5.12 represent the convergence results from dynamic systems with parametric uncertainties controlled by the distributed controllers in Theorem 5.2. For the dynamic controllers in with parametrical uncertainties in Theorem 5.3, the desired pattern and communication graph are shown in Figure 5.13 and Figure 5.14. Figure 5.15 to Figure 5.18 represent the convergence results from dynamic systems with parametric uncertainties controlled by the distributed controllers in Theorem 5.3.

Figure 5.1 is the desired formation.

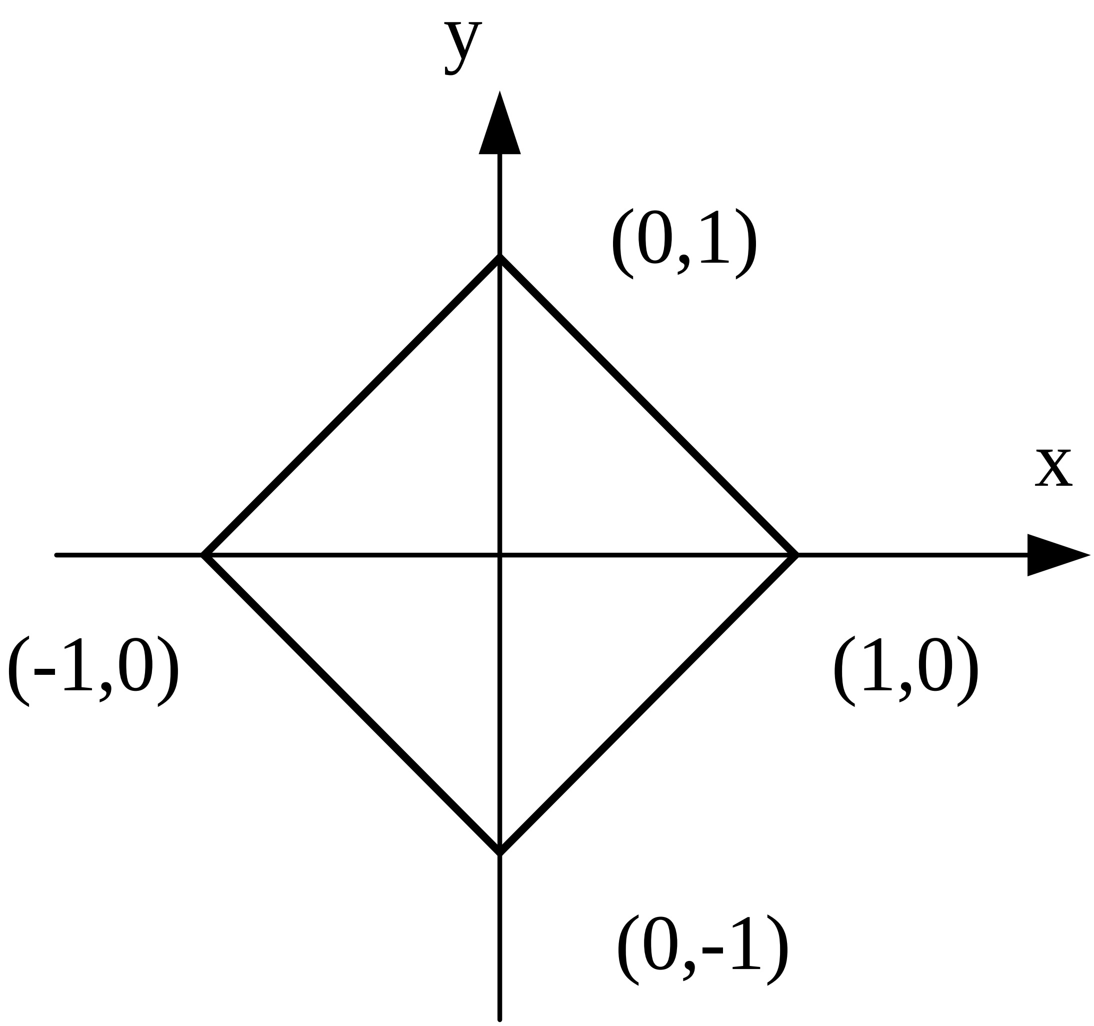

Figure 5.1 Desired formation of four vehicles I

Figure 5.2 is the communication graph.

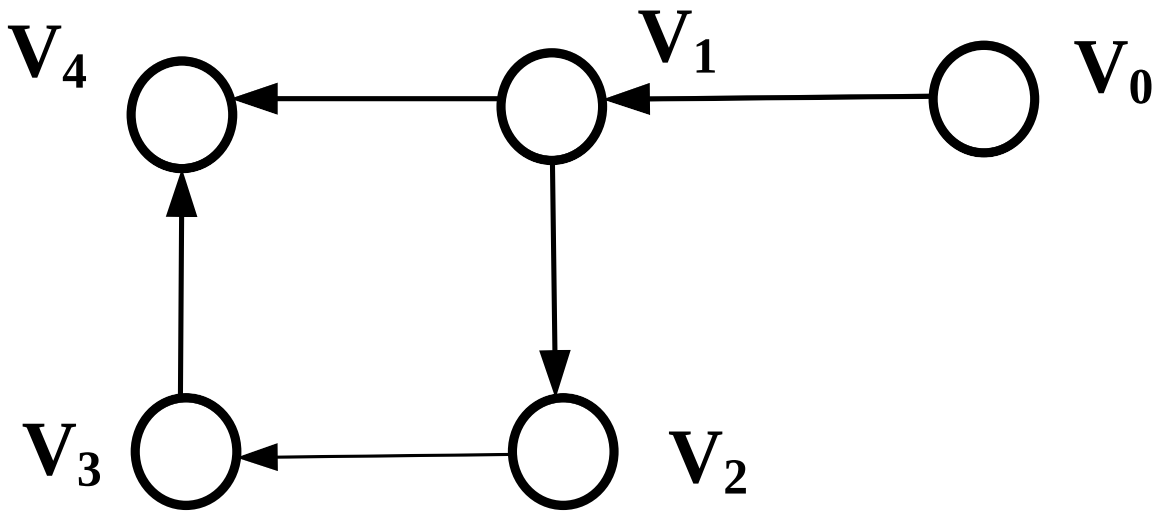

Figure 5.2 Information exchange topology III

For multiple dynamic systems without parametrical uncertainties, Figure 5.3 represents the centroid of x_j ($1 \leq j \leq 4$) and the desired trajectory x_0. Figure 5.4 represents the centroid of y_j ($1 \leq j \leq 4$) and the desired trajectory y_0. Figure 5.5 represents $(\theta_0 - \theta_j)$ ($1 \leq j \leq 4$), Figure 5.6 represents the formation tracking of four follower robots.

Figure 5.3 is the convergence result of x_j.

