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The Impacts of Supporting Productive Struggle Teaching Practice on Students' Conceptual Understanding, Procedural Fluency, and Strategic Competence: The Case of Quadratic Functions

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THE IMPACTS OF SUPPORTING PRODUCTIVE STRUGGLE TEACHING PRACTICE ON STUDENTS' CONCEPTUAL UNDERSTANDING, PROCEDURAL FLUENCY, AND STRATEGIC COMPETENCE: THE CASE OF QUADRATIC FUNCTIONS

A Dissertation

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ABSTRACT

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This quasi-experimental design study aimed to examine how exposure to supporting productive struggle teaching practice impacts students' conceptual understanding, procedural fluency, and strategic competence when solving high-demanding quadratic functions problem-solving tasks. Results suggested that students in the supporting productive struggle teaching practice group performed significantly better than students who received facilitated instruction in every three strands of mathematical proficiency (i.e., conceptual understanding, procedural fluency, and strategic competence). By examining the effectiveness of supporting productive struggle teaching practice, this study offers insight into the conceptions and strategies teachers can implement in their classrooms to improve students' mathematical proficiency.

DEDICATION

I dedicate my dissertation to my daughter, Busra. Thank you for being patient and waiting for mom to finish the assignments since you were born. Thank you, Veysel, for your endless support and encouragement for the entire doctorate program.

I also dedicate this dissertation to my family and many friends who supported me through this endeavor. My dear family, I have felt your support even though you are thousands of miles away from me. My dear friends, I will always appreciate what you have done.

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CHAPTER I

INTRODUCTION

The "struggle" is viewed as problematic in the United States mathematics classroom (Sherman et al., 2009) and conveys negative meaning (Hiebert & Wearne, 2003). Educators, parents, and policymakers seek solutions to overcome struggles because they view struggles as problematic (Adams & Hamm, 2008). However, Hiebert and Grouws (2007) suggest that struggling to make sense of mathematics is crucial in learning mathematics with understanding. The first standard of the *Common Core Standards for Mathematical Practice* (CCSSM) also refers to struggle as the process to "make sense of problems and persevere in solving them" (National Governors Association Center for Best Practices (NGA Center) and Council of Chief State School Officers (CCSSO), 2010, p. 9). Perseverance is a vital learner characteristic and an essential element in problem-solving and learning mathematics (Pasquale, 2016). Kapur (2010) argues that designing lessons for persistence are central to productive failure and effective mathematics learning. When students struggle but keep trying to make sense of a problem, they experience productive struggle (Pasquale, 2016). Besides, Peterson and Viramontes (2017) note that sense-making is an indicator of productive struggle. Warshauer (2011) also supports the connection between the idea of struggle and sense-making and reasoning. For students' productive struggle, I refer to the definition "effort to make sense of mathematics, to figure something out that is not immediately apparent" (Hiebert & Grouws, 2007, p. 287).

Common Core State Standards for Mathematics [CCSSM] were released in 2010 to ensure that all students graduate from high school as college and career ready (CCSSI, 2010). The adoption of CCSSM changed the focus of mathematics standards from only developing skills efficiency to rigorous thinking, conceptual understanding, and reasoning (Stein et al., 2017). However, CCSSM documents do not provide information about implementing these standards in the mathematics classroom. Therefore, The National Council of Teachers of Mathematics (NCTM, 2014) published the Principals to Actions book to help practitioners implement CCSSM successfully and ensure mathematics success for all students. NCTM (2014) identified eight research-informed high-leverage effective mathematics teaching practices as (a) establishing mathematics goals to focus on learning; (b) implementing tasks that promote reasoning and problem solving; (c) using and connecting mathematical representations; (d) facilitating meaningful mathematical discourse; (e) posing purposeful questions; (f) building procedural fluency from conceptual understanding; (g) supporting productive struggle in learning mathematics; and (h) eliciting and using evidence of student thinking. These mathematics teaching practices provide a framework to improve teaching and learning mathematics.

Supporting productive struggle in learning mathematics is listed as the seventh effective mathematics teaching practice. "Effective mathematics teaching consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships" (NCTM, 2014, p. 48). In this teaching practice, students' struggles became an opportunity to delve into the problem's structure and understand the relationship among mathematical ideas instead of only focusing on finding the correct answer (NCTM, 2014). By using the productive struggle teaching practice, students come to realize that they can do well in mathematics with effort and perseverance. Therefore, the implementation of the Common Core State Standards Initiative (CCSSI) (2010) requires teachers to go beyond facts and procedures and design lessons that allow applying strategies in problem-solving and understanding mathematical ideas (Zeybek, 2016). Thus, teachers need to set classroom norms that struggling to make sense of mathematics is seen as a natural and essential part of learning (Zeybek, 2016).

Productive struggle is a crucial element in learning mathematics conceptually, so students should be given opportunities to struggle productively (Hiebert & Grouws, 2007; Warshauer, 2015). Learning mathematics, referred to as mathematical proficiency, has been defined as developing five intervened strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick et al., 2001). The five strands provide a framework to discuss knowledge, abilities, skills, and beliefs that construct mathematical proficiency (Kilpatrick et al., 2001). This framework is similar to the recent National Assessment of Educational Progress (NAEP) assessments. Still, NAEP's framework focuses on three mathematical abilities: conceptual understanding, procedural fluency, and strategic competence (Kilpatrick et al., 2001). This study examined the impacts of supporting productive struggle on these three strands of mathematical proficiency. By incorporating supporting productive struggle teaching practice, students have an opportunity to make sense of and understand important mathematics (Warshauer, 2015). When teachers observe that students struggle or they appear confused, they need to use it as an opportunity to deepen students' understanding of mathematics (Warshauer, 2015).

When students struggle to apply learning in new and challenging ways, they are more likely to probe for novel or unexpected connections, consider multiple ways to solve problems, and wrestle with the underlying differences between correct and incorrect solutions, all hallmarks of long-term retention (Pasquale, 2016). They also develop resilience, complex reasoning skills, and how to set and achieve goals while creating a healthy attitude toward making mistakes.

Statement of the Problem

Students in the United States rated classroom teaching more effective if the instruction is less challenging and leads to higher immediate performance (Kornell & Hausman, 2016). Students often believe that low-effort studying strategies are the most effective favoring passive learning strategies such as memorization, highlighting, rereading, and listening to lectures (Deslauriers et al., 2019). Researchers discovered that students preferred lectures to challenge activities like hands-on experiments and group problem-solving. Unfortunately, the lectured students scored lower than their counterparts on their follow-up tests. Kapur (2008) also found that 7^{th}-grade students who regularly practice solving complex open-ended problems became sophisticated thinkers outscoring their peers from a more traditional lecture-based classroom by 57 percent. Temporary confusion and frustration are not necessarily things to be avoided; instead, they are a precursor to deeper, more durable learning (Deslauriers et al., 2019; Kapur, 2008).

Peterson and Viramontes (2017) argued that supporting productive struggle empowers a growth mindset. Students with a growth mindset believe that intelligence can be developed through effort, so they see the struggle as a learning opportunity (NCTM, 2014). Students with a fixed mindset believe that intelligence or math ability is an innate trait and learning mathematics should come naturally, so they are more likely to give up (Dweck, 2008). There is a belief that mathematics is something one is born with (Pasquale, 2016). You are good at math or not, so the struggle is not observed as an opportunity to learn. Students have a fixed mindset toward mathematics compared to other subjects (Dweck, 2008). However, mindset can be changed by valuing students' perseverance, efforts, varied strategies in solving problems, and willingness to ask questions (Dweck, 2008).

Studies have shown that providing opportunities to engage in productive struggle improves students' performance and conceptual understanding of mathematics. For example, Kapur (2008) found that engagement, persistence, motivation, and struggle lead to developing a deeper understanding. Jonsson et al.'s (2014) findings support Kapur's results. The authors stated that students who are encouraged to struggle productively and create their methods perform better than those who use instructed procedures. Bjork and Bjork (2011) also found that the conditions that challenge learners may lower the rate of apparent learning but maximize long-term retention and transfer. Teaching with productive struggle has long-term benefits as applying their learning to new problem situations (Kapur, 2010).

Kapur (2011) examined the role of productive failure in solving speed problems. Miller (2020) also found that productive struggle experiences enhance high school students' ability to progress through challenges in an engineering course. Kapur (2011) and Miller (2020) suggested future researchers extend the study to other subject areas. After reviewing high-ranked mathematics education journals, I did not come across any study which used quadratic functions as a concept regarding productive struggle in learning mathematics. Hence, I selected and designed the tasks related to the quadratic functions.

Preceding studies support a link between students' engagement in productive struggle and meaningful mathematics learning. Warshauer (2015) explored what students' productive struggle looks like, how teachers respond to students' struggles, and what teacher actions are productive in student engagement and conceptual understanding in a middle school mathematics classroom. The author recommended assessing how productive struggle contributes to student learning to future researchers. In my study, I aim to investigate how student exposure to productive struggle learning experiences contributes to students' conceptual understanding, procedural fluency, and strategic competence in solving high-demanding problem-solving questions in quadratic functions. The participants of previous studies were generally middle school students (Kapur, 2011; Warshauer, 2015) and pre-service teachers (Warshauer et al., 2021; Zeybek, 2016). Thus, this study will contribute to the literature by examining the role of productive struggle on high school students.

The school district where I conducted my study is a charter public school located in the south-central region of the United States. The district and school administrators are more concerned about test scores than developing mathematical proficiency. Most mathematics teachers in the school use traditional teaching methods that provide direct instruction, and students take notes and practice. Implementing supporting productive struggle teaching practice might be eye-opening for teachers and stakeholders in my school district to promote meaningful mathematics learning. This study contributes to mathematics education and learning environments enhanced by supporting productive struggle in learning mathematics. Mathematics teachers, coaches, curriculum developers, students, and parents would benefit from supporting productive struggle teaching practice to improve mathematical proficiency.

Purpose of the Study

The purpose of the study is to examine how exposure to supporting productive struggle teaching practice impacts students' conceptual understanding, procedural fluency, and strategic competence in the topic of quadratic functions.

I focused my study on the following research question.

Research Question

What are the differences regarding high school students' conceptual understanding, procedural fluency, and strategic competence when solving high-demanding quadratic functions problem-solving tasks between students exposed to productive struggle teaching practice and those who received facilitated instruction?

Null Hypotheses (H_0). The mean of the posttest scores for conceptual understanding, procedural fluency, and strategic competence is equal for the treatment and comparison groups after controlling for the effects of the pretest scores.

Alternative Hypotheses (H_1). The mean of the posttest scores for conceptual understanding, procedural fluency, and strategic competence is not equal for the treatment and comparison groups after controlling for the effects of the pretest scores.

Significance of the Study

Dominant cultural beliefs about teaching and learning mathematics hinder the implementation of effective teaching and learning practices (Philipp, 2007). Many teachers and parents believe that students should be taught as they were taught (NCTM, 2014). According to the video studies conducted by Trends in Mathematics and Science Study (TIMMS) in 1995 and 1999 (as cited by Peterson & Viramontes, 2017), a typical math class involves reviewing the homework, teacher-directed instruction, and practice. This scene is often observed in American classrooms for decades and continues to be apparent (Hiebert, 2013).

Stigler and Hiebert (2009) videotaped mathematics lessons in seven countries that performed high on international mathematics assessments to compare teaching in higher-achieving and lower-achieving countries. U.S. mathematics instruction does not support asking questions to reason and think mathematical ideas compared to high-achieving countries (Banilower et al., 2006). The authors stated that high achieving countries' common practice was "student engagement in an active struggle with core mathematics concepts and procedures" (p. 27). However, U.S. teachers wanted to rescue the students by doing the work for them. They ignored discussing the conceptual aspects of problems, so teachers did not allow students to struggle to solve the problem. Instead, they excessively scaffolded the tasks and took away students' thinking. The national assessment results show that these teaching methods are not providing evidence of high performance. Only 6.9% of 17-year-old students scored at or above proficient level according to the National Assessment of Educational Progress (NAEP) (National Center for Education Statistics, 2007).

A typical U.S. mathematics classroom involves direct teaching where students observe the teacher and are assigned practice questions to solve using demonstrated methods (Kapur, 2011). Having students practice the concept after explicitly modeling is expected in the U.S. mathematics classroom rather than supporting productive struggle in learning mathematics practice (Peterson & Viramontes, 2017). Hiebert and Wearne (2003) also noted that U.S. teachers rarely engage students in productive struggle. However, when students have a risk-free classroom structure and teachers emphasize mistakes as a learning opportunity, they will be motivated to persist (Kapur, 2011).

Warshauer (2015) suggested a range of teacher support when students struggle to advance students toward a resolution of understanding. Warshauer (2015) illustrated four strategies to support students' struggle productively by asking questions about their thinking and the source of their struggle, encouraging students to reflect on their work and supporting their struggle to explain their thinking, giving time to struggle but helping students to manage their struggle without lowering the demand and stepping in to explain everything, acknowledging students that struggle is a natural part of learning. Stigler and Hiebert (2009) described such support as providing students with opportunities to think more deeply about mathematical concepts. The teacher's responses to students' struggles have the power to enhance the level of learning depending on the goal of the task, students' prior knowledge, and students' willingness (Warshauer, 2015).

Warshauer (2015) examined 186 episodes of struggle visible to teachers in middle school and identified four types of struggles: getting started, carrying out a process, giving a mathematical explanation, and expressing misconceptions and errors. The author did not examine how supporting productive struggle impacted students' mathematics understanding and suggested future researchers explore the impacts of productive struggle on students' mathematics understanding. Empirical research on productive struggle and how to address struggle productively is limited (Warshauer, 2015). Therefore, it is crucial to know what productive struggle is and how supporting productive struggle contributes to students' learning (Peterson & Viramontes, 2017). This study aims to contribute to the literature regarding how engaging students in productive struggle promotes students' mathematics understanding.

CHAPTER II

REVIEW OF LITERATURE

The literature review examined the existing research supporting productive struggle teaching practice and its impacts on mathematical proficiency. The literature review began by defining productive struggle and then discussed constructivism as a broad theory of learning. This discussion included a brief description of both constructivism and social- constructivism, educational possibilities of the theory, highlighting how the theory is used in the mathematics classroom, as well as the connection between productive struggle and social constructivism. Then, the chapter examined the teaching strategies that support the productive struggle and the role of tasks in supporting the productive struggle. Finally, the role of productive struggle in mathematical proficiency: conceptual understanding, procedural fluency, and strategic competence is presented in this section.

Productive Struggle

Warshauer (2011) defined productive struggle as "a particular kind of phenomenon that may occur as students engage in a mathematical activity or problem that is challenging but reasonably within the students' capabilities, possibly with some assistance" (p. 10). The author noted that struggle or difficulties push the students in their thinking and deepen their understanding. Warshauer (2011) pointed out that struggle may or may not be visible as students engage in a mathematical task. The struggle may be manifested externally, so it is observable or may not be visible to teachers or observers. Warshauer (2011) stated that the manifestation of internal struggle is independent sense-making. The high-demand task provokes struggle and provides more incidence of struggle. One can see how internal thought is crucial in self-reflection when the struggle is considered. Internal thoughts could be affected by external factors like the presence of others.

Hiebert and Grouws (2007) defined productive struggle as an "effort to make sense of mathematics, to figure something out that is not immediately apparent" (p. 387). According to Granberg (2016), the struggle involves dealing with insufficient prior knowledge and interpreting new information to construct new knowledge. Peterson and Viramontes (2017) indicated that struggle in the mathematics classroom occurs when students are asked to solve a problem that does not have an obvious way to solve and connect mathematical ideas and strategies. The authors also pointed out that struggle does not mean that students will be given complex problems and let them struggle and get frustrated.

