12-2023

How an instructor's noticing for equity can foster students' sense of belonging and mathematical confidence

Sthefania Espinosa
The University of Texas Rio Grande Valley

HOW AN INSTRUCTOR’S NOTICING FOR EQUITY CAN FOSTER STUDENTS’ SENSE OF BELONGING AND MATHEMATICAL CONFIDENCE

A Thesis

by

Sthefania Espinosa

Submitted in Partial Fulfillment of the

Requirements for the Degree of

MASTER OF SCIENCE

Major Subject: Mathematics

The University of Texas Rio Grande Valley

December 2023

HOW AN INSTRUCTOR’S NOTICING FOR EQUITY CAN

FOSTER STUDENTS’ SENSE OF BELONGING

AND MATHEMATICAL CONFIDENCE

A Thesis
by
Sthefania Espinosa

COMMITTEE MEMBERS

Kaitlyn Stephens Serbin
Chair of Committee

Luis M. Fernandez
Committee Member

Hortensia Alzaga Elizondo
Committee Member

Younggon Bae
Committee Member

December 2023

ABSTRACT

Espinosa, Sthefania., How an instructor's noticing for equity can foster students' sense of belonging and mathematical confidence. Master of Science (MS), December, 2023, 99 pp., 10 tables, 12 figures, 76 references, 55 titles.

There are many aspects a teacher can notice inside the mathematics classroom, and the more a teacher notices, the more difficult it is to teach. In this study, I particularly focus on *noticing for equity,* which describes the role of the teacher in attending to students' mathematical thinking through an equity lens that can allow the instructor to notice the aspects of classroom mathematical activity that can make students feel less or more empowered in their mathematical practices (van Es et al., 2017). There exists few research about how students perceive their instructor's effort to promote equity and inclusion inside the classroom. Therefore, I observed an instructor's active learning Abstract Algebra course to analyze the pedagogical moves she enacts to make everyone feel included and comfortable sharing their ideas. Then, I interviewed students about their experience in this course, particularly about their perceptions of the pedagogical moves their instructor enacted, relating these moves with how they contributed towards their sense of belonging in mathematics and their mathematical confidence. In my discussion of the results, I provide an insight into how a teacher's noticing for equity proved to be beneficial to foster students' sense of belonging and mathematical confidence but how it does not necessarily guarantee an equitable experience for all.

DEDICATION

Para mi familia, mis maestros y mis ángeles en el cielo,

gracias por siempre creer en mí.

ACKNOWLEDGMENTS

I would like to thank my family for being one of the main reasons I can achieve this accomplishment. My parents have been an example of hard work and that is reflected in the person I am today.

I want to thank my advisor, Kaitlyn Serbin, for being so supportive and patient with my work. This would not be possible without your feedback, ideas and your constant motivation. I am so lucky to have had you as my advisor!

Thanks to Rafael who was with me along this process of writing, keeping me focused and encouraged.

To my professors who shared their knowledge with me and kept me interested in learning more.

Special thanks to the members of my Thesis Defense Committee: Tenchita, Younggon and Luis. I appreciate the feedback you gave me to improve my work.

And thank you to all the people who have inspired me and believed in me.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Page

CHAPTER I

INTRODUCTION

Fostering equitable practices is one of the most important goals mathematics educators can have for their teaching. This goal consists of making mathematics accessible to all students, without making assumptions on the student's achievement merely based on their characteristics such as gender, race or socio-economic status (Gutierrez, 2002). However, researchers argue that the focus of equity is deeper than students just being able to access knowledge; it is also how they learn it (Esmonde, 2009; Yolcu, 2019). The NCTM (2000) states that equity should demand "reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (p. 12). Several studies regarding equity have been published focusing on the ways that teachers can promote equity and what factors need to be considered to achieve it.

Active learning has been presented as a teaching method that can foster equity, and it has proven to be an effective method to improve students' mathematical achievements (Lugosi & Uribe, 2022) and promote social interaction and collaboration (Esmonde, 2009; Lahdepera & Nieminen, 2020). However, research has shown that implementing active learning can lead to inequities during these instances of collaboration, i.e., group work (Shah & Lewis, 2019) and class discussions (Reinholz, 2018) that can cause unfair learning opportunities (Yolcu, 2019;

Reinholz & Shah, 2018). To account for all the different lenses equity could be seen through, Boaler (2008) proposes *relational equity* as a term that considers how students treat each other inside the classroom. This focus is particularly important to consider inside active learning classrooms, as power and identity are factors that could contribute to the inequities during classroom interaction. Gutierrez (2011) argues that equity should focus not only on access and achievement, but on what she defines as the "critical axis" for equity, involving students' identity and power. She suggests that equity should be concerned with whether the students can identify themselves culturally and linguistically within the curriculum to find mathematics as a meaningful thing for their lives, as well as with how power works inside the classroom by studying whose contributions are heard and valued.

One way that teachers can promote equity in their classroom is through the equitable distribution of opportunities for students to participate (Esmonde, 2009). Informed by the relational equity construct, Shah & Lewis (2019) refer to this as *participatory equity*. The teacher plays a critical role in facilitating class discussions and deciding who gets to participate (Reinholz & Shah, 2018) and could achieve this by recognizing students' cultural and linguistical diversity (Yolcu, 2019) and by making sure "the amount of participation proportionally aligns with the demographics of the class" (Shah et al., 2016, p. 1260). However, we argue that this is not an easy role for the instructor, as they need to account for students' gender, race, ethnicity, beliefs, and even social class. An example of this is Lubienski's (2002) study, where she found relations on how higher socioeconomic status students perceived class discussions to be a way they could express their mathematical ideas. This is contrary to the lower socioeconomic status students, who viewed class discussions as merely giving the right answers to problems, reflecting how factors outside of class can impact the way students act in the classroom. This helps to show one of the many reasons it is critical for teachers to attend to equity while noticing students' mathematical thinking during class discussions and responding to their thinking in a way that gives students access and authority over their learning.

van Es and Hand (2017) describe *noticing for equity* as the teacher perceiving the connection of mathematical and interpersonal activity and attending to how these affect the participation of the students. Patterson Williams et al. (2020) argue that noticing for equity

> "addresses persistent patterns of difference in educational opportunities due to historical barriers to access, and responds to and educates about racism, linguicism, and other forms of bias that manifest in classroom interactions, curriculum, pedagogy, and the cultural infrastructure of a classroom" (p. 509).

We hypothesize that noticing for equity can help empower students, give them confidence in their mathematics, and contribute to their development of a sense of belonging to the mathematics community.

Although the mentioned theoretical constructs have been useful to address the issues of equity to an extent, there exists a gap in the literature to answer the question of how exactly it is that these equitable practices affect our students. As van Es (2017) states, there is not a lot of research investigating how mathematics teachers "notice aspects of classroom mathematical activity that have consequences for whether or not particular groups of students feel more or less empowered to take up these practices" (p. 252). Drawing on this idea, the purpose of this study is to consider the students' perceptions about their teacher's noticing for equity practices. Specifically, we want to investigate how the students' sense of belonging and their mathematical confidence is influenced by how their teacher notices through an equity lens during class discussions.

CHAPTER II

LITERATURE REVIEW

Equity and Inequity in Mathematics Education

Attending to equity has become an important focus for mathematics educators who are concerned about the accessibility for every student according to their specific needs. Gutierrez (2002) emphasizes the goal of equity as "being unable to predict student patterns (e.g., achievement, participation, the ability to critically analyze data or society) based solely on characteristics such as race, class, ethnicity, sex, beliefs and creeds, and proficiency in the dominant language" (p. 153). Addressing the inequities in mathematics education has become a concern, not only to researchers and mathematics educators, but even policy makers who argue that equitable practices could help maintain national security and global economic competitiveness (Adiredja & Andrews-Larson, 2017). However, to this day, there still exist issues of diversity, inclusion and equity in the mathematics community, as there still exists underrepresentation of racial and ethnic minorities, as well as females, in the STEM fields (Sparks & Pole, 2019).

When attempting to promote equity, there are multiple factors that teachers need to acknowledge. Teachers should specifically focus on how their teaching practices attend to students' identity and power as doers of mathematics. Studies suggest that taking a sociopolitical stance (Adiredja & Andrews-Larson 2017; Gutierrez, 2002; Parks & Schmeichel, 2012) is essential in order to fully address inequities, by recognizing identity and power issues that affect the students' experiences in learning mathematics. This sociopolitical turn will help identify assumptions that have been imposed on students who come from historically marginalized groups and ultimately achieve an anti-deficit stance (Adiredja, 2019), where there are no assumptions about the students' understanding of mathematics related to their race and gender.

