8-2022

Tracking Control of Quadrotors

Shihab Ahmed
The University of Texas Rio Grande Valley

TRACKING CONTROL OF QUADROTORS

A Thesis

by

SHIHAB AHMED

Submitted in Partial Fulfillment of the

Requirements for the Degree of

MASTER OF SCIENCE IN ENGINEERING

Major Subject: Electrical Engineering

The University of Texas Rio Grande Valley
August 2022

TRACKING CONTROL OF QUADROTORS

A Thesis
by
SHIHAB AHMED

COMMITTEE MEMBERS

Dr. Wenjie Dong
Chair of Committee

Dr. Weidong Kuang
Committee Member

Dr. Jun Peng
Committee Member

August 2022

ABSTRACT

Ahmed, Shihab, Tracking Control of Quadrotors. Master of Science in Engineering (MSE), August, 2022, 65 pp., 19 figures, references, 48 titles.

In this thesis, the tracking control problem of a 6 DOF quadrotor is considered, and different control methods are proposed considering optimal control, parametric and nonparametric uncertainty, input saturation, and distributed formation control. An optimal control approach is developed for single quadrotor tracking by minimizing the cost function. For uncertainties of the dynamic system, a robust adaptive tracking controller is proposed with the special structure of the dynamics of the system. Considering the uncertainty and input constraints, a robust adaptive saturation controller is proposed with the aid of an auxiliary compensated system. Decentralized formation control method for quadrotors is presented using a leader-follower scheme using the proposed optimal control method. Virtual leader is employed to drive the quadrotors to their desired formation and ultimately track the trajectory defined by the virtual leader. Sliding mode estimators have been implemented to estimate the states of the virtual leader. The control method is designed considering switching communication topologies among the quadrotors. Simulation results are provided to show the effectiveness of the proposed approaches.

DEDICATION

To my parents for their unconditional love and support.

ACKNOWLEDGMENTS

It has been pleasure working on this thesis and learning under Dr. Wenjie Dong, the chair of the thesis committee, who shared his valuable expertise and experience to guide me in proper direction. His patience and support have been very helpful throughout my Master's study. I would like to thank Dr. Weidong Kuang and Dr. Jun Peng, the thesis committee members, for their valuable time and support serving the committee. I learned about different ideas and skills attending the courses taught the faculty members would like to express my gratitude . I would like to thank my fellow learners for their support during my stay at the UTRGV. I will always be grateful to the University of Texas Rio Grande Valley for providing me this opportunity to learn. Finally, I would like to acknowledge the funding provided by the National Science Foundation (NSF grant no. ECCS-2037649). The opinions expressed in this thesis are solely those of the author(s), and do not necessarily represent those of the NSF.

TABLE OF CONTENTS

Page

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Significant development in control and deployment of unmanned aerial vehicles (UAV) in numerous real-world applications have been very popular recently. The UAVs have their applications in supervision, monitoring, and different military operations. The vertical takeoff and landing (VTOL) quadrotors can take off without the need for any runway-like space, possess hovering capability, take smaller region in space, they offer tremendous flexibility in their operations. They are adapting to new fields of usage. So, the operator has access to places where human approach is difficult and dangerous. Modern operations include simultaneous participation of multiple quadrotors to carry out a single task. This feature allows carrying out complex operations with the aid of these autonomous vehicles. Hence, proper control strategies need to be adopted to maintain desired position, orientation, and tracking of the reference to conduct the required task in near perfect manner. Also, to maintain constraints on power consumption, weight, actuator limitations, proper optimal control strategy is desirable. Since the coupled dynamics of the quadrotor are nonlinear in nature and the system is underactuated having only four inputs for 6 degrees of freedom (DOFs), developing control laws for such system has been an active area of research.

For carrying out operations of different magnitudes, formation flying of several quadrotors have been recently studied in different works. Due to their practical application in construction, surveillance, search and rescue and engagement in complex tasks, formation flying has garnered attention from the research community. With the growing complexity of operations, coordination among multiple UAVs plays a crucial role. With cooperative control, a single agent can be replaced by an equivalent group of multiple agents in a multiagent system. The group of agents will be able to complete tasks that are beyond the capability of a single agent. For this feature to work, information flow between the agents should be properly defined using necessary network topology.

In this work, the control problem of quadrotor UAVs is considered. The contribution of this work is twofold. In the first part, the kinematics and dynamics of a quadrotor is introduced, and an optimal control approach has been proposed to solve the attitude tracking control problem for a quadrotor. In the second part, formation control problem of a group of quadrotors is discussed and a control approach has been introduced for the group by extending the results of the first part. Simulation results have been provided to validate the proposed approaches.

1.1 Literature Review

1.1.1 Tracking Control of a quadrotor

There has been a vast amount of research activities for tracking control of quadrotors and UAVs in general. Mainly they aim to define control strategy to track the time-varying reference trajectories. The existing works can be divided in linear and nonlinear control strategies. Despite being a nonlinear system, basic linear control approaches like PID and LQR have been extensively used in quadrotor control. To apply linear control techniques, nonlinear dynamics of the quadrotor have been linearized around some operating point and design the appropriate linear control system. Since the linear controllers provide a comparatively simpler design process, they have been used in UAVs in lots of works [1], [2]. These approaches are most suitable for operating in a small region. Linear control methods such as PID and LQ have been useful in such cases. The approach in [2] finds a near-optimal trajectory to be followed while minimizes a performance index. Further works have been conducted to improve robustness in these linear control designs [3], [4]. The comparative analysis between PID and LQR in [1] shows better performance of LQR than that of PID. Feedback linearization has been mostly used to design the controller for the nonlinear system where the nonlinear model of the quadrotor is transformed into an equivalent linear system. [5] provides a feedback linearization approach implemented for quadrotor path following. The feedback linearization controller is relatively simpler but doesn't have a good profile under noisy conditions. Sliding mode control has been used as a nonlinear control approach without simplifying the model dynamics and showed reliable tracking for quadrotor in [6] where it handles the cascaded model by subdividing the quadrotor model into a fully actuated and an under-actuated subsystem. Using only sliding mode control itself has its disadvantage with chattering causing significant energy loss. Model Predictive Control has also been used in the linearized models which showed reliable performance despite slower response in the operating region [7] although MPC takes significant amount of computational time. For most of the similar control problem scenarios, applying nonlinear control techniques achieve better performance than the linear ones in terms of stability and robustness due to their ability to handle the nonlinearity of the system more effectively. [8] provides an adaptive sliding mode controller focusing on the underactuated property of the quadrotor and a feedback linearization controller. It provides a comparative analysis between both controllers. Since feedback linearization involves higher order derivatives, its susceptible to noise. Backstepping technique has been widely used for trajectory tracking that involves developing virtual control signals in different steps [9], [10]. In backstepping control, the control scheme is divided into several steps and focuses on stability of each subsystem. In [11], quaternions have been used to achieve global attitude tracking and command filtered backstepping controller is proposed. A quaternion extraction algorithm has been used in [12] to develop adaptive control of VTOL UAVs. Quaternion is helpful for computing stability and avoiding singularity. But to increase robustness of backstepping control, further modifications are required. The optimal use of input energy and time for driving the quadrotor has always been a concern. Optimization problem is formed taking these components as design variables. The objective here is to get a set of values for the variables that minimizes the cost function. The set of variables allows the selection of values from multiple options. The LQ based approach in [1] minimizes a cost function while solving for optimal control problem for a simplified quadrotor model. [13] provides LQ tracking control strategy for quadrotors using time-variant control gains. An adaptive LQ attitude tracking control is proposed in [14] employed Kalman filtering to perform well against sensor noises.

1.1.2 Tracking Control with Uncertainty and Disturbances

In practical scenarios, there is always uncertainty regarding quadrotor dynamic model. In [15] the position and attitude tracking control of a quadrotor was considered with inertia parameter uncertainty. Adaptive tracking controllers were proposed with the aid of the cascade structure of the dynamics of the system and the immersion and invariance technique. In [16], [17] the disturbances have been estimated by sliding mode observers and sliding mode tracking controllers have been proposed. The sliding mode technique was also applied to compensate for the unmodeled dynamics and robust adaptive tracking controllers were proposed in [18], [19]. In [20], composite learning controllers were proposed by using the terminal sliding mode for a quadrotor with unknown dynamics and time-varying disturbances. In [20]–[22], the immersion and invariance technique was applied to design adaptive controllers for a quadrotor with parametric uncertainty. In [23] the adaptive backstepping technique and command-filter compensation were applied and adaptive tracking controllers were proposed without computation of derivatives of signals. In [24], [25] controllers were proposed with the aid of model predictive control. In [26], robust adaptive controllers were proposed in the presence of wind disturbance with the aid of the singular-perturbation technique.

For a VTOL UAV, there is always constraint on its inputs. For the tracking control of VTOL UAVs with input constraints, a nested tracking controller was proposed with the aid of the nested saturation control in [26] if there is no uncertainty and disturbance. However, the selection of the control parameters is intricate, and the stability of the closed-loop system cannot be guaranteed if there is uncertainty in the model of the system. To overcome this, the tracking control problem of a quadrotor with parametric uncertainty and input constraints was studied in [27]. and an adaptive tracking controller was proposed. However, the closed-loop tracking error system is semi-globally stable, and the non-parametric uncertainty was not considered. In [28] the trajectory tracking control was studied for a VTOL aircraft with a simplified model under an input constraint. Since the proposed controller is based on a 3 DOF model, it cannot be extended to deal with a 6 DOF model of VTOL vehicles. In [29] the trajectory tracking control of a 6-DOF quadrotor UAV with an input constraint was studied. Position tracking controllers were proposed with the aid of backstepping techniques and a Nussbaum function under the assumptions that the inertia parameters are exactly known and the disturbances in the dynamics are constants. However, in practice these assumptions are not true.

1.1.3 Formation control of multiple quadrotors

There exists a vast amount of literature in the field of formation control due to the existence of numerous applications of formation control of multiagent systems. In such systems, one way or the other, the control strategy aims to achieve some sort of consensus among the agents. This consensus is achieved as the control system is designed with locally available information of each agent. Most available methodologies can be broadly categorized in behavioral approach [30], leader-follower strategy [31][32][33], virtual structure approach [34][35] and graph theory method [36], [37]. Several good literatures are mentioned in [30] while shows practical results for implementing a specific military movement behavior. There have been realistic animal behavior modeling and control for robotic works noted in [30]. Behavior based approach has its limitation in offering flexibility. In virtual structure method, the agents are considered as particles of a virtual rigid entity. The virtual structure can be reconfigured to reposition the robots, merge, or divide the formation dynamically. A specific bidirectional control flow has been maintained between the virtual rigid structure and the robots in [30]. The leader-follower approach has been widely used in lots of robotic applications[38][39][40][41]. The basic strategy is to make one or more follower agents to converge into a desired state or a trajectory. Since each follower can also be a leader, this feature offers greater scalability. Leader-follower approach employed in formation flying of UAVs described in [39] uses a Model Predictive Control to maintain the flight envelope constraints. Leader-follower approach has been used in [40] where control has been achieved with a distributed estimator. In [41] a specific formation is considered where the consensus among the quadrotors is realized by a proposed weight matrix. In distributed control, the agents usually aim to reach a consensus. A detailed analysis about consensus algorithms in switching topology is reported in [42]. Most of formation control works emphasize on using decentralized control since it saves unnecessary communication cost among all the agents in a system. For most of the decentralized control methods, some sort of state estimation is necessary since the agents are mostly unaware of their relative states compared to others. Accurate estimation saves extra control efforts by reducing the error between the estimated states and the states of the agent itself. Historically, Kalman filtering has been used in numerous estimation applications, especially in spacecrafts [43]. Kalman filter has been modified from time to time to make it more feasible for the system under consideration. Kalman filters are well-suited for linear systems. There has been Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) to cope with the nonlinearity to some extent. The work described in [44] uses Optimal Kalman Filter (OKF) for state estimation of a quadrotor which performs well under noisy environment. A decentralized sliding mode estimation approach is proposed in [45] that guarantees accurate estimation of position and velocity in finite time and well suited for nonlinear systems like a quadrotor. In [46] a similar distributed control has been proposed and solidified with Lyapunov function analyses.

1.2 Thesis Content

In this work, the tracking control for a single quadrotor is considered, where cost for the control effort and state penalty have been included in the cost function. The weight matrices can be manipulated to make proper adjustment of the cost function. The proposed approach has been implemented for formation control theory for multiple quadrotors. Distributed estimation has been made about the states of the virtual leader. The proposed sliding mode estimator from [45] is modified to implement for the estimation of each quadrotor. The control method makes quadrotors to follow the estimated trajectory. The position and attitude tracking control problem with parametric and nonparametric uncertainty and the tracking control problem with uncertainty and input constraints for a quadrotor have been considered. For the first problem, quaternion-based robust adaptive controllers is developed and for the second problem, a robust adaptive saturation controller is proposed such that the tracking errors are uniformly ultimately bounded.

In chapter 1, the applications of quadrotors are discussed in brief. Some literatures regarding the control approaches for single and multiple quadrotors have been studied. The overview of the work and summary of the contributions are provided.

In chapter 2, the dynamics of the quadrotor is presented, and the tracking control problem is formed. The cost function formulations for optimal control are provided. The developed control strategy is provided along with the simulation results.

In chapter 3, considering the parametric and non-parametric uncertainties in the dynamics of systems, a robust adaptive tracking controller is proposed with the aid of the special structure of the dynamics of the system. Considering the uncertainty and input constraints, a robust adaptive saturation controller is proposed with the aid of an auxiliary compensated system. Simulation results show the effectiveness of the proposed algorithms.

In chapter 4, distributed estimation is discussed for a multiagent system. Some graph theory preliminaries are provided. The proposed controller from chapter 2 is used for distributed control using the distributed estimator. The communication graphs and simulation results are presented.

Chapter 5 concludes the works done in the thesis. Some ongoing and possible future works are also included.

1.3 Thesis Contribution

The contributions of this work include the development of an optimal control strategy that is simple and convenient to apply for tracking control of both single quadrotor and a group of quadrotors with the aid of decentralized estimation. The special cascaded feature of the quadrotor is considered, and separate control inputs are developed. For tracking control of a quadrotor, uncertainty and input constraint have been considered and two different control approaches have been proposed. The contributions of this thesis are summarized as the followings.

1. An optimal control method of a 6-DOF quadrotor for position and attitude tracking to produce necessary thrust and torque vector is proposed.

2. A robust adaptive controller and a robust adaptive saturation controller is proposed for dealing with parametric and non-parametric uncertainty and input saturation.

3. A distributed multi-quadrotor formation control scheme is proposed by extending the single quadrotor tracking control.

4. The proposed control method in conjunction with the distributed estimator is used and a leader follower-based formation control method proposed.

CHAPTER II

OPTIMAL TRACKING CONTROL OF QUADROTOR

This chapter focuses on developing control strategy of a 6-DOF quadrotor. The control input is designed to produce optimal thrust and torque for the quadrotor to track the desired trajectory. An optimal control problem has been formulated for quadrotor tracking of the desired position and orientation. The approach has been provided step by step where cost functions have been formulated to calculate optimal weights on the control gain and the states under consideration. The cost function provides a measure of penalty for selecting control input and the corresponding states. Simulation results are provided to analysis performance of the overall approach.

2.1 Modeling of a quadrotor

Let $F: O - XYZ$ be an earth-fixed inertia frame. The origin O implies a fixed point on the ground. X axis points to the north and Z axis represents the vertical axis. Y axis is determined by the right hand. The unit vectors in the three axes are defined as $e_1 = [1, 0, 0]^T$, $e_2 = [0, 1, 0]^T$, and $e_3 = [0, 0, 1]^T$, respectively. For a quadrotor, a body fixed frame $F_b : O_b - X_b Y_b Z_b$ is rigidly attached to it. Its origin O_b is located at the center of the mass of the quadrotor shown in Figure 1. Axes X_b and Y_b lie in the plane defined by the centers of the four rotors. The Z_b axis is defined by the right hand. The coordinate of O_b in the inertia frame is defined as $p = [x, y, z]^T$. Let $\Theta = [\varphi, \theta, \psi]$T be the Euler angles of frame F_b with respect to frame F, the attitude of the quadrotor

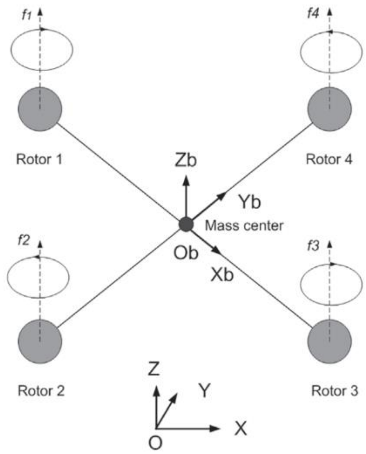


Figure 2.1: Configuration of a quadrotor

is uniquely defined by Θ. The attitude of the system can also be defined by the rotation matrix $R \in SO(3)$ from the inertia frame to the body frame

$$R = \begin{bmatrix} c\theta c\psi & s\theta c\psi s\varphi - s\psi c\varphi & s\theta c\psi c\varphi + s\psi s\varphi \\ c\theta c\psi & s\theta s\psi s\varphi + c\psi c\varphi & s\theta s\psi c\varphi - c\psi s\varphi \\ -s\theta & c\theta s\varphi & c\theta c\varphi \end{bmatrix} \tag{2.1}$$

where cθ denotes cosθ and sθ denotes sinθ. Under some assumptions, the kinematics, and dynamics of the quadrotor are defined by

$$\dot{p} = v \tag{2.2}$$

$$\dot{v} = -ge_3 + \frac{1}{m} fRe_3 \tag{2.3}$$

$$\dot{R} = RS(\omega) \tag{2.4}$$

$$J\dot{\omega} = S(J\omega)\omega + \tau \tag{2.5}$$

where g is the gravitational acceleration, v is the velocity of the mass center in the inertia frame F, $\omega = [\omega_1, \omega_2, \omega_3]^{\mathrm{T}}$ is the angular velocity of the quadrotor in the body frame F_b, J is the inertia matrix of the quadrotor. $S(\cdot)$ is a skew-symmetric matrix defined by

$$S(\omega) = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ -\omega_3 & 0 & -\omega_1 \\ \omega_2 & \omega_1 & 0 \end{bmatrix}$$

$f \in \Re$ is the total thrust produced by the four rotors along Z_b axis, and $\tau = [\tau_1, \tau_2, \tau_3]^T$ is the torque generated by the four rotors.