I adopted Hiebert and Grow`s (2007) definition of productive struggle, which is an effort to make sense of mathematics, to figure something out that is not immediately apparent. Some learning theories refer to struggle as a cognitive process internal to the learner. Dewey (1933) pointed out the process of struggle is necessary to construct a deep understanding. It starts with engaging in perplexity and doubt and continues as learners put things together to make sense and resolve the dilemma. Dewey (1933) suggested providing opportunities for students to muddle through the process of solving challenging problems rather than practice and drills. Besides, Piaget (1977) referred to struggle as disequilibrium, which challenges our thinking to restore equilibrium by building new information on existing knowledge. Disequilibrium occurs between internal schemas and the external environment. Productive struggle fits within the constructivist approach

(Miller, 2020). I base the concept of struggle on the constructivist theory. Constructivism and its' connection to mathematics learning and teaching and the relationship between constructivism and supporting productive struggle teaching practice were reviewed in the following sections.

Constructivism

Constructivism is the combination of multiple theories, and it is assimilated from behaviorism and cognitivism (Amineh & Asl, 2015). Mahoney (2005) points out that the words "constructivism," "constructivist," or related terms became popular in the second half of the 20th century. The most popular educational constructivism types are Jian Piaget's cognitive constructivism and Lev Vygotsky's social constructivism (Amineh & Asl, 2015).

The constructivist approach started with Jean Piaget, whose work emphasized understanding child development and learning as a construction process (Pritchard & Woollard, 2013). Bada and Olusegun (2015) defined constructivism as a learning theory that explains how human beings learn and gain knowledge. The constructivist approach describes learning as the process of constructing meaning (Caffarella & Merriam, 1999). People build new knowledge upon prior knowledge and experiences by asking questions and assessing the current knowledge (Bada & Olusegun, 2015).

Amineh and Asl (2015) stated that constructivism explains how learners can sense material and how materials can be thought effectively. Piaget (1977) claims that learning is an active process. Piaget explains that disequilibrium occurs when we encounter a situation that challenges our thinking, and we should change our thinking to restore equilibrium by building new information on existing knowledge. Constructivist learning is an active rather than passive process, so students should have opportunities to actively construct new knowledge (Bada &

Olusegun, 2015). Constructivist pedagogy focuses on student-centered instruction, autonomy, real-world applications, social interaction, activating students' prior knowledge, and continuous formative assessment (Abdulwahed et al., 2012).

The constructivist approach emphasizes the importance of student-centered instruction and understanding students' thinking processes (Even & Tirosh, 2002). Students build new knowledge upon their current knowledge and experiences, so each student has a different way of learning a new concept (Eraslan, 2005). Therefore, teachers need to create a learning atmosphere where students can construct their learning through authentic activities. Having students construct their knowledge does not mean acting like they are experts. Students need guidance to develop background knowledge and scaffold new information (Krahenbuhl, 2016). When students are exposed to a lot of new information, they may feel overwhelmed, and effective learning may not occur (Krahenbuhl, 2016). Teachers need to consider and be aware of students' deficiencies and guide them through the process of knowledge construction (Krahenbuhl, 2016). Teachers have a crucial role in checking students' prior knowledge and applying what they learned in class (Amineh & Asl, 2015). Mvududu and Thiel-Burgess (2012) also suggest that teachers consider students' knowledge and provide opportunities to apply the knowledge to practice. The teachers' part is being aware of students' prior knowledge and challenging and supporting students during learning (NCTM, 2000).

On the other hand, Amineh and Asl (2015) defined social constructivism as a theory of knowledge that learning occurs through coordinating with other people. Vygotsky contributed to the concept of teaching and learning in several ways, pointing out the zone of proximal development and scaffolding (Goodman, 2010). Vygotsky (1980) introduced the zone of proximal development (ZPD), which is the distance between actual development and potential development

(Connery & Curran, 2010). The actual development (child can perform alone) and the potential development (child can do with help) should be distinguished to understand the relationship between learning and development (Palincsar, 1998). Connery and Curran (2010) stressed that ZPD connects current individual conception to an advanced understanding level through collaboration. Productive interactions direct instruction toward the zone of proximal development (Palincsar, 1998).

Vygotsky (1980) claims that cognitive skills develop on a social level first and are then internalized by individuals. Students may start learning with collaborative or group work and then individually contribute to the world of expertise (Amineh & Asl, 2015). Internal development occurs when students interact with their peers or other individuals during the learning process (Ginga & Zakariya, 2020). Learning warns of different internal development processes that can be activated if the learner has an opportunity to interact with others (Vygotsky, 1980). Interacting with others through group discussions improves students' higher-order thinking skills (Palincsar, 1998). Hence, students need interaction with their classmates or teachers to develop conceptual understanding and internalize knowledge (Ginga & Zakariya, 2020).

Schunk (2012) listed several methods that use social constructivism: instructional scaffolding, reciprocal teaching, peer collaboration, apprenticeships, cooperative learning, and jigsaw methods. For instance, instructional scaffolding is defined as a procedure to design tasks beyond students' capabilities. The teacher is more active and models a skill to support the learners at the beginning. And then, the teacher reduces scaffolding to let the learners develop skills independently. The primary factor of scaffolding is keeping the learner in the zone of proximal development.

Learning and Teaching Mathematics from Social-Constructivist Perspective

Educators are concerned about low mathematics performance, so it is necessary to examine innovative teaching methods (Ginga & Zakariya, 2020). Implementing teaching methods derived from learning theories to enhance mathematics instruction and learning effectiveness is crucial (Ginga & Zakariya, 2020). Trends in teaching and learning altered from behaviorism to cognitivism and then constructivism (Abdulwahed et al., 2012). Therefore, the new focus requires some differences in mathematics teaching techniques, assessment methods, and classroom practices (Eraslan, 2005).

Ernest (1998) points out that the social aspect of learning is disregarded in epistemology and philosophy. Individual knowers, cognizing the subject, and objectivized knowledge is some of the main focus of traditional epistemologists. The psychology of mathematics education's primary issue might be seeking a theory of learning mathematics to guide teaching and learning (Ernest, 1998). Piaget's Stage Theory affected the research on mathematics learning and conceptual development in the 1980s (Ernest, 1998). Then, radical constructivism became more popular due to its construction of meaning (Ernest, 1998). However, radical constructivism focuses on the individual aspect of learning rather than the social part of learning (Ernest, 1998). That's why social constructivist theory plays a crucial role in acknowledging both the social process and individual sense-making in learning mathematics (Ernest, 1998). Ernest adopted the Vygotskian roots of constructivism to develop his social constructivist theory of learning mathematics.

Ernest (1998) stated that the philosophy of mathematics considered knowledge as a product rather than the process of coming to know. Ernest (1998) used the term conceptual continuity in mathematics to refer to that mathematical knowledge is constructed from various meaningful elements such as symbols, notations, problems, explanations, proofs, and more. Learning a new concept requires combining some of these components, so most of the original concept is similar to pre-existing elements (Ernest, 1998). If mathematics is seen as a social construct, it would significantly impact goals, content, teaching methods, assessment, and teachers' beliefs (Ernest, 1998). The view of mathematics could be redesigned to give everybody access and power. If mathematics is not a finished product, and it keeps developing by human beings, we need to see the reflection of these ideas on school mathematics curriculums (Ernest, 1998).

However, the typical U.S. classroom environment involves teachers presenting a topic to the class, and students practice using the demonstrated methods (Stigler & Hiebert, 2009). Teacher-centered instruction makes students passive and does not improve their problem-solving and critical thinking skills (Koc & Demirel, 2002). Obioma (2011) also pointed out how poor teaching strategies negatively affect students' understanding of mathematics. Even assessing the students differs from traditional teaching to a constructivist teaching approach. In the traditional teaching approach, teachers use tests to measure students' knowledge; however, it does not provide information about students' mathematical understanding (Eraslan, 2005). Students' view and thinking is the way of assessment in the constructivist approach (Eraslan, 2005). Open-ended questions and problem-solving tasks became central for evaluating mathematical knowledge from the constructivist perspective (Ginsburg, 1997).

Recent studies showed that the social constructivist teaching approach enhanced students' performance in mathematics. For example, Ginga and Zakariya (2020) found that the treatment group that experienced a social constructivist instructional approach performed significantly better than the conventional teaching group. Similar results were obtained by Bay et al. (2012) and Ilyas et al. (2013). Bay et al. (2012) conducted a study on pre-service teachers to investigate the impacts of authentic task-based socio-constructivist instruction on participants' problem-solving and metacognitive skills. The social constructivist instruction group improved problem-solving and metacognitive skills more than the comparison group (Bay et al., 2012). Ilyas et al. (2013) studied how Vygotsky's social constructivist approach impacts students' understanding of algebra. The social constructivist approach provided peer-interaction opportunities, sharing ideas, listening, observing others' viewpoints, and a friendly collaborative learning atmosphere. While the treatment group is taught by social constructivist instruction, the comparison group is taught by one-way teaching. While the pretest scores were parallel, the treatment group performed significantly better than the comparison group in the post-test. These studies' findings show that the social constructivist approach enhances learning outcomes and creates a collaborative learning environment.

Productive Struggle and Social-Constructivist Theory

Both personal constructions and social interaction play crucial roles in mathematics understanding (Cobb, 2000). Most of the constructivist teaching methods match with the National Council of Teachers of Mathematics (NCTM) Principles and Standards (2000) (Hennessey et al., 2012). NCTM standards support classroom activities that provide meaningful peer interaction and improve students' reasoning skills (Hennessey et al., 2012). Driscoll (1994) claims that the constructivist learning theory supports students' conceptual and logical development. Knowing students' prior knowledge and implementing different representations of information are some of the other corresponding practices of the constructivist approach and NCTM (2000) standards (Hennessey et al., 2012). NCTM (2000) standards also state that students must learn mathematics with conceptual understanding by constructing new knowledge on prior knowledge and experience.

The social-constructivist theory also aligns with the principles of effective mathematics teaching practices proposed by NCTM (2014). NCTM (2014) suggested constructing mathematics knowledge from the learners' perceptions and experiences by participating in informed exploration and cooperative learning activities. Besides, one of the students' actions of supporting productive struggle teaching practice refers to collaborative learning as helping one another without giving the answer or explaining what to do to solve the question (NCTM, 2014). Recent research also supports that mathematic learning is an active process in that students construct their knowledge from their experience by cooperating with their peers, teachers, and other adults (Hiebert & Grouws, 2007). Learners should experience constructing knowledge socially through discourse, activity, and interaction related to meaningful problems (NCTM, 2014).

NCTM's (2014) Principals to Action book will remain only ideas until all stakeholders take action using research-informed decisions. The most important stakeholder is the teacher because teachers are the ones who can implement effective mathematics teaching practices proposed by NCTM (2014) in the classroom. The classroom and learning environment should allow students to engage in meaningful tasks that enhance mathematical understanding actively, problem-solving, and reasoning (NCTM, 2014). Interaction with peers or teachers, making sense of mathematics, comparing different ways to solve problems, and verifying solutions are some of the classroom routines that should be observable to ensure mathematics success for all students (NCTM, 2014). In such classrooms, students may take hours or even days to solve a problem by working cooperatively with their classmates. Teachers use questioning techniques effectively and provide opportunities for productive struggle. Teachers should implement lesson plans that empower a productive disposition toward mathematics learning, curiosity, and perseverance (NCTM, 2014). Setting risk-free classroom environments where students can externalize their struggles and obtaining the wrong answer is seen as an opportunity to explore and grow support and motivates students to persist and struggle (Carter, 2008).

A Vygotskian perspective suggests that internal mental functioning and social interactions direct students' struggle toward understanding (Warshauer, 2015). For instance, when students struggle and make mistakes, the teacher uses this as an opportunity to question, explain, justify, and extend their ideas to their peers (Hoffman, 2009). Such classroom activities promote productive struggle and sense-making (Warshauer, 2015).

Teaching Strategies to Support Productive Struggle

Teaching mathematics effectively requires teachers to be skilled at teaching methods that develop mathematics learning for all students (NCTM, 2014). NCTM (2014) provided a table about teacher and student actions to support productive struggle in learning mathematics. Teacher actions could be summarized in four categories: anticipating what students might struggle with, giving time to struggle, helping students to realize that mistakes are a natural part of learning, and praising students' efforts. In exposure to the productive struggle learning experience, student actions could be struggling, asking questions about their struggle, persevering in problem-solving, and helping others without giving the answer.

Mathematics lessons should be designed to promote student engagement by discussing the tasks that will improve problem-solving and reasoning skills (NCTM, 2014). Student learning is mainly based on what happens inside the classroom and their interactions with peers and teachers over the curriculum (Ball & Forzani, 2011). When preparing their lesson plans, teachers need to consider the struggles and misconceptions that can arise (NCTM, 2014). Thinking and planning about possible struggles allow the teacher to support students without removing all the challenges in the task.

Teachers impact students' perception and approach toward struggle in the mathematics classroom (NCTM, 2014). Teachers must see the struggle as an integral part of learning mathematics, convey this message to students, and provide ample time and opportunities for students to keep trying (NCTM, 2014). The teacher's role and beliefs are critical in preparing students to be persistent in problem-solving and productive failure (Kapur, 2011). The author states that teachers must emphasize using multiple representations and methods even if it does not lead to correct solutions. Secondly, teachers must resist stepping in and explaining to students what to do. Teachers may encourage the students to try harder and solve the question with the group members. Kapur (2011) argues that it is more productive to delay help by providing time for students to find the solution by themselves. If teachers help students see mistakes as learning and sense-making sources, the struggle will lead to a productive engagement in mathematics class (Warshauer, 2015).

Warshauer (2015) pointed out the importance of teacher actions to resolve students' struggles. However, the level of support should not deprive students of the chance to think for themselves. Telling the answer or providing direct guidance that will guide students' thinking to the answer without making connections and reasoning removes or lowers the cognitive demand. Probing guidance and affordance do not lower cognitive demand because participants have more time to delve into the concept (Zeybek, 2016). Warshauer (2015) suggested that teachers respond with examples that connect students' thinking with their prior knowledge to provide skills and techniques to solve the task. The author also stated that teachers might use *revoicing* to help students clarify their solutions.

Hiebert and Wearne (2003) found that asking more questions, having students make connections between critical mathematical ideas, and providing wait time improved students' computation and understanding more than students who practiced demonstrated procedures. Asking questions directs students' thinking and helps them organize their ideas (Sorto et al., 2009). If the questions are scaffolded on students' ideas, they allow a productive exchange and enhance students' learning (Warshauer, 2015).

Lai et al. (2017) pointed out that the lessons that allow productive struggle generally start with a problem-solving or critical-thinking activity before receiving any instruction related to a new concept. Teachers must be available in this process to guide students through their struggles without giving answers to students (Miller, 2020) and to praise their students for their efforts to make sure students persevere through challenges (NCTM, 2014). Teachers should provide feedback to students that value their effort of trying different methods in solving problems, asking questions about specific parts of the tasks, and their attempts to explain the procedure and answer precisely using mathematical language.

When students feel frustrated or are stuck, the teacher should provide support without lowering the task's cognitive demand (NCTM, 2014). When teachers observe students struggle, they want to rescue their students by breaking down the task and explaining the concept step by step. However, such rescuing undermines students' effort, lowers the task's cognitive demand, and reduces students' chances to engage in making sense of mathematics (Stein et al., 2009).

