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# An Application of Matrices to the Spread of the COVID 19

Selena Suarez
*The University of Texas Rio Grande Valley*

# AN APPLICATION OF MATRICES TO THE SPREAD OF THE COVID 19

A Thesis

by

SELENA SUAREZ

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THE COVID 19

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SELENA SUAREZ

COMMITTEE MEMBERS

Dr. Jasang Yoon
Chair of Committee

Dr. Paul Bracken
Committee Member

Dr. William Heller
Committee Member

Dr. Hyung Won Kim
Committee Member

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ABSTRACT

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We represented a restaurant seating arrangement using matrices by using 0 entry for someone without covid and 1 entry for someone with covid. Using the matrices we found the best seating arrangements to lessen the spread of covid. We also investigated if there was a factor needed to create a formula that could calculate the matrix that shows who would be affected with covid with each seating arrangement. However, there did not seem to be a clear pattern within the factors. Aside from covid applications, we also investigated the symmetries in seating arrangements and the possible combinations with these arrangements taking into consideration the equivalent matrices. We realized that the most equivalent matrices possible was 8, which happened when there were no symmetries in the matrices. There were 4 equivalent matrices when there was diagonal, vertical, or rotational symmetry. There were 2 equivalent matrices when there was diagonal and rotational or vertical and rotational symmetry. There was only 1 equivalent matrix where there was all symmetries present. This knowledge was used to create a new method for the 4 by 4 case in order to find all possible combinations of seating arrangements and this new method made the process much more efficient. After concluding our results using this method, we realized there was symmetry for possible combinations and for ways of spreading covid which can be seen in the list of tables.

DEDICATION

I dedicate this to my beloved husband, Jose Olivarez, and to my mom and dad.

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I would like to say thank you to Dr. Yoon for continuously meeting with me and providing timely feedback. I appreciate the encouragement and enthusiasm shown with this thesis. Thank you for making the process easier and for asking the right questions to guide this research. I would also like to thank my husband for providing me with the support and allowing me to devote time to my thesis. Thank you for always encouraging me to do my best in the work that I do. Additionally, I want to thank my parents and family for always believing in me and supporting me throughout everything I do. You all made this possible.

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# CHAPTER I

## INTRODUCTION

The purpose of this study is to show how the powers of adjacency and connectivity matrices and inite (undirected) graphs can be used to investigate the spread of Covid-19 between people at a restaurant. We also show the best seating arrangements for restaurants given the number of customers that have Covid. We investigate those seating arrangements further to see if there is a formula that will track the spread of Covid-19. We first recall the notion of graphs; most of the discussion in the rest of this section is taken from **GJSW** and **Hog1**. We will denote by $G = (V, E) = (V(G), E(G))$ a finite (undirected) graph. The set $V(G)$ of *vertices* is finite, and the set $E(G)$ of *edges* is a subset of the set $\{\{i, j\} : i, j \in V(G)\}$. We allow that $E$ may contain loops, i.e., $i$ may equal $j$ for an edge $\{i, j\} \in E$. Two vertices connected by an edge are said to be *adjacent.* Notice that two vertices may be connected by more than one edge, that a vertex need not be connected to any other vertex, and that a vertex may be connected to itself (a loop). The *order* of $G$ is the number of vertices of $G$. An adjacency matrix can be made if we have any given seating arrangement. The seating arrangement is made into a graph, $G$. The vertices would be the people sitting at the table and the edges would be representing the spread of Covid through the interaction between those vertices. The $(i, j)$ entry of $m \times n$ adjacency matrix is 1 if the person at the position with COVID, and 0 if the person without COVID. From this we can ask the following:

Problem 1: What is the total number of ways Covid can be spread from person A to person D through n interactions? Problem 2: For $n \in \mathbb{N}$, $n$ people with covid, we need to find the best seating arrangement to reduce the spread of covid.