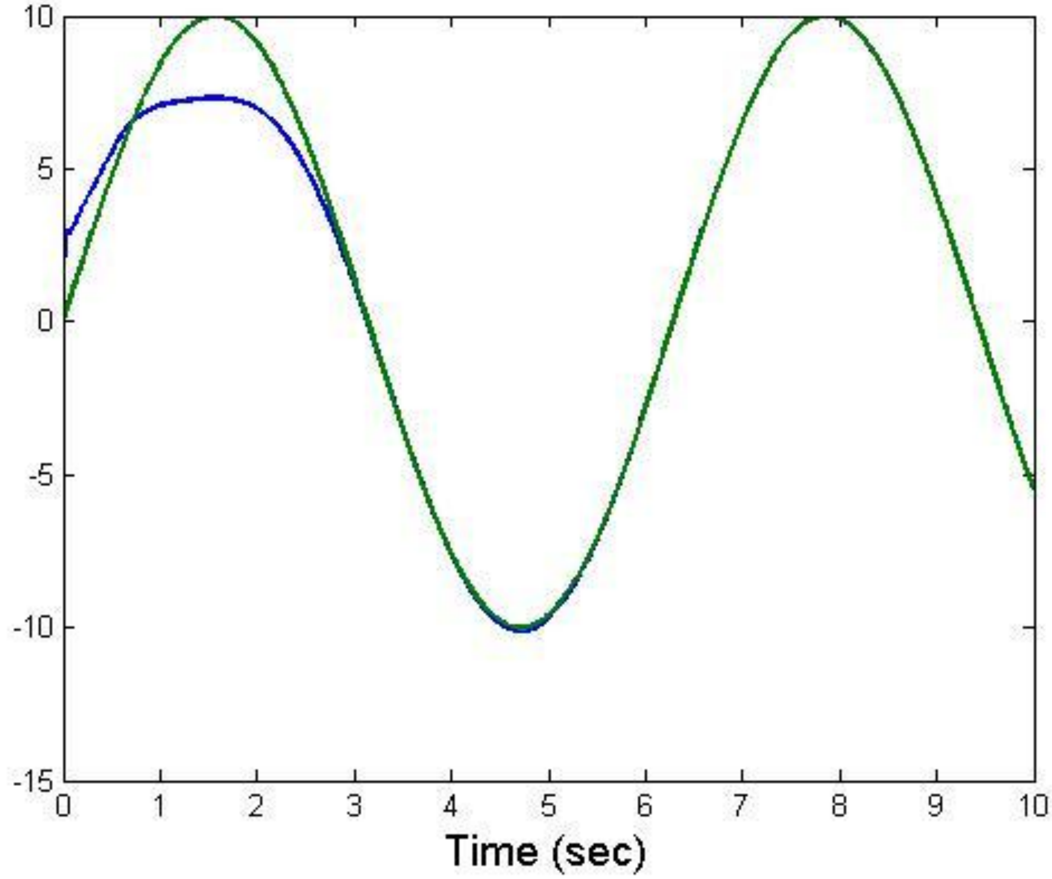

Figure 5.3 The centroid of x_j ($1\leq j \leq 4$) and the desired trajectory x_0

Figure 5.4 is the convergence result of y_j

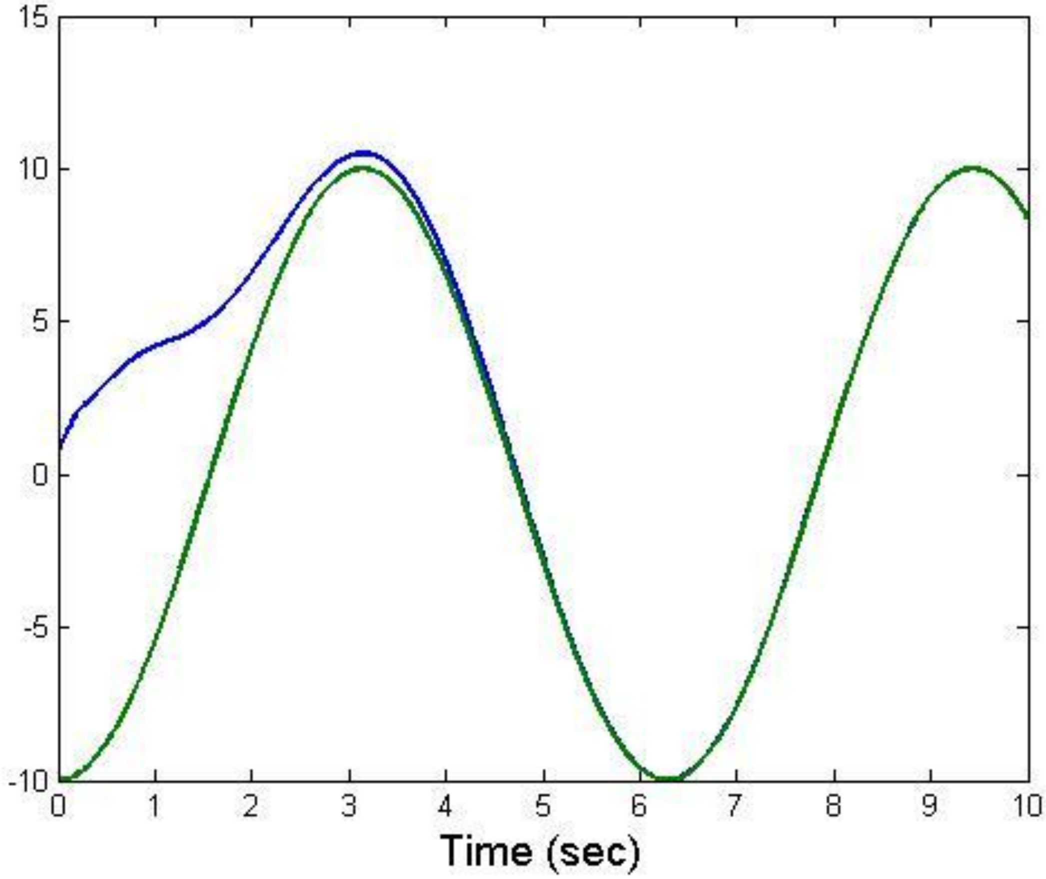

Figure 5.4 The centroid of y_j ($1 \leq j \leq 4$) and the desired trajectory y_0

Figure 5.5 is the converge result of θ_j.

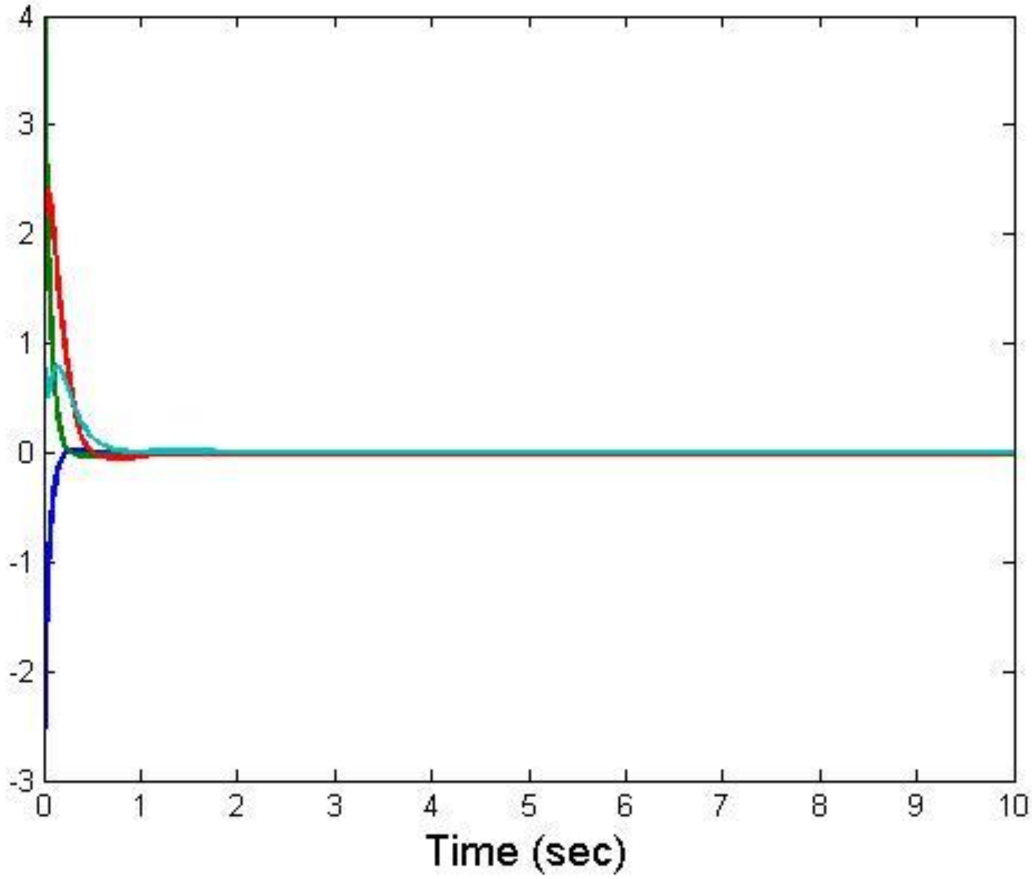

Figure 5.5 $\theta_j - \theta_0$ ($1 \leq j \leq 4$)

Figure 5.6 is the formation tracking of multi-vehicle system.

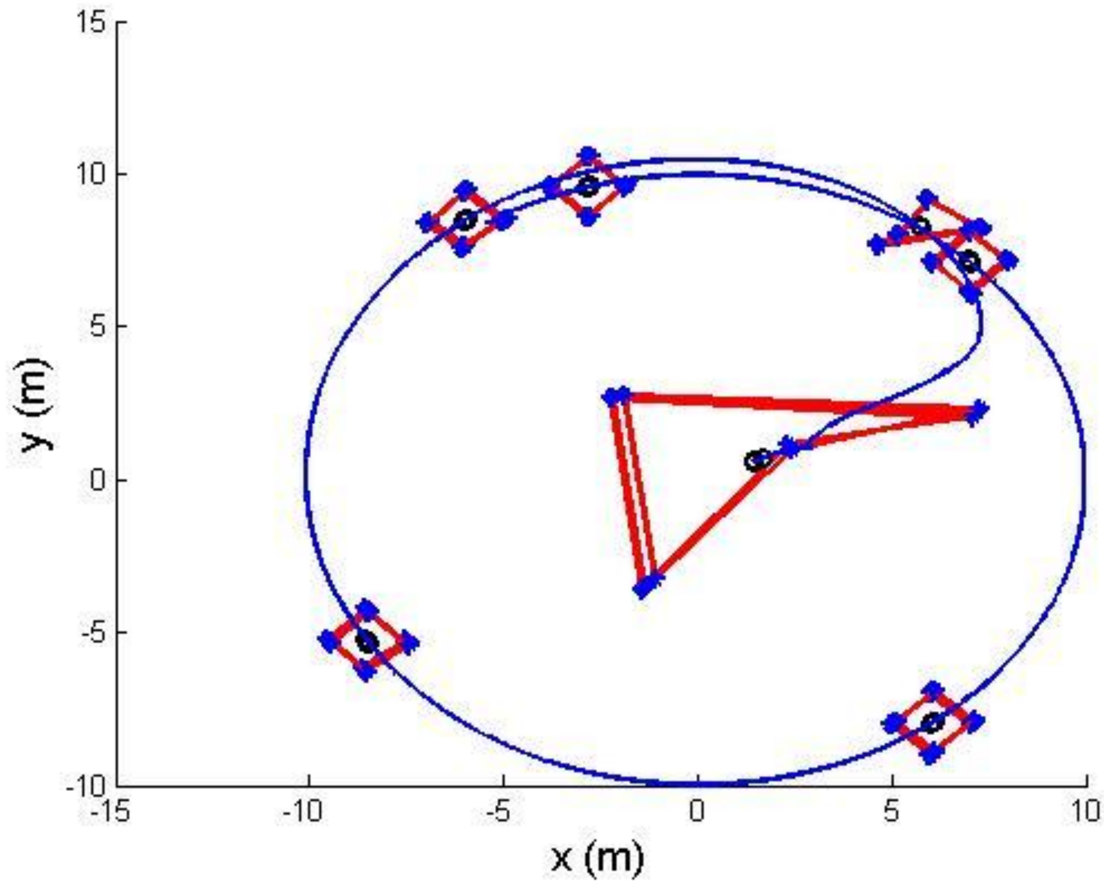

Figure 5.6 Formation tracking of four follower robots I

Figure 5.7 is the desired formation.