Mathematics teachers need to give students time to struggle and become self-reliant problem solvers (Peterson & Viramontes, 2017).

The Role of Task in Supporting the Productive Struggle

Stein et al. (2009) identified four task levels: cognitive demand as memorization, procedures without connections, procedures with connections, and doing mathematics. Memorization involves practicing learned facts and formulas and reproducing demonstrated materials. The mathematics tasks that have procedures without connections are algorithmic and focus on producing the correct answer instead of meaningful mathematics learning or justification. The tasks with procedures with connections allow multiple representations and a deeper understanding. Doing mathematics type of tasks requires complex thinking and exploration. The mathematical task is vital to creating a classroom environment where mathematical discourse will occur, and students will interact with their peers and teachers (Wilkie, 2022).

Zeybek (2016) also pointed out that the task's selection and construction are among the most critical aspects of pedagogical decisions because tasks open or close the doors for meaningful mathematics learning. The author found that implementing a high demanding task motivated pre-service teachers to persevere in solving the problem. High cognitive level tasks allowed pre-service teachers to connect mathematical ideas to construct new knowledge and validate possible solutions. Students should be provided enough time to solve challenging tasks and develop curiosity (Goldenberg et al., 2015). Thus, the task should have multiple solutions, allow students to make sense of the concept, and acquire conceptual knowledge through interaction and discussion with peers or teachers (Zeybek, 2016).

Kapur (2008, 2011) used ill-defined tasks with multiple solutions and unknown parameters and required assumptions and well-defined tasks with fewer parameters that are not challenging to complete in the context of productive failure. The author found that students who practiced ill-defined tasks performed worse than students who practiced well-defined tasks during the intervention. However, students who practiced ill-defined tasks performed better on well-defined and ill-defined tasks than those who practiced well-defined tasks during the posttest. Kapur (2011) argued that the challenging and divergent tasks helped students develop problem-solving structures. Thus, the task design is crucial to enhancing mathematical reasoning and conceptual learning (Jonsson et al., 2014).

Hiebert and Wearne (2003) argued that challenging problems should be integrated into teaching to make the struggle productive. When students are given complex problems within reach of students and have tools to solve them, productive struggle occurs (Peterson & Viramontes, 2017). High cognitive demanding tasks that allow connecting concepts, justification, and explanation enable students to develop conceptual knowledge and understanding (Boston & Smith, 2009).

Peterson and Viramontes (2017) listed three elements to ensure that struggle is productive. Firstly, the tasks need to be highly demanding but within reach of students (Hiebert & Grouws, 2007) and in the zone of proximal development (Vygotsky, 1980). Students may not solve the problem immediately but can solve it by interacting with their peers and teachers. The struggle is not productive when the problem is not within reach or students are not used to such cognitive work (Peterson & Viramontes, 2017). Secondly, teachers must ensure that mathematics is central to the learning goals. Lastly, it is crucial to observe if students employ sense-making. Kapur (2010) also classified three factors that promote beneficial struggle: choosing the appropriate level of challenging task, encouraging students to explain what they are doing, and having students compare and contrast good and bad solutions.

The Role of Productive Struggle in Mathematical Proficiency

Kilpatrick et al. (2001) used the term mathematical proficiency to refer to learning mathematics successfully. Mathematical proficiency has five elements: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The authors pointed out that these five elements are interwoven, so there is a powerful insight into traditional mathematics classrooms to improve students` knowledge, ability, and skills. Being proficient requires applying knowledge from one setting to another (NCTM, 2000). Factual knowledge, procedural fluency, and conceptual understanding are essential components of mathematical proficiency (NCTM, 2000). This study examined how student exposure to productive struggle learning experiences contributed to students' conceptual understanding, procedural fluency, and strategic competence. Therefore, I focused on these three strands of mathematical proficiency.

Hiebert and Grouws (2007) defined conceptual understanding as mental connections among mathematical facts, procedures, and ideas. NCTM's (2014) definition involved comprehension and connecting concepts, operations, and relations. Conceptual understanding has become more critical in today's world because students can handle most arithmetic and algebra concepts with calculators, so understanding the number concepts and modeling gained more interest (NCTM, 2000). Conceptual understanding is an integrated and functional comprehension of mathematical ideas (Kilpatrick et al., 2001). Conceptual understanding lets students connect the facts and methods rather than see isolated facts (Kilpatrick et al., 2001). Students with conceptual understanding know why a mathematical idea is important and when and how to use it (Kilpatrick et al., 2001). They can organize the information by connecting the new knowledge with prior knowledge (Kilpatrick et al., 2001). When students conceptualize a concept, their knowledge is organized in a coherent whole, so they learn new ideas by connecting them to existing knowledge (Mutambara et al., 2019). Retention is another crucial aspect of conceptual understanding because students learn the facts and methods with understanding so they can remember easily (Kilpatrick et al., 2001).

Conceptual understanding is necessary to develop procedural fluency, defined as meaningful and flexible use of procedures (NCTM, 2014). Conceptual understanding and procedural fluency are critical in effective mathematics teaching (NCTM, 2014). Additionally, the National Mathematics Advisory Panel (2008) points out the importance of balancing the conception and procedures in mathematics learning. Effective mathematics teaching helps students be fluent in the process if the teacher creates a learning environment that encourages conceptual understanding (NCTM, 2014). When students deepen their conceptual understanding, they become more fluent in computation procedures (Kilpatrick et al., 2001). Procedural fluency is significant because being fluent lets students choose the most appropriate and productive method (Martin, 2009). Students who are fluent in procedures may focus on different aspects of problems to grasp new information (Kilpatrick et al., 2001). The Common Core State Standards [CCSSM] sets the guidelines and grade-level expectations that incorporate conceptual understanding and procedural fluency.

Strategic competence (problem-solving) is the ability to formulate, represent, and solve mathematical problems (NCTM, 2014). Problem-solving is one of the five process standards of NCTM (2000), and it is also widely emphasized in the Common Core State Standards in the USA (CCSSI, 2010). There is a solid supportive connection between strategic competence, conceptual understanding, and procedural fluency (Kilpatrick et al., 2001). The authors noted that students need to understand the quantities and the relationship between them to develop problem-solving strategies. Strategic competence arises in every step in developing procedural fluency in computation.

When the literature is reviewed to explore the impacts of supporting productive struggle on students' conceptual understanding, procedural fluency, and strategic competence, studies suggest that struggling to make sense of mathematics is essential in learning mathematics with understanding (Hiebert & Grouws, 2007). Kapur (2008) found that engaging in productive failure improved conceptual understanding to solve complex math problems. Miller (2020) also found that supporting productive struggle in learning enhances high school students' problem-solving skills. Peterson and Viramontes (2017) claimed five benefits of productive struggle: a sense of accomplishment, knowledge, understanding, high achievement, improved achievement, mastery, and long-term retention. Kapur (2016) found that students who engaged in productive failure perform higher and deepen their mathematical understanding.

Summary

This chapter envisioned incorporating productive struggle teaching practice to promote mathematical proficiency. The researcher discussed how exposing students to productive struggle teaching practice provides hopes for high-quality mathematics education. Dewey (1933) and Piaget (1977) referred to struggle as a cognitive process for deeper understanding. As a broad philosophy of learning, Piaget's cognitive development and Vygotsky's approach regarding the zone of proximal development and cooperative learning were discussed. The discussion of Ernest`s (1998) constructivist theory of learning mathematics adopted the Vygotskian roots of constructivism were presented in this chapter. Most of the constructivist approaches are aligned with NCTM's (2014) effective mathematics teaching practices, so the connection between social-constructivism and productive struggle was reported in this section.

The teaching strategies that support productive struggle highlighted how these effective mathematics teaching practices are oriented to foster procedural fluency, conceptual understanding, and strategic competence. In addition, this paper highlighted the role of tasks in supporting productive struggle teaching practice. High cognitive demanding tasks within students' reach allow students to develop a conceptual understanding (Boston & Smith, 2009).

CHAPTER III

METHODOLOGY

Research Design

Quasi-experimental designs are widely used to assess policy and programs (Dong & Maynard, 2013). Since the researcher cannot artificially create the groups, participants are not randomly assigned to groups (Creswell, 2015). When it is impossible to create a randomized sample, researchers may use a quasi-experimental design (Gay et al., 2012). Therefore, a quasi-experimental design was appropriate for this study. This study took place in March and April of 2022 over five weeks in a charter public high school.

Context and Sample

The population selected for this study is a public high school located in the south-central region of the United States. The school is classified as a public charter school that serves 447 9^{th}-grade through 12^{th}-grade students. The school population comprises 78 Gifted and Talented students, 85 English learners, 228 bilingual students, and 282 economically disadvantaged students who receive free or reduced lunch. About 86% of the students are Hispanic, and 14% are White, African American, Asian, and others. The school promotes science, mathematics, and educational technology.

Students learn quadratic functions in the Algebra II course at our school. That is why the students currently enrolled in the Algebra II class were selected for the sample. Four sections took Algebra II in the 2021-2022 school year. All four sections had the same teacher to accommodate the differences in characteristics of the instructor. The explicit child consent and parent assent forms in writing were obtained before participating in the study (see Appendix E and F). The subjects selected for this study were 74 tenth and eleventh-grade students from four Algebra 2 classrooms.

During the participant selection process, it is crucial to determine the sample size (Creswell, 2015). It is vital to have adequate statistical power to choose the minimum detectable effect size (Dong & Maynard, 2013). Sample size formulas determine the number of individuals to study (Creswell, 2015). Some excellent tools are developed to detect a sufficient sample size to achieve the desired statistical power level (Dong & Maynard, 2013). Hence, a power analysis was used to decide an appropriate number of participants.

The sample included 74 students, 42 were male, and the rest were female. The sample size is larger than the desired sample size determined by power analysis to be an adequate sample size for this study. Power analysis for a two-level and one covariate ANCOVA was conducted to determine a sufficient sample size using an alpha of 0.05, a power of 0.80, a large effect size ($f = 0.40$), and two tails (Faul et al., 2013). Leppink (2019) suggested using a power of at least 0.80 to detect an impact of interest in the experiment and replicate meaningful research. There was a close allocation of participants into each group. Based on the assumptions mentioned above, the desired sample size is 64. Since the sample size of the current study was 74, it was an adequate sample size. Participants' age ranged from 15 years old to 19 years old.

Around 91% of the students who participated in this study received free or reduced lunch. 77% of the participants were Hispanic, 10% Black, 7% Asian, and 6% White.

The teacher is a certified mathematics teacher who taught middle and high school mathematics courses for nine years. He is 38 years old and has a master's degree in mathematics. Students must be exposed to productive struggle learning experiences to turn their struggle into productive struggle. That's why the teacher was trained before the study about strategies to support students' productive struggle. The teacher and I attended NCTM one-day virtual workshop called supporting students` productive struggle. We read and discussed several articles to understand supporting productive struggle teaching practice. We also watched the video recordings in the NCTM`s (2014) principals to actions book. The teacher practiced these strategies with one of the classes who did not participate in the study.

Conditions

Two classes from 10th grade and two from 11th grade enrolled in Algebra II in the 2021-2022 school year. The students were divided into two groups: the treatment group and the comparison group. In a quasi-experimental design, pre-existing groups receive different treatments instead of randomized groups (Stuart & Rubin, 2010). It may not be feasible to randomize the sample in social sciences due to practical and ethical concerns (Stuart & Rubin, 2010). One 10th grade and one 11th grade class were in the comparison group, and the other 10th grade and 11th-grade classes were in the treatment group. The same teacher taught all four classes.

While the comparison group consisted of 35 students, the treatment group consisted of 39 students. A Chi-square goodness of fit test was conducted to examine whether the group was equally distributed across the comparison and treatment groups. The results of the test were not significant based on an alpha value of .05, $\chi^2(1) = 0.22$, $p = .642$. The observed and expected frequencies were not significantly different for the comparison and treatment groups. Table 1 presents the results of the Chi-Square goodness of fit test.

Table 1

Chi-Square Goodness of Fit Test for Group

Level	Observed Frequency	Expected Frequency
Comparison	35	37.00
Treatment	39	37.00

Note. $\chi^2(1) = 0.22$, $p = .642$.

All four classes took the pre-test. According to the pretest scores, two classes were selected for the comparison group, and the other two classes became the treatment group. Convenient sampling was used for the sample because participants were not randomly assigned to the comparison and treatment groups. There are many experimental situations in education where researchers need to use convenient groups due to participant availability and the inconvenience of forming artificial groups (Creswell, 2015). Convenient sampling allows researchers to select willing and available participants to participate in the study (Creswell, 2015). Even though the sample may not represent the population, the sample can reveal helpful information about the research question (Creswell, 2015).

Treatment Group (Productive Struggle Group (P.S.))

The treatment group comprised two sections; a 10th-grade and an 11^{th}-grade class (n = 39). Students worked on tasks cooperatively in small groups. The teacher was provided with the lesson plan (see Appendix D) for each lesson and practiced teacher actions to support students' productive struggle. Warshauer (2015) identified four types of teacher responses to students' struggles: telling, directed guidance, probing guidance, an affordance that is determined by evaluating their effect on the cognitive demand of the task, and how they address students' struggles and students' thinking. While telling and directed guidance lower the cognitive demand, probing guidance and affordance maintained cognitive demand (Warshauer, 2015). In accordance with Washauer's findings, when students asked for help, the teacher did not help students by telling them what to do or explaining the procedure. Instead, the teacher used probing guidance and affordance to support persistence in the process. In addition, NCTM's (2014) teacher actions and student actions (see table 2) that support productive struggle in learning mathematics were observable in the treatment group. The lesson plans were designed so that when the teacher followed the lesson plan, students were exposed to supporting productive struggle teaching practice. I video recorded the teacher and the classes to verify the use of supporting productive struggle teaching techniques in the treatment group.

Table 2

Support Productive Struggle in Learning Mathematics adapted from NCTM, 2014

What are teachers doing?	What are students doing?
● Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. ● Giving students time to struggle with tasks and asking questions that scaffold students' thinking without stepping in to do the work for them. ● Helping students realize that confusion and errors are a natural part of learning by facilitating discussions on mistakes, misconceptions, and struggles. ● Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	● Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle. ● Asking questions that are related to the sources of their struggles will help them make progress in understanding and solving tasks. ● Persevering in solving problems and realizing that it is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. ● Helping one another without telling their classmates the answer or how to solve the problem.

Comparison Group (Facilitated Instruction (F.I.) Comparison Group)

The comparison group comprised two sections; a 10th-grade and an 11^{th}-grade class (n = 35). The instruction of the comparison group was designed to be the same as the treatment group but with one significant exception. While the treatment group was facilitated by probing guidance and affordance, the comparison group was facilitated by telling and directed guidance, based on Warshauer's (2015) four types of teacher responses. Students worked in small cooperative groups. Students received instruction as a whole class, as a small group, and individuals. Whole class instruction was designed as mini-lectures on critical aspects of the lesson. Individual and group-level facilitation was on-demand. This facilitation could be clarification, pointing out students' mistakes, providing corrective feedback, and taking students' attention to essential aspects of the concept. NCTM's (2014) supporting productive struggle teacher and student actions were not observable in the comparison group.

The Content

I have selected the quadratic function unit to collect data. Quadratic functions and equations are a particular case of functions taught in high school math courses (Memnun et al., 2015). Quadratic functions are the extension of linear functions and can provide information about students' generalizations about other function families (Ellis & Grinstead, 2008). It is crucial to learn quadratics because various aspects of quadratics are used in higher mathematics courses and real-world settings such as paths of projectiles, suspension bridges, automobile headlights, satellite dishes, radio telescopes, maximizing profit, and more (Parent, 2015). Santia (2019) emphasizes the importance of quadratic functions and the lack of appreciation of quadratic functions in daily life.