In recent years, the focus to make students become engaged and productive in their learning was cultivated, and active learning was shown to be a way to promote it. Freeman et al. (2014) collected 338 written definitions to collectively define *active learning* as the engagement of students when learning through activities and discussions that emphasize higher-order thinking. Active learning consists of facilitating class-discussions and promoting group work. Cooperative learning is a common practice used to promote equitable learning (Esmonde, 2009), and delegating mathematical authority in these instances is a productive way to ensure equity (Dunleavy, 2015). When compared to traditional lecturing, active learning has resulted in lower failure rates (Freeman et al., 2014), improvement in students' content learning (Minner et al., 2010), and "deep engaging with meaningful mathematics and collaborative processing of mathematical ideas" (Laursen et al., 2014). Although these studies show that active learning is contributing to the students' performance in doing math, it does not necessarily ensure success for all students. For example, when looking at the performance of inquiry-oriented classes versus non-inquiry-oriented courses, Johnson et. al (2020) found that men outperformed women students in inquiry-oriented based classrooms.

Given that the learning environment of an active learning classroom is set to have discussions where students are expected to outspokenly share their ideas and work collaboratively, without careful attention, inequities will be most likely occur (i.e., microaggressions during student-student interactions) (Reinholz et al., 2022). Teachers have an important role in making sure that active learning is implemented in a way that improves equity, but this will depend on the teacher's culture, beliefs (Matthews, 2019), position (Rubel, 2017), stereotypes (Reinholz & Wilheim, 2022), and attitude (Leedy et al., 2003). It could be the case that teachers become responsible for the inequities inside the classroom as their beliefs about students, specifically students from minorities, influence their choice of how they facilitate classroom participation (Rubel, 2017). For example, Copur-Gencturk et al. (2020) studied teacher's biases on the mathematical ability of students and found how teachers of color could hold the belief that White students could be mathematically more capable due to internalized stereotypes they have been subjected to in their lives, which could negatively affect their students of color. Thus, this shows that although having teachers of color in the faculty could help students feel identified and represented, it still does not ensure an equitable experience for students of color due to the culture of power and the positioning of nondisabled cisgender white men as the standard model that others can be compared to (Reinholz & Wilhelm, 2022). Furthermore, some teachers can base their decisions on their position. Rubel (2017) describes this as teachers who adopt ideological principles – meritocracy, color-blindness, and "I can't relate" stance – focusing only on student achievement and ignoring the systemic barriers, impacts of race, and the lack of attention to student's social realities. Teachers could have assumptions about certain student groups such as having a deficit view (Jong, 2017) by setting low expectations for students from minorities in hyper-segregated schools (Rubel, 2017), implicit biases (Reinholz et. al, 2022), and stereotypes such as the negative perceptions about Latinx students' ability to speak in the language of instruction that prevent them from being called on to participate in class discussions (Reinholz & Wilhelm, 2022). Teachers also have attitudes towards students, positioning girls who succeed in math as hard workers but boys who succeed in math as talented, leading them to treat boys superior to girls. (Leedy et al., 2003). Having those underlying assumptions, attitudes, and stereotypes about students who come from historically marginalized groups cause them to receive less opportunities to participate (Esmonde, 2009) and contribute to inequities that prevent students from engaging in discussions and mathematical discourse (Leyva, 2017). To help mitigate these inequities, mathematics education researchers have proposed mathematical teaching practices that have the goal of improving equity within the classroom.

Equitable Teaching Practices

Moschkovich (2011) defines equitable teaching practices as those that "support mathematical reasoning and mathematical discourse and broaden participation for students from non-dominant communities" (p. 100). These practices seek to eliminate oppressive norms that mathematics education has imposed on historically marginalized groups (Chao et al., 2014). Bartell et al. (2017) describe nine equitable practices where they highlight the importance of establishing classroom norms that cultivate equitable participation, attending to student's racial and cultural identities, "monitoring how students position each other" (p.11) and engaging in conversations on sociopolitical topics using mathematics. In addition to these practices, teachers need to attend to how often students participate in class discussions in various ways. Teachers have an important role in soliciting students' participation and responding in equitable and inclusive ways. Boaler (2008) proposes that teachers should foster *relational equity*, which focuses on the relationship amongst students and how they treat each other. This type of equity is important to make students feel confident when they are called on to share their contributions and could help support their participation to feel validated and respected by their peers, allowing them to position students as capable learners, while “addressing issues of status inside the classroom and holding students accountable for each other’s learning (Hand et al., 2015, p. 238). Informed by this construct, Shah & Lewis (2019) presented *participatory equity*, which concerns how the provision of opportunities to participate and actual participation in class are fairly distributed among the students. In their study, they observed and analyzed participatory equity during collaborative work and concluded that the emergence of inequities is fluctuating, and thus, assumptions should not be made about whether one scenario is more equitable than another. They suggested that the teacher should constantly monitor the students’ collaboration to possibly attenuate inequities. Therefore, attention to participation structures is key for students to actively engage in collaborative mathematical discussions (Hunter & Hunter, 2022), and such participation can support students in building knowledge that can eventually foster their achievement and confidence in mathematics (Serbin et al., 2020).

As a way to further investigate how participation emerges and whether or not the opportunities to participate are fairly distributed, Shah et al. (2016) suggest Equity Quantified In Participation (EQUIP) as an equity analytic tool that quantitively measures participation patterns in terms of distribution among the classroom’s gender and/or race. The goal of EQUIP is to answer “(a) who participates, (b) what is the nature of that student participations, and (c) what opportunities do teachers make available to support the participation?” (Reinholz & Shah, 2018, p. 148). EQUIP (Reinholz & Shah, 2018) is a great tool that can help instructors reflect on their teaching and make them mindful of how they attend to equity in mathematical discourse. When using EQUIP to investigate gender inequity in an inquiry-based classroom, Reinholz et al. (2022) concluded that women’s participation was correlated with their performance, whereas men’s participation was not. I use this EQUIP analytic tool in this study to analyze an instructor's (in)equitable provision of opportunities for students to verbally participate in class. .

Although these equitable practices are designed to be enacted by teachers, the practices are focused to give students mathematical authority over their knowledge. Fostering equity in the classroom involves positioning students as competent doers of mathematics. Gehrtz et al. (2022) proposed leveraging student thinking as a way to engage students and promote equitable participation opportunities that could help construct students' conceptual understanding. To do so, the teachers can facilitate class discussions where students have the opportunity to express their ideas, and teachers can prompt their thinking by asking follow-up questions that can help build on their knowledge (Serbin et al., 2020). McDuffie et al. (2014) suggest revoicing students' contributions to "clarify, call attention to, and/or refine language or ideas" (p. 116). Therefore, teachers have an important role in eliciting, interpreting, and building on students' contributions in ways that attribute competence to students and give them the opportunity to build their mathematics identity and sense of power. We claim that the practice of teacher noticing could be a way to achieve these goals.

Noticing Students' Mathematical Thinking in Class Discussions

Teacher noticing is a teaching practice interpreted in the literature as "how teachers make sense of complex classroom environments" (Jacobs et al., 2010, p. 170) and how they manage the in-the-moment information (Sherin et al., 2011). Through the practice of teacher noticing, teachers can influence the quality of their teaching of mathematics by the way they pay attention and make sense of what happens in the classroom (Lee & Choy, 2017). Schoenfeld (2011) argues that what a teacher notices or not will inform what they do and what they do not do. A specific type of teacher noticing, noticing students' mathematical thinking consists of three interrelated skills: attending, interpreting, and deciding how to respond to student's mathematical thinking (Jacobs et al. 2010). Jacobs et al. (2010) define attending as how a teacher pays attention to the "mathematical details in children's strategies" (p. 172) and "gathers evidence of a student's thinking as it happens in the moment" (Floro & Bostic, 2017, p. 76). The interpreting aspect consists of making sense of the students' strategies (Jacobs et al., 2010) by connecting their strategies with research on children's understanding (Teuscher et al., 2017).

Deciding is referred to how the teacher collects the information of the student's mathematical thinking to make an informed decision on how to respond to the student's contribution/idea (Floro & Bostic, 2017, p. 76) and "involves reasoning about classroom interactions" (Louie, 2017, p. 59). Altogether, Krupa et al. (2017) describe the premise of teacher noticing as how

> "Teachers must notice (attend to) the specific mathematical ideas evident in students' written work or verbal responses in order to make sense of (interpret) that thinking. Interpretations of students' thinking can then be used to inform teachers' next steps (respond)" (p. 50).