2.2 Problem Statement

The dynamic model in (2.2)-(2.5) is a 6-DOF model. Four of them can be controlled independently due to the coupling between the position and attitude. In this work, we formulate the following tracking control problem.

Tracking Control Problem: Given the desired trajectory $p^d(t)$ and the desired yaw angle $\psi^d(t)$, if the states (p, v, Θ, ω) of the system is known and the inertia parameters m and J are unknown, the control problem is to design state feedback control inputs f and τ such that

$$\lim_{t\to\infty}\left(p(t) - p^d(t)\right) = 0 \tag{2.6}$$

$$\lim_{t\to\infty}\left(\psi(t) - \psi^d(t)\right) = 0 \tag{2.7}$$

For the desired trajectory, the following assumptions are made.

Assumption 2.1: p^d is smooth and $||p^d|| \le M_d$.

Assumption 2.2: ψ^d is smooth. $\dot{\psi}^d$ and $\ddot{\psi}^d$ are bounded.

2.3 Controller Design

By simple algebraic calculation, (2.4) can be written as

$$\dot{\Theta} = W(\Theta)\omega \tag{2.8}$$

where

$$W(\Theta) = \frac{1}{cos\theta}\begin{bmatrix} cos\theta & sin\varphi\ sin\theta & cos\varphi\ sin\theta \\ 0 & cos\varphi\ cos\theta & -sin\varphi\ cos\theta \\ 0 & sin\varphi & cos\varphi \end{bmatrix}$$

and $det(W(\Theta)) = \frac{1}{cos\theta}$. W is nonsingular if $\theta \neq \frac{(2k-1)\pi}{2}$ for any integer *k*. Our controller will be designed based on equations (2.2)-(2.3), (2.8), and (2.5).

Considering the structure of the system in (2.2)-(2.5), for position tracking control, the error dynamics is developed, and a virtual input is chosen for the error dynamics. Considering the virtual input and position and velocity errors as the states, a cost function is formulated. two different weighted matrices have been used for implementing LQ optimal control approach. For the orientation tracking control, the error dynamics for orientation leads to formulation of another cost function, which has been optimized in a similar way. The optimal values for the weight matrices are selected by solving the algebraic Riccati equation.

The control design procedure can be summarized in the following steps.

Step 1: Let $e_p = p - p^d$ and $e_v = v - \dot{p}^d$, we have

$$\dot{e}_p = e_v \tag{2.9}$$

$$\dot{e}_v = \frac{1}{m} fRe_3 - ge_3 - \ddot{p}^d = \nu \tag{2.10}$$

We find ν such that the cost function

$$J_1 = \int_0^\infty (e^T Q_1 e + \nu^T R_1 \nu) d\tau \tag{2.11}$$

is minimized, where Q_1 and R_1 are positive definite matrices, and $e^{\mathrm{T}} = [e_p^{\mathrm{T}}, e_v^{\mathrm{T}}]$.

The system described in the problem is linear. Hence, the system can be rewritten in state space form

$$\dot{e} = A_1 e + B_1 v$$

The optimal value of v that minimizes the cost function J_1 is formulated as

$$v^* = -K_1 e \tag{2.12}$$

We can write

$$K_1 = {R_1}^{-1} {B_1}^T P_1 e \tag{2.13}$$

where, P_1 must satisfy the Riccati equation,

$$A_1{}^T P_1 + P_1 A_1 - P_1 B_1 {R_1}^{-1} {B_1}^T P_1 + Q_1 = 0 \tag{2.14}$$

Step 2: In this step, we find f and virtual control inputs φ_d and θ_d for φ and θ. We choose

$$f = m||v + ge_3 + \ddot{p}^d|| \tag{2.15}$$

Let

$$fe_3 = R^{\mathrm{T}}(\Theta_d)\alpha$$

where

$$\alpha = m(v + ge_3 + \ddot{p}^d)$$

Simple calculation gives

$$\alpha_1 c\theta_d c\psi_d + \alpha_2 c\theta_d s\psi_d - \alpha_3 s\theta_d = 0 \tag{2.16}$$

$$\alpha_1(s\theta_d c\psi_d s\varphi_d - s\psi_d c\varphi_d) + \alpha_2(s\theta_d s\psi_d s\varphi_d + c\psi_d c\varphi_d) + \alpha_3 c\theta_d s\varphi_d = 0 \tag{2.17}$$

$$\alpha_1(s\theta_d c\psi_d c\varphi_d + s\psi_d s\varphi_d) + \alpha_2(s\theta_d s\psi_d c\varphi_d - c\psi_d s\varphi_d) + \alpha_3 c\theta_d c\varphi_d = f \tag{2.18}$$

where $c\theta_d = cos\theta_d$ and $s\theta_d = sin\theta_d$. From (18) we have

$$\theta_d = \arctan\left(\frac{\alpha_1 cos\psi_d + \alpha_2 sin\psi_d}{\alpha_3}\right) \tag{2.19}$$

$(20) \times \sin\varphi_d - (19) \times \cos\varphi_d$ yields

$$f sin\varphi_d = \alpha_1 sin\psi_d - \alpha_2 cos\psi_d \,.$$

From this equation, we choose

$$\varphi_d = \arcsin\left(\frac{\alpha_1 sin\psi_d - \alpha_2 cos\psi_d}{||\alpha||}\right) \tag{2.20}$$

Step 3: In this step, we design the virtual control for ω such that (6)-(7) are satisfied. To this end,

$$\ddot{\tilde{\Theta}} = \ddot{\Theta} - \ddot{\Theta}^d = W\dot{\omega} + \dot{W}\omega = WJ^{-1}(S(J\omega)\omega + \tau) + \dot{W}\omega - \ddot{\Theta}^d \tag{2.21}$$

We let

$$\tau = JW^{-1}\left(u - \dot{W}\omega + \ddot{\Theta}^d\right) - S(J\omega) \tag{2.22}$$

then

$$\ddot{\tilde{\Theta}} = u$$

Let $z^T = \left[\tilde{\Theta}^{\mathrm{T}}, \dot{\tilde{\Theta}}^{\mathrm{T}}\right]^{\mathrm{T}}$, then

$$\dot{z} = A_2 z + B_2 u$$

where

$$A = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix}, B = \begin{bmatrix} 0_{3\times3} \\ I_{3\times3} \end{bmatrix}$$

We define a cost function

$$J_2 = \int_0^\infty (z^T Q_2 z \ + \ u^T R_2 u) d\tau \tag{2.23}$$

We find u such that the cost function J_2 is minimized.

Following the similar steps from (3)-(6) we can write the optimal control u as

$$u^* = -K_2 z \tag{2.24}$$

where

$$K_2 = \ {R_2}^{-1} {B_2}^T P_2 z \tag{2.25}$$

and the Riccati equation

$${A_2}^T P_2 + P_2 A_2 - P_2 B_2 {R_2}^{-1} {B_2}^T P_2 + Q_2 = 0 \tag{2.26}$$

Based on the controller design procedure described above, we construct the following theorem.

Theorem 2.1: For the system described in (2.2) - (2.5) and the desired trajectory p^d and ψ^d, the control inputs (f, τ) in (2.15) and (2.22) with optimal gains from (2.13) and (2.25) ensure that (2.6) - (2.7) are satisfied. Furthermore, the thrust force f is larger than zero at any time and is bounded.

2.4 Simulation

Simulation results are presented to illustrate the effectiveness of the proposed controller. We consider a quadrotor modeled as a rigid body with mass $m = 3.5$kg and inertia tensor $J =$ diag([0.13,0.13,0.13]) kg m^2. In this case, the desired trajectory p^d and ψ_d are chosen as

$$p^d(t) = \begin{bmatrix} 10(1 - cos\frac{\pi t}{360}) \\ 10sin\frac{\pi t}{180} \\ 10 - 9\, exp(-0.2t) \end{bmatrix}$$

$$\psi^d = \frac{\pi}{2} sin(0.1t)$$

For the proposed control method in theorem 2.1 with the aid of (2.16) and (2.23) we choose the initial conditions as $p(0) = [0,0,0]^T$, $v(0) = [0,0,0]^T$, $\Theta = [0,0,0]^T$, $\omega = [0,0,0]^T$ for the simulation. The values for Q_1, R_1 and Q_2, R_2 define the cost function that the developed control system tries to minimize. This provides the values for the gain K_1 and K_2. The tracking errors $x - x^d$, $y - y^d$ and $z - z^d$ have been shown in accordance with their time response in Figure 2.2 which represents the convergence of quadrotor trajectory with the desired one. The thrust response for the quadrotor represented in Figure 2.3 shows that f settles around a value that maintains the trajectory tracking. It shows that f is bounded and is larger than zero for any time. Figure 2.4 shows the yaw angle error response, and the torque input is represented in Figure 2.5. The simulation results show proper trajectory tracking by the proposed approach in theorem 1.

2.5 Conclusion

This chapter considers the attitude tracking control problem of a quadrotor UAV. The dynamics of the quadrotor is shown, and the error dynamics have been developed, where a virtual control input has been chosen. The optimal value of the control input is chosen based on the developed cost function. The cost function provides the mean for selecting optimal values for the control input and the errors. More aggressive control approach can be chosen incurring larger cost for the control inputs. The optimization scope of the cost function has been provided using LQ optimal control. The simulation results show efficient tracking performance of the quadrotor described by the dynamic equations.

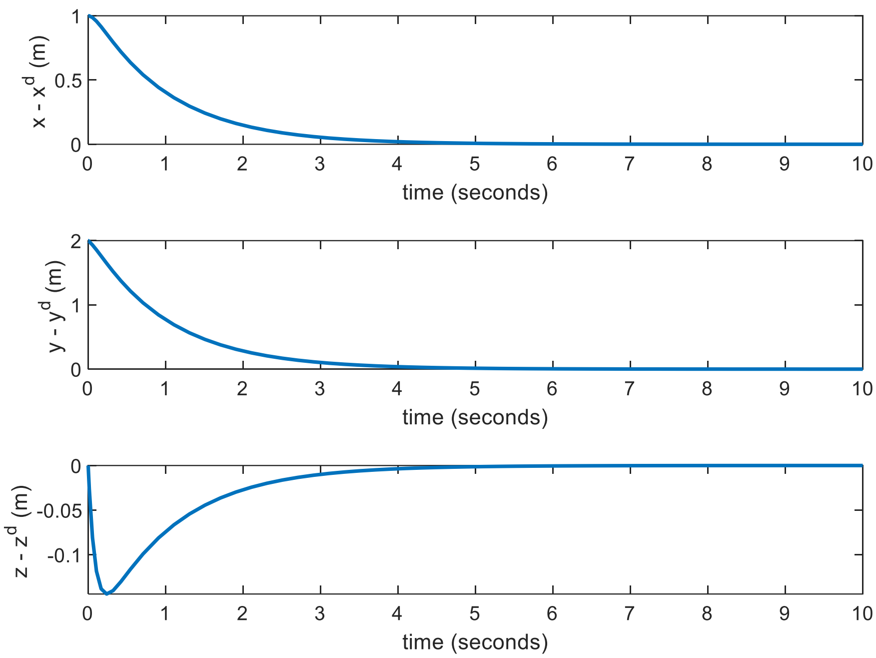


Figure 2.2: Time response of $p - p^d$

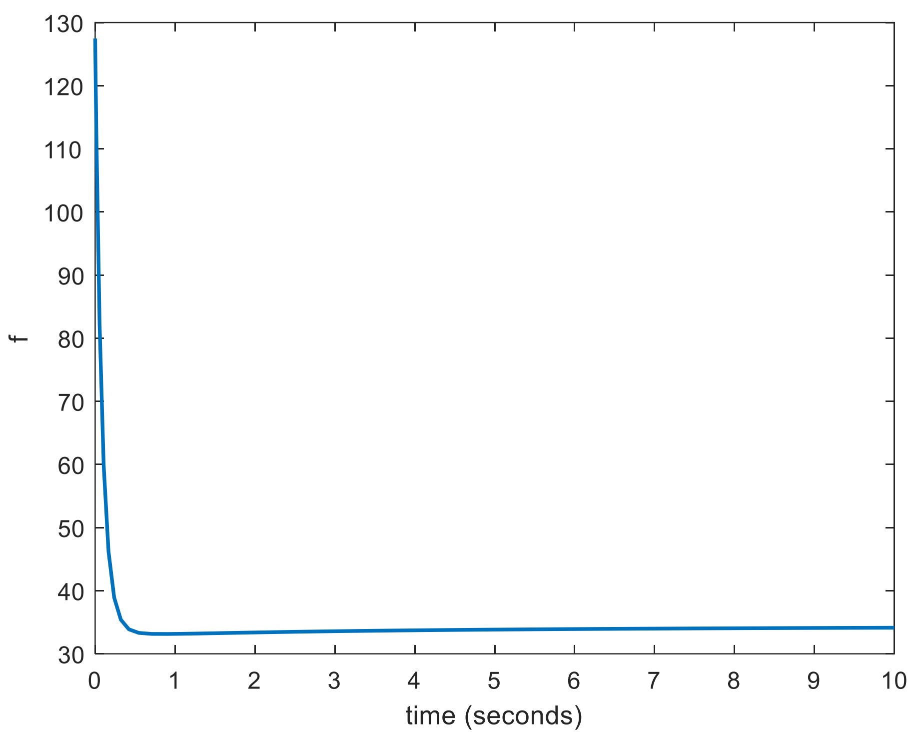


Figure 2.3: Time response of f

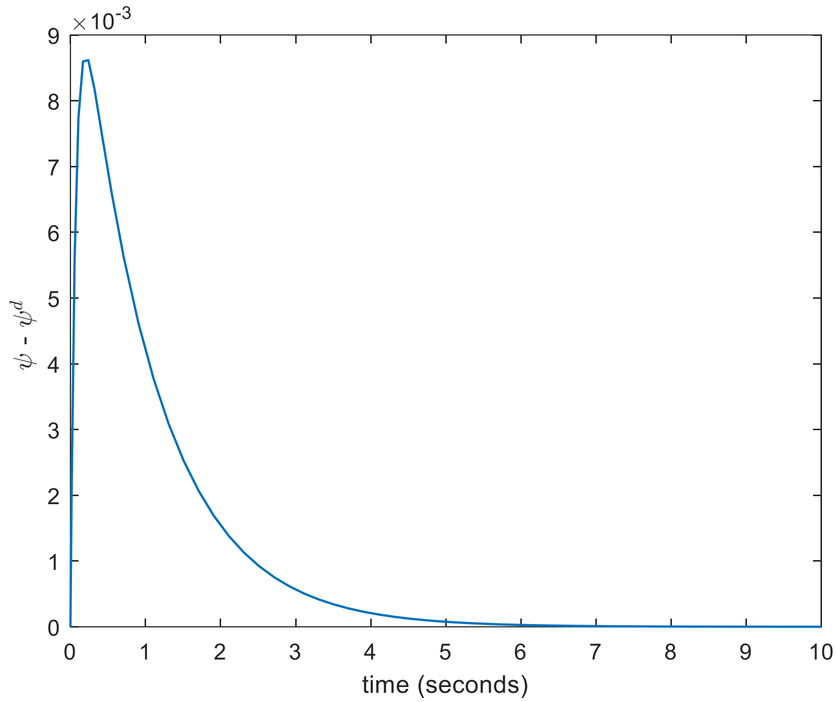


Figure 2.4: Time response of $\psi - \psi^d$

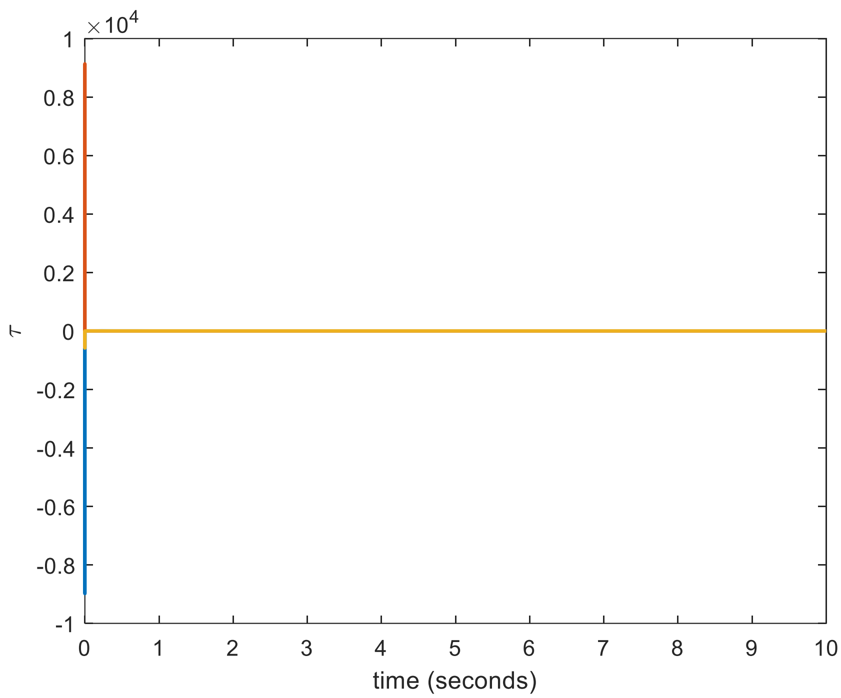


Figure 2.5: Time response of τ

CHAPTER III

TRACKING CONTROL OF QUADROTORS WITH UNCERTAINTY AND INPUT CONSTRAINTS

This chapter studies two control problems of 6-DOF virtual takeoff and landing (VTOL) quadrotors. The tracking control problem is associated with uncertainty and input saturations to deal with. In most practical implementation, presence of some sort of uncertainty is common. In this chapter, one problem is the position and attitude tracking control of quadrotors with both parametric and non-parametric uncertainty. The other problem is the position and attitude tracking control of quadrotors with both uncertainty and input constraint. For the first problem, with the aid of backstepping techniques and saturation control a new quaternion-based robust adaptive controller is proposed such that the position and the attitude converge to their desired values, respectively. For the second problem, by introducing an auxiliary compensated system a new robust adaptive saturation controller is proposed such that the tracking errors of the position and the attitude are uniformly ultimately bounded (UUB). The thrust force is guaranteed to be positive, and the desired attitude is guaranteed to be well defined at any time. Proper assumptions have been made for the proposed approach to work under reasonable conditions. Simulation results have been provided for both controllers which shows the response of the system and the control inputs.