Moreover, linear and quadratic functions are the first topics students deal with in higher-order thinking involving graphing (Parent, 2015). Hoon et al. (2018) conducted a study to investigate the correlation between students' level of knowledge in functions and quadratic functions. If students understand the key features of quadratic functions and their applications, they can better understand other functions and more complex concepts (Hoon et al., 2018).

The College Career Math Ready course's quadratic function unit, designed by the Southern Regional Education Board (SREB), was adopted in this study. SREB is one of the leading organizations that encourages states to develop a plan to have high school graduates be college and career-ready (Barnett et al., 2013). The quadratic functions unit comprised ten lessons that allowed students to explore quadratic functions through application and conceptual problems by focusing on the interplay of multiple representations of equations in various forms such as tables, graphs, and written forms. The lessons were designed to support conceptual understanding, procedural fluency, and problem-solving (strategic competence) (Barnett et al., 2013). Mathematical Actions and Processes (MAP) are process standards for the Oklahoma Academic Standards for mathematics. Seven MAPs leverage both NCTM (2000) process standards and five strands of mathematics proficiency (NRC, 2001) to support students in accessing and understanding mathematics for life and workspace (Oklahoma Academic Standards for Mathematics, 2016).

MAPS are listed below and integrated into the lessons (see Table 3).

MAP 1: Develop a Deep and Flexible Conceptual Understanding

MAP 2: Develop Accurate and Appropriate Procedural Fluency

MAP 3: Develop Strategies for Problem Solving

MAP 4: Develop Mathematical Reasoning

MAP 5: Develop a Productive Mathematical Disposition

MAP 6: Develop the Ability to Make Conjectures, Model, and Generalize

MAP 7: Develop the Ability to Communicate Mathematically.

Table 3

Quadratic Functions Unit Outline

Quadratic Functions	Lesson Details	MAP
Lesson 1: Key Features of Quadratic Functions	Students began this lesson by launching projectile motion and modeling the flight of an angry bird by using tongue depressors, gummy bears, and rubber bands. Students learned the characteristics of the parabola.	MAP 1 MAP 6 MAP 7
Lesson 2: The effect of coefficients on the standard form of the quadratic function	Students conceptualized the projectile motion equation and explored the effects of graphically manipulating the structure of the coefficients with technology.	MAP 1 MAP 4 MAP 6 MAP 7
Lesson 3: Making sense of the structure of the three forms of quadratic functions	This lesson was a modification of the NCTM illuminations Egg Launch Contest. Students used the three forms of quadratic (tables, graphs, and algebraic equations) to determine the winner of the egg launch competition. Students also worked on the Skeleton Tower task (see Appendix D) adopted from MARS.	MAP 2 MAP 3 MAP 4 MAP 6 MAP 7
Lesson 4: Forming Quadratics	Forming Quadratics formative assessment lesson from Mathematics Assessment Resource Center (MARS) was used to garnish information from the structure of the forms of quadratics (standard form, vertex form, and factored form). Students were provided domino cards, matched the graphs with the equations, and completed the equation's missing form.	MAP 1 MAP 2 MAP 3 MAP 4 MAP 5 MAP 6
Lesson 5: Same story, different equation-moving between the forms	This lesson focused on moving between forms by drawing on skills of multiplying and factoring. This lesson involved some projectile motion questions. Students were also given three forms (standard, vertex, and factored form) of the same quadratic function, and then they proved that the equations were equal.	MAP 2 MAP 3 MAP 4 MAP 5 MAP 6 MAP 7
Lesson 6: Getting to vertex form-completing the square	Students used algebra tiles to discover what is meant spatially by "completing the square." This lesson built a conceptual understanding of the process of completing the square.	MAP 1 MAP 2 MAP 3 MAP 6 MAP 7

Table 3, cont.

Quadratic Functions	Lesson Details	MAP
Lesson 7: Transformation of quadratic functions	Students used technology to investigate the effects of changing parameters on the resulting graph. Students used an online interactive tool to explore how each parameter (slider) changes the graph.	MAP 1 MAP 2 MAP 4 MAP 6 MAP 7
Lesson 8: Solving Quadratics	This lesson first concentrated on solving a quadratic by exploring a graphical and tabular approach. Students made connections to the terminology of solving roots and x-intercepts.	MAP 2 MAP 3 MAP 4 MAP 6 MAP 7
Lesson 9: Solving Quadratics Comparing Methods	Knowing what to do with each form of quadratic structure leads to strategic competence in efficiently solving quadratic equations. The investigation focused on choosing the most appropriate method. At the end of the lesson, students were given a problem and ask them to come up with a convincing argument for their method.	MAP 2 MAP 3 MAP 4 MAP 5 MAP 6 MAP 7
Lesson 10: Generalizing Solving-The quadratic formula	The quadratic formula is a way to express repeated reasoning of solving quadratics in vertex form. Students explored this pattern and arrived at the quadratic formula. Students generalized the process of completing the square as the quadratic formula.	MAP 1 MAP 2 MAP 3 MAP 4 MAP 6 MAP 7

Students started the unit by launching projectile motion and modeling the motion. They modeled an angry bird's flight to identify their graph's critical features. They were engaged in some activities to use technology such as graphing calculators and spreadsheets to explore how parameter change affects the quadratic function's graph and key features. One of the NCTM illumination activities, named as egg launch contest (see Appendix C), was also adapted to make sense of the structure of the three forms of the quadratic functions (i.e., table, algebraic equation, and graph). Students participated in small group activities and whole-class discussions during the unit. Students used algebra tiles, which were meant spatially by completing the square, to build a conceptual understanding of completing the square. Each group was given lesson booklets that included the students' activities and workspace. These class works were used to gather information about students' methods and representations.

The same content was covered for both the comparison and treatment groups. Both groups used the same instructional materials and resources. While the comparison group received facilitated instruction, the treatment group was exposed to supporting productive struggle teaching practice. The quadratic function unit lasted approximately five weeks; students had five 45-minute periods per week.

Data Collection

In this study, I examined the difference regarding high school students' conceptual understanding, procedural fluency, and strategic competence when solving high-demanding quadratic functions problem-solving tasks between students exposed to supporting productive struggle teaching practice and those who received facilitated instruction. I used a quasi-experimental design.

Instrument

Pretest and posttest were used to check students' performance before and after the treatment. The pretest and posttest design approach could be applied in quasi-experimental design (Creswell, 2015). The researcher administers a pretest for comparison and treatment groups, conducts the treatment activities for the treatment group only, and uses the posttest to evaluate the differences between groups (Creswell, 2015).

Researchers need to make any possible effort to create groups equivalent to reducing the threats and strengthening the study (Gay et al., 2012). A pretest was implemented before beginning the study to ensure that the groups' level was close. Mathematics Understanding Rubric designed by Telese (1994) and revised by Aguilar and Telese (2018) was used to score pretests for all four classes. After scoring the pretest, an Analysis of Variance (ANOVA) was conducted if the mean of the pretest scores was significantly different between the factor levels of the treatment and comparison group. The ANOVA is appropriate when the research goals involve identifying significant differences in a continuous variable between two or more discrete groups (Gay et al., 2012).

After the quadratic function unit was completed, which took around five weeks, a posttest was given to both groups to compare students' Conceptual Understanding (C.U.), Procedural Fluency (P.F.), and Problem-Solving-Strategic Competence (PS-SC) of quadratic functions by using Mathematics Understanding Rubric (Aguilar & Telese, 2018). Students' solutions to each pretest and posttest problem were rated using Mathematics Understanding Rubric. The reliability of the rubric was tested by using the Generalizability theory. The G-coefficients were 0.86, 0.92, and 0.88 for C.U., P.F., and PS-SC, respectively.

I have selected open-response, demanding problem-solving tasks for both pretest and posttest (see Appendix A). Both instruments involved the same six open-response questions, which had multiple parts. Open-response questions were selected to get more information about students' conceptual understanding, procedural fluency, and strategic competence in quadratic functions. Mathematics Understanding Rubric designed by Aguilar and Telese (2018) was used to assess students' responses. Table 4 presents the name of each problem in both pretest and posttest and the webpage where the question was retrieved. The test items were selected to assess students' conceptual understanding, procedural fluency, and strategic competence.

Table 4

Pretest and Posttest Question Breakdown

Question #	Pretest and Posttest Questions
Question 1	Bunny Rabbit Population Problem (www.shelovesmath.com)
Question 2	Quadratics Trajectory (Path) Problem (www.shelovesmath.com)
Question 3	Small Steel Frame (www.mathsisfun.com)
Question 4	Optimizing of Area Problem (www.shelovesmath.com)
Question 5	I Rule! (www.mathematicsvisionproject.org)
Question 6	NAEP (1996) Pattern Problem (www.nationsreportcard.gov)

Validity

The researchers get assistance from judges or experts to determine whether the questions are valid (Creswell, 2015). This form of validity works well if the items' possibilities are easily identified and well-known (Creswell, 2015). Five high school mathematics teachers who have taught Algebra II or higher-level mathematics courses and two professors from a university mathematics education department validated the instrument by checking the difficulty, clarity, language, and applicability to measure students' conceptual understanding, procedural fluency, and strategic competence. Their suggestion was considered to adjust before administering the instrument.

Reliability

When researchers administer the same test multiple times, the instrument could be considered a reliable tool if the instrument's scores are stable and consistent (Creswell, 2015). I scored all pretest and posttest by myself to accommodate the inter-rater reliability. Inter-rater reliability is a concern in research studies due to ensuring raters' consistency in the performance level (Graham et al., 2012).

The test's reliability was evaluated by administering the test multiple times to a small group of students (n = 35) who did not participate in the study. Cronbach's alpha value was calculated to measure the instrument's internal consistency. The Cronbach's alpha coefficient was evaluated using the guidelines suggested by George and Mallery (2019) where > .9 excellent, > .8 good, > .7 acceptable, > .6 questionable, > .5 poor, and ≤ .5 unacceptable. The lower and upper bounds of Cronbach's α were calculated using a 95.00% confidence interval. The items for conceptual understanding had a Cronbach's alpha coefficient of .70, indicating acceptable reliability. Table 5 presents the results of the reliability analysis.

Table 5

Reliability Table for Conceptual Understanding

Scale	No. of Items	α	Lower Bound	Upper Bound
Conceptual Understanding	6	.70	.56	.83

The items for procedural fluency had a Cronbach's alpha coefficient of .74, indicating acceptable reliability. Table 6 presents the results of the reliability analysis.

Table 6

Reliability Table for Procedural Fluency

Scale	No. of Items	α	Lower Bound	Upper Bound
Procedural Fluency	6	.74	.62	.85

The items for strategic competence had a Cronbach's alpha coefficient of .74, indicating acceptable reliability. Table 7 presents the results of the reliability analysis.

Table 7

Reliability Table for Strategic Competence

Scale	No. of Items	α	Lower Bound	Upper Bound
Strategic Competence	6	.77	.65	.88

Data Analysis

Data analysis aimed to identify the differences regarding high school students' conceptual understanding, procedural fluency, and strategic competence when solving high-demanding quadratic functions problem-solving tasks between students who experienced supporting productive struggle teaching practice and those who received facilitated instruction. The independent variable in the analysis was the teaching method, which consists of two levels: Supporting productive struggle teaching practice and facilitated instruction. The dependent variable is the participants' posttest scores. Students' solutions for each posttest question were rated using Mathematics Understanding Rubric (Aguilar & Telese, 2018).

Descriptive statistics was used initially to generate meaningful information about raw data. The descriptive analysis provides the initial details on study outcomes and participants' responses (Creswell, 2015). Descriptive statistics were implemented to determine the mean scores and standard deviations of pretest and posttest for comparison and treatment groups.

After the descriptive analysis, the comparison and treatment group outcomes are compared to answer the hypothesis and the research question (Creswell, 2015). The author listed ANCOVA as one of the comparing statistics to compare the groups. Since the comparison and treatment group participants were not randomly assigned, there was a possibility of a difference between pretest scores. If a significant difference is identified, an analysis of covariance (ANCOVA) can be considered to statistically equate the groups (Gay et al., 2012). The author suggested that covariance (ANCOVA) analysis is one of the forms of analysis of variance (ANOVA). They also further stated that ANCOVA mainly controls extraneous variables and enhances statistical test power by reducing within-group variance. Since ANCOVA reduces the random sampling error, it improves the significance test's power (Gay et al., 2012). The authors suggested that ANCOVA adjusts posttest scores by controlling the differences between pretest scores.

Therefore, an Analysis of Covariance (ANCOVA) was conducted to assess if differences exist in students' conceptual understanding, procedural fluency, and strategic competence in solving high-demanding quadratic functions tasks (posttest scores) between the comparison and treatment groups after controlling pretest scores. ANCOVA revealed the impacts of supporting productive struggle teaching practice on students' conceptual understanding, procedural fluency, and strategic competence in solving high-demanding quadratic functions problems while controlling pretest scores co-vary with the posttest scores. Pretest scores were selected as a covariate because students' initial knowledge and background information about quadratic functions may affect their posttest performance. Controlling the pretest scores helped the researcher identify whether the teaching method affected students' posttest performance.

Summary

Relying on a quasi-experimental design, this study answered the following: What is the difference regarding high school students' conceptual understanding, procedural fluency, and strategic competence when solving high-demanding quadratic functions problem-solving questions between students who experienced productive struggle teaching practice and those who received facilitated instruction. This study used a pretest-posttest quasi-experimental research design. Analysis of covariance (ANCOVA) evaluated if there was a statistically significant difference between posttest scores of the treatment and comparison groups after controlling pretest scores.

CHAPTER IV

RESULTS

This study used a quasi-experimental design and collected quantitative data to explore how exposure to supporting productive struggle teaching practice impacts students' conceptual understanding, procedural fluency, and strategic competence in the topic of quadratic functions.

Data Analysis

Data analysis focused on exploring the differences regarding high school students' conceptual understanding, procedural fluency, and strategic competence on high-demanding quadratic functions problem-solving questions between students exposed to productive struggle teaching practice and those who received facilitated instruction. The Statistical Package for Social Sciences (SPSS, version 27) was used for computing all tests at a .05 significance level.

The analysis of the data was organized according to the research question. The two groups were: supporting productive struggle teaching practice was the treatment group, and facilitated instruction was the comparison group. For both groups, pretests were scored by using Mathematics Understanding Rubric designed by Aguilar and Telese (2018). The rubric evaluates three categories as Conceptual Understanding (CU), Procedural Fluency (PF), and Strategic Competence (SC) in five levels from 0 to 4. Since the pretest had six questions, the minimum test score could be 0, and the maximum test score could be 24 in each category. ANOVA was calculated separately for CU, PF, and SC.

Pretest Performance Differences Between Treatment Conditions

First**,** A one-way ANOVA was performed to compare both groups' pretest scores in conceptual understanding (CU). A one-way ANOVA revealed that there was no statistically significant difference in the pretest conceptual understanding mean scores (CU) between the treatment and the comparison groups, $F(1, 72) = 0.18$, $p = .669$, indicating the differences in CU among the levels of the group were all similar (Table 8). The means and standard deviations are presented in Table 9.