A teacher's noticing is informed by the teacher's knowledge and experiences (Kilic, 2016), professional vision, pedagogical commitments, disposition, years of teaching (Patterson Williams et al., 2020), beliefs (Stahnke et al., 2016) and their expectations and perspectives (van Es et al., 2017). Therefore, noticing is a particularly useful but complex practice, given that attending, interpreting, and responding to student's mathematical thinking happen in the moment during classroom discourse and although the teacher has the role of eliciting student's mathematical thinking, the teacher must also account for the contributions that are not productive for the mathematical goals class.

The teacher noticing skills that can support teachers in making sense of student mathematical thinking in the moment can be informed by Leatham and colleagues' (2015) MOST analytic framework – Mathematically Significant Pedagogical Opportunity to Build on Student Thinking (p. 91). Teuscher et al., (2017) describe a MOST as "an instance of student mathematical thinking that, if made the object of classroom discussion, would help the mathematical understanding of the students in the class move forward" (p. 33). The framework consists of the teacher first identifying whether a classroom situation could be considered as an instance of student mathematical thinking that includes a mathematical point related to the student's mathematics. Once the student's mathematical thinking is identified, the next step consists of finding out if this instance is mathematically significant. This is done by identifying whether the mathematical point is appropriate and central. The appropriateness will depend on whether the idea is adequate for the student's understanding but not yet understood and the central aspect by whether the mathematical point is a learning goal for the classroom. When the student's mathematical thinking is mathematically significant, the last step is to decide if it would help build a pedagogical opportunity, which could occur depending on two criteria – opening and timing. Opening refers to whether the student's mathematical thinking created an intellectual need (Harel, 2013) in the students to create new knowledge, leading them to understand the mathematical point. Timing depends on whether it is an opportune moment to satisfy that intellectual need or build on such mathematical point during class. Therefore, if the teacher identifies an instance of a student's mathematical thinking that is mathematically significant and can create a pedagogical opportunity, then the instance can be considered a

MOST (Leatham et al., 2015). This MOST framework is inherently related to the teacher noticing practice. After attending to and interpreting students' mathematical thinking, the MOST framework can help the teacher in recognizing the students' contributions that would be productive for the mathematics they are teaching and in deciding how to respond to or build on those contributions during class.

Peterson et al. (2017) point out that making decisions on whose ideas are taken up or put aside should not be made based on the characteristics or demographics of the student who is sharing their idea. Teachers should pay particular attention to the way they decide whose contributions are productive and whose contributions are unrelated to the conversation given that putting aside these tangential student contributions (Peterson et al., 2017) could be perceived as an act of inequity. Thus, it should be a goal for teachers to notice these contributions in an equitable way, focusing beyond students' mathematical thinking (Louie, 2017), that can help "provide all students with meaningful learning opportunities and experiences (Jong, 2017, p. 208). In order to manage equitable class discourses, van Es et al. (2017) recommend noticing for equity.

The teacher noticing skills that can support teachers in making sense of student mathematical thinking in the moment can be informed by Leatham and colleagues' (2015) MOST analytic framework – Mathematically Significant Pedagogical Opportunity to Build on Student Thinking (p. 91). Teuscher et al., (2017) describe a MOST as "an instance of student mathematical thinking that, if made the object of classroom discussion, would help the mathematical understanding of the students in the class move forward" (p. 33). The framework consists of the teacher first identifying whether a classroom situation could be considered as an instance of student mathematical thinking that includes a mathematical point related to the student’s mathematics. Once the student’s mathematical thinking is identified, the next step consists of finding out if this instance is mathematically significant. This is done by identifying whether the mathematical point is appropriate and central. The appropriateness will depend on whether the idea is adequate for the student’s understanding but not yet understood and the central aspect by whether the mathematical point is a learning goal for the classroom. When the student’s mathematical thinking is mathematically significant, the last step is to decide if it would help build a pedagogical opportunity, which could occur depending on two criteria – opening and timing. Opening refers to whether the student’s mathematical thinking created an intellectual need (Harel, 2013) in the students to create new knowledge, leading them to understand the mathematical point. Timing depends on whether it is an opportune moment to satisfy that intellectual need or build on such mathematical point during class. Therefore, if the teacher identifies an instance of a student’s mathematical thinking that is mathematically significant and can create a pedagogical opportunity, then the instance can be considered a MOST (Leatham et al., 2015). This MOST framework is inherently related to the teacher noticing practice. After attending to and interpreting students’ mathematical thinking, the MOST framework can help the teacher in recognizing the students’ contributions that would be productive for the mathematics they are teaching and in deciding how to respond to or build on those contributions during class.

Peterson et al. (2017) point out that making decisions on whose ideas are taken up or put aside should not be made based on the characteristics or demographics of the student who is sharing their idea. Teachers should pay particular attention to the way they decide whose contributions are productive and whose contributions are unrelated to the conversation given that putting aside these tangential student contributions (Peterson et al., 2017) could be perceived as an act of inequity. Thus, it should be a goal for teachers to notice these contributions in an equitable way, focusing beyond students' mathematical thinking (Louie, 2017), that can help "provide all students with meaningful learning opportunities and experiences (Jong, 2017, p. 208). In order to manage equitable class discourses, van Es et al. (2017) recommend noticing for equity.

Noticing for Equity

The noticing for equity framework (van Es et al., 2017) grounds in the noticing for students' mathematical thinking (Jacobs et al., 2010) in a way that it suggests attending to students' mathematical thinking. However, noticing for equity focuses on how this students' mathematical activity connects to the students' interpersonal activity and has the goal of checking how this connection affects the way in which students participate in the classroom. The pedagogical commitment to noticing for equity is taking an anti-deficit stance (van Es et al., 2017) that can make historically marginalized groups feel more empowered by their mathematical practices. Noticing for equity consists of "creating individual and collective safety and support by attending and responding to equitable distribution of talk and engaging larger historical and socio-political issues impacting the present discursive moment" (Patterson-Williams et al., 2020, p. 509). Teachers can achieve this by attending to issues of status and positioning (Louie, 2017), as well as the dimensions of classroom activity: participation, access, and opportunity with the goal of narrowing the gaps in classroom participation (van Es et al., 2017). This aligns with Shah et al.'s (2016) conceptualization of an equitable classroom, where the participation discourse is equitably distributed among the students, proportional to the class's demographics. Thus, teachers have the key role of challenging privileged discourses through their implementation of equitable teaching practices to face the underrepresentation of females and minorities in STEM (Spark & Lewis, 2019).

To address issues of equity in the classroom, van Es et al. (2017) studied four secondary mathematics teachers committed to equitable teaching and identified five teaching practices among them associated with noticing for equity (see Table 1): *leave students to grapple with mathematical ideas, make norms explicit for doing mathematics, support students in developing mathematical identities*, *connect with students to honor individual strengths, and making system of schooling explicit* (p. 259). In their descriptions of these practices, they point out how the teachers in their study not only attended for student's mathematical activity but also the "interpersonal aspects of the student's participation" (p. 260) such as how students got along during group work and how the student's participation related to their social and cultural communities. These practices were useful to notice different aspects of the classroom activity, such as how well students understood a task, how well students worked in groups and how well students followed classroom norms.

Table 1: van Es et al.'s noticing for equity teaching practices (2017).

Noticing for equity practice	**Description**
Leave students to grapple with mathematical ideas	Giving students sufficiently scaffolded high-level mathematical ideas and establishing classroom norms around making sense of mathematical ideas either individual or in groups.
Make norms explicit for doing mathematics	Establishing and making norms explicit for how to do mathematics in their classrooms.
Support students in developing mathematical identities	Create opportunities for students to express what they know and understand both about the mathematics and their worlds and use what they learned about students to create meaningful learning spaces.

Table 1: cont.

Connect with students to honor individual strengths	Connecting with each student in sincere ways that recognize and honor who they are as people.
Make system of schooling explicit	Attending to student participation within a lesson and over the course of a year and inferring how their participation would have potential impact on their future selves.