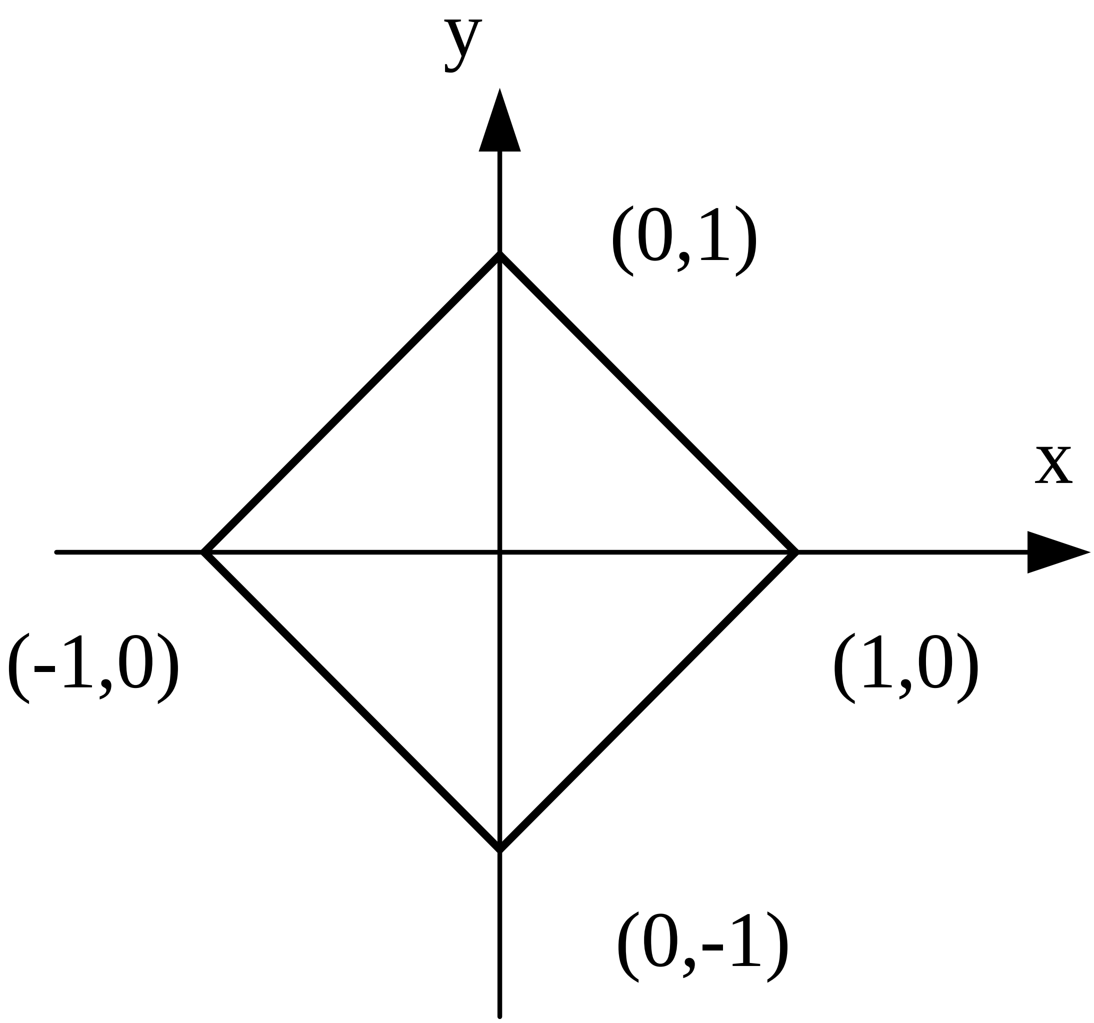

Figure 5.7 Desired formation of four vehicles II

Figure 5.8 is the communication graph.

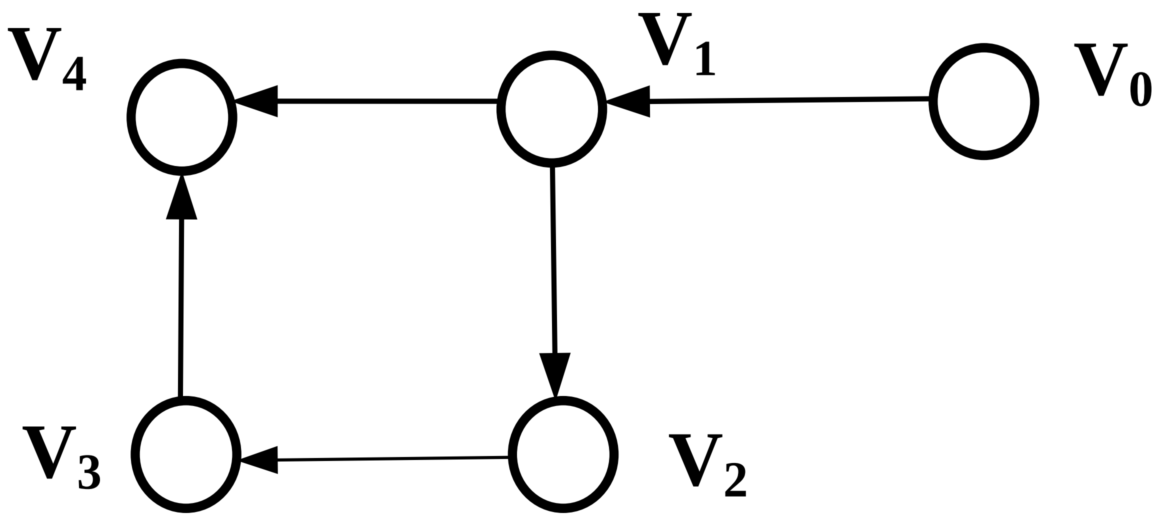

Figure 5.8 Information exchange topology IV

For multiple dynamic systems with parametrical uncertainties controlled by the distributed controllers in Theorem 5.2, Figure 5.9 represents the centroid of x_i (1≤ i ≤4) and the desired trajectory x_0. Figure 5.10 represents the centroid of y_i (1≤ i ≤4) and the desired trajectory y_0. Figure 5.11 represents $(\theta_0 - \theta_i)$ (1≤ i ≤4). Figure 5.12 represents the formation tracking of four follower robots.

Figure 5.9 is the convergence result of x_i.

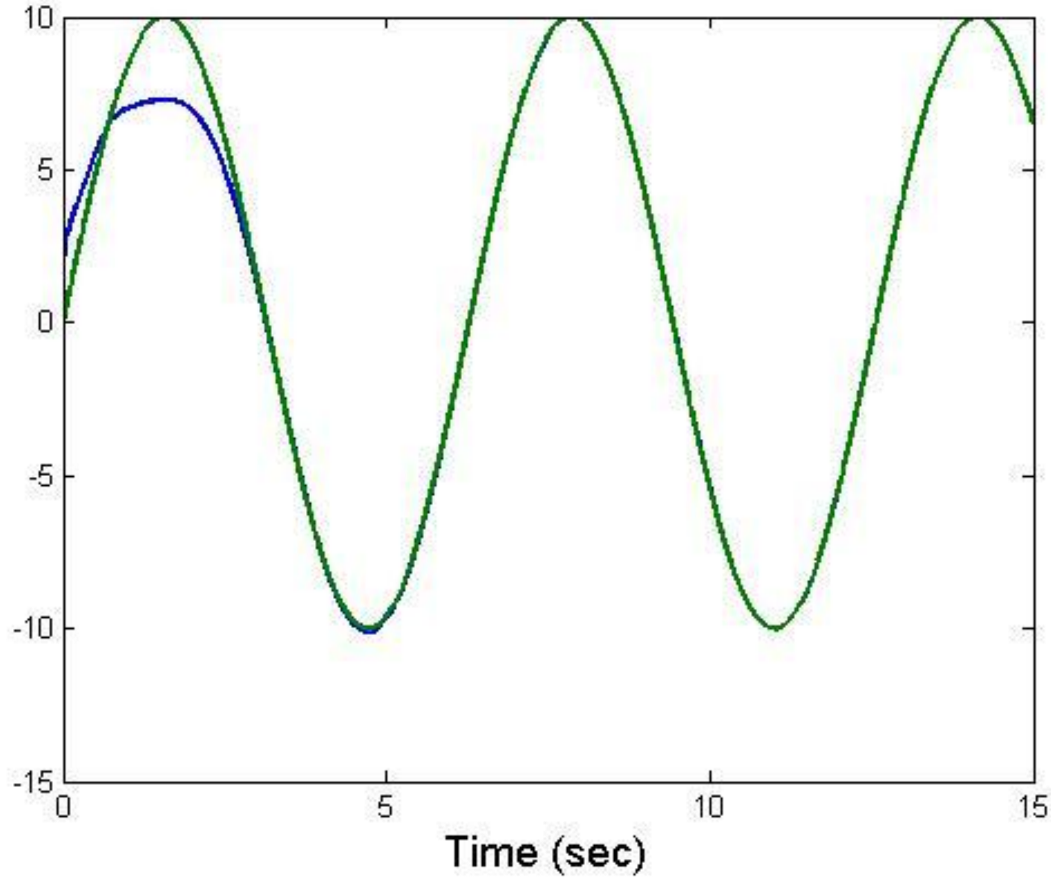

Figure 5.9 The centroid of x_i ($1 \leq i \leq 4$) and the desired trajectory x_0

Figure 5.10 is convergence result of y_i.