# CHAPTER II

## NOTATION AND PRELIMINARIES

We first consider the following example of a restaurant seating arrangement $R$:

$$\text{Consider}\begin{pmatrix} \text{Booth A} & & \\ \text{Booth B} & \text{Booth C} & \text{Booth D} \end{pmatrix}$$

with $d(A,B), d(A,C), d(B,C), d(C,D) < 6$, where $d(A,B)$ means the distance between Booth A and Booth B. Let

$$\begin{aligned} a &: \quad \text{person at Booth A} \\ b &: \quad \text{person at Booth B} \\ c &: \quad \text{person at Booth C} \\ d &: \quad \text{person at Booth D} \end{aligned}$$

and $R$ :

$$\begin{array}{ccc} a & \longleftrightarrow & c \\ \updownarrow & \nearrow\swarrow & \updownarrow \\ b & & d \end{array} \tag{2.1}$$

Suppose you want to know how many interactions are required to spread COVID from person $a$ to $d$. We let

$$m_{ij} = \begin{cases} 1, & \text{if COVID can be spread from person } i \text{ to person } j \\ 0, & \text{otherwise.} \end{cases}$$

Then, we have $M_R = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ and $M_R^2$ means that the total number of ways COVID can be spread from person $i$ to person $j$ through 2 interactions. Thus, $M_R^k$ means that the total number of ways COVID can be spread from person $i$ to person $j$ through $k$ interactions.

$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} =$

| | | |
|---|---|---|
| X | O | X |
| X | X | O |
| X | X | X |

Figure 2.1: The above matrix is demonstrating a seating arrangement with 9 tables

$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} =$

| | | | |
|---|---|---|---|
| X | X | X | X |
| X | O | X | X |
| X | X | O | X |
| X | O | X | X |

Figure 2.2: The above matrix is demonstrating a seating arrangement with 16 tables.

$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$

| | | | |
|---|---|---|---|
| O | O | X | X |
| X | O | X | X |
| X | X | X | X |
| X | X | X | X |

O O → X X

↓

X ← O → X X

↓

X X X X

X X X X

Figure 2.3: 5 ways of spreading Covid to the people around the ones that have Covid.

Next, we have:

$$\mathit{Rotational\ only}: \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \mathit{Diagonal\ only}: \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathit{Vertical\ only}: \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 2.4: The new method used for the 4x4 cases 1.

$$\mathit{Diagonal\ and\ Rotational\ Symmetry}:$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathit{Vertical\ and\ Rotational\ Symmetry}:$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Figure 2.5: The new method used for the 4x4 cases 2.

$All\ 3\ Symmetries$:

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Figure 2.6: The new method used for the 4x4 cases 3.

# CHAPTER III

## MAIN RESULTS

$n$ **tables with only the first row and column are seated,** where $n \geq 3$ :

**Theorem 1.** *Let* $A_1 = \begin{pmatrix} 0 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 0 \end{pmatrix} \in M_n(\mathbb{R})$ *and* $s_{n-2} = \begin{pmatrix} -1 & -1 & \cdots & -1 \\ -1 & -1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & -1 \end{pmatrix} \in M_{n-2}(\mathbb{R})$, *where* $n \geq 3$. *Let* $S_n = 0 \oplus s_{n-2} \in M_n(\mathbb{R})$. *Then, we have*

$$A_1^2 = \begin{pmatrix} n-1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & \cdots & 1 \end{pmatrix} \text{ and}$$

$$A_1 + A_1^2 + S_n = \begin{pmatrix} n-1 & 1 & \cdots & 1 & 1 \\ 1 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & 0 & \cdots & 0 \\ 1 & 1 & \vdots & \ddots & \vdots \\ 1 & 1 & 0 & \cdots & 0 \end{pmatrix}.$$

**Remark.** *If* $2n-2$ *tables are seated, the best is determined by the seating that has the least ways of spreading.*

$n$ **tables with only the first row and column and the diagonal tables are seated,** where $n \geq 6$ :

**Theorem 2.** *Let* $B_1 = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 1 \end{pmatrix} \in M_n(\mathbb{R})$ *and* $t_{n-2} = \begin{pmatrix} 0 & -1 & \cdots & -1 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 \\ 0 & \cdots & 0 & 0 \end{pmatrix} \in M_{n-2}(\mathbb{R})$, *where* $n \geq 6$. *Let* $T_n = \begin{pmatrix} 0 & t_{n-2} \\ 0 & 0 \end{pmatrix} \in M_n(\mathbb{R})$ $(n \geq 6)$. *Then, we have*