Although there is not a specific list of pedagogical moves and strategies that can guarantee equity in the mathematics classrooms, researchers have listed different ways noticing equitably looks like. The teacher has the role of finding the practices that work for them and their students. Some of these practices are to create social norms for the classroom that fosters respect among the students when working collaboratively and sharing ideas. Teachers who practice noticing tend to infer students' mathematical thinking rather than just looking for correctness and completeness. Hand et al. (2015) suggest organizing students' mathematical contributions to "help bring student's sense-making into the flow of classroom conversations" (p. 264). The teacher orchestrates the students' ideas, so these become productive and direct the mathematical flow of the lesson (Sherin, 2002). By doing this, students who are deemed incompetent or often perceive themselves as misunderstood can feel like their contributions matter and are important to construct knowledge in the classroom. It is important to consider that tangential contributions will be likely to happen that do not work for the flow of the class, and thus, the instructor has to be careful in how they tackle those contributions by putting them aside in a nice manner (i.e., responding the student with "Thank you for sharing") (Peterson et al., 2017). Particular attention should be paid to which students are dominating the classroom discourses and how students are positioning themselves. Such positioning of students, especially in group work settings, can cause inequities that determine who is getting to share their ideas and whose contributions are viewed as valuable (Patterson et al., 2020). Another thing teachers should notice are power differentials that can occur during group work and class discussions that will vary according to the context where the teaching takes place, as there can be specific local power differentials that the teacher might not be accounting for.

Thus, noticing for equity is a complex practice as it focuses on factors beyond students' mathematical understanding. Given the importance noticing for equity has in making sure that the student's contributions are heard and valued, I argue that this practice could positively affect the student's perceptions of mathematics, particularly, the student's sense of belonging to the mathematics community and their mathematical confidence.

Mathematical confidence

Mathematical confidence is the student's belief that they are able to do mathematics (Parson et al., 2009). When Hardy (2008) asked students to describe confident mathematics students, their answers involved students who speak out and take risks and those who are willing to give answers and ask for help. Parsons et al. (2009) developed a survey where they categorized three domains of mathematical confidence: Overall Confidence in Mathematics, Topic Confidence for particular mathematics topics, and Applications Confidence. They define Overall Confidence in Mathematics as the student's belief of being able to do "any or all mathematics" (p. 55), Topic Confidence is the students' belief of being able to do a particular topic of mathematics, and Applications Confidence is whether they feel capable of applying mathematics. In their study, they found that students' confidence in doing mathematics matters and is related to students' performance. Burton (2004) points out that teachers and students have different views of confidence, where teachers see it as a construct defined by students' behavior and willingness, whereas students described confidence focusing on their feelings towards mathematics and how the classroom setting affects these feelings. Although mathematical confidence can be described from different perspectives, this construct has been associated with students' success in mathematics (Burton, 2004) and the retention of students in STEM fields (Ellis et al., 2016).

Several studies have shown that gender gaps in mathematics education have a negative effect on women's mathematical confidence. Girls tend to have lower confidence in their ability in doing mathematics than boys because of their parents' and teachers' negative expectations of their performance (Gutbezahl, 1995) which can discourage them from pursuing advanced mathematics courses. When investigating the likelihood of students dropping out of a STEM major after taking Calculus I, Ellis et al. (2016) found that women's mathematical confidence was a major factor that contributed to their decision to not pursue a STEM major anymore. They found that female and male students arrive to college with different levels of confidence, being that women have less confidence in their mathematical abilities than men. Similarly, Ganley and Lubienski (2016) showed that girls are less confident and less interested in mathematics than men during third to eight grades, due to the students', their parents', and their teachers' views about mathematics being a male domain (Leedy et al., 2003). This points out how outside factors can negatively affect the way women perceive themselves as capable of doing mathematics inside the classroom. Although the teacher does not necessarily have the ability to change the students' parents' gender-biased beliefs and attitudes about mathematics, the teacher has an important role in supporting students, especially women, in becoming more confident about their abilities. Another factor that can contribute to the student's mathematical confidence is their perception of mathematics. Most students find math to be a hard subject that can contribute to students feeling incapable of even attempting it. Furner (2017) describes math anxiety as one of the main reasons students can have a negative perception about mathematics. He suggests that teachers can have behaviors that create students’ math anxiety such as the quality of instruction, gender biases, unrealistic expectations, embarrassing students in front of peers, and language barriers, among others, that could lead to lowering students’ mathematical confidence. Therefore, teachers should be aware of these behaviors and be intentional about not conveying them to their students.

It is essential to foster students’ mathematical confidence in active learning classes. Some studies have found that student achievement is correlated to the student’s confidence in their abilities to do mathematics. Thus Parson et al. (2009) argue that students’ achievements and experiences could be improved if their teachers boosted their confidence in doing mathematics. It is therefore critical for researchers to identify ways in which teachers can support the improvement of students’ mathematical confidence. One such way is by implementing inquiry-based learning. Laursen et al. (2014) found inquiry-based learning to be an effective way to increase women’s collaboration gains and to improve their confidence of sharing and teaching mathematical ideas to their peers. Another way to promote students’ mathematical confidence is through teachers’ provision of feedback. Zander et al. (2020) inferred that immediate criterion-oriented feedback after testing students could narrow the confidence gap between girls and boys since they had previously found that girls had lower self-efficacy than boys.

Researchers have also studied how mathematical confidence and self-efficacy can be improved by fostering students’ growth mindset and fostering inclusive social norms in class. Growth mindset (Dweck, 2006) refers to the capability a student has to develop and improve their skills through time and grow from their mistakes, whereas a fixed mindset believes that intelligence is a given and students who are not inherently smart cannot achieve the knowledge an intelligent student has. Boaler (2013) talks about how a fixed mindset can cause inequalities that contribute to the marginalization of women and students from minorities. To cultivate this growth mindset, Sun (2018) suggests setting norms that remind students that “growth often requires making mistakes, struggling, or experiencing failure might also convey growth-mindset messages about mathematics ability” (p. 336). Furthermore, to contribute to the improvement of students’ mathematical confidence, the teacher can foster social norms that enable opportunities to participate. Serbin et al. (2020) propose that teachers should implement social norms of having students work individually on tasks to later share their contributions with their peers and evaluate each other’s responses. The students, especially women, can benefit from these socials norms as “sharing contributions and explaining reasoning can also serve to position women as mathematically competent learners, which can foster women’s confidence in mathematics” (p. 8). Furner (2017) highlights the teacher’s role in creating a “learning environment conducive to learning with compassion, has high expectations for all students without regard to gender, race, or language barriers, and uses a variety of assessment methods and teaching styles to best reach all students” (p. 7). Overall, fostering such a learning environment in the classroom has the potential to support students in improving their mathematical confidence.

Darragh (2013) found that the students’ mathematical confidence and sense of belonging are interrelated and thus, encouraging students to be confident may help them develop a more robust sense of belonging and mathematics identity. However, from her results, she argues that in order for a student to feel confident they must also feel belonging to their classroom. In the current study, I leverage this construct of sense of belonging, which will help to understand the students' perceptions of their teacher's noticing for equity fostering a sense of belonging among students.

Sense of belonging

Good et al. (2012) define *sense of belonging* in mathematics as "one's personal belief that one is an accepted member of an academic community whose presence and contributions are valued" (p. 701). Sense of belonging also refers to how students perceive themselves as important to others (Lahdepera & Nieminen, 2020) and as an "active participant in all aspects of the learning process" (Ames, 1992, p. 263). Research has shown that students' sense of belonging can be related to their academic performance and their decision in pursuing higher education (Good et al., 2012; Lahdepera & Nieminen, 2020). Learning collaboratively can positively influence the way students perceive themselves as mathematicians and part of the math community. Thus, active learning is a beneficial teaching method that could help build students' sense of belonging in mathematics. Lewis et al. (2017) describe working in groups as a "psychological vaccine that protects people from the negative effects of identity threat and serves to boost sense of belonging" (p. 421). Similarly, Good et al. (2012) talk about how students who actually feel a sense of belonging to the mathematics community are more likely to be actively participating in class rather than wanting to disappear and not get noticed. However, this will depend on how the instructor manages peer collaboration and their disposition for equitable instruction (Hand, 2012). There might also exist the case of students who do not want to belong being that their only goal is to just get their degree (Lahdepera & Nieminen, 2020). But even if that is the case, teachers should still make sure that they are positively fostering students' sense of belonging.