Table 8

Analysis of Variance Table for CU by Group

Term	*SS*	*df*	*F*	*p*	$\eta_p 2$
Group	1.52	1	0.18	.669	0.00
Residuals	595.34	72			

Table 9

Mean, Standard Deviation, and Sample Size for CU by Group

Combination	*M*	*SD*	*n*
Comparison	3.20	3.30	35
Treatment	3.49	2.44	39

Secondly, an analysis of variance (ANOVA) was conducted to determine whether there were significant differences in procedural fluency (PF) in comparison and treatment groups. The ANOVA was examined based on an alpha value of .05. The difference between the groups on pretest procedural fluency scores was not significant, $F(1, 72) = 0.40$, $p = .527$, indicating the results in PF among the treatment and comparison groups were all similar (Table 10). The means and standard deviations are presented in Table 11.

Table 10

Analysis of Variance Table for PF by Group

Term	*SS*	*df*	*F*	*p*	$\eta_p 2$
Group	5.69	1	0.40	.527	0.01
Residuals	1,011.72	72			

Table 11

Mean, Standard Deviation, and Sample Size for PF by Group

Combination	*M*	*SD*	*n*
Comparison	4.91	4.22	35
Treatment	4.36	3.27	39

Finally**,** ANOVA was conducted based on an alpha value of .05 to determine whether there were significant differences in Strategic Competence (SC) among the comparison and treatment groups. The results of the ANOVA suggested that there was no statistically significant difference in the pretest strategic competence (SC) mean scores between the treatment and the comparison groups, $F(1, 72) = 0.03$, $p = .873$, indicating the differences in SC among the levels of the group were all similar (Table 12). The means and standard deviations are presented in Table 13.

Table 12

Analysis of Variance Table for SC by Group

Term	*SS*	*df*	*F*	*p*	η_p2
Group	0.20	1	0.03	.873	0.00
Residuals	568.84	72			

Table 13

Mean, Standard Deviation, and Sample Size for SC by Group

Combination	*M*	*SD*	*n*
Comparison	3.77	3.16	35
Treatment	3.67	2.45	39

An Analysis of Variance (ANOVA) revealed no statistical differences in the pretest scores between productive struggle and the facilitated instruction group for each category. There was no statistically significant difference in the model. As a result, posthoc comparisons were not conducted.

Posttest Performance Differences Between Treatment Conditions

An analysis of covariance (ANCOVA) was conducted to assess whether conceptual understanding, procedural fluency, and strategic competence posttest scores significantly differ across levels of the independent variable while controlling for pretest scores. ANCOVA provided information about whether mean outcome scores differed across the comparison and treatment groups. ANCOVA has several assumptions that need to be evaluated.

Independence. The scores of the posttest are independent. The posttest questions were open-response questions, and students were not allowed to communicate with each other or share their answers with their classmates during testing. Therefore, the independence of the observation assumption was met.

Normality. The assumption of normality was assessed by plotting the quantiles of the model residuals against the quantiles of a Chi-square distribution, also called a Q-Q scatterplot (DeCarlo, 1997). For the assumption of normality to be met, the quantiles of the residuals must not strongly deviate from the theoretical quantiles. Strong deviations could indicate that the parameter estimates are unreliable. Figures 1, 2, and 3 present a Q-Q scatterplot of model residuals for conceptual understanding (CU), procedural fluency (PF), and strategic competence (SC) in order.

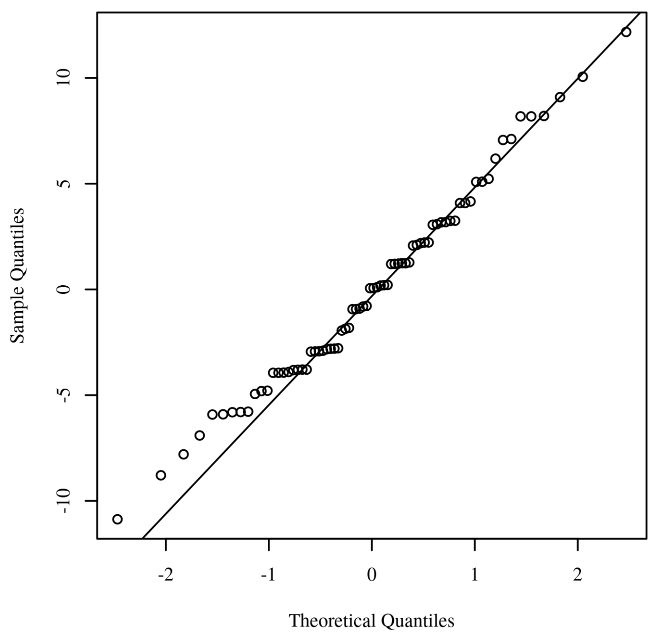

Figure 1: CU Q-Q scatterplot for normality of the residuals for the regression model

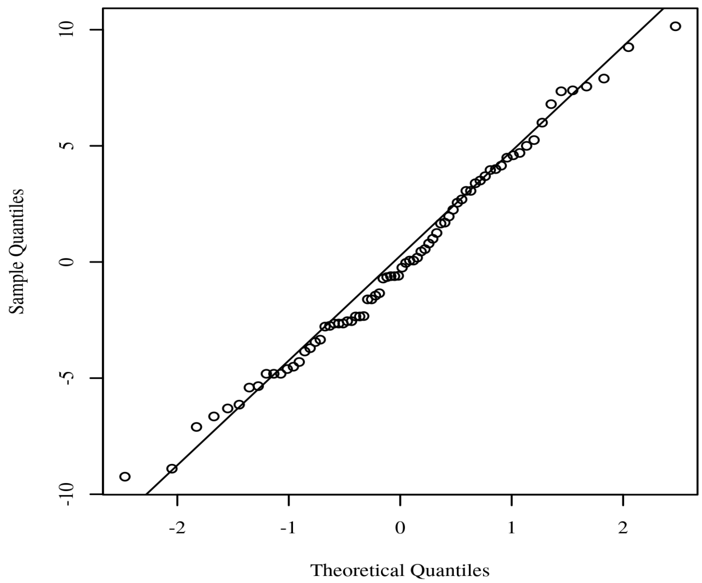

Figure 2: PF Q-Q scatterplot for normality of the residuals for the regression model

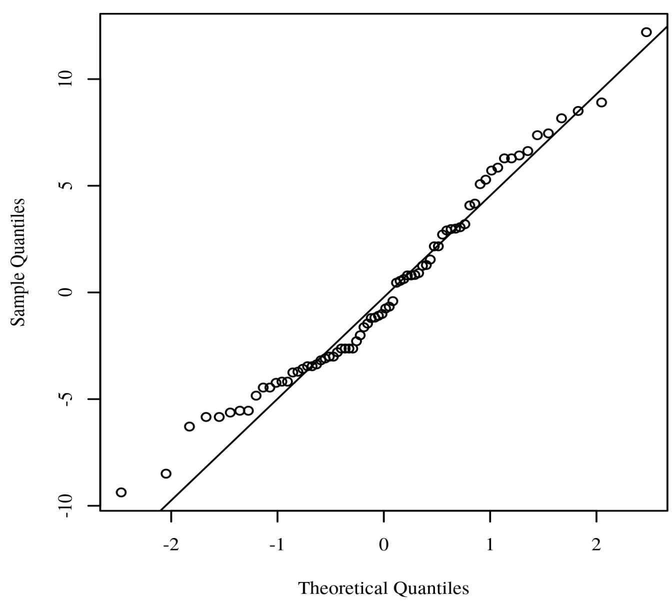

Figure 3: SC Q-Q scatterplot for normality of the residuals for the regression model

Homoscedasticity. Homoscedasticity was evaluated by plotting the residuals against the predicted values (Field, 2017). The assumption of homoscedasticity is met if the points appear randomly distributed with a mean of zero and no apparent curvature. Figures 4, 5, and 6 present a scatterplot of predicted values and model residuals for conceptual understanding (CU), procedural fluency (PF), and strategic competence (SC) in order.

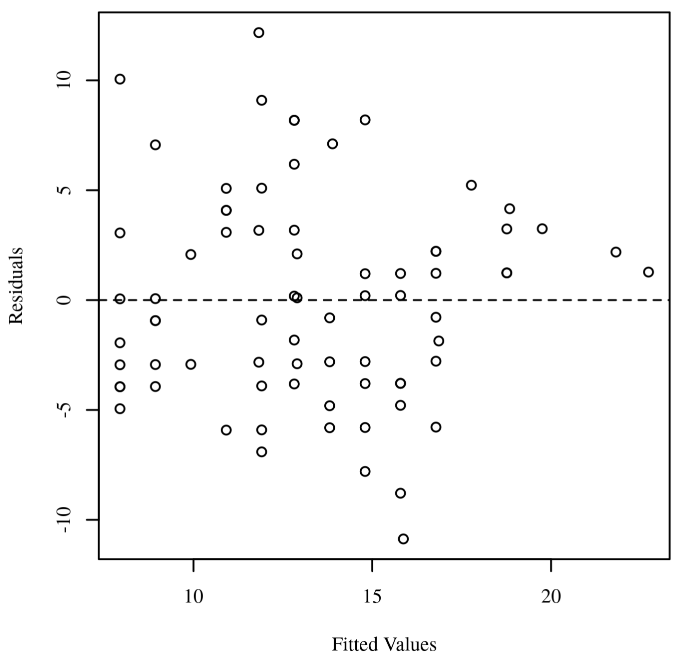

Figure 4: Residuals scatterplot testing homoscedasticity for CU

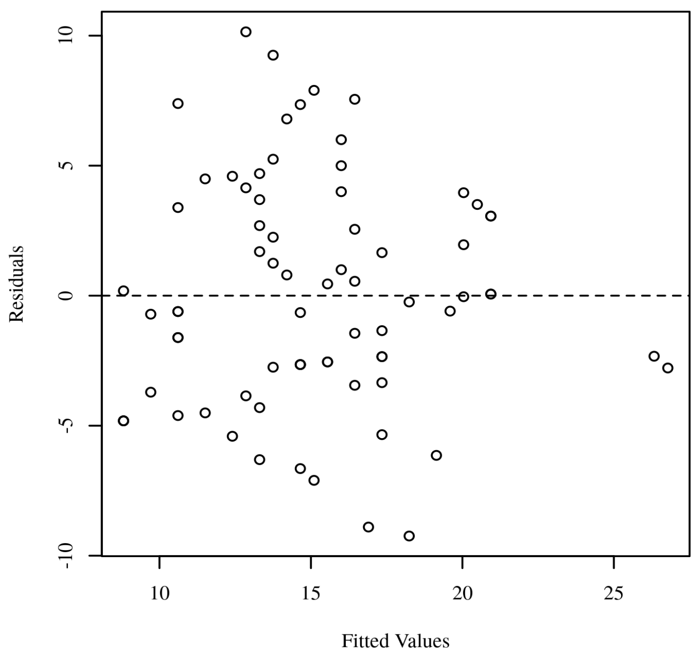

Figure 5: Residuals scatterplot testing homoscedasticity for PF

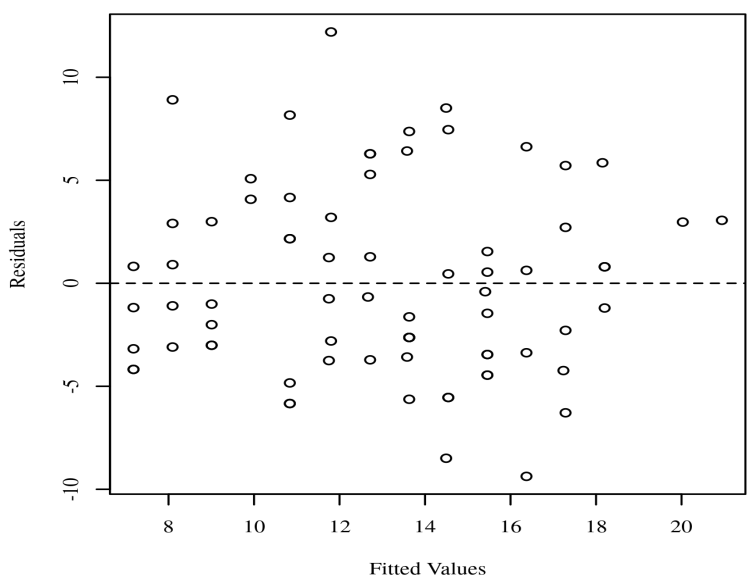

Figure 6: Residuals scatterplot testing homoscedasticity for SC

Outliers. Studentized residuals were calculated, and the absolute values were plotted against the observation numbers to identify influential points (Field, 2017; Pituch & Stevens, 2015). Studentized residuals are calculated by dividing the model residuals by the estimated residual standard deviation. An observation with a studentized residual greater than 3.21 in absolute value, the 0.999 quantiles of a *t* distribution with 73 degrees of freedom, was considered to influence the model's results significantly. Figure 7, 8, and 9 presents the studentized residuals plot of the observations for conceptual understanding (CU), procedural fluency (PF), and strategic competence (SC) in order. Observation numbers are specified next to each point with a studentized residual greater than 3.21.