$$B_1^2 = \begin{pmatrix} n+1 & 2 & 2 & \cdots & 2 \\ 2 & 2 & 1 & \cdots & 1 \\ 2 & 1 & 2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 1 \\ 2 & 1 & \cdots & 1 & 2 \end{pmatrix} \text{ and}$$

$$B_1 + B_1^2 + T_n^* + T_n = \begin{pmatrix} n+4 & 3 & 3 & \cdots & 3 & 3 & \cdots & 3 \\ 3 & 3 & 1 & \cdots & 1 & 1 & \cdots & 1 \\ 3 & 1 & 3 & \ddots & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & 1 & \ddots & \ddots & \vdots \\ 3 & 1 & \ddots & 1 & 3 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 & \ddots & 3 & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & 3 & 1 \\ 3 & 1 & 0 & \cdots & 0 & 1 & 1 & 3 \end{pmatrix}.$$

**Example 1.** $3 \times 3 = 9$ *tables with* 9 *people and* 2 *with COVID.*

*By using permutations of rows and columns, we observe that there are* 8 *possible combinations of seating arrangements where* 2 *people have COVID.*

*We first consider* $C(3)_{21}$*, then* $C(3)_{24}$ *and* $C(3)_{25}$ *and so on.*

*Next we consider* $C(3)_{26}$*, then* $C(3)_{27}$ *and* $C(3)_{23}$ *and so on. Finally, we consider* $C(3)_{28}$*. Therefore,* *we have the following:*

$$
\begin{aligned}
C(3)_{21} &= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{22} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{23} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\
C(3)_{24} &= \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{25} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{26} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \\
C(3)_{27} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{28} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.
\end{aligned}
$$

**Remark.** *By a simple calculation, we can observe that $C(3)_{21}$ is the best to lessen the spread of the $2$ people with COVID because this is the seating arrangement with 3 ways of spreading where as all the others have more than 3 ways of spreading.*

**Example 2.** $3 \times 3 = 9$ *tables with* $9$ *people and* $3$ *with COVID.*
*By using permutations of rows and columns, we observe that there are* $16$ *possible combinations of seating arrangements where* $3$ *people have COVID.*

$$
C(3)_{31} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{32} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{33} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
$$

$$C(3)_{34} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{35} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; C(3)_{36} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix};$$

$$C(3)_{37} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; C(3)_{38} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{39} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix};$$

$$C(3)_{310} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{311} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{312} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$C(3)_{313} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}; C(3)_{314} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}; C(3)_{315} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$C(3)_{316} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

**Remark.** *By a simple calculation, we can observe that $C(3)_{31}$ is the best to lessen the spread of the 3 people with COVID because it only has 3 ways of spreading.*

**Example 3.** $3 \times 3 = 9$ *tables with* 9 *people and* 4 *with COVID.*

*By using permutations of rows and columns, we observe that there are* 23 *possible combinations of seating arrangements where* 3 *people have COVID.*

$$C(3)_{41} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{42} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{43} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Next, we have:

$$C(3)_{44} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}; C(3)_{45} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{46} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix};$$

$$C(3)_{47} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{48} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; C(3)_{49} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix};$$

$$C(3)_{410} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}; C(3)_{411} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; C(3)_{412} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix};$$

$$C(3)_{413} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}; C(3)_{414} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}; C(3)_{415} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix};$$

$$C(3)_{416} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}; C(3)_{417} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}; C(3)_{418} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$C(3)_{419} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}; C(3)_{420} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}; C(3)_{421} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$C(3)_{422} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}; C(3)_{423} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

**Remark.** *By a simple calculation, we can observe that $C(3)_{41}$ and $C(3)_{45}$ are the best to lessen the spread of the 4 people with COVID.*

**Example 4.** $3\times 3 = 9$ *tables with* 9 *people and* 5 *with COVID.*

*By using permutations of rows and columns, we observe that there are* 23 *possible combinations of seating arrangements where* 3 *people have COVID.*

$$C(3)_{51} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; C(3)_{52} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{53} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix};$$

$$C(3)_{54} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}; C(3)_{55} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; C(3)_{56} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix};$$

$$C(3)_{57} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}; C(3)_{58} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}; C(3)_{59} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix};$$

$$C(3)_{510} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{511} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; C(3)_{512} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix};$$

$$C(3)_{513} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}; C(3)_{514} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}; C(3)_{515} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix};$$

$$C(3)_{516} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}; C(3)_{517} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}; C(3)_{518} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix};$$

$$C(3)_{519} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}; C(3)_{520} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}; C(3)_{521} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

$$Next, we have: C(3)_{522} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}; C(3)_{523} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