3.1 Model of a Quadrotor with Uncertainties

The dynamic model of the quadrotor from (2.2)-(2.5) can be rewritten as

$$\dot{p} = v \tag{3.1}$$

$$\dot{v} = -ge_3 + \frac{1}{m} fRe_3 + d_1 \tag{3.2}$$

$$\dot{R} = RS(\omega) \tag{3.3}$$

$$J\dot{\omega} = S(J\omega)\omega + \tau + d_2 \tag{3.4}$$

where, d_1 and d_2 denote non-parametric uncertainty which include un-modeled dynamics, friction, and disturbance.

3.2 Problem Statement

The following tracking control problems are now formulated.

Tracking control with uncertainty: It is given a desired trajectory $p^d(t)$ and a desired unit vector $b_2^d(t)$. If m, J, d_1, and d_2 are unknown, the control problem is to design a state feedback controller (f, τ) such that

$$\lim_{t \to \infty} \left(p(t) - p^d(t) \right) = 0 \tag{3.5}$$

$$\lim_{t \to \infty} \left(b_2(t) - b_2^d(t) \right) = 0 \tag{3.6}$$

Tracking control with uncertainty and input saturation: It is given a desired trajectory $p^d(t)$ and a desired unit vector $b_2^d(t)$. If m, J, d_1, and d_2 are unknown, the control problem is to design a state feedback controller (f, τ) such that (3.5)-(3.6) are satisfied and

$$0 < f \leq M_f \tag{3.7}$$

$$\left|\tau_j\right| \leq M_\tau, 1 \leq j \leq 3 \quad (3.8)$$

where M_f and M_τ are appropriate positive constants.

In order to solve the above problems, the following assumptions are made.

Assumption 1: The mass m is an unknown constant and $m \in [\underline{m}, \bar{m}]$ where $\underline{m}$ and $\bar{m}$ are known constants.

Assumption 2: The inertia matrix J is an unknown diagonal constant matrix (i.e., $J = diag([J_1, J_2, J_3])$) and $J_i \in [\underline{J}, \bar{J}]$ for $1 \leq i \leq 3$, where $\underline{J}$ and $\bar{J}$ are known constants.

Assumption 3: d_1 and d_2 are bounded and $|d_{1j}| \leq D_1$ and $|d_{2j}| \leq D_2$ for $1 \leq j \leq 3$, where D_1 and D_2 are known constants, and d_{1j} and d_{2j} are the j-th elements of d_1 and d_2, respectively.

Assumption 4: $p^d(t) = [p_1^d(t), p_2^d(t), p_3^d(t)]^\top$ is smooth, $|\ddot{p}_j^d(t)| \leq M_p$ ($1 \leq j \leq 3$) for any time.

Assumption 5: $b_2^d(t)$ is smooth. $\dot{b}_2^d$ and $\ddot{b}_2^d$ are bounded. $b_2^d(t) \times b_3^d(t) = 0$ for any time where $b_3^d(t) = \frac{\ddot{p}^d(t) + g e_3}{\|\ddot{p}^d(t) + g e_3\|}$ and $\times$ denotes the cross product of two vectors.

Assumption 6: $M_f > \frac{\sqrt{3}(D_1 + M_p) + g}{\underline{m}}$.

In (3.7), M_f and M_τ should be large enough such that there exist controllers which make (3.5)-(3.6) satisfied.

3.3 Quaternions

The attitude of a VTOL UAV can be defined by a unit quaternion $q = [\eta, \epsilon^\top]^\top$ where $\eta \in \Re$ and $\epsilon \in \Re^3$. The relation between q and R is defined by

$$R = R(q) = I + 2\eta S(\epsilon) + 2S^2(\epsilon).$$

For any rotation matrix R, there are exactly two unit quaternions, $\pm q$, such that $R = R(q) = R(-q)$. For two unit quaternions $q_1 = [\eta_1, \epsilon_1^T]$ and $q_2 = [\eta_2, \epsilon_2^T]$, the multiplication of q_1 and q_2 is defined by

$$q_1 \otimes q_2 = \begin{bmatrix} \eta_1\eta_2 - \epsilon_1^T\epsilon_2 \\ \eta_1\epsilon_2 + \eta_2\epsilon_1 + S(\epsilon_1)\epsilon_2 \end{bmatrix}$$

The identity quaternion is $\mathbf{1} = [\mathbf{1}, \mathbf{0}, \mathbf{0}, \mathbf{0}]^{\boldsymbol{T}}$. Each unit quaternion $q = [\eta, \epsilon^T]^T$ has an inverse, $q^{-1} = [\eta, -\epsilon^T]^T$, such that $q^{-1} \otimes q = q \otimes q^{-1} = \mathbf{1}$.

With the aid of the unit quaternion, (3.3) can be written as

$$\dot{q} = \frac{1}{2}A(q)\omega \tag{3.9}$$

where

$$A(q) = [-\epsilon, \eta I + S^\top(\epsilon)]^\top \tag{3.10}$$

In this work, some saturation functions will be applied. It is given a positive constant M, a function $\sigma{:} \rightarrow \Re$ is said to be a smooth monotonically increasing saturation with M if it is a smooth function satisfying:

(a). $s\sigma(s) > 0$ for all $s \neq 0$;

(b). $|\sigma(s)| \leq M$ for all $s \in \Re$;

(c). $\sigma(s)$ is monotonically increasing.

For given $M_i > 0$ $(1 \leq i \leq 3)$, the smooth function $\sigma(s)$ with M_i is denoted as $\sigma_i(s)$. If $s = [s_1, \dots, s_n]^\top$, $\sigma_i(s) = [\sigma_i(s_1), \dots, \sigma_i(s_n)]^\top$ for $1 \leq i \leq 3$.

3.4 Controller Design with Uncertainty

We design a controller such that (3.5)-(3.6) hold when there is uncertainty. Considering the structure of the system in (3.1)-(3.4), a modified backstepping approach will be proposed as follows:

Step 1: Let $e_p = p - p^d$ and $e_v = v - \dot{p}^d$, we have

$$\dot{e}_p = e_v \tag{3.11}$$

$$\dot{e}_v = -ge_3 - \ddot{p}^d + \frac{1}{m} fRe_3 + d_1 \tag{3.12}$$

Consider fRe_3 as a virtual control input, we design it such that (3.5) is satisfied. Noting the special structure of the system in (3.11)-(3.12), we choose a Lyapunov function

$$V_1(\Lambda_1, \Lambda_2, \tilde{\beta}) = \int_0^{\Lambda_1} \sigma_1(s)^\top ds + \int_0^{\Lambda_2} \sigma_2(s)^\top ds + \frac{k_1}{2} e_v^\top e_v + \frac{\gamma_1^{-1}}{2} \tilde{\beta}^2$$

where $0 = [0,0,0]^\top$, $k_1 > 0$, $k_2 > 0$, γ_1 is a positive constant, and

$$\Lambda_1 = k_1 e_p + k_2 e_v, \qquad \Lambda_2 = k_2 e_v, \qquad \tilde{\beta} = \frac{1}{m} - \beta$$

where β is an estimate of $\frac{1}{m}$. It can be proved that V_1 is a positive definite function of e_p, e_v, and $\tilde{\beta}$. Furthermore, $V_1 = 0$ if $e_p = 0$, $e_v = 0$, and $\tilde{\beta} = 0$. The derivative of V_1 is

$$\dot{V}_1 = k_1 \sigma_1(\Lambda_1)^\top e_v + (k_2 \sigma_1(\Lambda_1) + k_2 \sigma_2(\Lambda_2) + k_1 e_v)^\top \times (-ge_3 - \ddot{p}^d + \beta fRe_3 + d_1)$$
$$- \gamma_1^{-1} \left(\frac{1}{m} - \beta\right) (\dot{\beta} - \gamma_1 (k_2 \sigma_1(\Lambda_1) + k_2 \sigma_2(\Lambda_2) + k_1 e_v)^\top fRe_3)$$

To make $\dot{V}_1$ as small as possible, we choose the virtual control input for fRe_3 and update law of β as

$$\alpha = [\alpha_1, \alpha_2, \alpha_3]^\top$$

$$= \frac{1}{\beta}(-\sigma_1(\Lambda_1) - \sigma_2(\Lambda_2) - D_1 h(G, \delta) + g e_3 + \ddot{p}^d) \quad (3.13)$$

$$\dot{\beta} = \mathrm{P}roj_{\Omega_m}(\gamma_1 G^\top f R e_3) = \mathrm{P}roj_{\Omega_m}(H)$$

$$= \begin{cases} H, & if \beta \in \left(\frac{1}{\overline{m}}, \frac{1}{\underline{m}}\right), \\ & or\ if\ \ \beta = \frac{1}{\overline{m}}\ \ and\ \ H > 0, \\ & or\ if\ \ \beta = \frac{1}{\underline{m}}\ \ and\ \ H < 0 \\ 0, & otherwise \end{cases} \quad (3.14)$$

where

$$\Omega_m = \left[\frac{1}{\overline{m}}, \frac{1}{\underline{m}}\right]$$

$$H = \gamma_1 G^\top f R e_3 \quad (3.15)$$

$$G = [G_1, G_2, G_3]^\top$$

$$= k_2 \sigma_1(\Lambda_1) + k_2 \sigma_2(\Lambda_2) + k_1 e_v \quad (3.16)$$

$$h(G, \delta) = \left[\frac{G_1}{\sqrt{G_1^2 + \delta(t)^2}}, \frac{G_2}{\sqrt{G_2^2 + \delta(t)^2}}, \frac{G_3}{\sqrt{G_3^2 + \delta(t)^2}}\right]^\top \quad (3.17)$$

$\delta(t) = e^{-\delta_1 t}$ and $\delta_1 > 0$. Then, if $fRe_3 = \alpha$, we have

$$\dot{V}_1 \le -k_2(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))^\top(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))$$

$$-\frac{k_1}{k_2}\sigma_2(\Lambda_2)^\top \sigma_2(\Lambda_2) + 3D_1\delta(t) \quad (3.18)$$

It should be noted that α is well defined since $\beta \geq \frac{1}{\bar{m}}$. For the defined α, we have the following lemma by simple calculation.

Lemma 1 If M_1 and M_2 are chosen such that

$$g - M_p - M_1 - M_2 - D_1 > 0 \tag{3.19}$$

then $\| \alpha \| > 0$ and

$$\| \alpha \| \leq \frac{\sqrt{3}(M_1 + M_2 + D_1 + M_p) + g}{\underline{m}} \tag{3.20}$$

Step 2: We find f and a virtual control input q_d for the unit quaternion q. We choose

$$f = \| \alpha \| \tag{3.21}$$

It is obvious that $f > 0$ for any time with the aid of Lemma 1.

Let $r_3 = \frac{\alpha}{\|\alpha\|}$, we define

$$r_2 = \frac{\left(r_3^\top b_3^d\right) b_2^d - \left(r_3^\top b_2^d\right) b_3^d}{\| \left(r_3^\top b_3^d\right) b_2^d - \left(r_3^\top b_2^d\right) b_3^d \|} \tag{3.22}$$

$$r_1 = \frac{r_2 \times r_3}{\| r_2 \times r_3 \|} \tag{3.23}$$

The desired attitude of R is chosen as

$$R_d = [r_1, r_2, r_3] \tag{3.24}$$

and the desired quaternion $q_d = [\eta_d, \epsilon_d^\top]^\top$ of q is calculated from (3.24) by the equations (166)-(168) in [47] which are omitted here. The desired angular velocity is calculated by

$$\omega_d = 2A(q_d)^\top \frac{dq_d}{dt} \tag{3.25}$$

It should be noted that q_d and ω_d are well defined because f is always positive.

With the aid of the virtual control input q_d,

$$fR_d e_3 = \alpha$$

and equation (3.12) can be written as

$$\dot{e}_v = -\sigma(\Lambda_1) - \sigma(\Lambda_2) - D_1 h + d_1 + \left(\frac{1}{m} - \beta\right) fRe_3 + \beta \parallel \alpha \parallel R_d(R_d^\top R - I_3)e_3 \tag{3.26}$$

Step 3: We assume ω is a virtual control input and design a virtual controller for ω such that (3.5)-(3.6) are satisfied. Let the difference between q and q_d be

$$\tilde{q} = q_d^{-1} \otimes q = [\tilde{\eta}, \tilde{\epsilon}^\top]^\top, \tag{3.27}$$

The derivative of $\tilde{q}$ is

$$\dot{\tilde{q}} = \frac{1}{2} A(\tilde{q})\left(\omega - \tilde{R}^\top \omega_d\right) \tag{3.28}$$

where $\tilde{R} = R_d^\top R$. (3.24) can be written as

$$\dot{e}_v = -\sigma_1(\Lambda_1) - \sigma_2(\Lambda_2) - D_1 h + d_1 + \left(\frac{1}{m} - \beta\right) fRe_3 - 2\beta \parallel \alpha \parallel R_d(\tilde{\eta} I_3 + S(\tilde{\epsilon}))S(e_3)\tilde{\epsilon} \tag{3.29}$$

Choose a Lyapunov function

$$V_2 = V_1 + 2(1 - \tilde{\eta}) = V_1 + \tilde{\epsilon}^\top \tilde{\epsilon} + (1 - \tilde{\eta})^2 \tag{3.30}$$

It can be proved that V_2 is a positive definite function of $(\Lambda_1, \Lambda_2, \tilde{\beta}, 1 - \tilde{\eta})$ and $V_2 = 0$ if $(\Lambda_1, \Lambda_2, \tilde{\beta}, 1 - \tilde{\eta}) = (0,0,0,0)$. The derivative of V_2 is

$$\dot{V}_2 \le -k_2(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))^\top(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2)) - \frac{k_1}{k_2}\sigma_2(\Lambda_2)^\top\sigma_2(\Lambda_2)\delta + 3D_1 +$$

$$\tilde{\epsilon}^\top(\omega - \tilde{R}^\top\omega_d$$

$$-2\beta \parallel \alpha \parallel S(e_3)^\top(\tilde{\eta}I_3 + S(\tilde{\epsilon}))^\top R_d^\top G)$$

To make $\dot{V}_2$ as small as possible, a virtual controller μ for ω can be chosen as

$$\mu = -k_3\tilde{\epsilon} + \tilde{R}^\top\omega_d + 2\beta \parallel \alpha \parallel S(e_3)^\top(\tilde{\eta}I_3 + S(\tilde{\epsilon}))^\top R_d^\top G \tag{3.31}$$

where k_3 is a positive constant. If ω were a real control input, i.e., $\omega = \mu$, then

$$\dot{V}_2 \le -k_2(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))^\top(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2)) - \frac{k_1}{k_2}\sigma_2(\Lambda_2)^\top\sigma_2(\Lambda_2) + 3D_1\delta - k_3\tilde{\epsilon}^\top\tilde{\epsilon}$$

Step 4: Since ω is not a real control input, ω cannot be μ. Let

$$\widetilde{\omega} = [\widetilde{\omega}_1, \widetilde{\omega}_2, \widetilde{\omega}_3]^\top = \omega - \mu,$$

then,

$$\dot{\tilde{q}} = \frac{1}{2}A(\tilde{\eta}, \tilde{\epsilon})(-k_3\tilde{\epsilon} + \widetilde{\omega} + 2\beta \parallel \alpha \parallel S(e_3)^\top(\tilde{\eta}I_3 + S(\tilde{\epsilon}))^\top R_d^\top G) \tag{3.32}$$

$$J\dot{\widetilde{\omega}} = \tau - (S(\omega)\Gamma(\omega) + \Gamma(\dot{\mu}))a + d_2 \tag{3.33}$$

where $\Gamma(\omega)$ denotes a diagonal matrix with its diagonal elements being the vector ω and

$$a = [a_1, a_2, a_3]^\top = [J_1, J_2, J_3]^\top.$$

Since a and d_2 are unknown, an adaptive robust control law will be proposed such that (3.5)-(3.6) are satisfied. To this end, we choose a Lyapunov function

$$V_3 = V_2 + \frac{1}{2}\widetilde{\omega}^\top J\widetilde{\omega} + \frac{\gamma_2^{-1}}{2}(a - \hat{a})^\top(a - \hat{a})$$

where γ_2 is a positive constant and $\hat{a}$ is an estimate of a which will be designed later. The derivative of V_3 along the solution of (3.29), (3.32) and (3.33) is

$$\dot{V}_3 \leq -k_2(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))^\top(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2)) - \frac{k_1}{k_2}\sigma_2(\Lambda_2)^\top\sigma_2(\Lambda_2) + 3D_1\delta - k_3\tilde{\epsilon}^\top\tilde{\epsilon} + \tilde{\epsilon}^\top\widetilde{\omega} + \widetilde{\omega}^\top(\tau - (S(\omega)\Gamma(\omega) + \Gamma(\dot{\mu}))\hat{a} + d_2) - \gamma_2^{-1}(a - \hat{a})^\top\big(\dot{\hat{a}} + \gamma_2(S(\omega)\Gamma(\omega) + \Gamma(\dot{\mu}))^\top\widetilde{\omega}\big)$$

To make $\dot{V}_3$ as small as possible, we choose the control law τ and the update law $\hat{a}$ as follows:

$$\tau = -k_4\widetilde{\omega} - \tilde{\epsilon} + \big(S(\omega)\Gamma(\omega) + \Gamma(\dot{\mu})\big)\hat{a} - D_2 h(\widetilde{\omega}, \delta) \tag{3.34}$$

$$\dot{\hat{a}} = \mathrm{P}roj_{\Omega_a}(-\gamma_2(S(\omega)\Gamma(\omega) + \Gamma(\dot{\mu}))^\top\widetilde{\omega}) \tag{3.35}$$

where $\Omega_a = \left[\underline{J}, \bar{J}\right]$ and k_4 is a positive constant. Then,

$$\dot{V}_3 \leq -k_2(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))^\top(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2)) - \frac{k_1}{k_2}\sigma_2(\Lambda_2)^\top\sigma_2(\Lambda_2) + 3D_1\delta + 3D_2\delta - k_3\tilde{\epsilon}^\top\tilde{\epsilon} - k_4\widetilde{\omega}^\top\widetilde{\omega} \tag{3.36}$$

Based on the above controller design procedure, we have the following results.