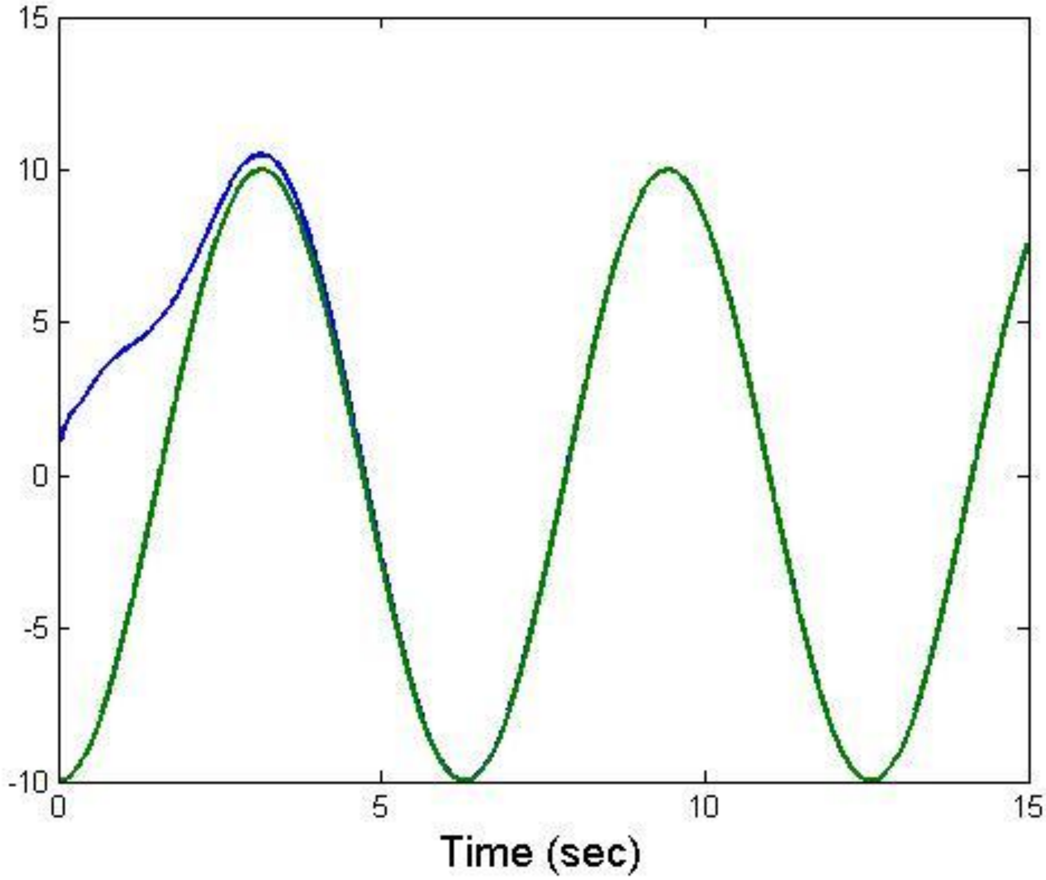

Figure 5.10 The centroid of y_i ($1 \leq i \leq 4$) and the desired trajectory y_0

Figure 5.11 is the convergence result of θ_i.

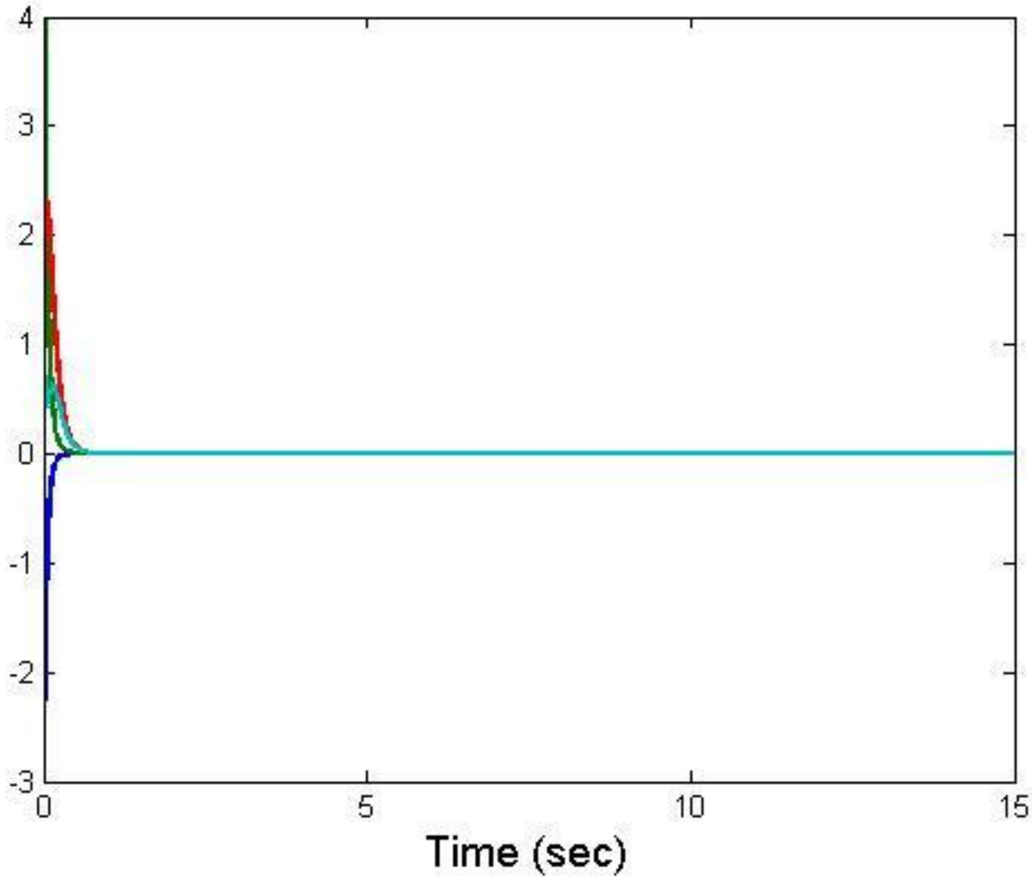

Figure 5.11 $\theta_i - \theta_0$ ($1 \le i \le 4$)

Figure 5.12 is the formation tracking of multi-vehicle system.

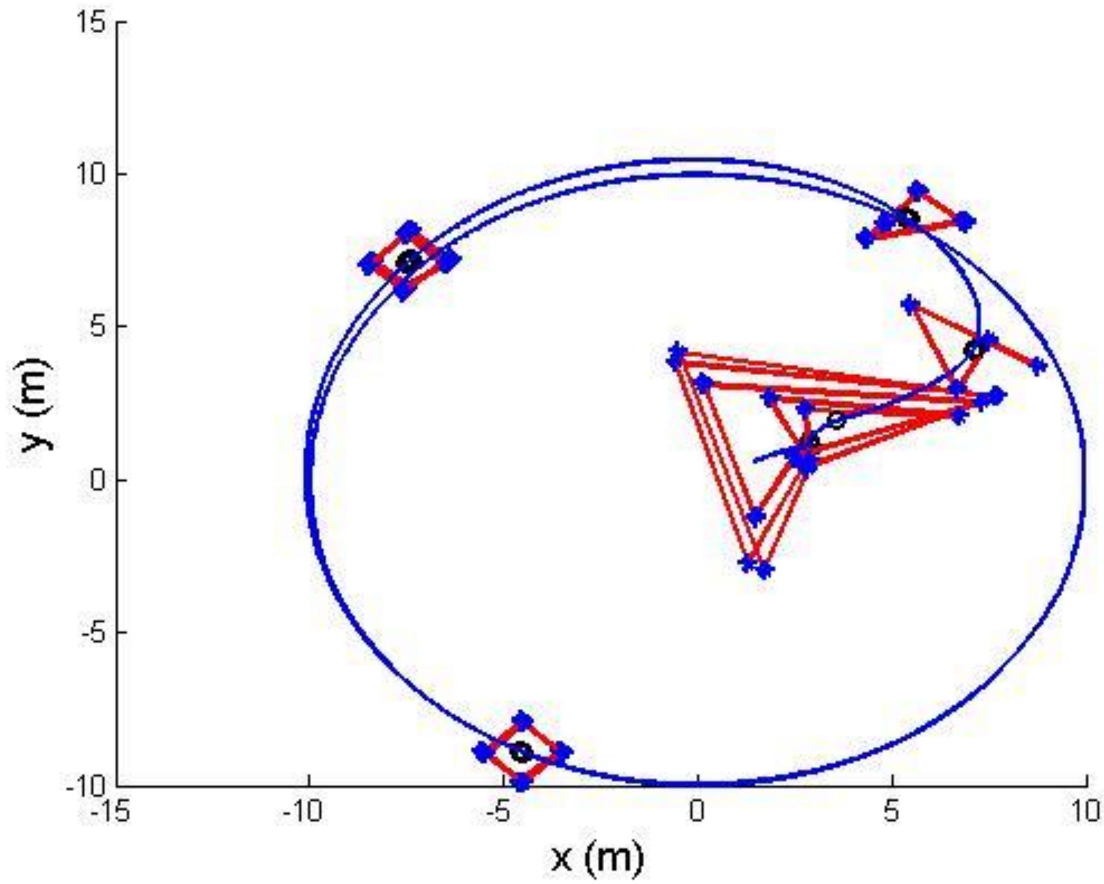

Figure 5.12 Formation tracking of four follower robots II

Figure 5.13 is the desired formation.