There exist multiple factors that can cause a lack of belonging to the mathematics community. When studying the factors that could contribute towards sense of belonging, Lahdepera & Nieminen (2020) studied four categories: interpersonal relationships, science identity, personal interest, and competence. They found that the lack of belonging could happen because of the student's personal characteristics such as being shy or socially anxious, as well as their competency to do mathematics. Other studies show that the lack of belonging occurs because of stereotypes imposed on women and minority student groups such as the belief that women have less ability than men (Good et al., 2012), which might be a factor that prevents them from getting access to the education they deserve. One such example can be found in Herzing's (2010) study which pointed out that women graduate students in STEM were less likely to receive mentoring from a male faculty than men students do which can contribute to women feeling left out and as if they do not belong. Matthews (2020) argues that teacher's preconceived beliefs contribute to the underservice of Black, Latinx, and economically disadvantaged students, which also negatively affects their sense of belonging. It is also important to consider that the classroom setting will also determine whether students have a sense of belonging. Another way in which students' sense of belonging can be affected is by the relationship they have with their peers. In a mathematics classroom where students are called on to participate, students are likely to feel pressured to give a correct answer. If the teacher fosters social norms in the classroom, where students' contributions are considered valid, regardless of being right or wrong, it can help students feel more confident in expressing their ideas, as well as feel that their contributions are accepted and valued. Creating a learning environment where the student can grow from their mistakes could lead them to feel part of the mathematics community (Hamm & Faircloth, 2005; Darragh, 2013).

To measure students' sense of belonging in mathematics, Good et al. (2012) developed the Sense of Belonging scale composed of five factors: Membership, Acceptance, Affect, Decide to Fade, and Trust. The scale has a Likert scale format from 1 to 8 with statements about students' sense of belonging. With this scale, they found how having a good sense of belonging in mathematics was associated with the students' intention to persist in mathematics and have less anxiety towards mathematics. Therefore, by leveraging a student's sense of belonging in math, the students can position themselves as competent doers of mathematics, developing a mathematics identity that may positively benefit their performance. Thus, it is an important factor to consider and foster in the classroom setting.

CHAPTER III

SUMMARY AND PURPOSE OF THE STUDY

We support Sherin's (2002) view about teaching being "a complex cognitive activity and that teachers hold multiple goals simultaneously" (p. 228). Being a mathematics teacher has its own difficulties regarding the content that is taught and thus, attending to more than students' mathematical thinking appears to be a complex but necessary thing to do. Noticing for equity seems to be a good option to address the urgency of providing equitable mathematics classrooms to students, especially for students who still struggle to get the access they deserve to their education. In the current study, I aim to investigate the way students perceive their teacher's efforts towards fostering equity in the mathematics classroom, as there is little to no research regarding the students' views towards equitable practices and their effect in the student's perception of mathematics. I will particularly focus on the previously mentioned constructs: sense of belonging and mathematical confidence. Thus, I hypothesize that an instructor noticing through an equity lens can contribute to a change in the way students perceive themselves in the mathematics community and helps build up their confidence in doing mathematics.

I address the following research questions:

- What pedagogical moves does the instructor enact to notice for equity?
- Do the identified pedagogical moves support an equitable distribution of participation opportunities among women and men students?

- What are the students' perceptions of their instructor's effort towards noticing for equity, and how do these efforts foster their sense of belonging in mathematics and their mathematical confidence?

CHAPTER IV

METHODOLOGY

Setting and Participants

This study takes place in a Hispanic-serving institution located in South Texas in an active learning Abstract Algebra for pre-service teachers course taught by Dr. Evans, a mathematics teacher educator and a mathematics education researcher with a commitment to fostering equity and inclusion in her class. The class consisted of 9 men and 11 women students who are undergraduate prospective middle school mathematics teachers. The class met two days a week in a hybrid modality, meeting in person one day of the week and on Zoom, an online conference platform, during the second day with each class session being approximately 75 minutes long. For the analysis of this study, data were collected during five days when the class met in person.

On the first day of class, the teacher educator invited students to establish a list of social norms for their classroom. The social norms consisted of being respectful to their classmates and their opinions, being collaborative and supportive with their peers, making sure no one is talking over someone else, and allowing individual private think time before discussing their ideas with their group. The construction of social norms between the teacher and the students helped to support an inclusive environment that allowed students to share their thoughts and ideas without being judged by their peers and instructor. In a typical class session, the students were expected to collaboratively work on tasks that aimed to reinvent abstract algebra concepts. The instructor would join the groups, online or in-person, to help leverage and support student thinking. After giving them time to work, the instructor called on students to discuss the tasks as a group.

Data Collection and Analysis of Students' Math Sense of Belonging and Mathematical Confidence

Data were collected by administering the Math Sense of Belonging Scale (Good et al., 2012), which is a survey that quantitatively measures the student's sense of belonging, as well as Parsons et al.'s (2009) math confidence questionnaire.

The Math Sense of Belonging scale and confidence questionnaire was administered during the first weeks of class and towards the end of the semester. This allowed me to compare the results for each participant and see whether their sense of belonging and mathematical confidence stayed the same, showed improvement, or lowered during the course. After filling out the scale and questionnaire for a second time towards the end of the semester, students were interviewed to further understand the results found and talked about their perceptions of how their teacher managed the classroom participation in an equitable manner. The results from both instruments were checked and compared to see if there is an increase, decrease, or if their score stayed the same to identify whether the teacher's attending to equity provoked any change in the students' sense of belonging and mathematical confidence.

During the first week of class I was given a few minutes of class to talk about my study to the classmates, without Dr. Evans being there. The students were invited to fill out the questionnaires if they agreed to participating in the study. I received responses from three students and they were chosen as participants. The selected participants, Allison and Raziel, were individually interviewed at the beginning and towards the end of the semester with their informed consent. Both students were informed that their participation on this study would not be known by their instructor until after the semester was over and in no way their opinions would affect their grade on the course. I conducted semi-structured interviews (Kallio et al., 2016), where a set of preliminary questions was previously designed to elicit student's perception about the main themes for this study: their instructor's equitable practices, mathematical confidence, and sense of belonging. Some of these questions were informed by the students' responses to their Sense of Belonging scale and Confidence questionnaire. For instance, in the Sense of Belonging students responded "When I am in a math setting I feel that I belong to the math community" in a likert scale from 1 to 8 (1 = Strongly Disagree, 8 = Strongly Agree). In the interview I would ask them to elaborate on why they reflected on their sense of belonging in the way they responded. The students were also asked about how confident they would describe themselves in doing mathematics, to which they responded in a Likert scale from 1 to 5 (1 = Not confident, 5 = Very Confident), during the interview I would follow up on this questions to ask why they considered their confidence to be rated as they responded. The order of the set of preliminary questions was developed in a way that students could feel comfortable in sharing their ideas where they talked first about their overall experience, then focusing on their thoughts about active learning, group work, their perceptions about inequities during group work, participation in class discussions, their perceptions about inequities during class discussions, leading to talk about how they perceived their mathematical confidence and sense of belonging as constructs that could benefit their learning. The first interview that took place during the first two weeks of class, was designed to learn more about the student's previous experiences with mathematics and their perception of their sense of belonging, mathematical confidence, and perceptions of how their previous instructors managed in-class participation before coming to

Dr. Evans’s class. The last and second interview was conducted two weeks before finals week, where the main focus was to understand if the experience of this particular course where the instructor fosters an equity-based classroom had an effect, positive or negative, in the students’ sense of belonging and mathematical confidence. The questions also elicited evidence on how the students perceived their instructor attending to equity during class discourse and whether they attributed changes in their sense of belonging and math confidence to their instructor noticing for equity. The semi-structured interview method was a great fit for this study as it allowed me to infer more into the student’s perceptions by asking follow-up questions that helped students elaborate more on their answers to help me get a deeper understanding on their thoughts and experiences. For these interviews, some follow-up questions were designed to support main theme questions, but other follow-up questions occurred spontaneously based on their answers to make them elaborate more. Both interview protocols were edited and revised to make sure the questions would be useful for my data collection. The final semi-structured interview questions for the pre-interview and post-interview can be found in the Appendix. The two interviews were conducted with each participant and had a duration of 90 minutes each. Participants were given the option to conduct the interview in-person or through Zoom, an online video conference platform. In both settings, the audio was recorded and transcribed to a word-processor.

Data Collection and Analysis of Equitable Distribution of Participation Opportunities

I recorded five days of class to have evidence of the teacher-student interactions during classroom discourse. I set a video recording camera in a tripod in the front part of the classroom so that both the teacher and all students were visible. The recordings took place during the days of the week the class met in person at school.

I analyzed the class data using EQUIP, that consisted of capturing participation sequences where there was a back-and-forth communication from teacher to student. A new sequence was recorded each time a new student speaks. These sequences were coded along two of the seven dimensions proposed by Reinholz and Shah (2018): teacher solicitation method and explicit evaluation (see dimensions in Table 2). Transcripts of every video were developed for further analysis and the data were also coded in terms of the class's demographics, in this case I studied the equity distribution of participation opportunities between women and men, and at the end both transcript and demographic data were combined to generate equity analytics. Finally, the total of interactions and how these interactions were categorized in the two dimensions of EQUIP, were used to measure the equity ratio where we can identify whether the actual participation aligns with the expected participation. The actual participation is the participation that was observed and recorded, whereas the expected participation is the one that aligns equitably based on the gender and race demographics in the classroom. For this class, 55% of the interactions were expected to be from women and 45% from men.