***Theorem 3.1*:** For the system in (3.1)-(3.4) and given desired trajectories p^d and b_2^d, the control inputs (f, τ) in (3.21) and (3.34) with the update laws in (3.14) and (3.35) ensure that (3.5)-(3.6) are satisfied and $(\beta, \hat{a})$ are bounded. Furthermore, the thrust force f is larger than zero at any time and is bounded.

3.5 Controller Design with Uncertainty and Input Saturation

Assume the input τ is subject to a saturation constraint, i.e.,

$$\tau = \sigma_3(\tau^d) \tag{3.37}$$

where $\tau^d \in R^3$ is a new control input without constraint and will be designed later. In order to compensate the effect of the input saturation, the following auxiliary compensated system is defined:

$$\dot{q}_a = \frac{1}{2} A(q_a)\big(\tilde{R}_a \omega_a - k_5 \epsilon_a\big) \tag{3.38}$$

$$\dot{\omega}_a = -k_4 \omega_a + \Gamma\big(\hat{\theta}\big)(\tau - \tau^d) \tag{3.39}$$

where $q_a = [\eta_a, \epsilon_a^\top]^\top$ is an auxiliary unit quaternion, $\omega_a \in R^3$ is an auxiliary angular velocity, $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3]^\top$ is an estimate of

$$\theta = [\theta_1, \theta_2, \theta_3]^\top = \left[\frac{1}{J_1}, \frac{1}{J_2}, \frac{1}{J_3}\right]^\top$$

and will be designed later, k_4 and k_5 are positive constants, and

$$R_a = I + 2\eta_a S(\epsilon_a) + 2S^2(\epsilon_a) \tag{3.40}$$

$$\tilde{R}_a = R_a^\top R \tag{3.41}$$

Define

$$\tilde{q}_a = q_a^{-1} \otimes q \tag{3.42}$$

then,

$$\dot{\tilde{q}}_a = \frac{1}{2} A(\tilde{q}_a)\big(\omega - \omega_a + k_5 \tilde{R}_a^\top \epsilon_a\big) \tag{3.43}$$

Let r_2 and r_1 be defined as in (3.22) and (3.23) and

$$\bar{\alpha} = R_a^\top \alpha \tag{3.44}$$

$$r_3 = \frac{\bar{\alpha}}{\| \bar{\alpha} \|} \tag{3.45}$$

where α is defined in (3.13). Define R_d as in (3.24). The unit quaternion corresponding to R_d is denoted as q_d. By (3.25), we can calculate ω_d.

Define

$$\tilde{q} = [\tilde{\eta}, \tilde{\epsilon}^\top]^\top = q_d^{-1} \otimes \tilde{q}_a \tag{3.46}$$

then,

$$\dot{\tilde{q}} = \frac{1}{2} A(\tilde{q})\big(\omega - \omega_a - \tilde{R}^\top \omega_d + k_5 \tilde{R}_a^\top \epsilon_a\big) \tag{3.47}$$

where $\tilde{R} = R_d^\top R_a^\top R$. We choose f as in (3.21). It can be shown that $f = \| \bar{\alpha} \|$. So, $f R_d e_3 = \bar{\alpha} = R_a^\top \alpha$ and equation (3.12) can be written as

$$\dot{e}_v = -\sigma(\Lambda_1) - \sigma(\Lambda_2) - D_1 h + d_1 + \left(\frac{1}{m} - \beta\right) f R e_3$$

$$-2\beta \| \bar{\alpha} \| R_a R_d \big(\tilde{\eta} I_3 + S(\tilde{\epsilon})\big) S(e_3) \tilde{\epsilon} \tag{3.48}$$

Choose a Lyapunov function

$$V_4 = V_1 + 2(1 - \tilde{\eta}).$$

It can be proved that V_4 is a nonnegative function of Λ_1, Λ_2, and $\tilde{\eta}$. The derivative of V_4 is

$$\dot{V}_4 \leq -k_2(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))^\top(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))$$

$$-\frac{k_1}{k_2}\sigma_2(\Lambda_2)^\top \sigma_2(\Lambda_2) + 3D_1\delta + \tilde{\epsilon}^\top(\omega - \tilde{R}^\top \omega_d$$

$$-\omega_a - 2\beta \parallel \alpha \parallel S(e_3)^\top(\tilde{\eta} I_3 + S(\tilde{\epsilon}))^\top (R_a R_d)^\top G + k_5 \tilde{R}_a^\top \epsilon_a).$$

To make $\dot{V}_4$ as small as possible, a virtual controller μ for ω can be chosen as

$$\mu = 2\beta \parallel \alpha \parallel S(e_3)^\top(\tilde{\eta} I_3 + S(\tilde{\epsilon}))^\top (R_a R_d)^\top G - k_3 \tilde{\epsilon}$$

$$+\tilde{R}^\top \omega_d - k_5 \tilde{R}_a^\top \epsilon_a \tag{3.49}$$

where k_3 is a positive constant. If ω were the real control input, i.e., $\omega = \mu$, then

$$\dot{V}_4 \leq -k_2(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))^\top(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))$$

$$-\frac{k_1}{k_2}\sigma_2(\Lambda_2)^\top \sigma_2(\Lambda_2) + 3D_1\delta + \tilde{\epsilon}^\top(-k_3\tilde{\epsilon} - \omega_a).$$

Since ω is not the real control input, ω cannot be μ. Let

$$\widetilde{\omega} = [\widetilde{\omega}_1, \widetilde{\omega}_2, \widetilde{\omega}_3]^\top = \omega - \mu - \omega_a$$

then,

$$\dot{\tilde{q}} = \frac{1}{2} A(\tilde{q})(-k_3\tilde{\epsilon} + \widetilde{\omega} + 2\beta \parallel \alpha \parallel S(e_3)^\top(\tilde{\eta} I_3 + S(\tilde{\epsilon}))^\top (R_a R_d)^\top G)$$

$$\dot{\widetilde{\omega}} = \Gamma(\Pi)B + \Gamma(\theta)(\tau + d_2) - \dot{\mu} + k_4\omega_a - \Gamma(\hat{\theta})(\tau - \tau^d)$$

$$= \Gamma(\hat{\theta})\tau^d + \Gamma(\theta - \hat{\theta})\tau + k_4\omega_a + \Gamma(\Pi)B + \Gamma(\theta)d_2 - \dot{\mu}$$

where

$$B = [B_1, B_2, B_3]^\top = [\omega_2\omega_3, \omega_1\omega_3, \omega_1\omega_2]^\top$$

$$\Pi = [\Pi_1, \Pi_2, \Pi_3]^\top$$

$$= [J_1^{-1}(J_2 - J_3), J_2^{-1}(J_3 - J_1), J_3^{-1}(J_1 - J_2)]^\top.$$

To make $\tilde{q}$ converge to the identity quaternion and $\widetilde{\omega}$ converge to zero, we choose a Lyapunov function

$$V_5 = V_4 + \frac{1}{2}\widetilde{\omega}^\top\widetilde{\omega} + \frac{\gamma_2^{-1}}{2}(\theta - \hat{\theta})^\top(\theta - \hat{\theta}) + \frac{\gamma_3^{-1}}{2}(\Pi - \widehat{\Pi})^\top(\Pi - \widehat{\Pi})$$

where, $\widehat{\Pi} = [\widehat{\Pi}_1, \widehat{\Pi}_2, \widehat{\Pi}_3]^\top$ is an estimate of Π, γ_2 and γ_3 are positive constants. The derivative of V_5 is

$$\dot{V}_5 \leq -k_2(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))^\top(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))$$

$$-\frac{k_1}{k_2}\sigma_2(\Lambda_2)^\top\sigma_2(\Lambda_2) + 3D_1\delta - k_3\tilde{\epsilon}^\top\tilde{\epsilon} + \widetilde{\omega}^\top(\Gamma(\hat{\theta})\tau^d$$

$$+\tilde{\epsilon} + \Gamma(\widehat{\Pi})B + k_4\omega_a + \Gamma(\theta)d_2 - \dot{\mu})$$

$$-\gamma_2^{-1}(\theta - \hat{\theta})^\top\left(\dot{\hat{\theta}} - \gamma_2\Gamma(\tau)\widetilde{\omega}\right)$$

$$-\gamma_3^{-1}(\Pi - \widehat{\Pi})^\top\left(\dot{\widehat{\Pi}} - \gamma_3\Gamma(B)\widetilde{\omega}\right).$$

To make $\dot{V}_5$ as small as possible, we choose

$$\tau^d = \Gamma^{-1}(\hat{\theta})\left(-k_4(\omega - \mu) - \tilde{\epsilon} - \Gamma(\widehat{\Pi})B + \dot{\mu} - \frac{D_2}{\underline{J}}h(\widetilde{\omega}, \delta)\right) \tag{3.50}$$

$$\dot{\hat{\theta}}_j = \mathrm{P}roj_{\Omega_\theta}(\gamma_2\tau_j\widetilde{\omega}_j), j = 1,2,3 \tag{3.51}$$

$$\widehat{\Pi}_j = \gamma_3 B_j\widetilde{\omega}_j, j = 1,2,3 \tag{3.52}$$

.

where $\Omega_\theta = \left[\frac{1}{\bar{J}}, \frac{1}{\underline{J}}\right]$. Then,

$$\dot{V}_5 \le -k_2(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))^\top(\sigma_1(\Lambda_1) + \sigma_2(\Lambda_2))$$

$$-\frac{k_1}{k_2}\sigma_2(\Lambda_2)^\top\sigma_2(\Lambda_2) + 3D_1\delta - k_3\tilde{\epsilon}^\top\tilde{\epsilon} - k_4\widetilde{\omega}^\top\widetilde{\omega} + \frac{3D_2\delta}{\underline{J}} \tag{3.53}$$

With the aid of the above procedure, we have the following results.

Theorem 3.2 For the system in (3.1)-(3.4) and given desired trajectories p^d and b_2^d, the control inputs (f, τ) in (3.21) and (3.37) with τ^d in (3.50) and the update laws in (3.14) and (3.51)-(3.52) ensure that

1) β, $\widehat{\Pi}$, and $\hat{\theta}$ are bounded,

2) e_p, e_v, $\tilde{\epsilon}$, and $\widetilde{\omega}$ converge to zero, and

3) $b_2 - b_2^d$ is uniformly ultimate boundedness (UUB).

Furthermore, f is larger than zero at any time and (7) are satisfied if M_1, M_2, and M_3 are chosen such that

$$M_1 + M_2 < \frac{\underline{m}M_f - g}{\sqrt{3}} - D_1 - M_p \tag{3.54}$$

$$M_3 = M_\tau \tag{3.55}$$

3.6 Simulation

Simulation results are presented to illustrate the effectiveness of the proposed controllers. We consider a VTOL UAV modeled as a rigid body with mass $m = 0.85$kg and inertia tensor $J = \mathrm{d}iag([4.856, 4.856, 9.801]) \times 10^{-2}$kg m^2 (see [23]). In the controllers, m and J are unknown. However, it is known that $m \in [0.7, 1]$kg, i.e., $\underline{m} = 0.7$kg and $\overline{m} = 1$kg. For disturbance, it is assumed that d_1 and d_2 are white noise with magnitudes $D_1 = D_2 = 0.05$.

In the simulation, the desired trajectory p^d and b_2^d are chosen as

$$p^d(t) = \begin{bmatrix} 100\cos(0.05t) \\ 100\sin(0.05t) \\ 10 - 10\exp(-0.1t) \end{bmatrix}$$

$$b_2^d(t) = \begin{bmatrix} \sin(0.05t) \\ -\cos(0.05t) \\ 0 \end{bmatrix}$$

If there is no input constraint, the robust adaptive controller is (3.21) and (3.34) with the aid of Theorem 1. In the control law, we chose

$$\sigma_i(x) = M_i \tanh(x)$$

where $M_i = 4$. Simulation was done for one group of control parameters. The time response of the tracking errors of $p_1 - p_1^d$, $p_2 - p_2^d$, and $p_3 - p_3^d$ are shown in Fig. 3.1 which shows they converge to zero. Fig. 3.2 depicts the response of the tracking error $\tilde{q}$. It shows that $\tilde{\eta}$ asymptotically converge to one and $\tilde{\epsilon}$ asymptotically converges to zero. Fig. 3.3 shows the total force f. It is obvious that f is bounded and is larger than zero at any time. Fig. 3.4 shows the input torque τ. The simulation results show the effectiveness of the results in Theorem 1.

If there are uncertainty and input constraints. The control laws can be obtained in (3.21) and (3.37) with the aid of Theorem 2. In the simulation, the bounds on the force and the toque are chosen as $M_f = 12$ N and $M_\tau = 0.05$. It can be shown that Assumption 6 is satisfied. Simulation was done for a set of chosen control parameters. The time response of the tracking errors of $p_1 - p_1^d$, $p_2 - p_2^d$, and $p_3 - p_3^d$ are shown in Fig. 3.5. Fig. 3.6 shows the response of the tracking error $q_d^{-1} \otimes q$. It is bounded by 0.05. The simulation results show the effectiveness of the results in Theorem 2. Fig. 3.7 shows the total force f. It is obvious that f is bounded and is

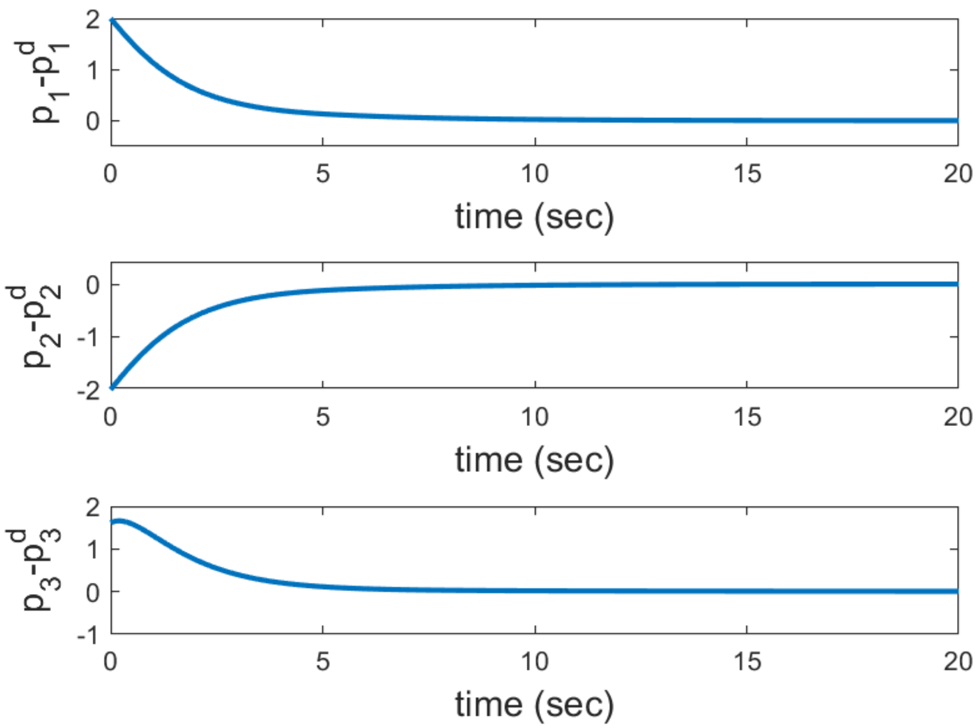


Figure 3.1: Time response of $p - p^d$

larger than zero at any time. Fig. 3.8 shows the input torque τ. The simulation results show the effectiveness of the results in Theorem 2.