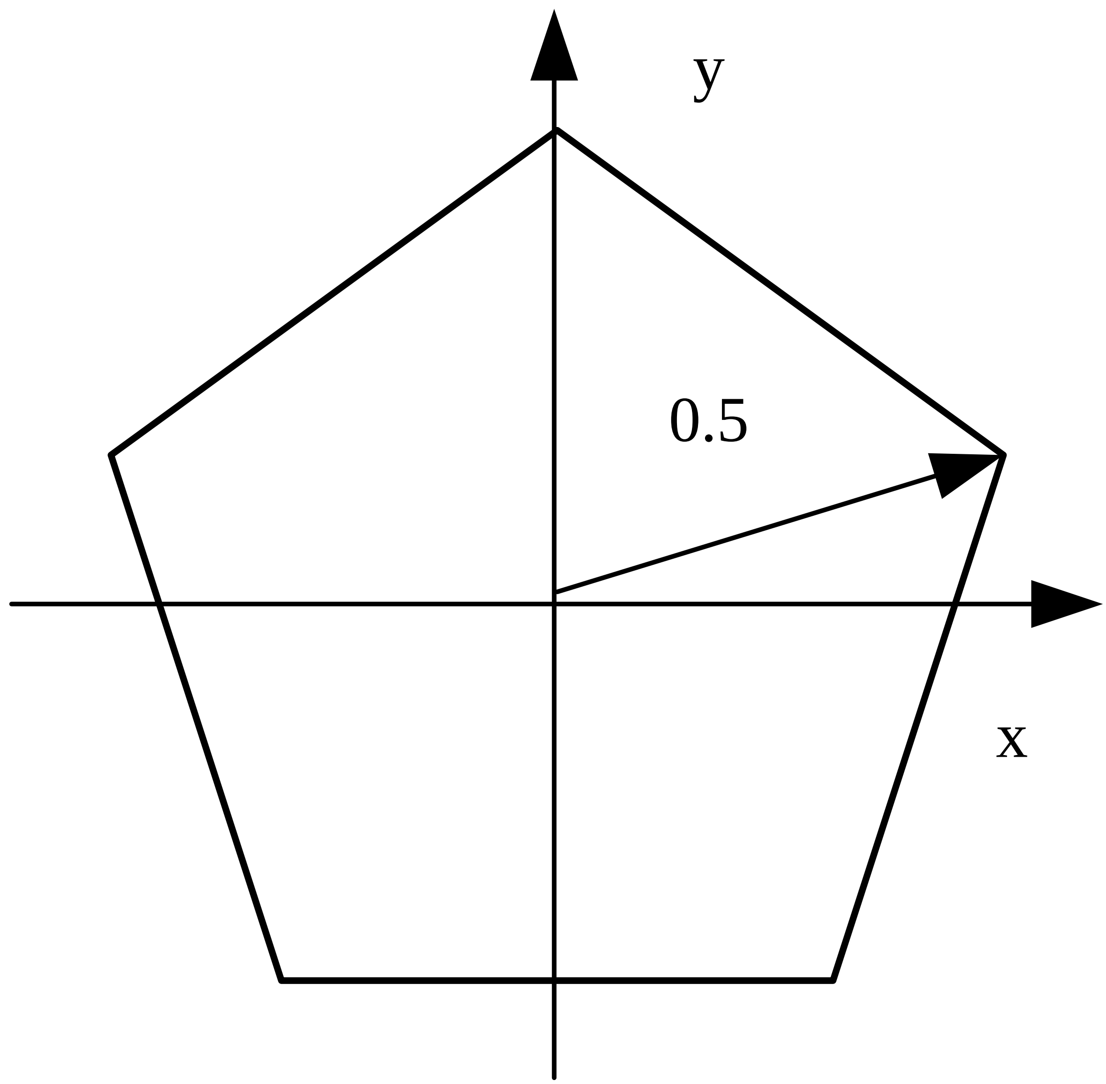

Figure 5.13 Desired formation of five vehicles III

Figure 5.14 is the communication graph.

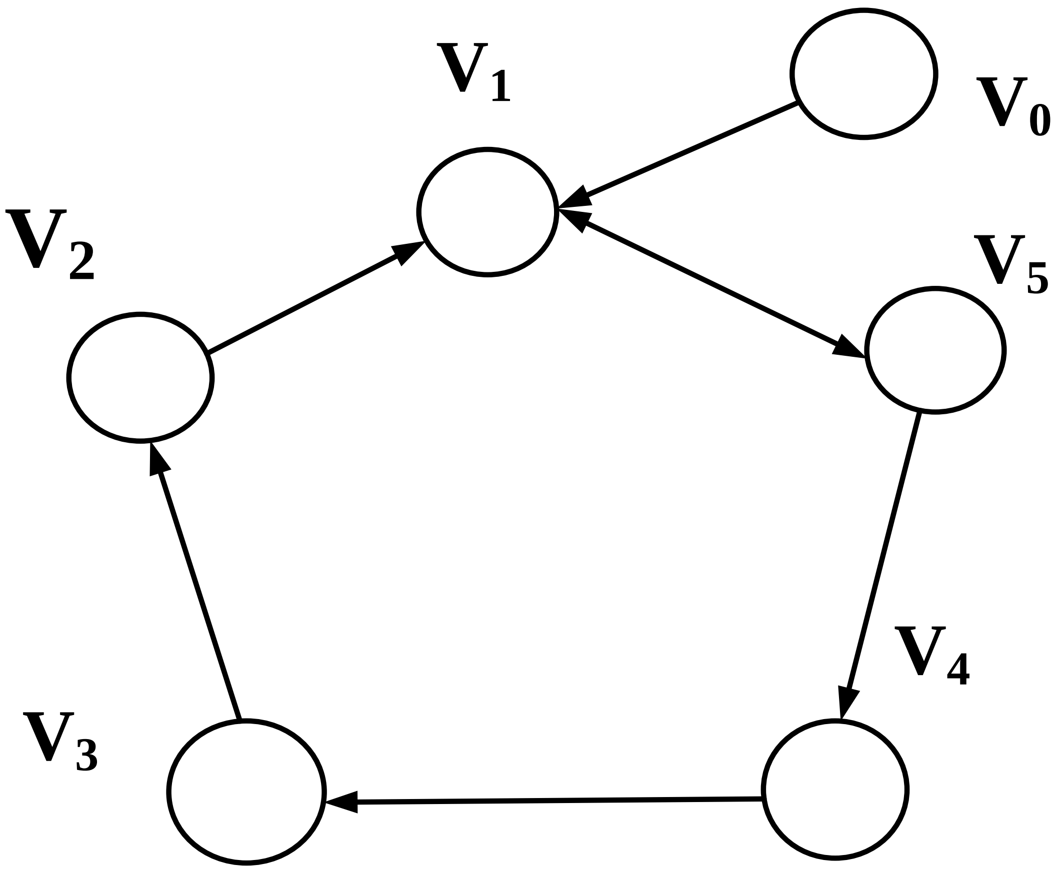

Figure 5.14 Information exchange topology V

For multiple dynamic systems with parametrical uncertainties controlled by the distributed controllers in Theorem 5.3, Figure 5.15 represents the centroid of x_k (1≤ k ≤5) and the desired trajectory x_0. Figure 5.16 represents the centroid of y_k (1≤ k ≤5) and the desired trajectory y_0. Figure 5.17 represents $(\theta_k - \theta_0)$ (1≤ k ≤5). Figure 5.18 represents the formation tracking of five follower robots.

Figure 5.15 is the convergence result of x_k.

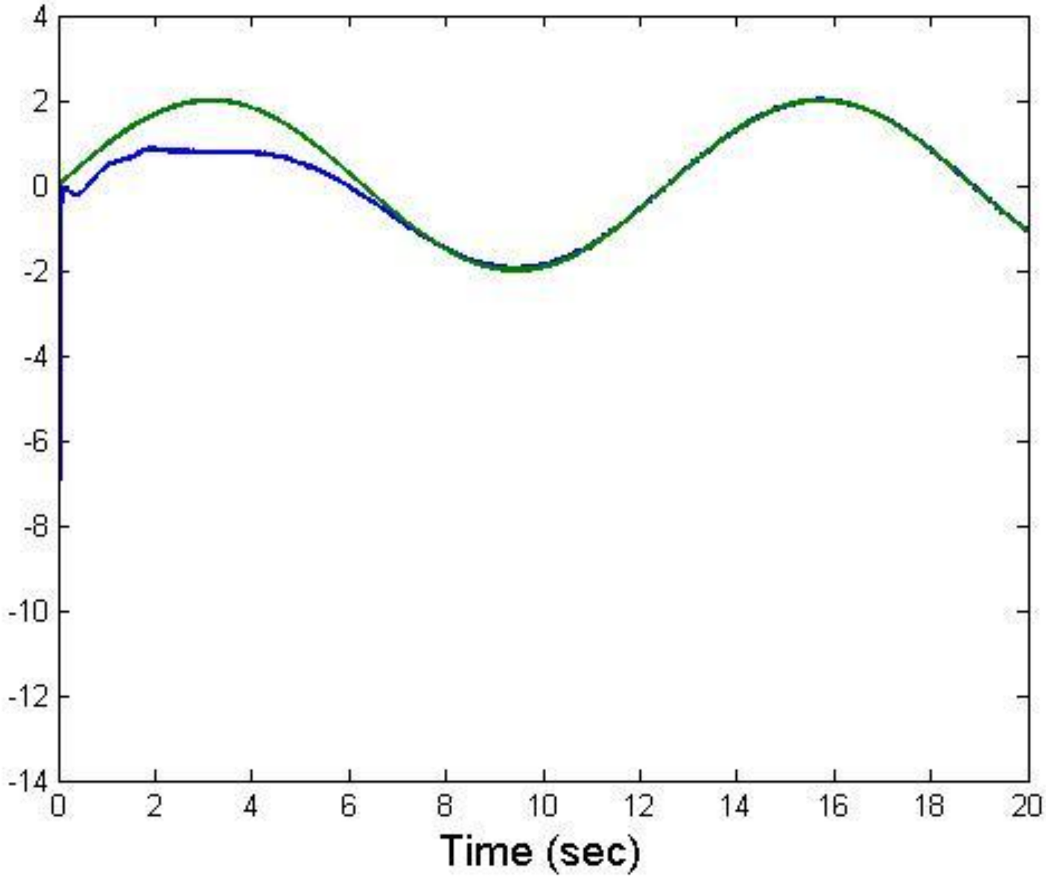

Figure 5.15 The centroid of x_k ($1 \leq k \leq 5$) and the desired trajectory x_0

Figure 5.16 is the convergence result of y_k.