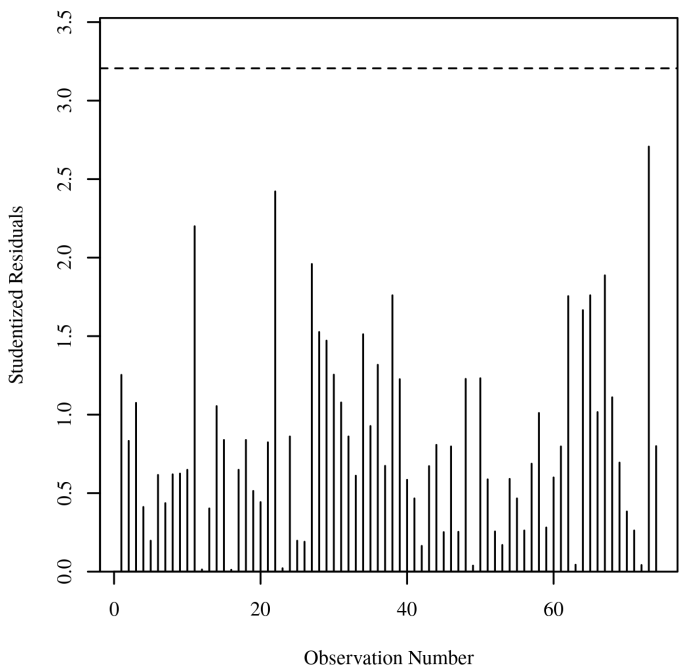

Figure 7: Studentized residuals plot for outlier detection for CU

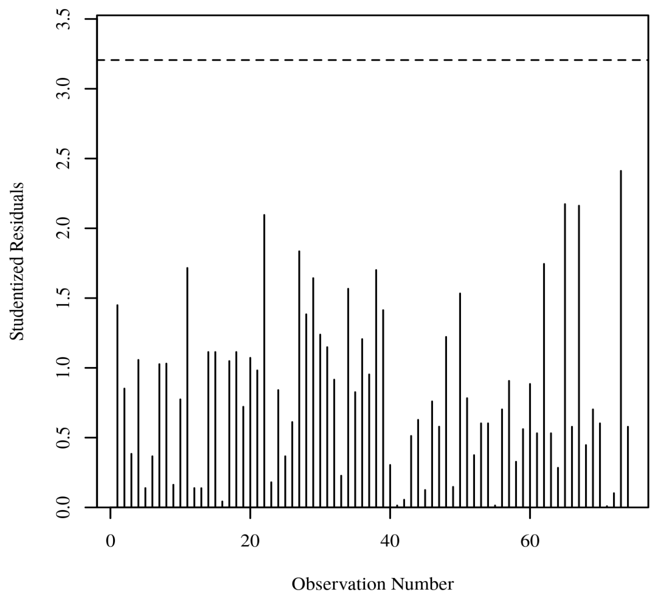

Figure 8: Studentized residuals plot for outlier detection for PF

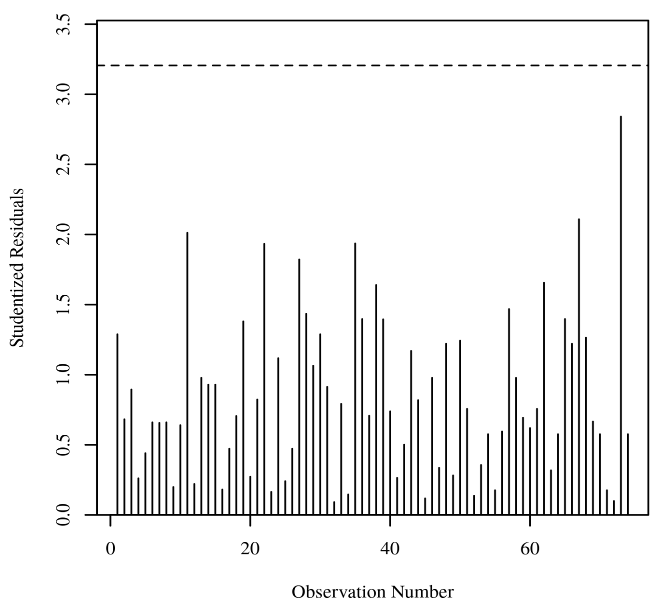

Figure 9: Studentized residuals plot for outlier detection for SC

Homogeneity of regression slopes. The assumption for homogeneity of regression slopes was assessed by rerunning the ANCOVA, but this time including interaction terms between each independent variable and covariate (Field, 2017; Pituch & Stevens, 2015). If the model with the covariate interaction terms explains significantly more variance than the original ANCOVA model, then there were significant interactions between the covariates and independent variables. The model with covariate-independent variable interactions did not explain significantly more variance for conceptual understanding posttest scores (PostCU) than the original model, $F(1, 70) = 0.21$, $p = .651$. The model with covariate-independent variable interactions did not explain significantly more variance for procedural fluency posttest scores (PostPF) than the original model, $F(1, 70) = 1.64$, $p = .205$. The model with covariate-independent variable interactions did not explain significantly more variance for strategic competence posttest scores (PostSC) than the original model, $F(1, 70) = 0.66$, $p = .419$. This implies that none of the covariates interacted with the independent variables, and the assumption of homogeneity of regression slopes was met.

Covariate-IV independence. Each independent variable and covariate must be independent (Miller & Chapman, 2001). Pretest was measured before starting the treatment to ensure that groups were as similar as possible using ANOVA as the results already presented for each category. There was no significant difference between the comparison and treatment in pretest scores before the implementation of the study. To assess independence, an ANOVA was conducted for each pair of numeric covariates and independent variables (Field, 2017). There were no significant models for any combination of covariates and independent variables based on an alpha of .05, indicating the assumption of independence between covariates and independent variables was met.

An analysis of covariance (ANCOVA) was conducted to determine whether there were significant differences in conceptual understanding posttest scores (PostCU) among the comparison and treatment groups while controlling for conceptual understanding pretest scores

(PreCU). The results of the ANCOVA were significant, $F(2, 71) = 19.60$, $p < .001$, indicating significant differences among the values of the comparison and the treatment groups (Table 14). The eta squared was 0.14, indicating group levels explain approximately 14% of the variance in PostCU. The means and standard deviations are presented in Table 15.

Table 14

Analysis of Covariance Table for PostCU by Group

Term	*SS*	*df*	*F*	*p*	η_p^2
Group	277.45	1	12.02	< .001	0.14
PreCU	584.39	1	25.31	< .001	0.26
Residuals	1,639.05	71			

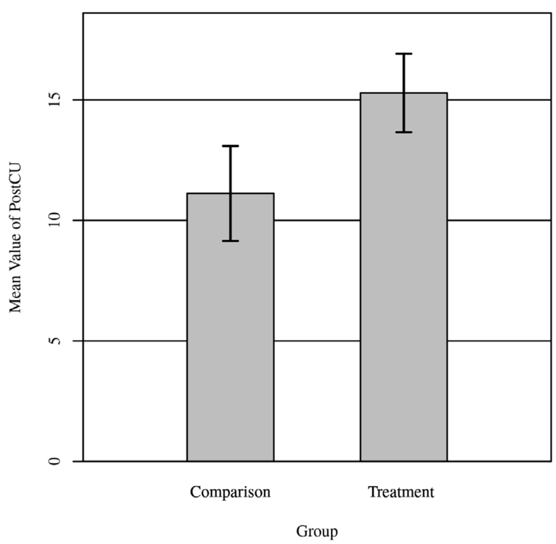

Figure 10: Mean value of PostCU by the levels of Group with 95.00% CI Error Bars

Table 15

Marginal Means, Standard Error, and Sample Size for PostCU by Group Controlling for PreCU

Combination	Marginal Means	*SE*	*n*
Comparison	11.26	0.81	35
Treatment	15.15	0.77	39

Estimated marginal mean contrasts were calculated to examine the differences between the level combinations using Tukey comparisons based on an alpha of .05. For the main effect of group, the mean of conceptual understanding posttest scores for the comparison group (M = 11.26, SD = 4.81) was significantly smaller than for treatment (M = 15.15, SD = 4.81), $p < .001$.

Second, ANCOVA was conducted to determine whether the group had significant differences in procedural fluency posttest scores (PostPF) while controlling for procedural fluency pretest scores (PrePF). The results of the ANCOVA were significant, $F(2, 71) = 26.48$, $p < .001$, indicating significant differences among the values of the comparison and the treatment groups (Table 16). The eta squared was 0.18, indicating group levels explain approximately 18% of the variance in PostPF. The means and standard deviations are presented in Table 17.

Table 16

Analysis of Covariance Table for PostPF by Group

Term	*SS*	*df*	*F*	*p*	η_p^2
Group	299.32	1	15.13	< .001	0.18
PrePF	816.56	1	41.28	< .001	0.37
Residuals	1,404.53	71			

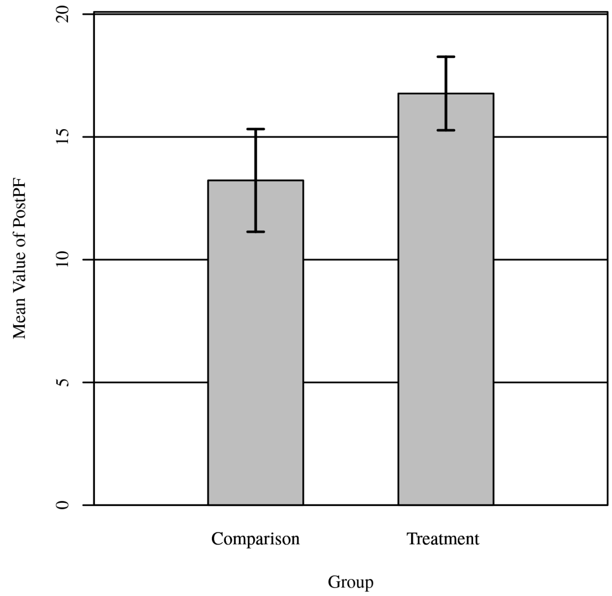

Figure 11: Mean value of PostPF by the levels of Group with 95.00% CI Error Bars

Table 17

Marginal Means, Standard Error, and Sample Size for PostPF by Group Controlling for PrePF

Combination	Marginal Means	*SE*	*n*
Comparison	12.97	0.75	35
Treatment	17.01	0.71	39

Estimated marginal mean contrasts were calculated to examine the differences between the level combinations using Tukey comparisons based on an alpha of .05. For the main effect of the group, the mean of PostPF for the comparison group (M = 12.97, SD = 4.45) was significantly smaller than for treatment group (M = 17.01, SD = 4.45), $p < .001$.

Finally, ANCOVA was conducted to determine whether there were significant differences in strategic competence posttest scores (PostSC) among comparison and treatment groups controlling for strategic competence pretest scores (PreSC). The results of the ANCOVA were significant, $F(2, 71) = 20.01$, $p < .001$, indicating significant differences among the values of groups (Table 18). The eta squared was 0.21, indicating that the group explains approximately 21% of the strategic competence posttest scores variance. The means and standard deviations are presented in Table 19.

Table 18

Analysis of Covariance Table for PostSC by Group

Term	*SS*	*df*	*F*	*p*	η_p^2
Group	393.76	1	18.47	< .001	0.21
PreSC	475.41	1	22.30	< .001	0.24
Residuals	1,513.84	71			

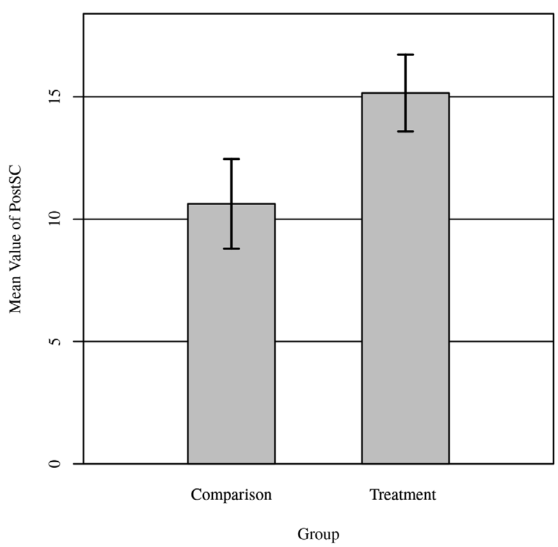

Figure 12: Mean value of PostSC by the levels of Group with 95.00% CI Error Bars

Table 19

Marginal Means, Standard Error, and Sample Size for PostSC by Group Controlling for PreSC

Combination	Marginal Means	*SE*	*n*
Comparison	10.58	0.78	35
Treatment	15.20	0.74	39

Estimated marginal mean contrasts were calculated to examine the differences between the level combinations using Tukey comparisons based on an alpha of .05. For the main effect of the group, the mean of PostSC for the comparison group ($M = 10.58$, $SD = 4.62$) was significantly smaller than for treatment group ($M = 15.20$, $SD = 4.62$), $p < .001$.

While pre-test scores were not significantly different, the posttest scores were statistically significant, as reported by an ANCOVA. An examination of the ANCOVA results revealed that there were statistically significant differences in posttest conceptual understanding, procedural fluency, and strategic competence mean scores between the treatment and comparison groups. The mean scores of the productive struggle teaching practice group for each category were higher than the facilitated instruction group.

Summary

The current study sought answers to the following research question:

Research Question

What are the differences regarding high school students' conceptual understanding, procedural fluency, and strategic competence when solving high-demanding quadratic functions problem-solving tasks between students exposed to productive struggle teaching practice and those who received facilitated instruction?

Null Hypotheses (H_0). The mean of the posttest scores for conceptual understanding, procedural fluency, and strategic competence is equal for the treatment and comparison groups after controlling for the effects of the pretest scores.

Alternative Hypotheses (H_1). The mean of the posttest scores for conceptual understanding, procedural fluency, and strategic competence is not equal for the treatment and comparison groups after controlling for the effects of the pretest scores.

In response to the research question, the results revealed statistically significant differences in the scores for conceptual understanding, procedural fluency, and strategic competence of the treatment group exposed to supporting productive struggle teaching practice, and the comparison group received facilitated instruction. Though the initial scores of the pretest were not significantly different, the treatment group performed significantly higher than the comparison group, as computed using an ANCOVA.

CHAPTER V

SUMMARY AND CONCLUSION

Research on the benefit of intentional struggle-filled learning has a long history involving Dewey (1933), Piaget (1977), and Vygotsky (1980), emphasizing that exposing children to challenging learning will allow them to construct their understanding (Vazquez et al., 2020). However, US mathematics instruction does not engage students in an active struggle with mathematics concepts (Banilower et al., 2006). Hiebert et al. (2005) analyzed 83 videos recorded from United States classrooms. They stated that even when students were presented with challenging problems and had a chance to struggle productively, teachers lowered the cognitive demand of the tasks and lessons by providing procedures.

The productive struggle has become a hot topic in mathematics education due to NCTM's (2014) effective mathematics teaching practice recommendations and CCSSM's (2010) standards of mathematics practices. Even though it is a hot topic, only recent studies have begun providing empirical evidence about productive struggle's benefits (Vazquez et al., 2020). This study examined the impact of supporting productive struggle teaching practice on students' conceptual understanding, procedural fluency, and strategic competence.

Supporting productive struggle in learning mathematics is one of the research-informed effective mathematics teaching practices identified and recommended by NCTM (2014), which is the world's most extensive and leading mathematics education organization for mathematics teachers in the United States. This study explored the impact of supporting productive instructions as anticipating students' struggles, giving time to struggle, helping students realize that confusion and mistakes are a natural part of learning, praising students for their effort, and probing guidance and affordance (Kapur, 2011; NCTM, 2014; Warshauer, 2015; Zeybek, 2016) on three strands of mathematical proficiency (i.e., conceptual understanding, procedural fluency, and strategic competence). The findings were interpreted by connecting them to the literature.

The following sections include the discussion of the results in the light of the reviewed literature as well as in light of current views and perspectives. The limitations of the study, implications, and future research directions are then discussed.

Discussion

Overall results of this study indicated that students in the supporting productive struggle teaching practice group performed significantly better than students who received facilitated instruction in every three strands of mathematical proficiency (i.e., conceptual understanding, procedural fluency, and strategic competence). These findings are supported by the Third International Mathematics and Science Study (TIMSS) video study. The goal of TIMSS was to assess the teaching methods that high-achieving countries like Japan and Hong Kong, for instance, used in 8th-grade math and science classes. They found that students in the high achieving countries provided chances to engage in struggle-inducing activities using their method and previous knowledge to solve problems (Hiebert et al., 2005). Kapur (2011, 2016) also found that engaging in productive failure, much like a productive struggle, improved material retention, conceptual understanding, and using various methods to solve complex math problems.

The studies in cognitive science also supported how productive struggle can enhance deeper learning and understanding. For example, engaging students in challenging activities help students build relevant associations (Carpenter, 2009) and improve their awareness of what they know and do not know (Toppino & Cohen, 2010).

Performance Differences between the Treatment and Comparison Groups

Research Question: What are the differences regarding high school students' conceptual understanding (CU), procedural fluency (PF), and strategic competence (SC) on high-demanding quadratic functions problem-solving questions between students exposed to productive struggle teaching practice and those who received facilitated instruction?

The research question explored the scores for conceptual understanding, procedural fluency, and strategic competence differences using ANOVA and ANCOVA. A summary of key findings followed by a discussion of intersections with recent research is presented below.

The students began with similar levels, as evidenced by pretest results. When controlling the pretest, both the comparison and treatment group improved their scores. The implementation of supporting productive struggle teaching practice resulted in the treatment group improving CU, PF, and SC more than the comparison group. The supporting productive struggle group participants outperformed the facilitated instruction group in each category (i.e., conceptual understanding, procedural fluency, and strategic competence). Students exposed to supporting productive struggle teaching experience had more correct answers than those who experienced facilitated instruction. Students' responses were scored by using Mathematics Understanding Rubric designed by Aguilar and Telese (2018). The results of ANCOVA were significant, indicating significant differences among the treatment conditions. The conceptual understanding posttest scores of the treatment group who exposed supporting productive struggle teaching practice scored 14 percent higher than the facilitated instruction group. The supporting productive struggle teaching practice group scored 18 percent higher in the procedural fluency posttest scores and 21 percent higher in the strategic competence posttest scores than the facilitated instruction group.