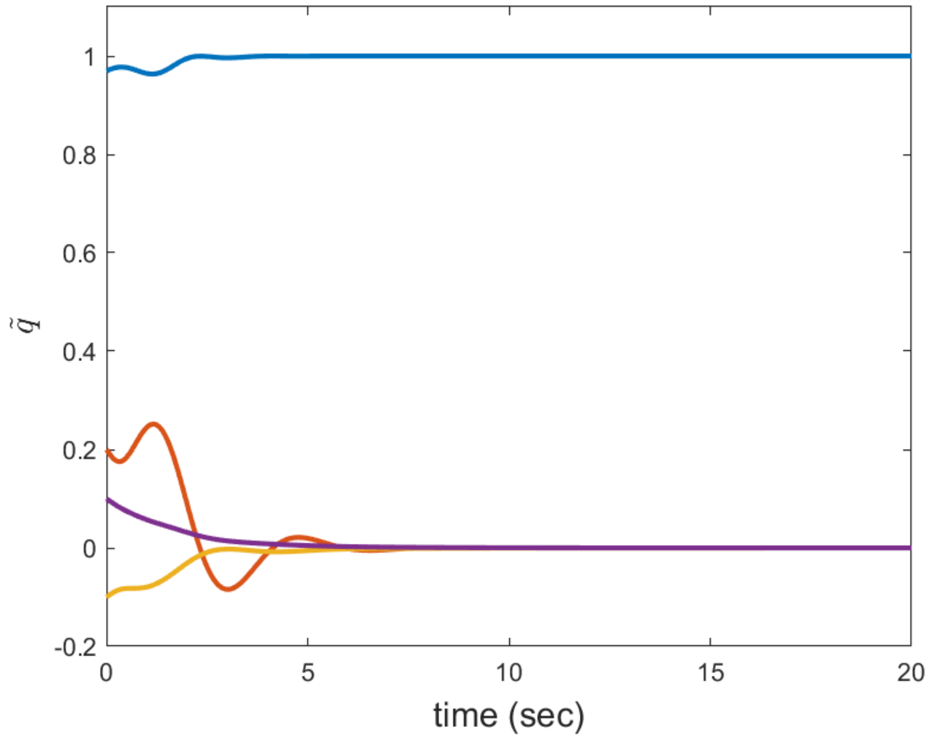


Figure 3.2: Time response of $\tilde{q}$

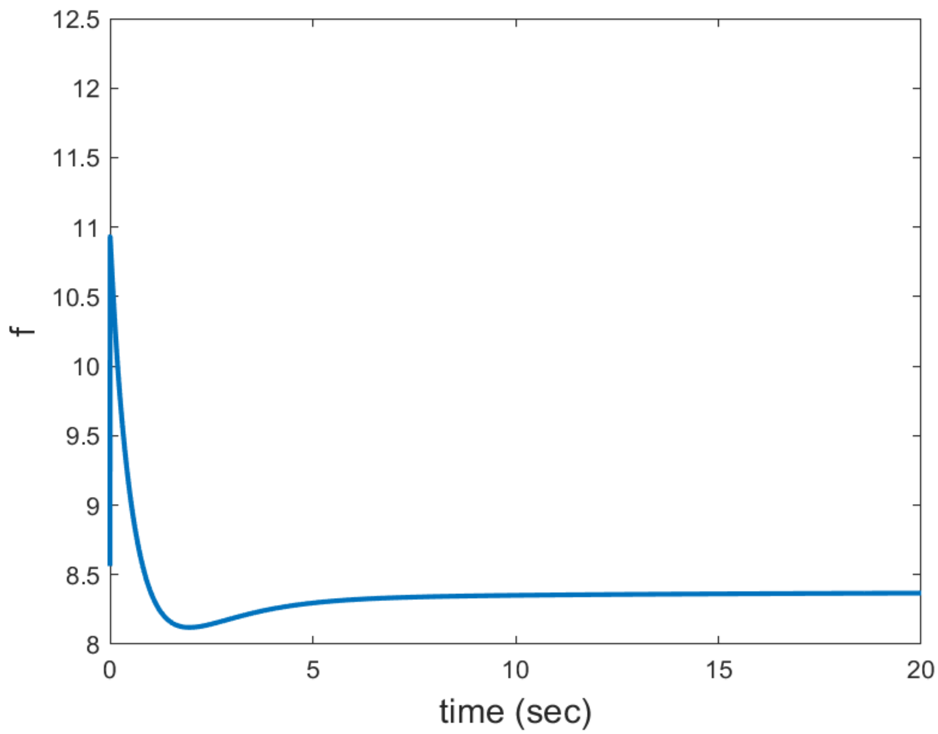


Figure 3.3: Time response of f

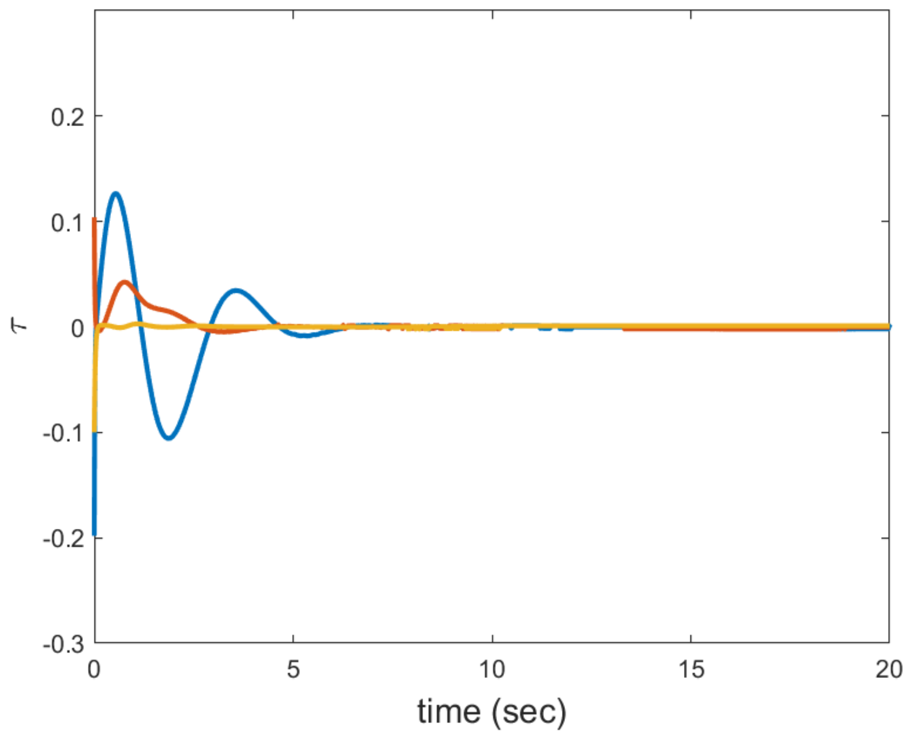


Figure 3.4: Time response of τ

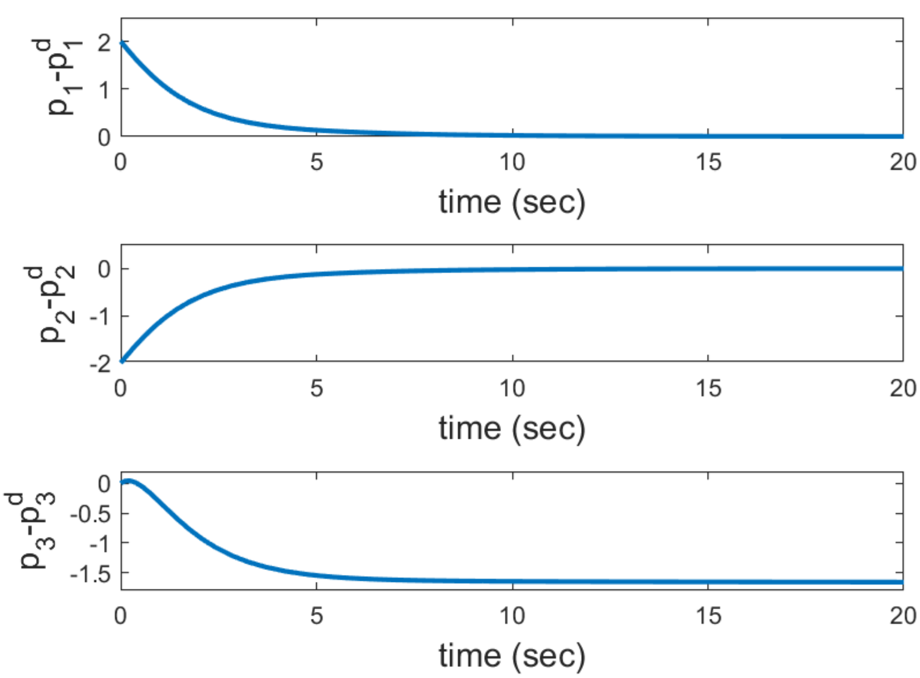


Figure 3.5: Time response of $p - p^d$

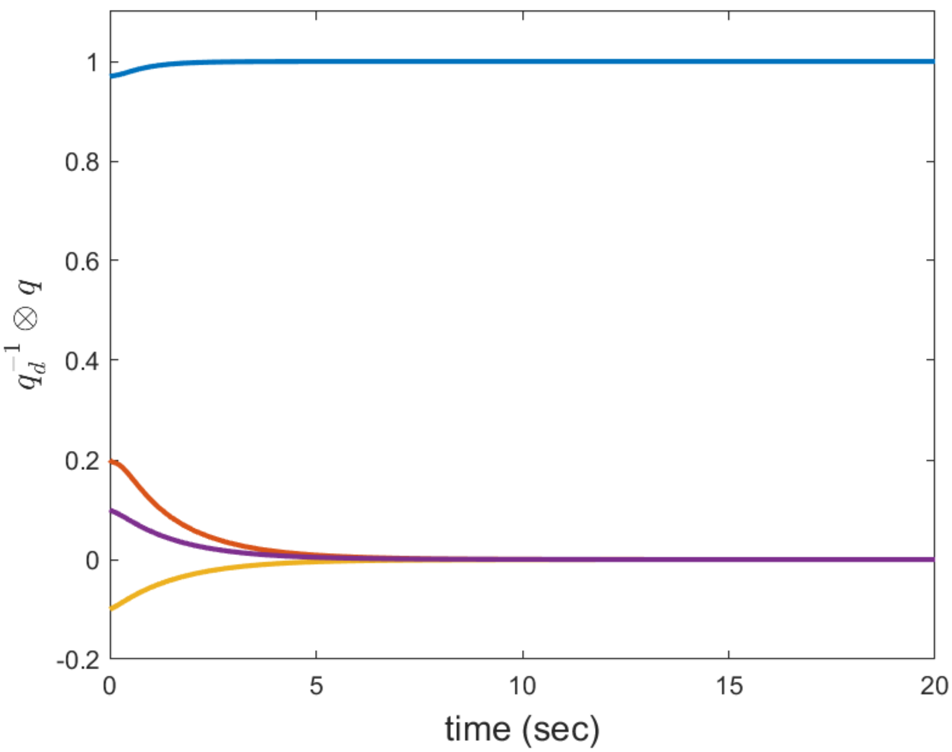


Figure 3.6: Time response of $\tilde{q}$

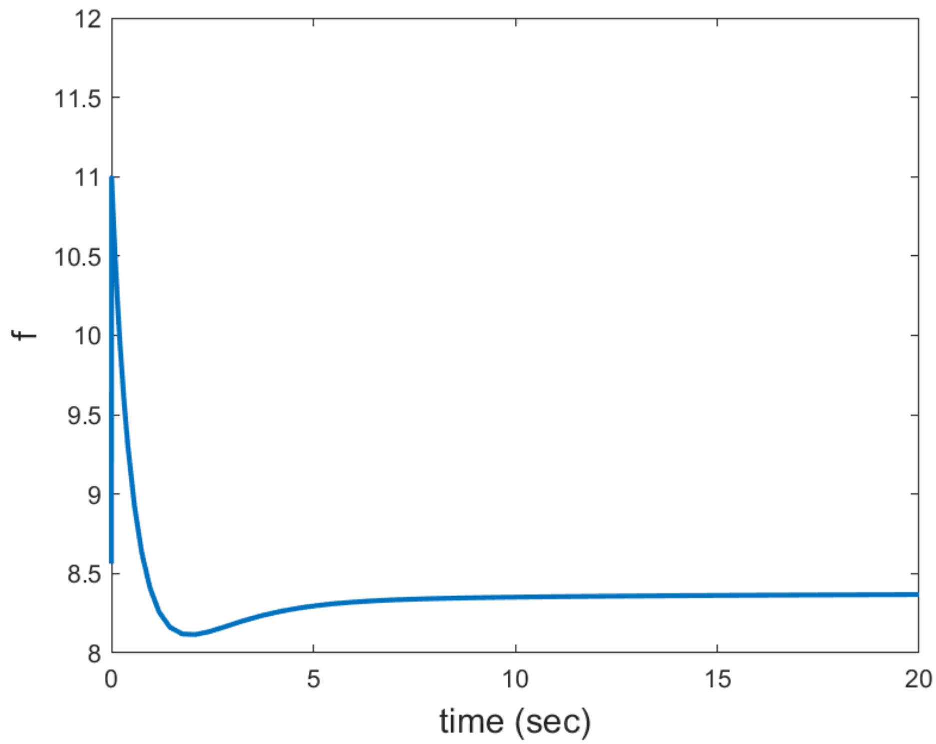


Figure 3.7: Time response of f

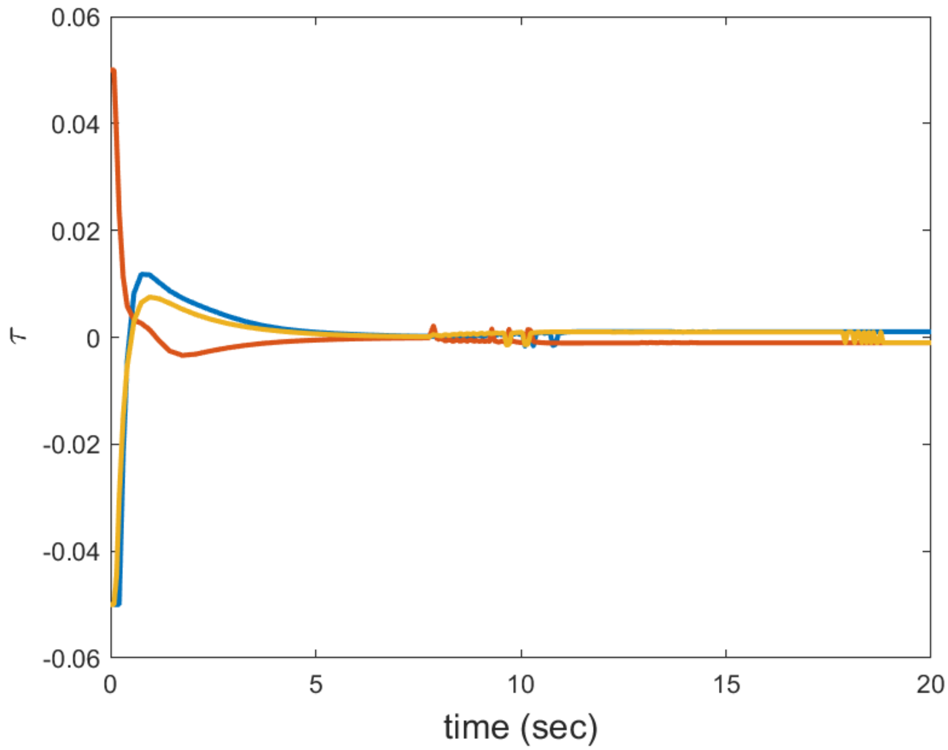


Figure 3.8: Time response of τ

3.7 Conclusion

This chapter deals with the tracking control problems of a quadrotor with uncertainty and input constraints. Considering the uncertainty in the dynamics of the system, a robust adaptive tracking controller was proposed such that the position and the attitude of a quadrotor asymptotically converge to their desired value with the aid of the backstepping technique. Considering the uncertainty and input constraints, a saturation robust adaptive controller was proposed with the aid of an auxiliary compensated system. Simulation results show the effectiveness of the proposed controllers.

CHAPTER IV

FORMATION CONTROL OF QUADROTORS

Features like low cost, high adaptivity and easy maintenance can be achieved, by cooperative control of multiple agents. This chapter provides a formation control scheme following a virtual leader. A distributed second order sliding mode estimator has been used for using the distributed control. Proper assumptions have been made to implement the estimators for the quadrotor system.

Here for the n-agent system, an agent is called a *leader* if the agent has no neighbor. An agent is called follower if the agent has neighbor(s) [48]. Only a subset of the followers has access to the information of the leader owing to constraints in communication variables. Each agent updates its state using the information of the neighbors and of its own. The agreement between the agents in a common group on a common state can be defined as consensus. Providing desired separation among the agents, specific geometric formations can be achieved. So distributed estimation is made about the virtual leader's position and orientation. This leads to development of distributed leader-follower formation control strategy.

In this work, the consensus among the quadrotor is achieved under a switching network topology. For the cascaded feature of quadrotor dynamics, the sliding mode estimator has been used separately for the position tracking and the attitude tracking of the virtual leader. The control laws from chapter 2 are used with the estimations to design the distributed control for the group of quadrotors. The communication digraph among the agents is realized using the graph theory notions.

4.1 Graph Theory

Graph theory implements the communication network among the corresponding agents. For explaining the graph used in this work, some preliminaries are needed to be introduced. A graph basically consists of vertices and their connecting edges. If there is at least one way to reach from one vertex to all other vertices, then the graph is connected. If even one of the vertices is not reachable from all other vertices, then the graph is disconnected. For every node of the graph, the *degree* is defined as the number of edges entering or leaving the node. We define the *degree matrix* D as the diagonal matrix as

$$D = diag(d_1, d_2, \dots, d_n)$$

where, d_i is the degree for *i*th vertex. For a directed graph, the *adjacency matrix* A is a symmetric $n \times n$ matrix with a_{ij} such that $a_{ij} = 1$ if $(v_j, v_i) \in \mathcal{W}$. Otherwise, $a_{ij} = 0$. The matrix $L = D - A$ is defined as the *graph Laplacian* for the graph G. An example is provided in Fig. 06 which consists of the five vertices and seven directed edges.