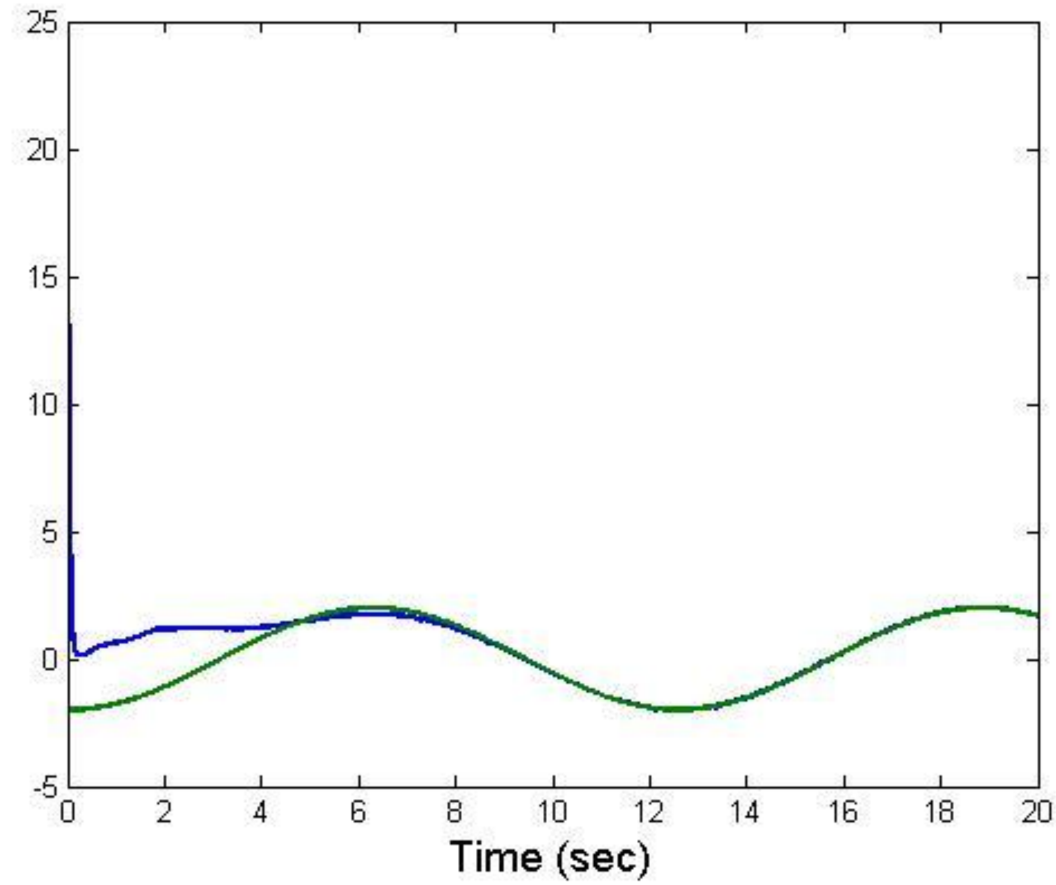

Figure 5.16 The centroid of y_k ($1\leq k \leq 5$) and the desired trajectory y_0

Figure 5.17 is the convergence result of θ_k.

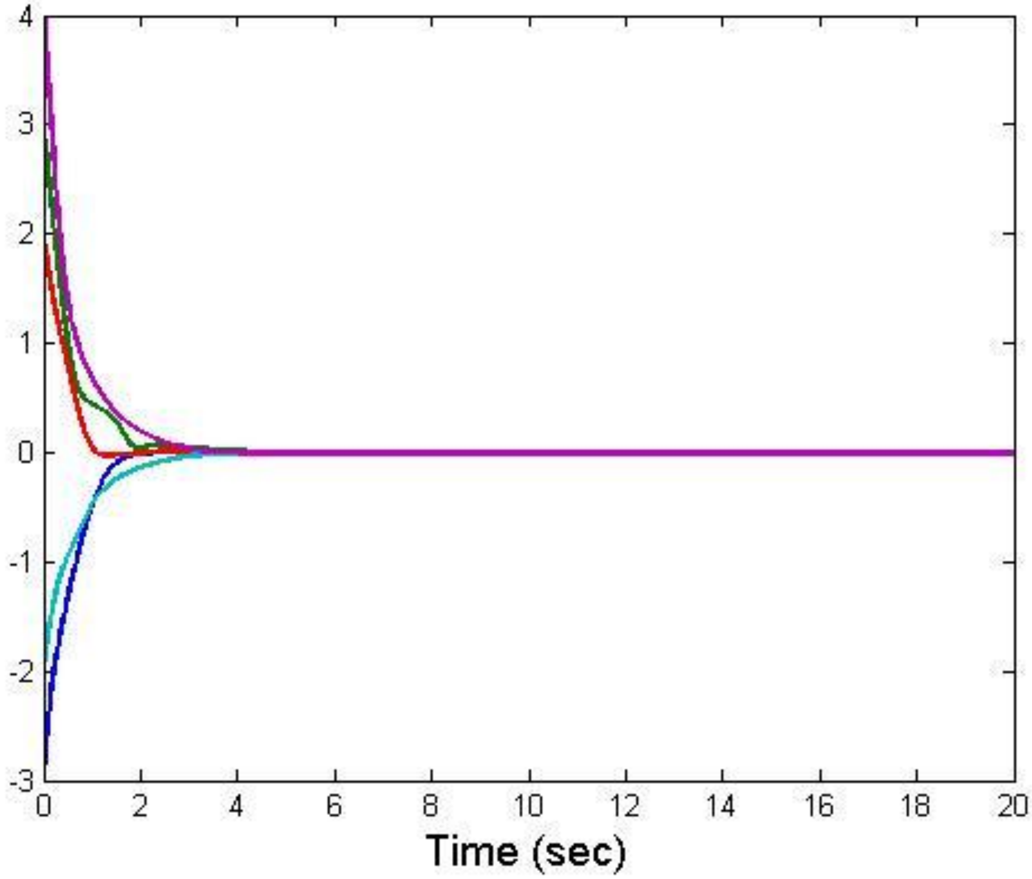

Figure 5.17 $\theta_k - \theta_0$ ($1 \leq k \leq 5$)

Figure 5.18 is the formation tracking of multi-vehicle system.

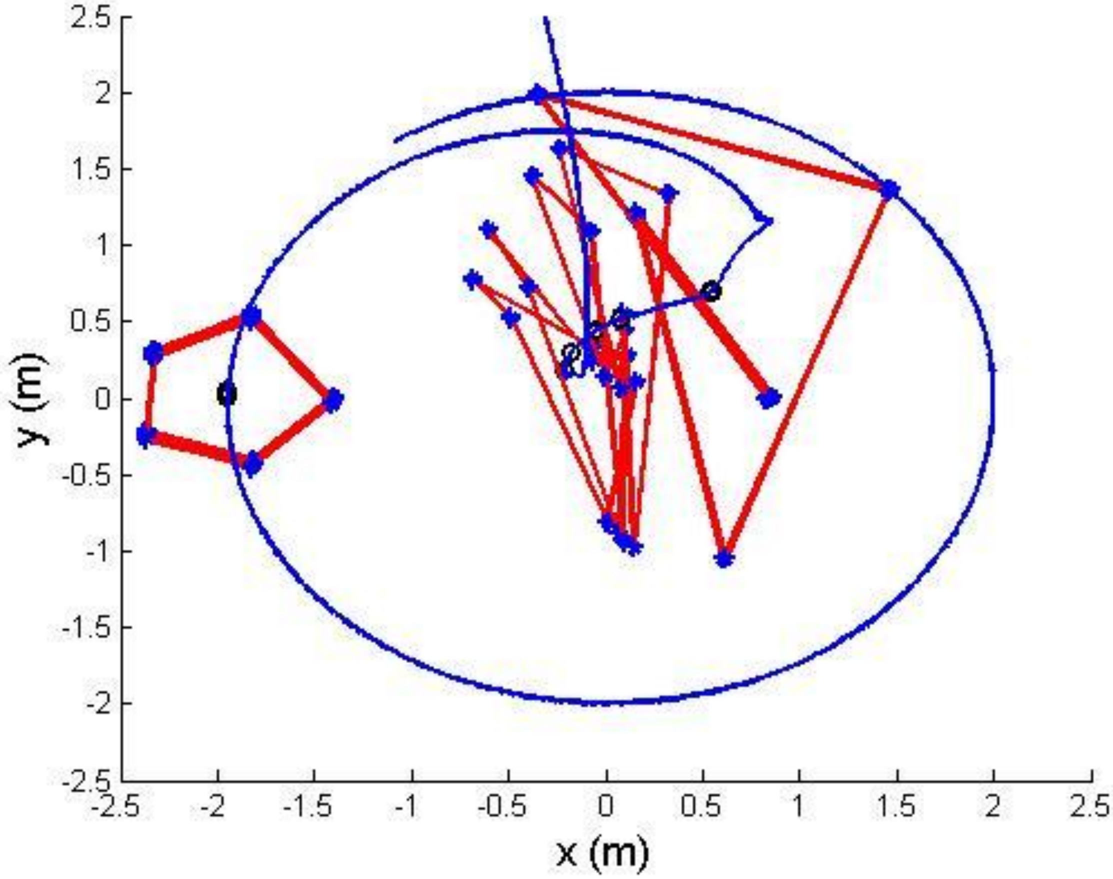

Figure 5.18 Formation tracking of five follower robots III

Figure 5.19 is the communication graph.