Table 2: Reinholz and Shah 's (2016) dimension for EQUIP

Dimension	**Description**
Discourse Type	Recognizes if classroom discourse is content-oriented or logical. Content-oriented involves student's mathematical thinking such as an explanation. Logical refers to discourse about classroom rules that give opportunity chance to participate.
Student Talk Length	This dimension evaluates the length of the students' contribution in words: 1-4 words, 5-20 words and 21+ words. This dimension helps demonstrate the important of students getting a chance to elaborate their thinking.

Table 2: cont.

Student Talk Type	Classifies students' statements on three levels: why statements, how statements and what statements.
Teacher Solicitation Method	Participation is called on by the instructor, random selection or not called on.
Wait Time	Describes the teacher gives their students before responding measured in: more than 3 seconds, less than 3 seconds or no times at all. Allowing more time to think can increase the quality of student talk.
Teacher Solicitation Type	Describes whether the instructor solicits cognitive demand by questioning: What, why, how, Other.
Explicit Evaluation	Describes whether the instructor evaluated the student's idea (yes) or the instructor does not evaluate the idea (no).

It is important to mention that the genders of Dr. Evans's students were assumed by me and thus, my results are restricted to these assumptions. I did not ask for the students' gender identity so it could be the case that student(s) who do not identify as cisgender were not considered to measure the distribution of participation opportunities. I recognize this as a limitation of the study because included another gender category could cause changes in the results found.

Data Analysis of Instructor's Noticing for Equity

Given that EQUIP mainly focuses on participatory equity and our focus is to learn about other teaching noticing for equity practices, I also deductively coded (Miles et al., 2014) our data based on van Es et al.'s (2017) noticing for equity practices (see Table 1). In this coding process, I assigned the code of one of van Es et al.'s practices to what the teacher was doing in class to determine how much of these practices the instructor enacted during class. Then I inductively coded (Miles et al., 2014) the pedagogical moves Dr. Evans used in those coded instances.

The *leaving the students to grapple with mathematical ideas* code was assigned to the moves where the instructor provided students with instructional tasks that would require them to make sense of the mathematics individually or in the group. The practice of making norms explicit for doing mathematics was coded to the activities where Dr. Evans would talk about the structure of class and the expected norms students would need to follow to create an inclusive environment. *Supporting students in developing mathematical identities* was coded whenever Dr. Evans gave space to the students of sharing their mathematical knowledge, taking up space in the classroom and she would guide her instructional task sequence based on students' contributions, regardless of the correctness of the ideas, by calling on students to participate, elaborating on their contributions, and responding to ideas that might seem irrelevant but turned out to be productive. The *connecting with students to honor individual strengths* was coded whenever Dr. Evans would engage with the students and provided them a space to "play around" within the mathematics context. Finally, the code for *making system of schooling explicit* was tied to the instances were Dr. Evans compared what they were learning in her course to the content they learned in secondary mathematics, giving students awareness of how both are interrelated but in abstract algebra they get to formalize what they previously learned, which could help students make meaningful connections with the class content. Once the interactions were coded and the pedagogical moves were identified, I inferred the intentions Dr. Evans had for each of the pedagogical moves and how they used them to advance the class's mathematical agenda.

Validity and Reliability

The validity and reliability of this study was assessed by consulting equity-informed mathematics educators that could check the analysis is not biased before making conclusions. I also had conversations with Dr. Evans regarding my analysis to verify my inferences about her pedagogical moves aligned with the intentions she had in enacting them. Due to time constraints, I could not interview her to learn if the pedagogical moves her students and I perceived were the only ones she intended to enact.

The instruments we used to collect data have shown to be validated and have reliability. Good et al.'s (2012) sense of belonging scale has a 0.87 test-retest correlation and validated their results by showing how their scale was a predictor factors such as intent to purse math or math anxiety. Parsons et al. (2009) developed and revised their questionnaire for several years in order to find the correlation between confidence, liking and motivation with the students' grades. Lastly, Reinholz and Shah explain that the dimensions of EQUIP are informed by construct in the literature about determining equity and consistency with other classroom observation tools. They also explain EQUIP was validated by the feedback from other researchers. Therefore, the analysis for this study was integrated by assessing students with the scale of sense of belonging, the questionnaire for mathematical confidence, the use of EQUIP and interviewing which yields to data triangulation. Triangulation helps to support the trustworthiness of my data collection as multiple data sources were used to address each of my research questions.

CHAPTER V

RESULTS

In this section I present ten pedagogical moves that Dr. Evans enacted during the five days of observed classes and how I connected them to van Es et al.'s (2017) noticing for equity practices (see Table 3). I describe each of these moves that Dr. Evans did during class discussions and use some excerpts from class to give context on when these pedagogical moves were used. At the end of the section, I show the results from analyzing Dr. Evans's classroom interactions with the students under two dimensions of EQUIP, teacher solicitation method and explicit evaluation, and how these interactions were distributed among the women and men students.

Table 3: Connections between noticing for equity practices and associated pedagogical moves.

Noticing for equity practice	**Pedagogical Move**
Make norms explicit for doing mathematics	• Fostering classroom norms
Leave students to grapple with mathematical ideas	• Give students a "moment of private think time" • Assigning group roles for collaborative work
Support students in developing mathematical identities	• Sharing group work's ideas with whiteboards // Gallery walks • Engaging students in the mathematical practices that mathematicians use.

Table 3: cont.

	• Recognizes students' individual contributions / Attributes contributions to individual student • Revoicing students' contributions
Support students in developing mathematical identities	• Attending to the productivity of student's contribution when it is not completely correct or precise
Connect with students to honor individual strengths.	• Engage in humor and play
Make system of schooling explicit	• Defining and understanding mathematical concepts in a formal way

Pedagogical moves that support noticing for equity

Listed above are the noticing for equity practices: make norms explicit for doing mathematics, leave students to grapple with mathematical ideas, support students in developing mathematical identities, connect with students to honor individual strength, and make system of schooling explicit. Each of these practices related to the pedagogical moves Dr. Evans enacted during five days of class. All the noticing for equity practices were found to be associated with Dr. Evans's efforts to foster equity and inclusion inside her classroom. Examples are shown of how these moves were enacted.

Make norms explicit for doing mathematics

This noticing for equity practice relates to way in which the instructor can notice how well students attend to classroom norms and practices and how they could support each other during group work. The pedagogical move of fostering classroom norms was identified in Dr. Evans's teaching. Classroom norms were important to establish given that the class was student-centered and required a lot of participation from the students. In the following description of the pedagogical move, I explain how Dr. Evans provided students the opportunity to contribute to the list of norms for the classroom.

Fostering classroom norms. During the first day of class, the instructor made small groups of students and asked them to write a list of things that would make them feel comfortable and included (see Figure 1). She explained,

> We want to make sure everyone feels comfortable sharing their ideas, especially if they're wrong. Being wrong is really productive because it's something that we can all learn from. So, for us to feel comfortable, being brave in front of others, not being embarrassed and being comfortable sharing, we want to come up with some social norms for us to feel safe in doing that.

Dr. Evans defined social norms as the expectations for behavior that everyone in the classroom would try to comply with.

Figure 1: Dr. Evan's task prompt for norm setting activity.

The students listed different norms for their classmates and for their instructor. The majority of the groups concluded that for them to feel included and comfortable in class, they needed their classmates to be respectful by not speaking over each other, being open-minded, not making fun of each other, and paying attention to the person who is speaking. As for the instructor, the class wrote down they wanted to her to be prepared and organized for class. They also asked for her to be inclusive, understanding and available (see list of norms in Figure 2).