Example 1. Consider the directed graph G in Fig. 06 that consists of five vertices and seven directed edges. Then as per the definition we have the degree matrix as

$$D = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

We have the adjacency matrix for graph G as

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Then we have the graph Laplacian L of the graph G as

$$L = D - A = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 3 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & -1 & 0 & -1 & 2 \end{bmatrix}$$

For the system under consideration in this thesis, n number of quadrotors are considered for a system, whose network topology can be described as a directed graph denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{W})$, where $\mathcal{V} = \{v_1, v_2, \cdots, v_n\}$ is the set of quadrotors and $\mathcal{W} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges connecting the quadrotors directionally. The node v_i represents the quadrotor i. A directed edge denoted as (v_i, v_j) indicates that the quadrotor j has access to the information of state of agent i. Hence, the quadrotor i is a neighbor of the quadrotor j. For a node v_i the set of neighbors is defined as

$$N_i = v_j \in \mathcal{V}: (v_i, v_j) \in \mathcal{W}.$$

A sequence of the edges in the form of $(v_1, v_2), (v_2, v_3), \ldots$ where $v_i \in \mathcal{V}$ comprises a directed path. If any pair of quadrotors are bidirectionally connected by an edge, that implies there is a directed path from any quadrotor to any other quadrotor in the same network, which constructs a complete directed graph. A directed spanning tree of $\mathcal{G}$ is a directed tree which contains all the nodes of the graph through directed paths. A directed spanning forest of $\mathcal{G}$ is a directed graph which contains one or several directed trees that contain all the nodes of $\mathcal{G}$, but not any pair shares any common node. If all the edges of $\mathcal{G}$ are also make the edges of a spanning tree, then $\mathcal{G}$ itself is a tree. If there exists a path from every node to a node j, then the node j is a

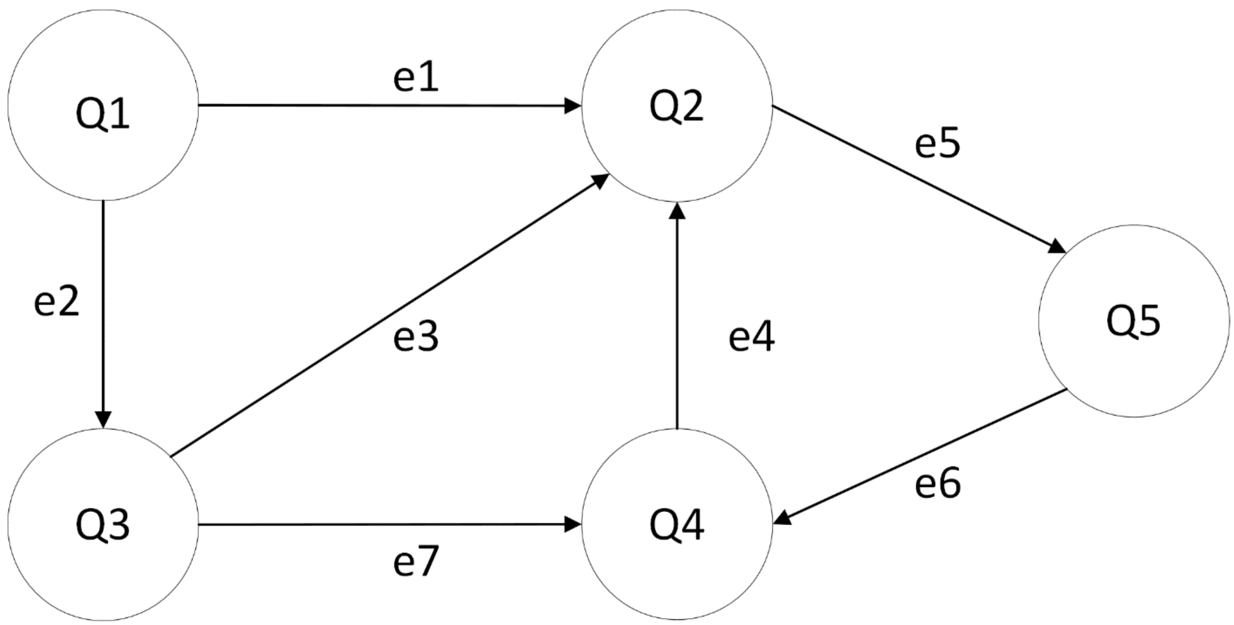


Figure 4.1: A directed graph $\mathcal{G}$

globally reachable node of $\mathcal{G}$. If there exists a path between any two distinct vertices, then $\mathcal{G}$ is called strongly connected.

The adjacency matrix used to represent the network topology is defined as $\mathcal{A} = [a_{ij}] \in R^{n \times n}$ where $a_{ij} > 0$ provided that $(v_j, v_i) \in \mathcal{W}$. Otherwise, $a_{ij} = 0$. Alternatively, Laplacian matrix can be used to represent the topology where the Laplacian matrix is defined as $\mathcal{L} = [l_{ij}] \in R^{n \times n}$ which can be related to the adjacency matrix as $l_{ii} = \sum_{j=1}^{n} a_{ij}$ and $l_{ij} = -a_{ij},\ i \neq j$.

4.2 Distributed Sliding Mode Estimation

The formation tracking problem is subdivided into two tasks: (i) decentralized estimation of states of the quadrotors, and (ii) desired state tracking of each quadrotor. Hence, the performance of the estimators will define to what extent the formation tracking control works with minimal error.

We assume the virtual leader whose position is given by $p_0(t) \in \Re^3$ and velocity is given by $v_0(t) \in \Re^3$ which are available to a number of quadrotors accumulating a subset of the *n* quadrotors. We also consider the attitude representation of the virtual leaders as $\Theta_0(\text{t}) \in \Re^3$ and the derivative $\dot{\Theta}_0(t) \in \Re^3$. Following assumptions are made regarding $v_0(t)$ and $\dot{\Theta}_0(t)$.

Assumption 4.1: $v_0(t)$ is differentiable.

Assumption 4.2: $sup_t|\dot{v}_0(t)| \leq \xi$ and

Assumption 4.3: $\Theta_0(\text{t})$ is differentiable.

Assumption 4.4: $sup_t\left|\ddot{\Theta}_0(t)\right| \leq \zeta$.

We write the quadrotor kinematics and dynamics as in equations (2.2) – (2.5) for the *i*th quadrotor as

$$\dot{p}_i = v_i \tag{4.3}$$

$$\dot{v}_i = -ge_3 + \frac{1}{m_i} f_i R_i e_3 \tag{4.4}$$

$$\dot{\Theta}_i = W(\Theta_i)\omega_i \tag{4.5}$$

$$J_i\dot{\omega}_i = S_i(J_i\omega_i)\omega_i + \tau_i \tag{4.6}$$

To estimate the position and velocity of the virtual leader in finite time, we consider the second order sliding mode estimator inspired by [45].

$$\dot{\hat{p}}_i(t) = \hat{v}_i(t) - \alpha_1\, sgn\left\{\sum_{j=0}^{n} a_{ij}(t)\left[\hat{p}_i(t) - \hat{p}_j(t)\right]\right\} \tag{4.7(a)}$$

$$\dot{\hat{v}}_i(t) = -\beta_1\, sgn\left\{\sum_{j=0}^{n} a_{ij}(t)\big[\hat{v}_i(t) - \hat{v}_j(t)\big]\right\}, \qquad i,j = 1,\dots,n \qquad (4.7(b))$$

where $\hat{p}_i(t)$ and $\hat{v}_i(t)$ are the estimates of the position and velocity of the *i*th quadrotor respectively, $a_{ij}(t)$ is the (i,j)th entry of the adjacency matrix $\mathcal{A}$ at time *t*.

where, $\widehat{\Theta}_i = [\hat{\varphi}_i, \hat{\theta}_i, \hat{\psi}_i]^T$. α and β are positive constants. (4.7) ensures that $\hat{p}_i(t) \longrightarrow p_0(t)$ and $\hat{v}_i(t) \longrightarrow v_0(t)$ in finite time.

When $\beta_1 > \xi$, perfect estimation of position and velocity will be generated for every

$$t \geq \bar{T}_{s_1} \qquad (4.8)$$

where $\bar{T}_s = 2t_1 + \frac{(\xi+\beta_1)t_1^2}{2\alpha} + \frac{(|\hat{p}_i(0)-p_0(0)|+|\hat{v}_i(0)-v_0(t)|)t_1}{\alpha_1}$, and

$$t_1 = \frac{\max_i\{|\hat{v}_i(0)-v_0(0)|\}}{\beta_1-\xi} \qquad (4.9)$$

Similar approach is taken for angular position and velocity estimation. Thus, we have,

$$\dot{\widehat{\Theta}}_i(t) = \hat{v}_i(t) - \alpha_2\, sgn\left\{\sum_{j=0}^{n} a_{ij}(t)\big[\widehat{\Theta}_i(t) - \widehat{\Theta}_j(t)\big]\right\} \qquad (4.10(a))$$

$$\ddot{\widehat{\Theta}}_i(t) = -\beta_2\, sgn\left\{\sum_{j=0}^{n} a_{ij}(t)\left[\,\dot{\widehat{\Theta}}_i(t) - \dot{\widehat{\Theta}}_j(t)\right]\right\}, \qquad i = 1,\dots,n \qquad (4.10(b))$$

We have similar condition on estimation generation time. So, when $\beta_2 > \zeta$, the estimation of orientation will be generated for every

$$t \geq \bar{T}_{s_2} \qquad (4.11)$$

we can write for $\bar{T}_{s_2}$ as

$$\bar{T}_{s_2} = 2t_1 + \frac{(\zeta + \beta_2)t_1^2}{2\alpha_2} + \frac{\left(\left|\widehat{\Theta}_i(0) - \Theta_0(0)\right| + \left|\dot{\widehat{\Theta}}_i(0) - \dot{\widehat{\Theta}}_0(t)\right|\right) t_1}{\alpha_2}$$

$$t_1 = \frac{\max_i \left\{\left|\dot{\widehat{\Theta}}_i(0) - \dot{\widehat{\Theta}}_0(0)\right|\right\}}{\beta_2 - \zeta} \tag{4.12}$$

4.3 Problem Statement

Based on the estimated states from the group of quadrotors using (4.7) and (4.10), we formulate the following tracking control problem.

Tracking Control Problem: Given the estimated trajectory $\hat{p}_i(t)$ and the estimated yaw angle $\hat{\psi}_i(t)$ for the *i*th quadrotor, if the states $(p_i, v_i, \Theta_i, \omega_i)$ of the system is known and the inertia parameters m_i and J_i are unknown, the control problem is to design state feedback control inputs f_i and τ_i such that

$$\lim_{t\to\infty}(p_i(t) - \hat{p}_i(t) - d_{i0}) = 0 \tag{4.13}$$

$$\lim_{t\to\infty}\left(\psi_i(t) - \hat{\psi}_i(t)\right) = 0 \tag{4.14}$$

where $p_i(t)$ and $v_i(t)$ are the position and velocity respectively for the *i*th quadrotor. $d_{i0} \in \mathbb{R}^3$ defines the distance vector. It provides the desired distance between agent *i* and the virtual leader. Using different values in d_{i0} leads to different formation of the agents. The formation can be constant or variable depending on whether d_{i0} is constant or not. In this case, constant values for d_{i0} is considered, making the structure of the formation structure constant.

4.4 Controller Design

The formation control problem is solved by coupling the estimation and the tracking tasks.

Step 1: We consider for the ith quadrotor as $e_{p_i} = p_i(t) - \hat{p}_i(t) - d_{i0}$ and $e_{v_i} = v_i(t) - \dot{\hat{p}}_i(t)$.

Then we have

$$\dot{e}_{p_i} = e_{v_i} \tag{4.15}$$

$$\dot{e}_{v_i} = \frac{1}{m_i} f_i R_i e_3 - g e_3 - \ddot{\hat{p}}_i = \nu_i \tag{4.16}$$

Which is similar to the error dynamics formed in Chapter I. So, we construct the cost function as the following.

$$J_{i_1} = \int_0^\infty \left(e_i^T Q_{1_i} e_i + \nu_i{}^T R_{1_i} \nu_i\right) d\tau \tag{4.17}$$

where, Q_{1_i} and R_{1_i} are positive definite matrices and $e_i^T = [e_{p_i}^T, e_{v_i}^T]$. We find ν_i such that the cost function is minimized. We can rewrite (2.12) for optimal value of ν_i as

$$\nu_i{}^* = -K_{1_i} e_i \tag{4.18}$$

where the value of K_{1_i} can be calculated using (2.14) and the algebraic Riccati equation (2.15). They are rewritten as

$$K_{1_i} = R_{1_i}{}^{-1} B_{1_i}{}^T P_{1_i} e_i \tag{4.19}$$

$$A_{1_i}{}^T P_{1_i} + P_{1_i} A_{1_i} - P_{1_i} B_{1_i} R_{1_i}{}^{-1} B_{1_i}{}^T P_{1_i} + Q_{1_i} = 0 \tag{4.20}$$

The terms have been defined in chapter 2 and contains same characteristics for a specific quadrotor i.

Step 2: We have developed the control approach for similar problem, and we can rewrite the thrust force expression as

$$f_i = m_i||v_i + ge_3 + \ddot{\hat{p}}_i|| \tag{4.21}$$

Each of *n* quadrotor will have its own trajectory to track based on its estimation. So, the tracking problem can be solved using (12) from chapter I.

Step 3: Similarly, we can write from (2.23)

$$\tau_i = J_i W_i^{-1}\left(u_i - \dot{W}_i\omega_i + \ddot{\Theta}_i^d\right) - S_i(J_i\omega_i) \tag{4.22}$$

with W_i and Θ_i are defined by (2.8). Similarly, the cost function for the *i*th quadrotor is formulated as

$$J_{i_2} = \int_0^\infty \left(z_i^T Q_{i_2} z_i \; + \; u_i^T R_{i_2} u_i\right) d\tau \tag{4.23}$$

where $z_i^T = \left[\tilde{\Theta}_i^T, \dot{\tilde{\Theta}}_i^T\right]^{\mathrm{T}}$, and $\ddot{\tilde{\Theta}}_i = u_i$. We design u_i such that the cost function J_{i_2} is minimized. For the optimal value of u_i, we can rewrite (2.25) as

$$u_i^* = \; -K_{2_i} z_i \tag{4.24}$$

where the value of K_{2_i} can be calculated using (2.26) and the algebraic Riccati equation (2.27). They are restated for *i*th quadrotor in general as,

$$K_{2_i} = \; R_{2_i}^{-1} B_{2_i}^T P_{2_i} z_i \tag{4.25}$$

$$A_{2_i}^T P_{2_i} + P_{2_i} A_{2_i} - P_{2_i} B_{2_i} R_{2_i}^{-1} B_{2_i}^T P_{2_i} + Q_{2_i} = 0 \tag{4.26}$$

We can summarize the overall procedure in the following theorem.

Theorem 4.1: Provided that the virtual leader has directed communication paths to every other vehicle at each time instant, the system described in (4.3)-(4.6), using estimations from (4.7) and (4.10), the control inputs (f_i, τ_i) in (4.21) and (4.22) with optimal gains from (4.19) and (4.25) ensure that (4.13) – (4.14) are satisfied.

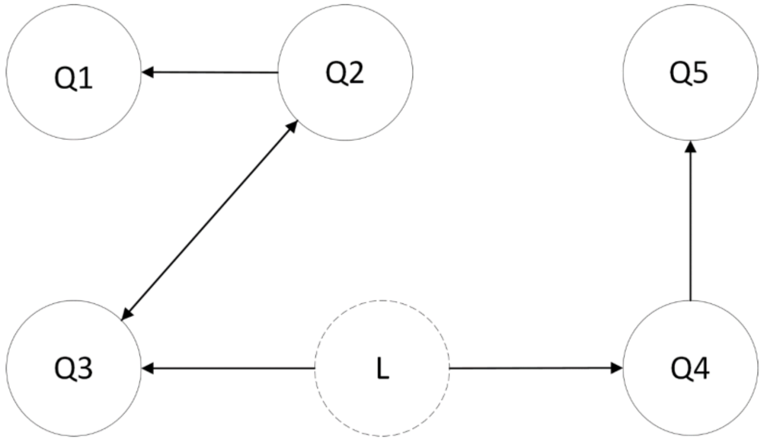
4.5 Simulation

Simulation has been carried out to validate the proposed theories. The simulation considers a group of five quadrotors. The quadrotors have their internal communication network topology defined by Figure 4.2. The directed quadrotor network topology switches each second from Figure 4.2(a) to Figure 4.2(b). As per the network topology, the two adjacent matrices have been formulated as shown in the example 1. L denotes the virtual leader and Q denotes the quadrotors. The virtual leader's trajectory $p_0(t)$ and $\psi_0(t)$ is chosen as

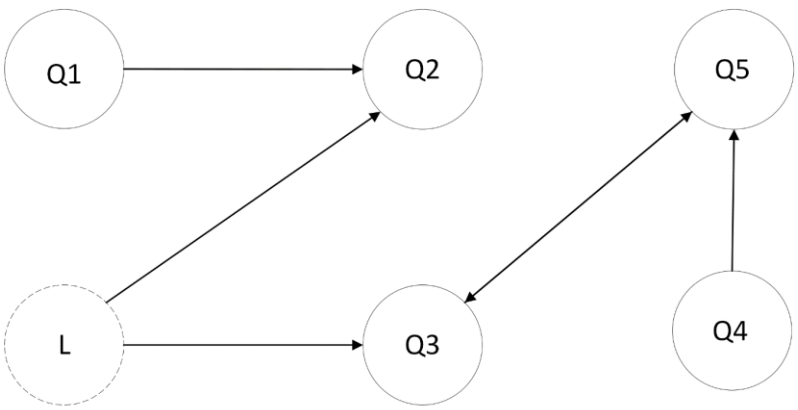
$$p_0(t) = \begin{bmatrix} sin(2t) + \frac{t}{5} \\ sin(1.5t) + 2t \\ cos(2t) \end{bmatrix}$$

$$\psi_0(t) = \frac{\pi}{2} sin(0.2t)$$

The initial quadrotor positions for the estimators embedded in each of the five quadrotors are given as $\hat{p}_1(0) = [4, 1, 1.5]^{\mathrm{T}}$, $\hat{p}_2(0) = [-2, 2, 2]^{\mathrm{T}}$, $\hat{p}_3(0) = [-3, 3, 0.2]^{\mathrm{T}}$, $\hat{p}_4(0) = [1, -3, 2.5]^{\mathrm{T}}$, and $\hat{p}_5(0) = [2, -1, 0]^{\mathrm{T}}$. Each quadrotor estimator starts from the initial states and makes decentralized estimation of the states of the virtual leader. Figure 4.3 shows the position estimates made by each quadrotor in finite time.