This finding aligns with that Kapur's (2016) findings. Kapur (2016) found that students engaged in productive failure perform higher and gain a deeper mathematical understanding than the direct instruction group. Thus, in concurrence with recent research findings (Kapur, 2016), data from this study highlighted significantly CU, PF, and SC performance differences between students receiving supporting productive struggle instruction than those facilitated instruction, while controlling the pretest.

This finding contributes to the development of a clearer understanding of previous research regarding supporting productive struggle in learning mathematics teaching practice. Several studies focus on various aspects of productive struggle as struggle types and teacher actions (Warshauer, 2015), preservice teachers' struggle types and how engaging in high-level non-routine tasks fosters productive struggle (Zeybek, 2016), parents' beliefs about productive struggle, and the relationship between parents' belief and their math homework help (Vazquez et al., 2020). However, none of these studies examined the impacts of productive struggle on students' learning.

After reviewing high-ranked mathematics education journals, which are presented in William et al.'s (2017) article such as Educational Studies in Mathematics (ESM), Journal for Research in Mathematics Education (JRME), Journal for Mathematical Behavior (JMB), and

International Journal of Math Education in Science and Technology (IJMEST), etc. the researcher could not find any study that directly examines how supporting productive struggle teaching practice impacts students mathematical proficiency. Some articles in practitioner journals review how productive struggle affects students' mathematical understanding. For example, Baker et al. (2020) unpacked the idea of productive struggle from a teacher and student perspective by using a specific scenario focused on a conversation between a student and a teacher. The student was assigned a problem that allowed productive struggle, and the teacher paid close attention to the students thinking by supporting productive struggle without immediately correcting the student's mistakes. Baker et al. (2020) found that struggle can become productive when the appropriate learning opportunities are available. They also emphasized the importance of recognizing and supporting the signs of productive struggle such as verbal, gestures, and frustration that will help students pursue deep learning.

Similarly, Taylor and Lee (2021) implemented a STEM task that engaged students in productive struggle. Teachers in this study reported that students developed an awareness of perseverance when students are not simply struggling but rather teachers have a structure to help them work toward mathematical success. These are some examples from the classroom that have a small sample. However, empirical research on productive struggle is limited (Warshauer, 2015), so this study contributed to the literature by examining how supporting productive struggle teaching practices impacts mathematical proficiency.

Teaching Strategies that Supported Students' Struggle

When students struggled and could not progress, the teacher asked manageable questions that helped students to move on to the next step and solve the question correctly. The result of this study coincides with the results obtained by Hiebert and Wearne (2003). They found that asking more questions improved students' understanding and computation rather than practicing demonstrated procedures. Other studies also supported that questioning helps students organize their thinking (Sorto et al., 2009), and scaffolding questions allow productive exchange and improve student learning (Warshauer, 2015).

Using multiple representations such as tables, graphs, models, and equations helped students to understand the concept better. Most students did not start solving tasks using tables or graphs. After discussing how they could progress on solving the tasks, they started making a table and were able to process. Using multiple representations allows students to comprehend math concepts from different perspectives (Duval, 2006). Smith's (2017) framework supported this finding by referring that engaging in a problematic situation, recording the data using multiple representations (e.g., table, graph, verbal), and finding patterns develop algebraic thinking.

Verifying answers helped the teacher to check if they knew what they were doing. For instance, if they knew what the variable represents in the equation. When students had a hard time getting a part of the question, they asked questions to the teacher, and the teacher answered the question without lowering the cognitive demand. Asking questions about their struggle is listed as one of the students' actions supporting productive struggle in learning mathematics by NCTM (2014).

Visualization using physical and virtual manipulatives was another strategy the teacher used to support students' productive struggle. For instance, students used algebra tiles in completing the square lesson. Algebra tiles clearly explain the notion of completing the square because available pieces could be used to form a perfect square (Miranda, 2010). Arcavi (2003) claimed that visualization is helpful and beneficial for both illustration and actual concept. A visual image analyzes the data and guides through the analytical development of a solution (Radford, 2010). The use of physical objects was stressed in teaching and learning algebra by NCTM standards (NCTM, 2000). Using physical or virtual manipulatives provides visual models of mathematical ideas (NCTM,2014). Aburime (2007) reported that manipulatives increased students` mathematics achievement.

Revoicing also helped students to resolve their struggles. The teacher pointed out something students had already figured out that enabled them to process the question. Warshauer (2015) also suggested revoicing to help students clarify their solutions. Students used the graphing calculator from time to time to graph, visualize, and calculate during the lessons.

Tools and technology also play a crucial role in meaningful mathematics learning because they support learners in exploring mathematics, making sense of the concept, and engaging students in mathematical reasoning (NCTM,2014). Graphing calculators were used in the classroom during the treatment for calculations and to explore different aspects of the tasks. For example, students used calculators to explore how a parameter change affects the graph. Ndlovu and Ndlovu (2020) conducted a mixed-method study to investigate the effects of the graphing calculator on eleventh-grade students' achievement in quadratic inequality problem-solving. They found that the treatment group that received instruction with graphing calculators performed significantly better than a comparison group taught without a graphing calculator. Additionally, students in the treatment group used more multiple solution strategies than the comparison group.

Implications

Overall, the results reported here should serve as positive news for researchers, teachers, and education reformers. Statistically, mathematical proficiency is significantly impacted by supporting productive struggle teaching practice. For the productive struggle to be effective, teachers should select the appropriate tasks that allow productive struggle by ensuring that tasks can be solved using multiple pathways, and teachers support and guide students without lowering the cognitive demand of the task. Students need sufficient time to solve challenging math problems and to develop curiosity (Goldenberg et al., 2015). Students' struggle does not cause frustration if the teacher provides feedback, asks manageable questions, have students make connections between multiple representations (Kapur, 2011; Warshauer, 2015).

Education reformers, decision-makers, and department coaches can consider ways to educate teachers on the purpose and efficacy of supporting productive struggle in learning mathematics approach for helping students to improve mathematics proficiency. Teachers do not have to change their teaching structure completely. For example, practicing procedural skills can still support productive struggle if novel problems are implemented in the lessons and students justify why a procedure works (Vazquez et al., 2020). However, US math education tends to convert highly structured problems designed for deeper thinking into routine exercises (Hiebert et al., 2005). Thus, teachers must resist stepping in and explaining to students everything step by step (Kapur, 2011).

Teacher training institutions should assume the lead in developing teachers on how to integrate supporting productive struggle teaching practice in their classes. Teachers should create a classroom structure in that struggle is viewed as a natural part of the learning process (Star,

2015) and highlights perseverance to make sense of learning mathematics (CCSSM, 2010). The idea of productive struggle may be unfamiliar and take time to become a classroom routine. Teachers may consider connecting with colleagues to observe and guide each other.

Learning the teaching strategies that support students' productive struggle may help teachers to integrate these strategies into their classrooms. For example, asking questions without giving the answer and lowering the cognitive demand helped the participants of this study to comprehend the concept and solve the tasks correctly. Teachers may try to anticipate students' struggles, design lessons accordingly, and guide students by asking questions. Learning strategies that support students' productive struggle may help educators or curriculum developers embed scaffold tasks that allow productive struggle. Also, I expect that, as students have more opportunities to engage in productive struggle, they will develop a deeper understanding of mathematics and become more responsible for their learning.

Limitations

This study has several limitations that are important to consider. This study was limited to a group of students from a single school selected using a non-random assignment of participants. Another limitation of this study was assigning participants to the treatment and comparison groups because the researcher used preexisting classrooms. The sample size was small. Although this allowed me to analyze the results deeply, I cannot make claims that can extend to all students.

Conclusion

This study explored how exposure to supporting productive struggle teaching experience impacts students' procedural fluency, conceptual understanding, and strategic competence. The treatment group exposed to supporting productive struggle teaching practice performed significantly better than the comparison group that received facilitated instruction. Based on the study results, it can be said that supporting productive struggle teaching practice is an effective method to improve students' conceptual understanding, procedural fluency, and strategic competence.

Future studies could examine how supporting productive struggle teaching practice impacts the other two strands of mathematical proficiency (i.e., adaptive reasoning and productive disposition). Also, the same research question could be examined with a larger population to confirm if the results are generalizable.

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APPENDIX A

APPENDIX A

QUADRATIC FUNCTIONS PRETEST AND POSTTEST

These questions require you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

Question 1: The observed bunny rabbit population on an island is given by the function $p = -0.4t^2 + 130t + 1200$, where t is the time in months since they began observing the rabbits.

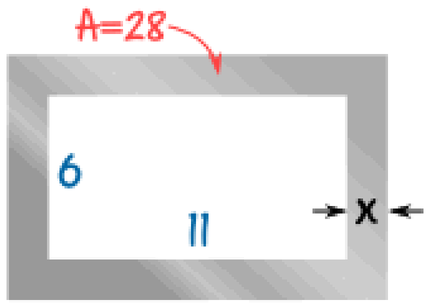
(a) When is the maximum population attained?

(b) What is the maximum population?

(c) When does the bunny rabbit population disappear from the island?

Question 2: Audrey throws a ball in the air, and the path the ball makes is modeled by the parabola $y - 8 = -0.018 (x - 20)^2$, measured in feet. What is the maximum height the ball reaches, and how far (horizontally) from Audrey is the ball at its maximum height? How far does the ball travel before it hits the ground?

Question 3:

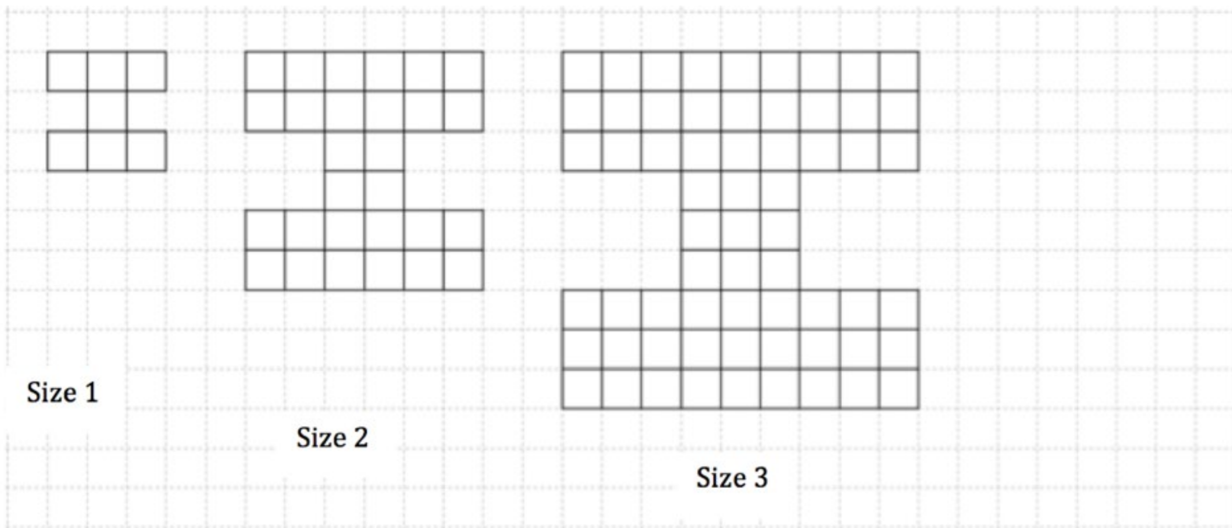


Your company is going to make frames as part of a new product they are launching. The frame will be cut out of a piece of steel, and to keep the weight down, the final area should be **28 cm^2**. The inside of the frame has to be **11 cm by 6 cm**. What should the width **x** of the metal be?

Question 4: A **rectangular** rose garden is being built against the back of a house with a fence around it, but we only have **120 feet** of fencing available. What would be the dimensions (length and width) of the garden, with one side attached to the house, to make the area of the garden **as large as possible?** What is this **maximum area**?

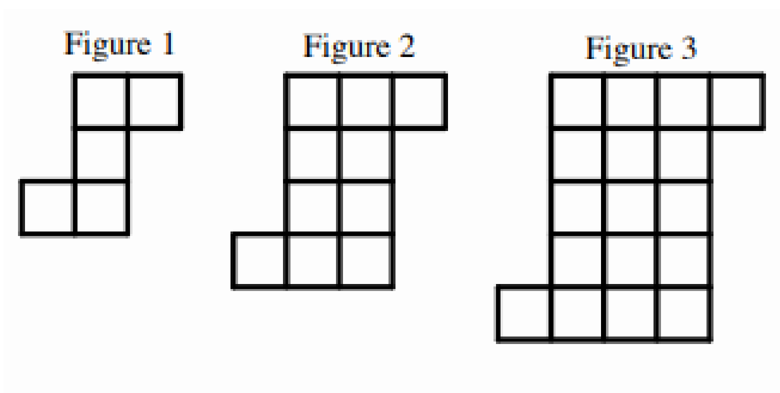
Question 5: Marco has started a new blog about sports at Imagination High School (mascot: the fighting unicorns) that he has decided to call "I Site". He created a logo for the website that looks like this:

He is working on creating the logo in various sizes to be placed on different pages on the website. Marco developed the following designs:



How many squares will be needed to create the size 100 logo? Develop a mathematical model for the number of squares in the logo for size *n*.

Question 6: The first 3 figures in a pattern of tiles are shown below. The pattern of tiles contains 50 figures.



Describe the 20th figure in this pattern, including the total number of tiles it contains and how they are arranged. Then explain the reasoning that you used to determine this information. Write a description that could be used to define any figure in the pattern.

APPENDIX B

APPENDIX B

Mathematics Understanding Rubric by Aguilar & Telese (2018)

Performance Level	Procedural Fluency	Conceptual Understanding	Problem Solving- Strategic Competency
0	No response.	Lack of evidence to determine knowledge, or no attempt made.	No response.
1	Incorrect or very limited use of operations, more than one major error or omissions.	Wide gaps in concept understanding, major errors made based on lack of conceptual knowledge.	Unworkable approach, incorrect or no use of mathematical representations, poor use of estimation, evidence for lack of understanding.
2	Some correct use of number operations but a major error or with several minor errors.	Some evidence of conceptual understanding, but difficulty in using models, diagrams, and symbols for representing concepts or translating from one mode to another mode. Some evidence of the concept's properties.	Appropriate approach, estimation used, implemented a strategy, possibly reasoned decision making, solution with observations.
3	Appropriate use of number operations with possible slips or omissions, but without significant errors.	Good evidence of conceptual knowledge. No major misconceptions; responses contain accurate use of models, diagrams, and symbols with evidence of translation from one mode to the other. Recognition of the meaning and interpretation of concepts. Some evidence of using concepts to verify or explain procedures.	Workable approach, used estimation effectively, mathematical representation used appropriately, reasoned decision-making inferred, judge reasonableness of solution.
4	Extended use of number operations without errors in calculations; appropriate use of models or representations.	Clear understanding of concepts and associated procedures. Effective use of models, diagrams, and symbols with broad translation from one mode to another. Recognition of the meaning and interpretation of concepts to explain or verify procedures or conclusions.	Efficient/sophisticated approach, estimation used effectively, extensive use of mathematical representations, explicit reasoned decision-making Solutions with connections, synthesis or abstraction.