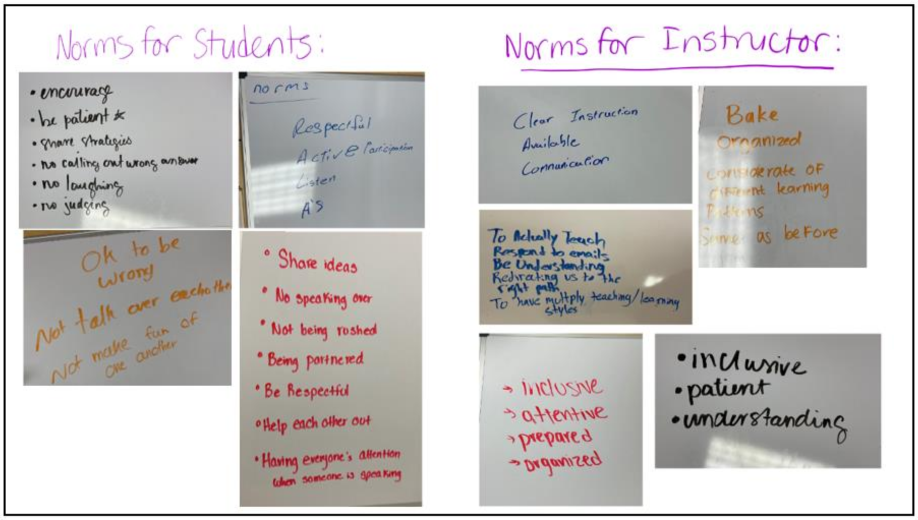

Figure 2: Students' norms for their classmates and instructor.

Fostering classroom norms was a key factor that helped maintain the flow of the class going forward as it provided guidelines that were to be followed during class discussions and group work. This pedagogical move seemed to allow the instructor make norms for doing mathematics explicit and helped her notice who was attending to them.

Leave students to grapple with mathematical ideas

Under this noticing for equity teaching practice are listed the pedagogical moves that I found to be contributors to students having authority over their mathematical activity. These following moves can seemed to help the instructor notice how well the students understand the tasks, how students are being resources of each other during group work and how they function as a group, as well as whether the students need more or less scaffolding with the task.

Give students a "moment of private think time". The "moment of private think time" was a short time where Dr. Evans would provide students a task and prompted them to individually read the question and reason about how to answer it. This time allowed students to first analyze what the problem is asking them to do and come up with ideas on how to solve it, allowing them to have authority over their own mathematical activity. During these moments of individual thinking, students found themselves with questions about the task itself and used this moment to clarify the task by asking the instructor about it. The following excerpt from class illustrates how Dr. Evans prompted students to take a moment of private think time before they worked in groups.

> **Dr. Evans**: I want you to start thinking about these questions. We're gonna be talking a lot about definitions in this class. Take a second to think: what is a definition, how do we use them as mathematicians, and if the level of difficulty depends on the level of school we are in. (She lets them have a moment of private think time)
> **Jaime**: (raises hand) What is a mathematical object?
> **Dr. Evans**: That's a great question. What is a mathematical object?
> **Jaime**: definition of cube
> **Instructor**: Basically, anything we have in math as a concept, we have a definition for it…

Once they had this individual time, the students shared their ideas with their respective groups or asked for help. The goal of the instructor was to make sure everyone has enough time to understand the task and come up with their own ideas, allowing them to have mathematical authority. Another reason could be to possibly avoid having only one of the students dominating the group work.

Another instance where the instructor gave students a "moment of private think time" was after every group in the classroom presented their ideas. She instructed them to take a moment to see if they had any questions about what they just talked about before they moved on to another task. This pedagogical move allowed the instructor to notice how students are receiving and understanding the information presented, which I infer helps her to enact the noticing for equity practice of *leaving students to grapple with mathematical ideas.* Given the structure of the class being mostly inquiry-oriented, it seemed important for Dr. Evans to have the students come up with the ideas. The tasks were designed in a way where students could first have an intuitive understanding of a mathematical structure, and the instructor would elicit and scaffold their thinking to refine and formalize it. By having students guiding the way the tasks developed, the students could be positioned as mathematics competent learners through engaging in tasks where the cognitive-demand level kept increasing as they advanced into each of the objectives of the course.

Assigning group roles for collaborative work. At the beginning of each class, Dr. Evans displayed on the projector an Excel spreadsheet with a random generator with the names of the students that assigns them to a group and gave each student the role they must follow during group work. In every group, there was a recorder, includer, speaker, and timekeeper. The groups and roles were different in every class, which allowed different students to participate each class.

The recorder would be the student who takes notes and writes down the group's ideas on the whiteboard. The includer's role is to make sure everyone in the group participates and feels comfortable sharing their ideas. The timekeeper makes sure the group is making good progress and is ready to present when Dr. Evans calls on their group, and the speaker is the student in charge of talking about their group's ideas to the whole class.

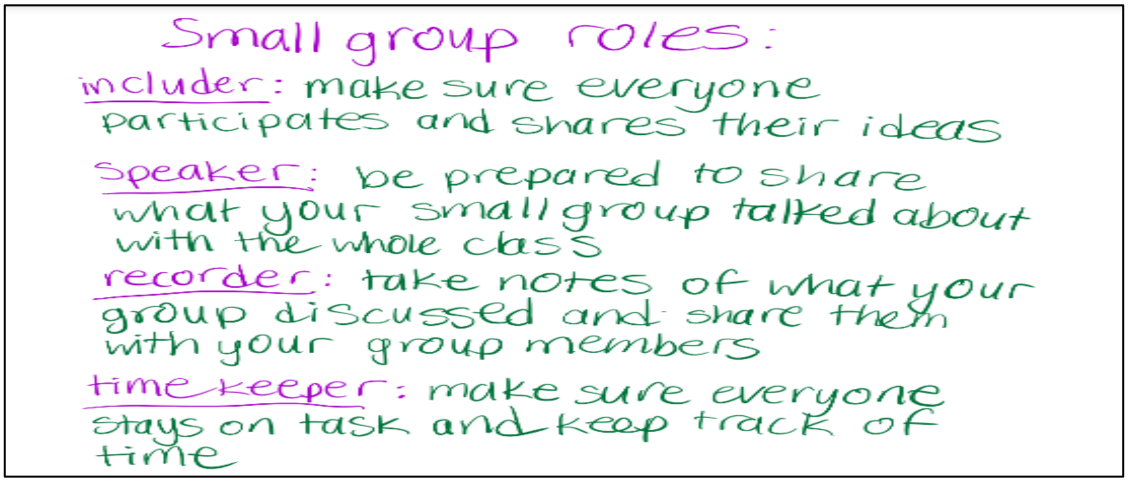

Figure 3: Description of group roles.

Working collaboratively creates the opportunity for students to be resources for each other to come up with ideas and share them with the members of the group. This pedagogical move allows the instructor to see who is taking up space in the group and can help her notice how well they work together. Assigning a role to every student in each group also makes the students accountable for the work they need to do during class.

1	Group	Roles	Students	
2	1	Includer		0.35660473
3	1	Speaker		0.88594635
4	1	Recorder		0.10843785
5	1	Time-Keeper		0.00280327
6	2	Includer		0.34418152
7	2	Speaker		0.34397247
8	2	Recorder		0.81530067
9	2	Time-Keeper		0.75856895
10	3	Includer		0.72745154
11	3	Speaker		0.14012393
12	3	Recorder		0.70473561
13	3	Time-Keeper		0.54957239
14	4	Includer		0.19740343
15	4	Speaker		0.84356642
16	4	Recorder		0.41893005
17	4	Participant		0.56213333
18	5	Includer		0.34585829
19	5	Speaker		0.35987278
20	5	Recorder		0.33229399
21	5	Time-Keeper		0.99851707
22	5	Participant		0.70017312

Figure 4: Random generator for group role assignment.

During small group work, Dr. Evans walked around the classroom stopping at each group’s table to hear their ideas and scaffold them to guide their thinking. After giving them time to work on a task, the instructor would go to each group and ask who the speaker is and allows them to start sharing their ideas.

> **Dr. Evans**: I'm going to start with you, I think you had a cool idea of explaining with the umbrella. Are you the speaker? I know it was your idea [to Carla].
> **Carla**: Yes, I'm the speaker.
> **Dr. Evans**: Perfect, Carla is gonna share. I'm gonna hold up your board. (holds up the board for everyone to see Group 5's work.)

This episode illustrates how Dr. Evans used assigned small group roles for students to work on mathematical tasks in group work. By using this pedagogical move, I infer she enacted the noticing for equity practice of leaving students to grapple with mathematical ideas.

Support students in developing mathematical identities

The following pedagogical moves were used to develop students' mathematical identities as students would be asked to "take up space" and share their ideas to the whole class. These moves could support Dr. Evans in noticing how well students understand the tasks, who is sharing ideas and how such ideas were guiding the flow of the class lessons.

Sharing group work's ideas with whiteboards // Gallery walks. Every group was given a whiteboard at the beginning of each class where they could write down their ideas when doing group work. After Dr. Evans gave time to each group to work on a particular task, the students were asked to present their work to the whole class. The students usually held up their boards and started explaining to the whole class the way they reasoned about the task. Dr. Evans would give feedback and, in some cases, would start a discussion from the group's understanding by elaborating on what the students said or posing follow-up questions for the class to consider.