(a)

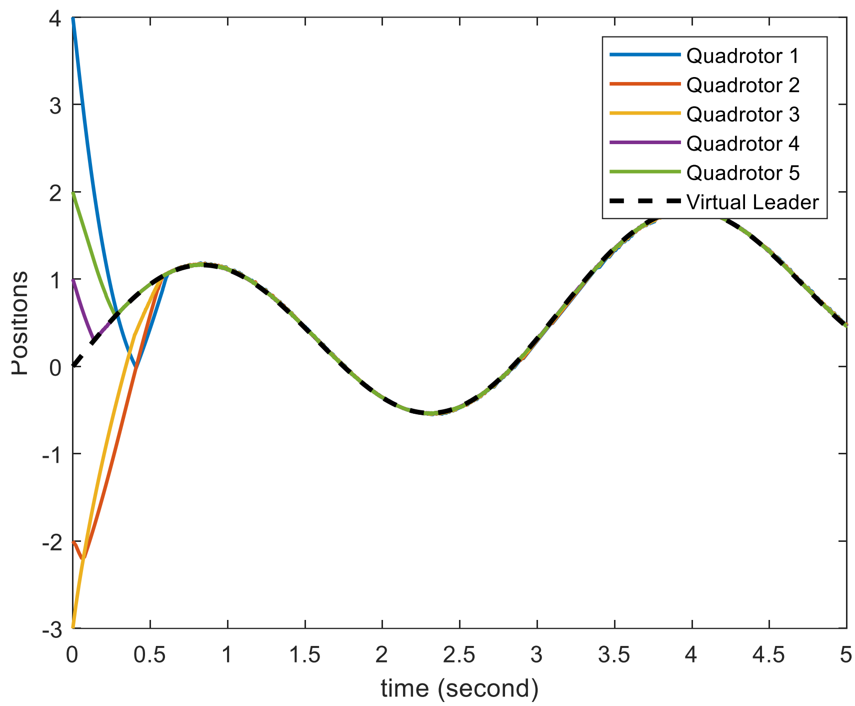


(b)

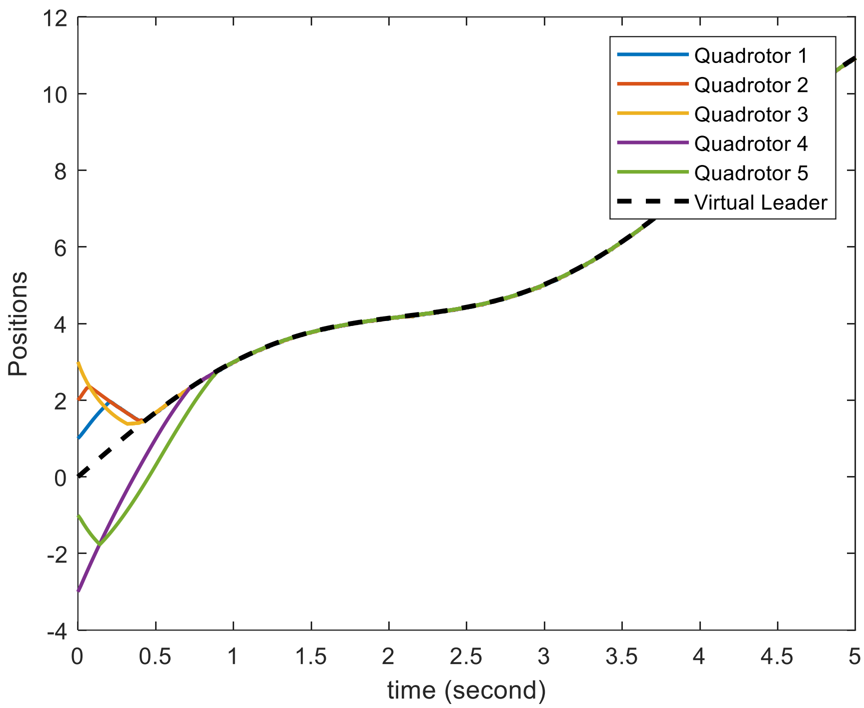
Figure 4.2: Communication topology for the quadrotor group with a virtual leader

(a) Graph 1, (b) Graph 2

The angular estimation result is visualized in Figure 4.5. Here, the initial values for the angles are selected as $\hat{\psi}_i(0) = [0.5; 0.2; -0.6; 0.4;\ 0.7]^{\mathrm{T}}, i\ =\ 1,2,\ldots 5$. From the simulation results, it is shown that the quadrotors can estimate the states of the virtual leader accurately in finite time for a given directed network topology with switching. With the estimation results, tracking control is employed for each quadrotor to track the estimated trajectories influenced by the virtual leader's states. Five different input signals make the trajectory of the respective quadrotor converge with the estimated values. The initial values for each quadrotor position are selected as $p_1(0) = [0.1, 0.2, 1]^{\mathrm{T}}$, $p_2(0) = [0.2, 0.1, 0.9]^{\mathrm{T}}$, $p_3(0) = [0.5, 0.4, 0.8]^{\mathrm{T}}$, $p_4(0) = [-0.6, 0.3, 1.2]^{\mathrm{T}}$, $p_5(0) = [0.7, 0.5, 1.4]^{\mathrm{T}}$. The initial values for the attitude are selected as $\Theta_i(0) = [0.2, 0.3, 1, 0.8, 0.5]^T, i = 1,2, \ldots 5$. The simulation result shown in the Figure 4.6 strengthens the proposed control method as it makes the multiple quadrotor system converge with the virtual leader's states.

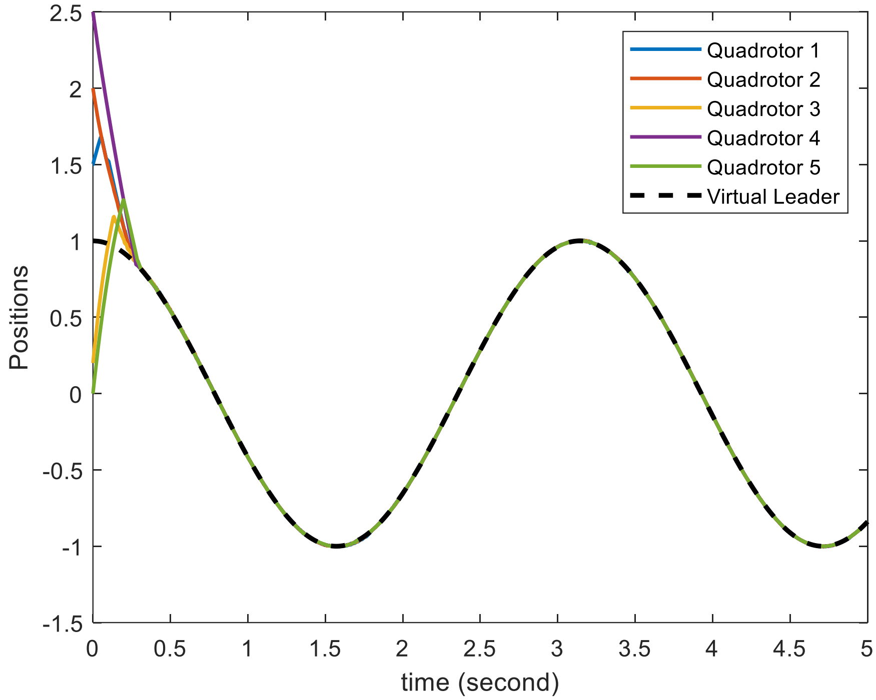


(a)



(b)

Figure 4.3: Position estimations of the virtual leader (a) x axis (b) y axis (c) z axis



(c)

Figure 4.3, cont.

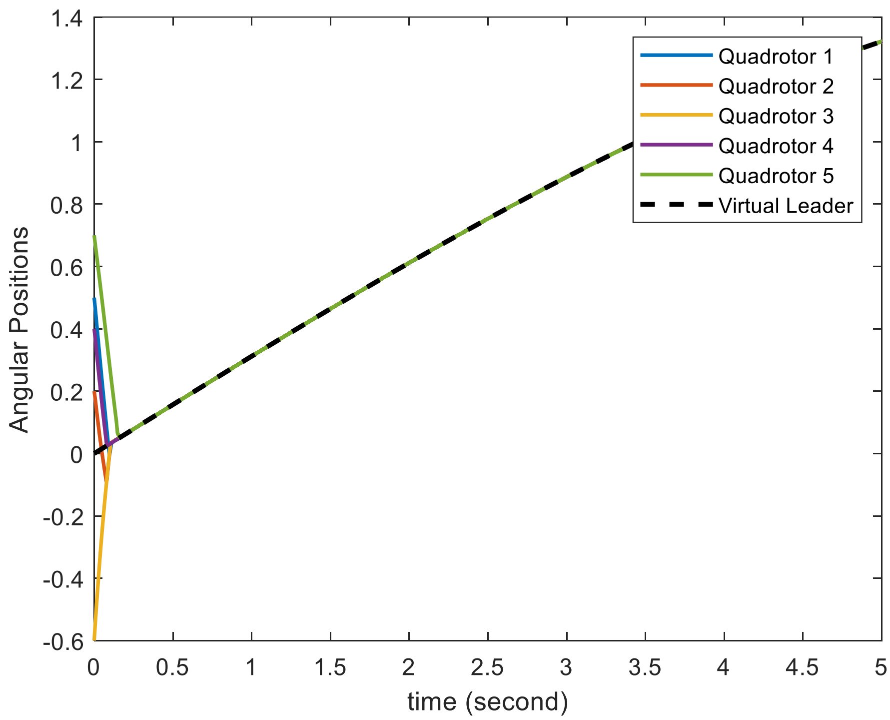
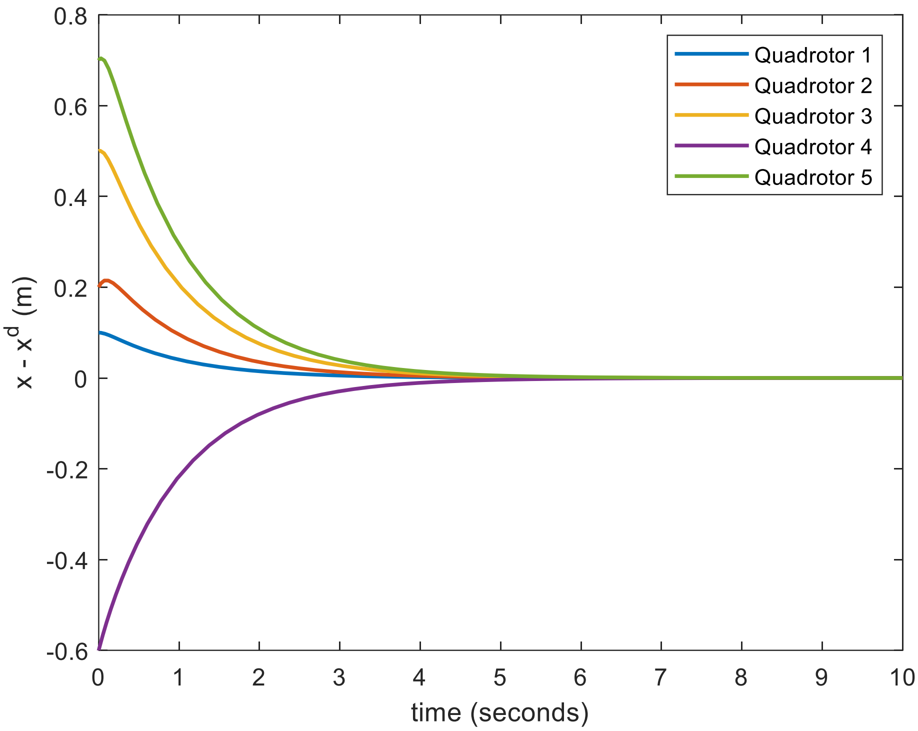
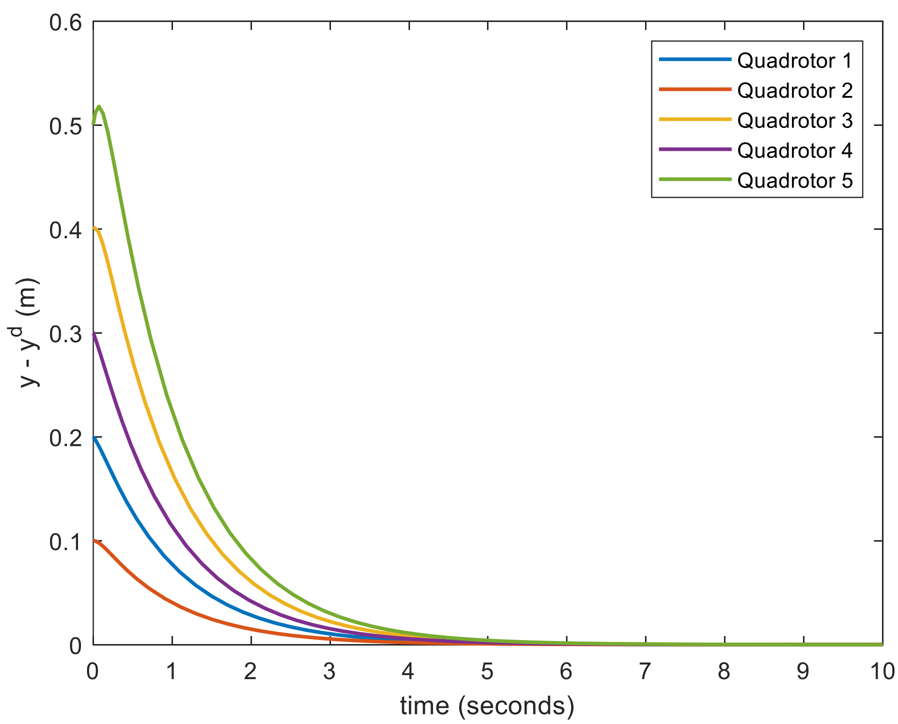


Figure 4.4: Yaw angle estimations of the virtual leader

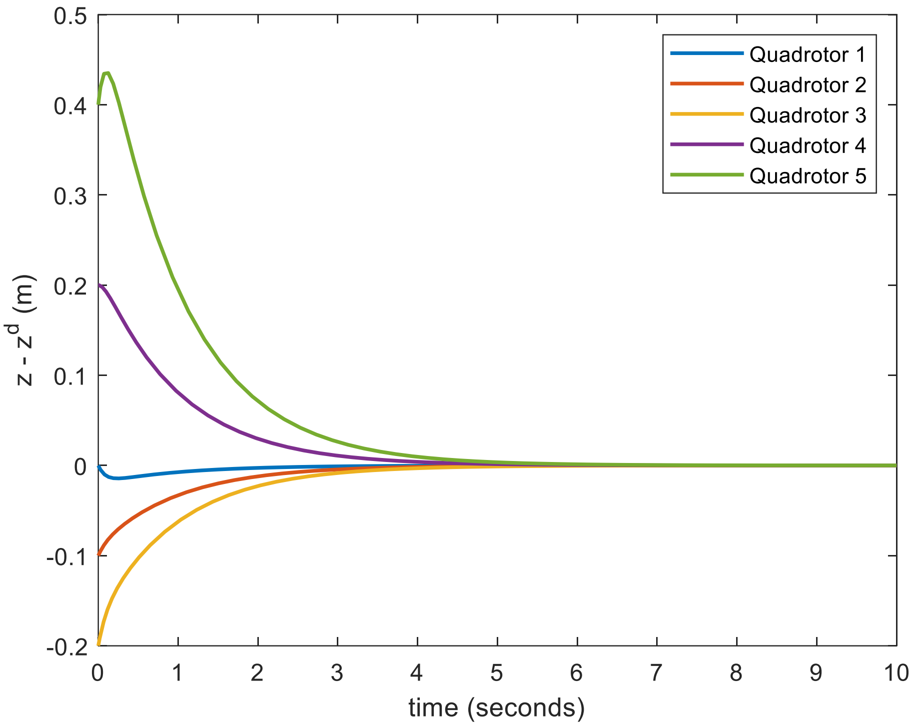


(a)



(b)

Figure 4.5: Position tracking errors of each quadrotor (a) x axis (b) y axis (c) z axis



(c)

Figure 4.5, cont.