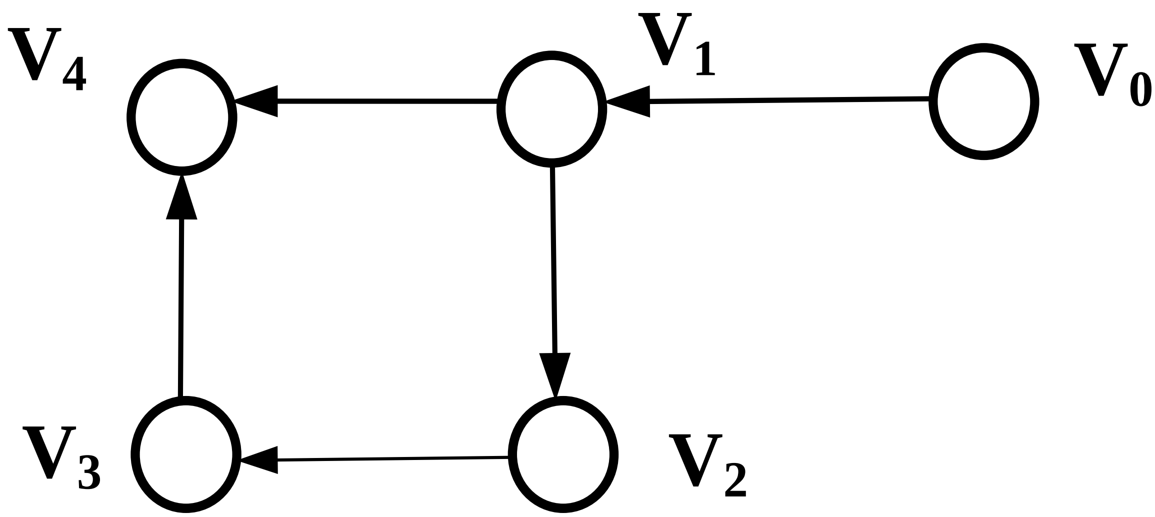

Figure 5.19 Information exchange topology G_1

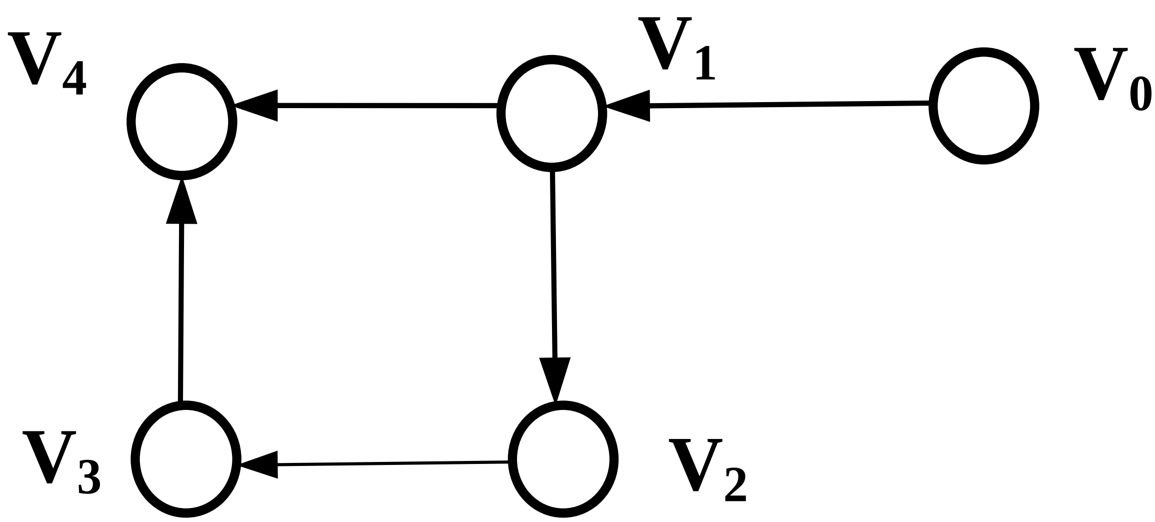

Figure 5.20 Information exchange topology G_2

For multiple dynamic systems with parametrical uncertainties under time-varying communication graph, the switching topology is defined by

$$G = \begin{cases} G_1, t - round(t) < 0 \\ G_2\ t - round(t) \geq 0 \end{cases}$$

Figure 5.21 represents the centroid of x_k ($1 \leq k \leq 4$) and the desired trajectory x_0. Figure 5.22 represents the centroid of y_k ($1 \leq k \leq 4$) and the desired trajectory y_0. Figure 5.23 represents $(\theta_k - \theta_0)$ ($1 \leq k \leq 4$). Figure 5.24 represents the formation tracking of four follower robots.

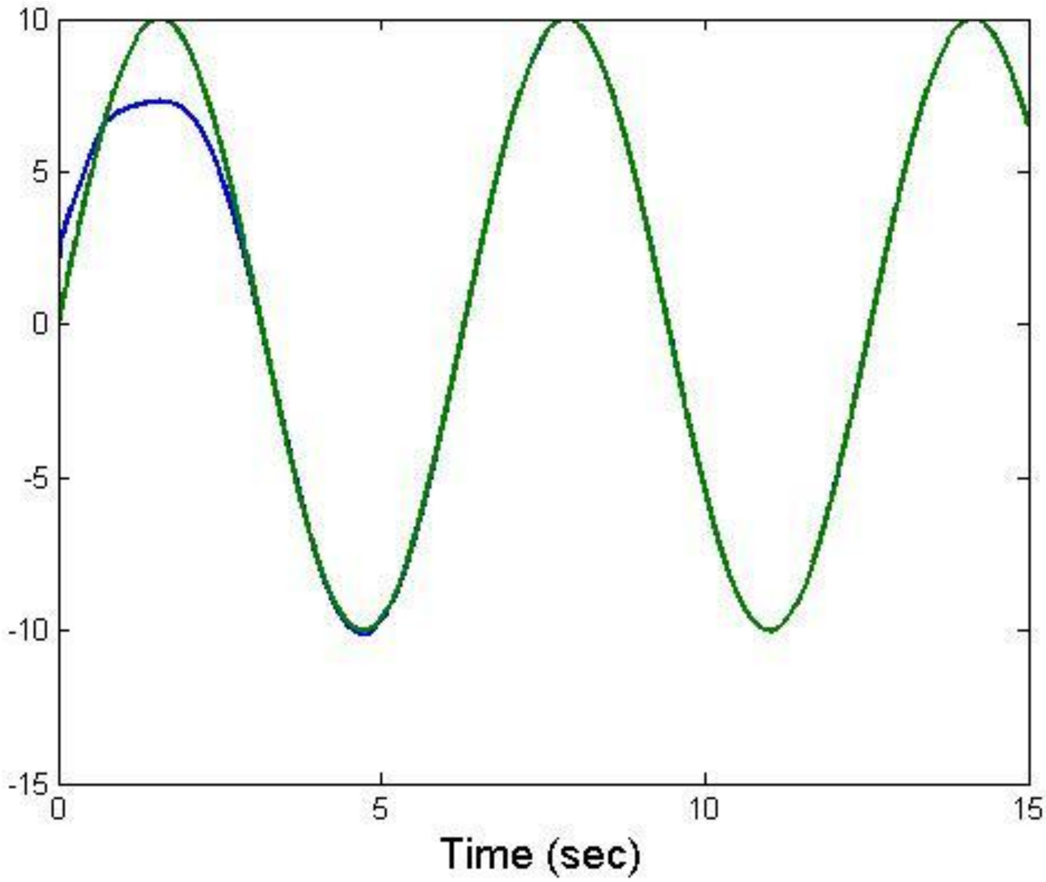

Figure 5.21 The centroid of x_k ($1 \leq k \leq 4$) and the desired trajectory x_0

Figure 5.22 is the convergence result of y_k.

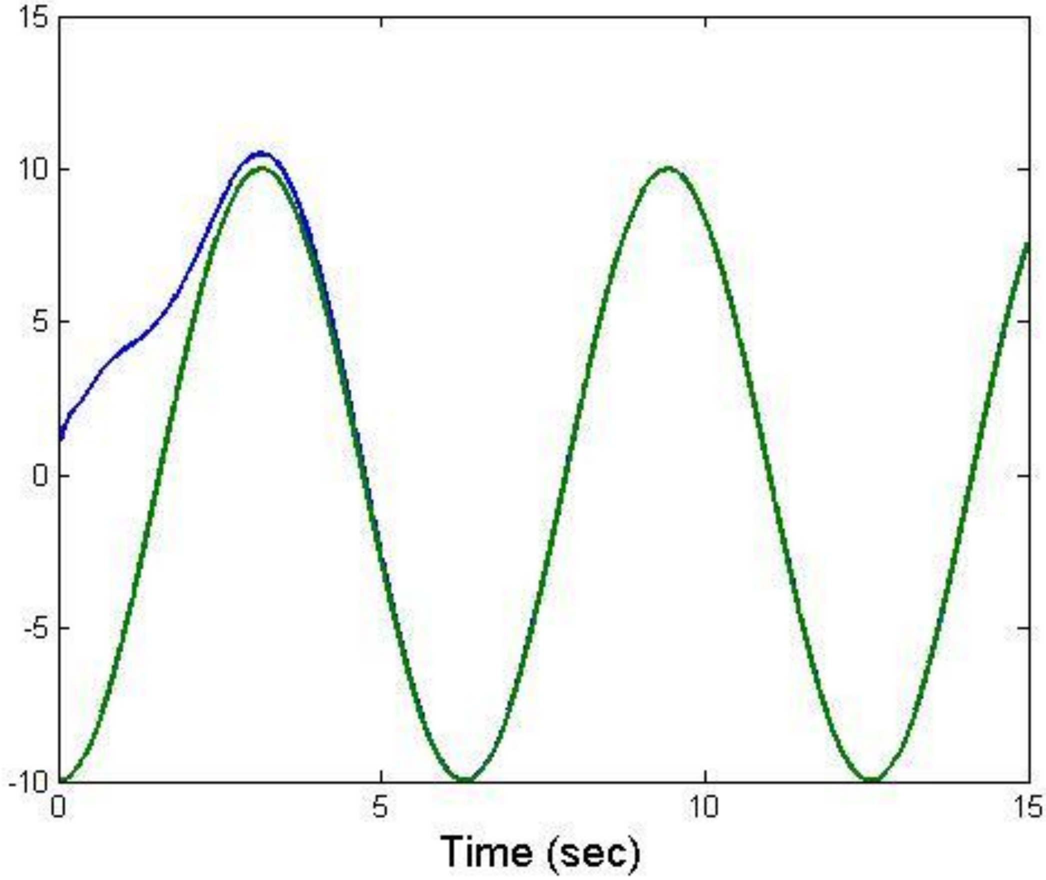

Figure 5.22 The centroid of y_k ($1 \leq k \leq 4$) and the desired trajectory y_0

Figure 5.23 is the convergence result of θ_k.