APPENDIX C

APPENDIX C

TASK 1: EGG LAUNCH CONTEST

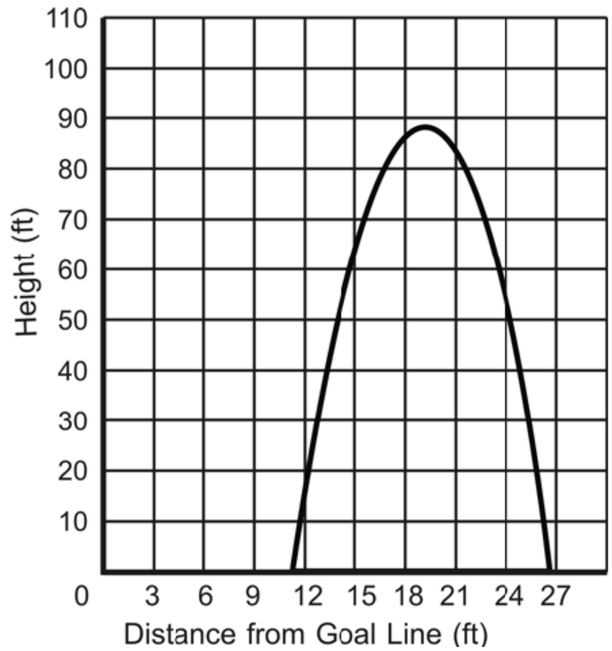
Mr. Rhodes' class is holding an egg launching contest on the football field. Teams of students have built catapults that will hurl an egg down the field. Ms. Monroe's class will judge the contest. They have various tools and ideas for measuring each launch and how to determine which team wins.

Team A used their catapult and hurled an egg down the football field. Students used a motion detector to collect data while the egg was in the air. They came up with the table of data below.

Distance from the Goal Line (in Feet)	Height (in Feet)
7	19
12	90
14	101
19	90
21	55
24	0

Team B's egg flew through the air and landed down the field. The group of students tracking the path of the egg determined that the equation $y = -0.8x^2 + 19x - 40$ represents the path the egg took through the air, where x is the distance from the goal line, and y is the height of the egg from the ground. (Both measures are in feet.)

When **Team C** launched an egg with their catapult, some of the judges found that the graph to below shows the path of the egg.



Which team do you think won the contest? Why?

APPENDIX D

APPENDIX D

SAMPLE LESSON PLAN

Engage	The teacher will pass out Egg Launch Contest handouts and have students read the activity sheet individually (**give time to struggle**). Students will make a prediction individually and write an explanation with reasoning. The task is a **high demanding problem-solving** task that is challenging and within reach of students. The teacher will anticipate possible solutions and difficulties that students may face. Students may graph Team A and Team B and compare with Team C. They may struggle to compare the quadratic function in three different forms. If students ask questions to the teacher, teacher will question them to reveal their thinking. The teacher will **guide students without giving the answer and lowering the demand of the task**. When students make a mistake, teacher will point out that **mistakes are natural part of learning**.
Explore	Students will work in groups to construct a viable argument for who they believe is the winner of the Egg Launch Competition. Students should grabble in groups with what information they have and what form would be most helpful. Students will be encouraged to be creative in thinking approaches and find the ways to evaluate the usefulness and correctness of their hypothesized solutions. The teacher **will not direct their thinking**. The teacher will **praise students for their efforts** and willingness to find different techniques to explain their thinking.

After students had time to construct a group argument, the teacher will select and sequence groups to present. Students will be **encouraged to critique the reasoning of others by asking questions and comparing methods**. The teacher can ask some questions to reveal students thinking and may be used to spark the discussion.

Anticipated possible questions could be:

How many points did it take to find the complete equation? Why is this so?

Was it easy to write all three forms? Explain.

Explain different strategies that could be used to find the maximum height of the egg? Which is the most efficient.

What group used the structure of the forms of quadratics to help write an equation.

Did another group use the same information in a different way?

The class will come to conclusion based on the winner of the competition for height and the winner for distance.

After students complete Egg Launch Contest activity, they will work on three forms of quadratic functions and making sense of them task. Students will be passed out the handout and work in groups of four. This task involves open-ended **problem-solving activities** some of the parts presented below.

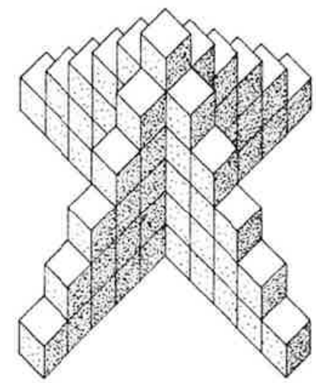
Task #5: Making Sense of the Three Forms of Quadratic Functions

Often times the standard form of a quadratic is used in projectile motion. For this particular situatio the equation $h(t) = -\frac{1}{2}gt^2 + v_o t + h_o$ gives the height of an object at time t for an object that has initia velocity, v_o and initial height of h_o. "g" is a gravitational constant and is either 9.8m/s² or 32ft/s². Of times a simpler form of the equations look like:

For Meters - $h(t) = -9.8t^2 + v_o t + h_o$

For Feet - $h(t) = -16t^2 + v_o t + h_o$

	1. A piece of paper and a hammer are dropped off the top of your school which is 90 feet high. They are both dropped from a still position ($V_0 = 0$). If we ignore the air resistance, which object (hammer or paper) hits the ground first? Provide a mathematical argument! 2. A potato is fired from a spud-gun at a height of 3m and initial velocity of 25m/s. How high the potato reaches and at what time does this occur? 3. Two competing catapults launch pumpkins. Catapult A launches from a starting height of 10ft and initial upward velocity of 45ft/sec. Catapult B launches from a starting height of 25ft and initial upward velocity of 40ft/sec. Which pumpkin, A or B, achieves a greater maximum height? Which pumpkin A or B is in the air longer? Is it possible from this scenario to determine the distance traveled horizontally by each pumpkin? Explain your choice and justify your answer.

Evaluate	Closing activity is Skeleton Tower task from MARS. **Task #7: Skeleton Tower** 1. How many cubes are needed to build this tower? Show your calculations 2. How many cubes are needed to build a tower like this, but 12 cubes high? Explain how you figure out your answer.  3. How would you calculate the number of cubes needed for a tower *n* cubes high? Students will have **time to individually think about the problem** followed by a **collaborative time with a partner**. As students work, the teacher will monitor their progress towards developing different forms of quadratics in order to select and sequence a class presentation. Students will make a conclusion that even though their expression for a tower of n cubes looks different, they are all equivalent.

APPENDIX E

APPENDIX E

CHILD ASSENT FORM

Purpose

I am studying how exposure to effective mathematics teaching practices (e.g., productive struggle) from the National Research of Council of Mathematics (NCTM) impacts students' conceptual understanding, procedural fluency, and strategic competence in solving quadratic functions problem-solving tasks. You can decide if you would like to participate in the study. There will be no consequences of any type, and you can drop from the study at any moment. I will be discussing this with your parents too. Your parents are not allowed to have you participate unless you agree.

Description of the Study

Throughout the quadratic functions unit, you will be engaged in high-quality mathematics tasks to help you understand quadratic functions. You will not do any extra assignment or activity other than the work you would already be doing in this class. As you learn math, I will video-record your teacher implementing the lesson plan. You will take a pretest at the beginning of the study and a posttest at the end. Neither your pretest nor posttest will impact your grading.

If you decide not to participate in the study, you will be attending your Algebra 2 class as you usually do, with the same instruction provided by the same teacher, but your data will not be included in this study.

Risks

Being in this study involves no greater risk than what you ordinarily encounter in daily life. Your participation in this research will be held strictly confidential, and only a code number will be used to identify your stored data.

Benefits

You will be exposed to an innovative teaching method that will help you to improve your mathematics skills. In addition, you will have an opportunity to experience how productive struggle enhances your mathematics learning and knowledge acquisition of topics related to quadratics function.

Mathematics teachers, administrators, policymakers, researchers in mathematics education will benefit from this study by better understanding the importance of supporting the use and implementation of productive struggle teaching practices on students' mathematics proficiency. The study results will reveal how productive struggle teaching practices could help students improve their mathematical proficiency.

Who to talk to about questions

If you have questions about the study, you can ask us now or later. Your parents have been given our contact information.

If you have questions about your rights in the study, contact The University of Texas Rio Grande Valley Institutional Review Board at (956) 665-3598 or irb@utrgv.edu.

I agree to take part in the study.

Child's Name	Signature	Date

APPENDIX F

APPENDIX F

PARENT/GUARDIAN PERMISSION FORM FOR CHILD PARTICIPATION IN RESEARCH

Study Title: The Impacts of Supporting Productive Struggle Teaching Practice on Students' Conceptual Understanding, Procedural Fluency, and Strategic Competence: The Case of Quadratic Functions.

Permission Form Name: Parent/Guardian Permission Form for child participation in research

Principal Investigator:	Sumeyra Karatas	Telephone: (956) 778-6211
Emergency Contact:	Sumeyra Karatas	Telephone: (956) 778-6211

Key points you should know

- I am inviting your child to be in a research study that I am conducting. Your child's participation is voluntary, and this means it is up to you and your child to decide if they want to be in the study. Even if you choose to have your child join the study, you are free to have them leave at any time if you change your mind.
- I want to invite your child to participate in my research study to learn how exposure to one of the effective mathematics teaching practices (e.g., productive struggle) from the National Council of Teachers of Mathematics (NCTM) impacts students' conceptual understanding, procedural fluency, and strategic competence in solving quadratic functions problem solving tasks.
- Why is your child being asked to be in this study?
    - Only students taking Algebra 2 during the 2021-22 school year will participate in the study.
- What will your child do if you agree for them to be in the study?
    - The participant students will receive high-quality mathematics instruction, be involved in high-leverage mathematical thinking activities, and develop a deep understanding of mathematical concepts. Two Algebra II classes will be selected as the comparison group, and the other two will be in the treatment group. Both comparison and treatment groups will be taught the same content and engaged in the same math tasks. The teacher will implement the same lesson content and tasks using her regular teaching style and standard pedagogy in the comparison group, whereas the teacher will be using the productive struggle teaching practice for the treatment group. Not providing the treatment will not deprive the

comparison group of quality instruction or content. If the treatment group outperformed the comparison group in the posttest, then the treatment will be offered to the comparison group students. Participants will be asked to take a pre and posttest at the beginning and the end of the study. Treatment group will be exposed to supporting productive struggle teaching practices during the quadratic functions unit for approximately five weeks. I will be video-recording the teacher's implementation of the lesson plan. The video recordings will only be used to ensure lesson plans are appropriately implemented. I will analyze students' pretests and posttests to determine their mathematical proficiency in the unit of quadratic functions. Please indicate whether you will allow us to do so by initialing one of the following:

  - _____(initials) Yes, I give permission for [videotaping/audiotaping]
  - _____ (initials) *No, I do not give permission for [videotaping/audiotaping]*

- Can your child be harmed by being in this study?
  - This project presents minimal risks, which may come from regular anxiety of being recorded during class time and knowing that the researcher will analyze the video. To minimize the anxiety, the teacher will explain to students that the videos will be analyzed to ensure that the lesson plans are implemented properly. There will not be any grading as a result of videos.
  - Risks to your child's personal privacy and confidentiality: Your child's participation in this research will be held strictly confidential, and only a code number will be used to identify their stored data. The data will be stored in the researcher's password-protected at UTRGV archives.
  - All information gathered in this research study will be stored in secure electronic and/or physical locations and protected to the extent afforded by law.
  - If we learn something new and important while doing this study that would likely affect whether you want your child to be in the study, we will contact you to let you know what we have learned.
- What are the costs of being in the study?
  - There is no cost of being in this study.
  - You will not receive any payments for taking part in this study.
- What other choices do you have if you decide not to have your child be in the study?
  - Your child will be attending his/her Algebra 2 class with the same instruction provided by the same teacher, but his/her data will not be included in this study.
- Could your child be taken out of the study?
  - Your child could be removed from the study if you or your child do not want to participate or discontinue at any time and without asking any questions.

Can the information we collect be used for other studies?

Information that could identify your child will be removed, and the information your child gave us may be used for future research by other researchers or us; we will not contact you to sign another consent form if we decide to do this.

We will not use or distribute information your child gave us for any other research by other researchers or us in the future. Data, with all identifying information removed, will be kept for three years and may be used for future research by the researchers in this study or by others.

What happens if I say no or change my mind?

- You can say you do not want your child to be in the study now, or if you change your mind later, you can stop their participation at any time.
- No one will treat your child differently, and your child will not be penalized.

How will my child's privacy be protected?

- We will share your child's information by assigning indirect identifiers as pseudonyms when reposting students' data.
- Your child's information will be stored with a code instead of identifiers (such as name, date of birth, email address, etc.).
- No published scientific reports will identify your child directly.
- If it is possible that your child's participation in this study might reveal behavior that must be reported according to state law (e.g., abuse, intent to harm self or others), disclosure of such information will be reported to the extent required by law.

Who to contact for research related questions

For questions about this study or to report any problems your child experiences as a result of being in this study, contact Sumeyra Karatas at (956) 778-6211 or sumeyra.karatas01@utrgv.edu.

Who to contact regarding your child's rights as a participant

This research has been reviewed and approved by the University of Texas Rio Grande Valley Institutional Review Board for Human Subjects Protections (IRB). If you have any questions about your child's rights as a participant, or if you feel that your child's rights as a participant were not adequately met by the researcher, please contact the IRB at (956) 665-3598 or irb@utrgv.edu.

Signatures

By signing below, you indicate that you voluntarily agree to have your child participate in this study and that the procedures involved have been described to your satisfaction. The researcher will provide you with a copy of this form for your own reference.

____ ____/_____/______

Parent/Guardian's Signature **Date**

Child's Full Name: _____________________________________

APPENDIX G

APPENDIX G

PARENT RECRUITMENT FORM

Hello! My name is Sumeyra Karatas; I am a doctoral student in mathematics education at the University of Texas Rio Grande Valley (UTRGV). I want to invite your child to participate in my research study to learn how exposure to one of the effective mathematics teaching practices (e.g., productive struggle) from the National Council of Teachers of Mathematics (NCTM) impacts students' conceptual understanding, procedural fluency, and strategic competence in the learning of topics related to quadratic functions.

The UTRGV Institutional Review Board for the Protection of Human Subjects (IRB) has reviewed and approved this research study.

Only students taking Algebra 2 during the 2021-22 school year will participate in the study. Participation in this research is entirely voluntary; your child may choose not to participate without penalty. The participant students will receive high-quality mathematics instruction, engage in high-leverage mathematical thinking activities, and develop a deep understanding of mathematical concepts.

Participants will be asked to take a pre and post-test at the beginning and the end of the study. They will be exposed to supporting productive struggle teaching practices during the quadratic functions unit for approximately five weeks. I will be video-recording the teacher's implementation of the lesson plan
If you let your child participate in this research study, please turn in the signed Child Assent Form and Parent/Guardian Permission Form.

If you have questions, please contact me by telephone at (956) 778-6211 or email at sumeyra.karatas01@utrgv.edu. You may also contact my faculty advisor, Dr. Jupp, at james.jupp@UTRGV.edu or Dr. Rodriguez at ignacio.rodriguez@utrgv.edu.

BIOGRAPHICAL SKETCH

Sumeyra Karatas received a bachelor's degree in mathematics education from Erciyes University in Kayseri, Turkey. She received a master's degree in master of education in curriculum and instruction in mathematics education from North American University in 2017. She has worked as a mathematics teacher in public schools for eleven years. She received her doctoral degree in curriculum and instruction in mathematics education from the University of Texas Rio Grande Valley in 2022. Email: sumeyra.karatas01@utrgv.edu