**Remark.** *By a simple calculation, we can observe that* $C(3)_{51}$ *and* $C(3)_{52}$ *are the best to lessen the spread of the* 5 *people with COVID with Selena factor* $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ *and* $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$*, respectively.*

**Example 5.** $3 \times 3 = 9$ *tables with* 9 *people and* 6 *with COVID.*

*By using permutations of rows and columns, we observe that there are* 16 *possible combinations of seating arrangements where* 3 *people have COVID.*

$$\begin{aligned} C(3)_{61} &= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}; C(3)_{62} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}; C(3)_{63} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}; \\ C(3)_{64} &= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; C(3)_{65} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}; C(3)_{66} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}; \\ C(3)_{67} &= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}; C(3)_{68} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}; C(3)_{69} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}. \end{aligned}$$

next, we have:

$$
\begin{aligned}
C(3)_{610} &= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}; C(3)_{611} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}; C(3)_{612} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}; \\
C(3)_{613} &= \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}; C(3)_{614} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}; C(3)_{615} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}; \\
C(3)_{616} &= \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}.
\end{aligned}
$$

**Remark.** *(i) Recall that two n by n matrices A and B are permutationally equivalent to each other, if there exists a permutation matrix $P_\pi$ such that $A = P_\pi^{-1} B P_\pi$*

| *COVIDs* | *possible combinations* | *Best seating* | *Selena factors* | *Ways of spreading* |
|---|---|---|---|---|
| *0* | *1* | 0 | *No* | *0* |
| *1* | *3* | $C(3)_{11}$ | $S(3)_{11}$ | *2* |
| *2* | *8* | $C(3)_{21}$ | $S(3)_{21}$ | *3* |
| *3* | *16* | $C(3)_{31}$ | $S(3)_{31}$ | *3* |
| *4* | *23* | $C(3)_{41}$,$C(3)_{45}$ | $S(3)_{41}$ ,$S(3)_{45}$ | *4* |
| *5* | *23* | $C(3)_{51}$,$C(3)_{52}$ | $S(3)_{51}$ ,$S(3)_{52}$ | *4* |
| *6* | *16* | $C(3)_{61}$ | $S(3)_{61}$ | *3* |
| *7* | *8* | $C(3)_{71}$ | *No* | *3* |
| *8* | *3* | $C(3)_{81}$ | *No* | *2* |
| *9* | *1* | $C(3)_{91}$ | *No* | *0* |

*where*

$$C(3)_{11}=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix},C(3)_{21}=\begin{pmatrix}1&1&0\\0&0&0\\0&0&0\end{pmatrix},C(3)_{31}=\begin{pmatrix}1&1&1\\0&0&0\\0&0&0\end{pmatrix}$$

$$C(3)_{41}=\begin{pmatrix}1&1&1\\1&0&0\\0&0&0\end{pmatrix},C(3)_{42}=\begin{pmatrix}1&1&0\\1&1&0\\0&0&0\end{pmatrix},C(3)_{51}=\begin{pmatrix}1&1&1\\1&1&0\\0&0&0\end{pmatrix},$$

$$C(3)_{52}=\begin{pmatrix}1&1&1\\1&0&0\\1&0&0\end{pmatrix}C(3)_{61}=\begin{pmatrix}1&1&1\\1&1&1\\0&0&0\end{pmatrix},C(3)_{71}=\begin{pmatrix}1&1&1\\1&1&1\\1&0&0\end{pmatrix},$$

$$C(3)_{81}=\begin{pmatrix}1&1&1\\1&1&1\\1&1&0\end{pmatrix},C(3)_{91}=\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix};S(3)_{11}=\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

$$S(3)_{21}=\begin{pmatrix}0&0&1\\1&1&0\\0&0&0\end{pmatrix},S(3)_{31}=\begin{pmatrix}0&0&0\\1&1&1\\0&0&0\end{pmatrix},S(3)_{41}=\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix},$$

$$S(3)_{45}=\begin{pmatrix}0&1&1\\1&0&0\\1&0&0\end{pmatrix},S(3)_{51}=\begin{pmatrix}0&1&1\\0&0&0\\0&0&0\end{pmatrix},S(3)_{52}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&-1\end{pmatrix}$$

$$S(3)_{61}=\begin{pmatrix}1&1&1\\0&0&0\\0&0&0\end{pmatrix}.$$

*(ii) We can find the possible combinations through the permutational equivalence. We can also find them using the vertical, diagonal, or rotational symmetry methods.*