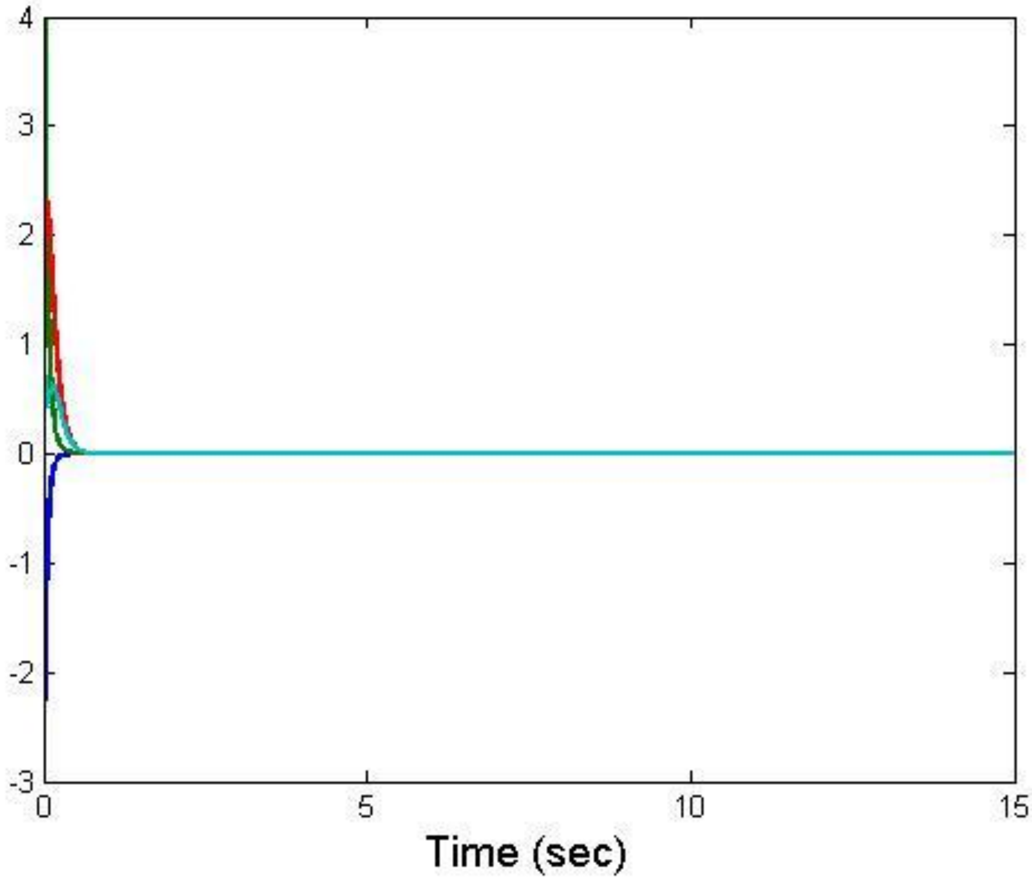

Figure 5.23 $\theta_k - \theta_0$ ($1 \leq k \leq 4$)

Figure 5.24 is the formation tracking of multi-vehicle system.

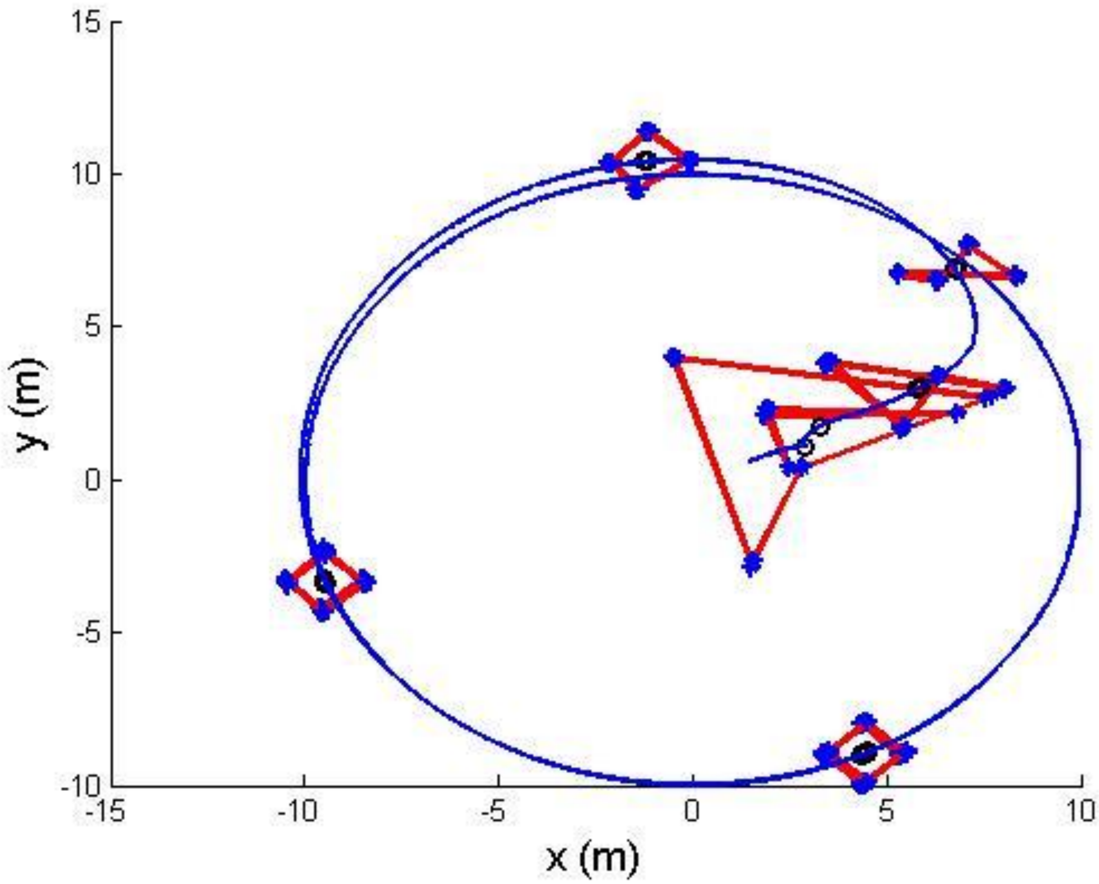

Figure 5.24 Formation tracking of four follower robots III

5.6 Summary

In this chapter, distributed formation tracking control of multiple vehicle dynamics is studied. Instead of designing controllers for velocities of vehicle kinematics, control laws are implemented on torques of vehicle dynamics which are usually the control inputs in real-life operations. Controllers of vehicle dynamics are based on the kinematic-based controllers designed in Section 4.4, Section 4.6 and backstepping methods. Dynamic models with parametrical and without parametrical uncertainties are both addressed with the aid of sliding mode control methods, adaptive control methods and linear feedback control methods. Time-varying communication topology resulting from vehicle disability or communication links change is considered, the proposed control algorithms are proved to guarantee consensus if the directed spanning tree exists in each graph union of every uniformly bounded time interval.

CHAPTER VI

CONCLUSION

In this thesis, formation tracking of multiple car-like robotic systems is studied. Each robot is modeled as a unicycle system and assumed to either receive or send states information from or to specific portion of robots. The reference trajectory is denoted by a virtual robotic system and regarded as a leader with its three generalized states and control velocities available to the follower robots which can directly receive the leader's information. The control goal is to design control laws for each follower robot with the aid of its own states information and its neighbor's information through communication. Formation tracking problem is transformed into consensus with the reference kinematics and stability of the induced nonlinear systems ensures the convergence. Unlike centralized control or solo-system control in which the control information is directly utilized, distributed control only utilize information of its own and its neighbor through communication. The communication topology between vehicles is characterized by Laplacian matrix from graph theory for mathematical analysis of the multiple kinematic systems. The original kinematics are transformed into chained-form systems. Equivalence between the stability of the original kinematics and that of the chained-form is proved thus by designing controllers for the transformed chained-form system the formation consensus can be achieved for the multiple robots. The chained-form systems are further rewritten in to a cascaded structure and exponential stability theory of cascaded systems is utilized to achieve the consensus of chained-form systems then the consensus of the original kinematic systems with the aid of Laplacian matrix and

Lyaponov methods. A novel kinematic controllers are designed based on the transformed chained-systems and are proved to realize the formation tracking goals. Compared with the controllers for the chained-form systems, the novel controllers have derivative terms which are usually not acquirable in real-life operations. Moreover, the controllers are more complex.

Consider that the controller of kinematics only aims at the control of velocities which are usually not directly implementable in real-life operation, dynamic controller is designed for the engine-generated torques with the aid of kinematic-based controller and backstepping methods. Since physical quantities of vehicle dynamics may not be available, both controllers of models without and with parametrical uncertainties are designed. The parametrical uncertainties of are estimated through sliding mode control and adaptive control for distributed controllers in Section 4.4 and Section 4.6 respectively. Eventually, time-varying communication topology case is addressed with the aid of SIA matrix and the consensus is proved with the condition of existence of directed spanning tree in each graph union of any non-overlapping , uniformly bounded time interval. Contributions of this thesis are

1. Much simpler and implementable distributed control algorithms are proposed based on the chained-form system and stability of cascaded system. Derivatives terms are removed in previous proposed control methods, which cannot be controlled in real-life operations through sensing. Moreover, the variable transformation in this thesis is much simpler with only chained-form transform thus the resulting control methods are much more concise.

2. Vehicle dynamics are considered. In previous distributed control laws, the controllers mainly aim at the vehicle kinematics, of which the velocities are the control inputs. However, in reality it is the engine-generate torques rather than the velocities that actually control the motion of the vehicle, velocities of kinematic models are actually intermediate states of the vehicle dynamics. In this thesis, vehicle dynamics with and without parametrical uncertainties are both addressed based on the kinematic-based controllers and backstepping methods. The parametrical uncertainties for unknown dynamics circumstances are estimated through sliding mode control.

In this thesis, distributed formation tracking control of multiple car-like robots is studied. There are still some improvements for the proposed distributed controllers. The proposed controllers have some limitations. Firstly, the vehicle model in this thesis is considered to be quasi-unicycle which has two identical wheels, it has simple structure and less controlling states and inputs, which is usually not the case of real vehicles. Secondly, the collision between vehicles during the convergence motion process is not considered. Improvements include

1. Design controllers for four-wheeled car-like robotic systems with more complex structure and controlling states. In some circumstances such hybrid electric vehicles, the ultimate control components are voltage or current sources. The engine system can be modeled by hybrid electrical circuit with internal resistance and motors then controllers require to be designed for the power sources.

2. Collision avoidance should be considered for designing the controllers. From the proposed control algorithms, it can be learned vehicles might collide during the motion since no collision avoidance restrictions are imposed onto the controllers. Functions which maintain minimum relative positions for neighbors of each vehicle should be devised to prevent the collision scenarios.

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VITA

Chunyu Chen was born in Xuzhou, Jiangsu Province, People's Republic of China. After completing his studies in Suining High School in 2008, he entered China University of Mining and Technology in the same year. He majored in Electrical Engineering and Automation during his undergraduate studies and graduated in June of 2012. In August of 2012, he came to the United States and was enrolled in University of Texas-Pan American in Edinburg, Texas. He is pursuing the degree of Master of Science in Electrical Engineering and expected to graduate in May, 2014.

Contact: chunyu539@gmail.com

This thesis was typed by the author.