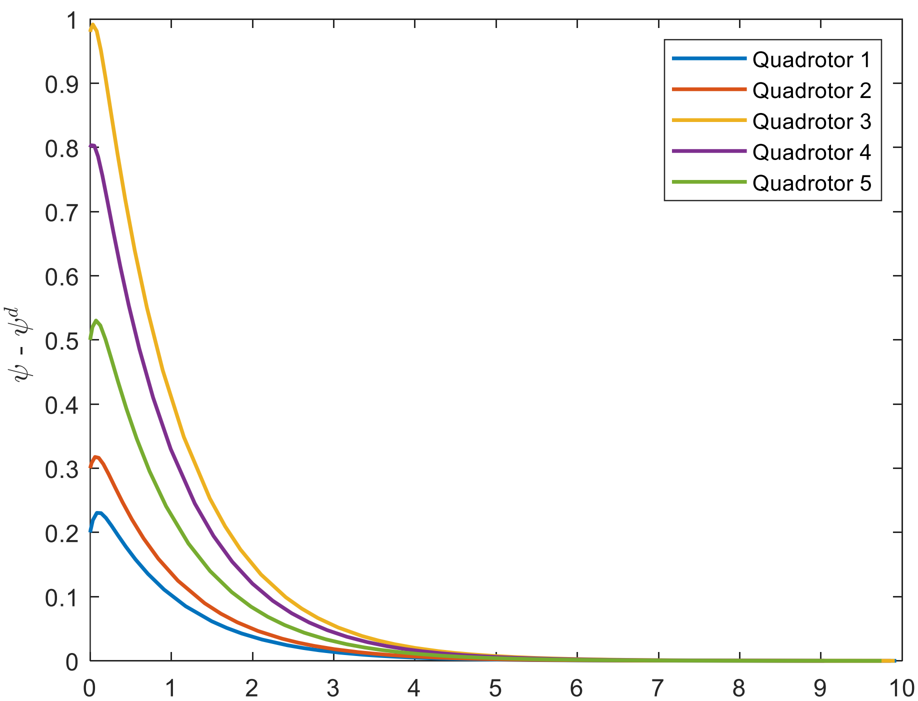


Figure 4.6: Attitude tracking errors of each quadrotor (a) x axis (b) y axis (c) z axis

CHAPTER V

CONCLUSIONS AND FUTURE WORK

5.1 Conclusion

In this thesis, single and multiple quadrotor formation control methods are studied and. For a 6-DOF quadrotor, optimal control approaches have been proposed to control the position and orientation of the quadrotor. With the provided assumptions, the proposed controller makes the quadrotor track the desired trajectory. This theory is extended in formation control of multiple quadrotors using a virtual leader-follower approach. A virtual leader is considered where only the neighbors to it has its information. The switching communication network topology is realized using graph theory. For dealing with uncertainty and constraints, a robust adaptive tracking control and a saturation robust adaptive tracking control have been proposed. Simulations have been modelled in MATLAB/SIMULINK environment.

The dynamics of the quadrotor has total thrust and torque as the control input. The control problem is to design the state feedback control inputs for converging the quadrotor system response to the desired trajectory. The error dynamics have been formulated to define the cost function to be optimized. Similar approach has been taken for attitude tracking control with another cost function.

For uncertainty in the system dynamics, a robust adaptive tracking controller is designed such that the position and attitude of the quadrotor still manage to converge to the desired values.

Quaternions have been used to define the orientation of the quadrotors and further modified to design the suitable control method. Input constraint has been considered and an auxiliary compensated system is developed to deal with the constraint. For input constraints along with the uncertainty, with the aid of an auxiliary compensated system, a saturation robust adaptive controller is designed.

For multiple quadrotor formation control, a virtual leader is considered where its locally known by its neighboring followers. The information topology switches every second. Provided the virtual leader's position, decentralized second order sliding mode estimators have been used to estimate the leader's position and attitude for every quadrotor in the system. The estimated trajectory is tracked using the optimal control theory provided for single quadrotor trajectory tracking.

The control gain has been provided by solving for the optimal control problem that optimizes the associated cost functions. For this purpose, linear quadratic approach has been implemented for each quadrotor. The decentralized nature of control provides flexibility and scalability for the application of the proposed controller in practice.

5.2 Future Works

Future theoretical work will investigate the performance of multi quadrotor system with the proposed controller under parametric and nonparametric uncertainties and noise. A detailed stability analysis will be performed. Adjustment will be made to make the controller more robust under such environment. For formation control, collision avoidance and communication delay will be studied to imposed stringent formation and tracking rules. The proposed approach will be extended for multirobot systems with uncertainty and input saturation. As for the estimation, reinforcement learning based approaches will be included in future works to make faster results and adjustments to changes in the system. Barrier function will be used to include the constraints.

REFERENCES

[1] S. Bouabdallah, A. Noth, and R. Siegwart, "PID vs LQ control techniques applied to an indoor micro Quadrotor," in *2004 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2004, vol. 3, pp. 2451–2456. doi: 10.1109/iros.2004.1389776.

[2] I. D. Cowling, O. A. Yakimenko, J. F. Whidborne, and A. K. Cooke, "Direct method based control system for an autonomous quadrotor," *Journal of Intelligent and Robotic Systems: Theory and Applications*, vol. 60, no. 2, pp. 285–316, Nov. 2010, doi: 10.1007/s10846-010-9416-9.

[3] I. D. Cowling, O. A. Yakimenko, J. F. Whidborne, and A. K. Cooke, "A prototype of an autonomous controller for a quadrotor UAV," in *Eur. Control Conf.*, 2007, pp. 1–4.

[4] T. Ryan and H. J. Kim, "LMI-based gain synthesis for simple robust quadrotor control," *IEEE Transactions on Automation Science and Engineering*, vol. 10, no. 4, pp. 1173–1178, 2013, doi: 10.1109/TASE.2013.2259156.

[5] A. Roza and M. Maggiore, "Path following controller for a quadrotor helicopter," in *Proceedings of the American Control Conference*, 2012, pp. 4655–4660. doi: 10.1109/acc.2012.6315061.

[6] R. Xu and Ü. Özgüner, "Sliding mode control of a quadrotor helicopter," in *Proceedings of the IEEE Conference on Decision and Control*, 2006, pp. 4957–4962. doi: 10.1109/cdc.2006.377588.

[7] R. Vilela Lopes, P. Henrique de Rodrigues Quemel Assis Santana, ieeeorg Geovany Araújo Borges, and unbbr João Yoshiyuki Ishihara, "Model Predictive Control applied to tracking and attitude stabilization of a VTOL quadrotor aircraft," 2012.

[8] D. Lee, H. J. Kim, and S. Sastry, "Feedback linearization vs. adaptive sliding mode control for a quadrotor helicopter," *International Journal of Control, Automation and Systems*, vol. 7, no. 3, pp. 419–428, Jun. 2009, doi: 10.1007/s12555-009-0311-8.

[9] E. Frazzoli, M. A. Dahleh, and E. Feron, "Trajectory tracking control design for autonomous helicopters using a backstepping algorithm," in *Proceedings of the American Control Conference*, 2000, vol. 6, pp. 4102–4107. doi: 10.1109/ACC.2000.876993.

[10] A. Das, F. Lewis, and K. Subbarao, "Backstepping approach for controlling a quadrotor using lagrange form dynamics," *Journal of Intelligent and Robotic Systems: Theory and Applications*, vol. 56, no. 1–2, pp. 127–151, Sep. 2009, doi: 10.1007/s10846-009-9331-0.

[11] S. Zhao, W. Dong, and J. A. Farrell, "Quaternion-based trajectory tracking control of VTOL-UAVs using command filtered backstepping," in *Proceedings of the American Control Conference*, 2013, pp. 1018–1023. doi: 10.1109/acc.2013.6579970.

[12] A. Roberts and A. Tayebi, “Adaptive position tracking of VTOL UAVs,” *IEEE Transactions on Robotics*, vol. 27, no. 1, pp. 129–142, Feb. 2011, doi: 10.1109/TRO.2010.2092870.

[13] E. C. Suicmez and A. T. Kutay, “Optimal path tracking control of a quadrotor UAV,” in *2014 International Conference on Unmanned Aircraft Systems, ICUAS 2014 - Conference Proceedings*, 2014, pp. 115–125. doi: 10.1109/ICUAS.2014.6842246.

[14] N. Koksal, M. Jalalmaab, and B. Fidan, “Adaptive linear quadratic attitude tracking control of a quadrotor uav based on imu sensor data fusion,” *Sensors (Switzerland)*, vol. 19, no. 1, 2019, doi: 10.3390/s19010046.

[15] W. Dong, M. N. A. Dony, and M. Rafat, “Tracking control of vertical taking-off and landing vehicles with parametric uncertainty,” *International Journal of Adaptive Control and Signal Processing*, vol. 34, no. 9, pp. 1294–1307, Sep. 2020, doi: 10.1002/acs.3143.

[16] L. Besnard, Y. B. Shtessel, and B. Landrum, “Quadrotor vehicle control via sliding mode controller driven by sliding mode disturbance observer,” *J Franklin Inst*, vol. 349, no. 2, pp. 658–684, Mar. 2012, doi: 10.1016/j.jfranklin.2011.06.031.

[17] R. Xu and Ü. Özgüner, “Sliding mode control of a class of underactuated systems,” *Automatica*, vol. 44, no. 1, pp. 233–241, Jan. 2008, doi: 10.1016/j.automatica.2007.05.014.

[18] B. Zhao, B. Xian, Y. Zhang, and X. Zhang, “Nonlinear robust sliding mode control of a quadrotor unmanned aerial vehicle based on immersion and invariance method,” *International Journal of Robust and Nonlinear Control*, vol. 25, no. 18, pp. 3714–3731, Dec. 2015, doi: 10.1002/rnc.3290.

[19] Y. Zou, “Nonlinear robust adaptive hierarchical sliding mode control approach for quadrotors,” *International Journal of Robust and Nonlinear Control*, vol. 27, no. 6, pp. 925–941, Apr. 2017, doi: 10.1002/rnc.3607.

[20] B. Xu, “Composite Learning Finite-Time Control with Application to Quadrotors,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 10, pp. 1806–1815, Oct. 2018, doi: 10.1109/TSMC.2017.2698473.

[21] B. Zhao, B. Xian, Y. Zhang, and X. Zhang, “Nonlinear Robust Adaptive Tracking Control of a Quadrotor UAV Via Immersion and Invariance Methodology,” *IEEE Transactions on Industrial Electronics*, vol. 62, no. 5, pp. 2891–2902, May 2015, doi: 10.1109/TIE.2014.2364982.

[22] J. Hu and H. Zhang, “Immersion and invariance based command-filtered adaptive backstepping control of VTOL vehicles,” *Automatica*, vol. 49, no. 7, pp. 2160–2167, Jul. 2013, doi: 10.1016/j.automatica.2013.03.019.

[23] Y. Zou and Z. Meng, “Immersion and Invariance-Based Adaptive Controller for Quadrotor Systems,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 11, pp. 2288–2297, Nov. 2019, doi: 10.1109/TSMC.2018.2790929.

[24] K. Alexis, G. Nikolakopoulos, and A. Tzes, “Switching model predictive attitude control for a quadrotor helicopter subject to atmospheric disturbances,” *Control Engineering Practice*, vol. 19, no. 10, pp. 1195–1207, Oct. 2011, doi: 10.1016/j.conengprac.2011.06.010.

[25] Z. Zongyu, “Adaptive Trajectory Tracking Control of a Quadrotor Unmanned Aircraft,” 2011.

[26] J. Xu, P. Shi, C. C. Lim, C. Cai, and Y. Zou, “Reliable Tracking Control for Under-Actuated Quadrotors with Wind Disturbances,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 10, pp. 2059–2070, Oct. 2019, doi: 10.1109/TSMC.2017.2782662.

[27] J. Yu, P. Shi, W. Dong, and C. Lin, “Command Filtering-Based Fuzzy Control for Nonlinear Systems with Saturation Input,” *IEEE Transactions on Cybernetics*, vol. 47, no. 9, pp. 2472–2479, Sep. 2017, doi: 10.1109/TCYB.2016.2633367.

[28] J. Ma, Z. Zheng, and P. Li, “Adaptive Dynamic Surface Control of a Class of Nonlinear Systems with Unknown Direction Control Gains and Input Saturation,” *IEEE Transactions on Cybernetics*, vol. 45, no. 4, pp. 728–741, Apr. 2015, doi: 10.1109/TCYB.2014.2334695.

[29] Y. Li, S. Tong, and T. Li, “Composite adaptive fuzzy output feedback control design for uncertain nonlinear strict-feedback systems with input saturation,” *IEEE Transactions on Cybernetics*, vol. 45, no. 10, pp. 2299–2308, Oct. 2015, doi: 10.1109/TCYB.2014.2370645.

[30] T. Balch and R. C. Arkin, “Behavior-based formation control for multirobot teams,” *IEEE Transactions on Robotics and Automation*, vol. 14, no. 6, pp. 926–939, 1998, doi: 10.1109/70.736776.

[31] H. G. Tanner, G. J. Pappas, and V. Kumar, “Leader-to-formation stability,” *IEEE Transactions on Robotics and Automation*, vol. 20, no. 3, pp. 443–455, Jun. 2004, doi: 10.1109/TRA.2004.825275.

[32] V. Roldão, R. Cunha, D. Cabecinhas, C. Silvestre, and P. Oliveira, “A leader-following trajectory generator with application to quadrotor formation flight,” *Robotics and Autonomous Systems*, vol. 62, no. 10, pp. 1597–1609, 2014, doi: 10.1016/j.robot.2014.05.002.

[33] J. Hu and G. Feng, “Distributed tracking control of leader-follower multi-agent systems under noisy measurement,” *Automatica*, vol. 46, no. 8, pp. 1382–1387, Aug. 2010, doi: 10.1016/j.automatica.2010.05.020.

[34] M. A. Lewis and K.-H. Tan, “High Precision Formation Control of Mobile Robots Using Virtual Structures,” Kluwer Academic Publishers, 1997.

[35] W. Ren and R. W. Beard, “Decentralized Scheme for Spacecraft Formation Flying via the Virtual Structure Approach,” *Journal of Guidance, Control, and Dynamics*, vol. 27, no. 1, pp. 73–82, 2004, doi: 10.2514/1.9287.

[36] J. A. Fax and R. M. Murray, “Information flow and cooperative control of vehicle formations,” *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1465–1476, Sep. 2004, doi: 10.1109/TAC.2004.834433.

[37] R. Olfati-Saber and R. M. Murray, “Distributed structural stabilization and tracking for formations of dynamic multi-agents,” in *Proceedings of the IEEE Conference on Decision and Control*, 2002, vol. 1, pp. 209–215. doi: 10.1109/cdc.2002.1184493.

[38] P. K. C. Wang, “Navigation Strategies for Multiple Autonomous Mobile Robots Moving in Formation.”

[39] B. Vanek, T. Péni, J. Bokor, and G. Balas, “Practical approach to real-time trajectory tracking of UAV formations,” in *Proceedings of the American Control Conference*, 2005, vol. 1, pp. 122–127. doi: 10.1109/acc.2005.1469919.

[40] J. Hu and G. Feng, “Distributed tracking control of leader-follower multi-agent systems under noisy measurement,” *Automatica*, vol. 46, no. 8, pp. 1382–1387, Aug. 2010, doi: 10.1016/j.automatica.2010.05.020.

[41] Z. Hou and I. Fantoni, “Interactive leader-follower consensus of multiple quadrotors based on composite nonlinear feedback control,” *IEEE Transactions on Control Systems Technology*, vol. 26, no. 5, pp. 1732–1743, Sep. 2018, doi: 10.1109/TCST.2017.2738602.

[42] R. Olfati-Saber and R. M. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004, doi: 10.1109/TAC.2004.834113.

[43] E. J. Leffens, F. L. Markley, and M. D. Shuster, “Kalman filtering for spacecraft attitude estimation,” *Journal of Guidance, Control, and Dynamics*, vol. 5, no. 5, pp. 417–429, 1982, doi: 10.2514/3.56190.

[44] J. J. Xiong and E. H. Zheng, “Optimal Kalman Filter for state estimation of a quadrotor UAV,” *Optik (Stuttg)*, vol. 126, no. 21, pp. 2862–2868, Nov. 2015, doi: 10.1016/j.ijleo.2015.07.032.

[45] Y. Cao, W. Ren, and Z. Meng, “Decentralized finite-time sliding mode estimators and their applications in decentralized finite-time formation tracking,” *Systems and Control Letters*, vol. 59, no. 9, pp. 522–529, Sep. 2010, doi: 10.1016/j.sysconle.2010.06.002.

[46] Y. Hong, G. Chen, and L. Bushnell, “Distributed observers design for leader-following control of multi-agent networks,” *Automatica*, vol. 44, no. 3, pp. 846–850, 2008, doi: 10.1016/j.automatica.2007.07.004.

[47] M. D. Shuster, “Survey of attitude representations,” *Journal of the Astronautical Sciences*, vol. 41, pp. 439–517, Oct. 1993

[48] Y. Cao and W. Ren, “Containment control with multiple stationary or dynamic leaders under a directed interaction graph,” in *Proceedings of the IEEE Conference on Decision and Control*, 2009, pp. 3014–3019. doi: 10.1109/CDC.2009.5399946.

BIOGRAPHICAL SKETCH

Shihab Ahmed started pursuing his MS in the Electrical Engineering Department at the University of Texas Rio Grande Valley in Spring 2021. He completed Bachelor's in Electrical and Electronic Engineering from Rajshahi University of Engineering and Technology, Rajshahi, Bangladesh, in 2019. Before joining UTRGV, he worked as an electrical engineer in the power generation sector in Bangladesh. Shihab Ahmed received the prestigious Presidential Graduate Research Assistantship (PGRA) to continue his higher study at the UTRGV where he worked as a Research Assistant under the supervision of Dr. Wenjie Dong. He was awarded the Master of Science in Engineering degree in August of 2022. For any information, anyone can reach out to him using the following email address: shihabdpu@gmail.com