*(iii)*

| *COVIDs* | *possible combinations* | *Best seating* | *Selena factors* | *Ways of spreading* |
|---|---|---|---|---|
| *3* | *77* | $C(4)_{31}, C(4)_{32}$ | $S(4)_{31}, S(4)_{32}$ | *4* |
| *4* | *252* | $C(4)_{41}, C(4)_{42}$ | $S(4)_{41}, S(4)_{42}$ | *4* |
| *5* | *567* | $C(4)_{51}, C(4)_{52}$ | $S(4)_{51}, S(4)_{52}$ | *5* |
| *6* | *1053* | $C(4)_{61}$ | $S(4)_{61}$ | *5* |
| *7* | *1465* | $C(4)_{71}$ | $S(4)_{71}$ | *5* |
| *8* | *1674* | $C(4)_{81}$ | $S(4)_{81}$ | *4* |
| *9* | *1465* | $C(4)_{91}$ | $S(4)_{91}$ | *5* |
| *10* | *1053* | $C(4)_{10,1}$ | $S(4)_{10,1}$ | *5* |
| *11* | *567* | $C(4)_{11,1}, C(4)_{11,2}$ | $S(4)_{11,1}, S(4)_{11,2}$ | *5* |
| *12* | *252* | $C(4)_{12,1}, C(4)_{12,2}$ | *No,* $S(4)_{12,2}$ | *4* |

$$C(4)_{31} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, C(4)_{32} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$S(4)_{31} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, S(4)_{32} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

*Next, we have:* $C(4)_{41} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $C(4)_{42} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$,

$S(4)_{41} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $S(4)_{42} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$,

$C(4)_{51} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $C(4)_{52} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$,

$S(4)_{51} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $S(4)_{52} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$,

$C(4)_{61} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $S(4)_{61} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$,

$C(4)_{71} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $S(4)_{71} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$,

$$C(4)_{81} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, S(4)_{81} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C(4)_{91} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, S(4)_{91} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

$$C(4)_{10,1} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, S(4)_{10,1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix},$$

$$C(4)_{11,1} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, C(4)_{11,2} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$S(4)_{11,1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}, S(4)_{11,2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix},$$

$$C(4)_{12,1} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}, C(4)_{12,2} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$S(4)_{12,2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix},$$

CHAPTER IV

# FUTURE WORK AND OPEN QUESTIONS

(i) Investigate cases where $n \geq 5$ We need to see if the symmetry in possible combinations and the ways of spreading continues to occur for the cases 5 by 5 and larger.

(ii) Investigate the Selena factors further. Possibly looking into further detail in the factors or seeing how they can be improved. In Theorem 1 and Theorem 2, there was a formula developed using the Selena factors, we want to expand this research in more detail.

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## BIOGRAPHICAL SKETCH

Selena Suarez (selena942012@gmail.com) graduated with her degree, Mathematics B.S., from California State University Bakersfield. She also received her Single Subject Mathematics Credential from CSUB in 2016. While she was in school, she devoted her time to tutoring students in mathematics at different grade levels. After receiving her degree, she became a middle school math teacher and taught for two years. She worked at Wonderful College Prep Academy, where she taught 6th grade. She was able to build trusting relationships with all her students, and she absolutely loved teaching. She also taught 7th grade students at Rio Bravo Greeley Union School District. She participated in school events and got to know the students, colleagues, and parents very well. After teaching for two years, she decided to work on getting a master's degree in order to teach students at a community college. She graduated from University of Texas Rio Grande Valley with a Master's of Science degree in Mathematics in 2022. While in graduate school, she was also accepted into the Faculty Diversification Fellowship Program for Bakersfield College. Through this program, she was able to work closely with two of her mentors and observe them teaching community college students. During the last semester of graduate school, she became an adjunct math professor for Bakersfield College.