To engage students in mathematical practices, some of the group activities consisted of asking the students to come up with their own definitions for some mathematical objects, such as identity and equivalence class. To contrast and compare all of the group's ideas, Dr. Evans did *gallery walks*, where every group would display their whiteboard in front of the class to look at the similarities and differences between every group's definitions. This is evident in the following class episode, in which the students placed their whiteboards at the front of the room, and Dr. Evans prompted students to compare the groups' written ideas.

> **Instructor**: What similarities do we see in all of this work? What are the main ideas?
> **Rosa**: Concept.
> **Gerard**o: Concept.
> **Instructor**: Mhmm. I see "meaning" a lot. I see this idea of solving problems. How does definitions help us solve problems?
> **Lino**: It helps us see which formula to use.
> **Instructor**: Mhmm, what other example can we think about?
> **Omar**: [*Shares example*]
> *Instructor nods.*
> **Instructor:** *[asking a particular group]* Tell us a little more on how we use definitions on proofs.
> **Gerardo:** Proving means to explain the reasoning why this is the way it is
> **Instructor:** Mhmm.. I love it when you said that proof is the way we can reason about why something is the way it is. We use definitions as our evidence to back up your claims, definitions help us back up our claims.

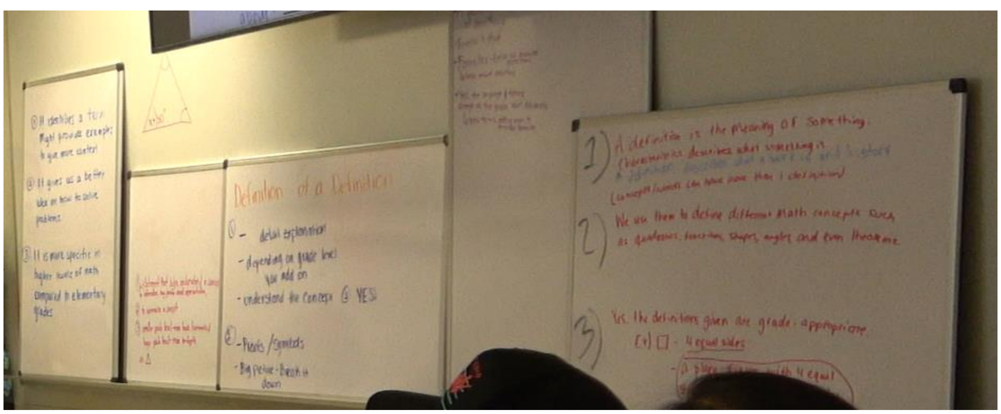

Figure 5: Gallery walk for defining the characteristics of a mathematical "definition".

This activity and pedagogical move allowed the instructor to build up a definition from everyone's ideas and correct student thinking if necessary but also create an environment where students can "take up space", sharing their ideas verbally and visually to the classroom. This pedagogical move seems to *support students in developing their mathematical identities* by giving students opportunities to be recognized as competent learners of mathematics as they share their board work and their ideas contribute to the overall class's understanding.

Engaging students in the mathematical practices that mathematicians use. Following the last example, one of the characteristics of Dr. Evans's instructional task sequence was to engage students in mathematical practices that mathematicians engage in, such as conjecturing, defining, symbolizing, and proving. She explicitly reminded students to identify themselves as mathematicians and has a section in the class notes where students can identify the practices in which they grew as mathematicians (see Figure 6).

Reflect: How did we grow as mathematicians in this objective 2 section? What mathematical practices did we engage in?

generalizing, symbolizing, defining, conjecturing

Figure 6: Reflection task about the mathematical practices students engaged in.

During class discussions, when a student shares an idea, but they are not sure if such idea is correct, Dr. Evans calls it a conjecture. She often encouraged students to share their "rough draft thinking" (Jansen, 2020) and conjectures with the class. She would ask the whole class what they think about it and then she would elaborate on why the conjecture is true or false. She often posed counterexamples to help students understand why a conjecture was not true. One such example was when a student conjectured that you could identify the identity from a Cayley table if an element is operated with itself and the output is that element.

> **Dr. Evans**: E is our identity. It's like the "do nothing element".
> **Raziel**: Can you also look at, when the element is operating with itself and it gives you back the identity?
> **Dr. Evans**: Oh, that's an interesting conjecture. Okay, so I'm gonna rephrase what Raziel is saying. If I operate an element with itself will we get E back? will we get our identity back? So that's, it is always true for identities but it's not only identities that it is true for. So on the homework, you're gonna see an example for that…
> **Dr. Evans:** If you operate an element with itself, like how you have E and E, and then you get that same element, can you just say that that's the identity?
> **Dr. Evans**: Oh, I see. If you operate E with E, you get back E.
> **Raziel**: Yeah, and it doesn't work for the rest. C and C is not C, A with A is not A. So only E with E gives back E.
> **Dr, Evans**: I'm trying to think of a counterexample for that, but I think it will work for most cases. If not, all cases.
> **Dr. Evans**: One way to say this is if $a * a = a$, then a is the identity. We could prove this using group axioms. (writes out proof for Raziel's conjecture) see Figure #. So we proved it, solid conjecture. Questions about Raziel's conjecture?

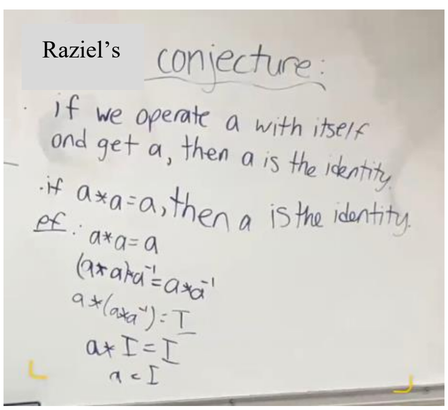

Figure 7: Raziel's conjecture and proof.

In this episode, we see how Dr. Evans attended to the student's thinking by first considering and evaluating their conjecture, and to determine whether it was true or false, she wrote out the proof for the whole class. At first, this contribution might have seemed tangential to the main point of the example, but it turned out to be a useful technique that students could use to find out what the identity was from the Cayley table. This contribution can be beneficial for the students who were still having trouble identifying it, and they will now have another perspective on how to point it out. For Raziel who shared this idea, having his conjecture be proven true could most definitely improve their identity as a mathematician and being given that credit for such idea can positively affect their confidence in sharing voluntary contributions. Overall, Dr. Evans used this pedagogical move of engaging students in mathematical practices that mathematicians use, such as conjecturing, defining, and proving to help students act like mathematicians and be recognized as mathematicians. This pedagogical move thereby could be used to enact the noticing for equity practice of supporting students in developing their mathematics identities.

Recognizes students' individual contributions. As mentioned earlier, working collaboratively was a big part of this course. The collaboration allowed students to discuss the groups' shared ideas to the whole class. The instructor played an important role in scaffolding their understanding to guide them into the intended knowledge, but during this process of attending to every group's idea, the instructor made sure to recognize the ideas and conversations each group had and who was the one sharing them. For example, in groups, every student had a role, and the speaker was the one in charge of communicating the group's idea to the class. However, Dr. Evans did attribute the ideas that were productive to the person who talked about them, regardless of them not being the speaker of the group. This is evident in the following transcript excerpt:

> **Dr. Evans**: "I'm going to start with you, I think you had a cool idea of explaining with the umbrella" "Are you the speaker? I know it was your idea [to Carla].
> **Carla**: Yes, I'm the speaker.
> **Dr. Evans**: Perfect, Carla is gonna share. I'm gonna hold up your board.
> (holds up the board for everyone to see Group 5's work.)

In this example, Dr. Evans made sure to give credit to the student who posed the "cool idea" as she prompted the student to share with the class. Another instance of attributing contributions to individual students was when, similarly as the in Figure 7, the instructor wrote out the student's ideas into the notes and labeled them with the name of the student who shared them (see Figure 8). Consider the following excerpt:

> **Dr. Evans**: Do we see a pattern?
>
> **Jaime**: That every multiple of 3 is green?
>
> **Dr. Evans**: Oh! Every multiple of 3 is green!
>
> **Jaime**: Except for zero, 3 times 0 is 0.
>
> **Dr. Evans**: Mhmm… so, zero is a multiple for any number. Every multiple of 3 is green, uh! (labels it as Jaime's conjecture in the notes) We got a conjecture alert! Do we agree with that conjecture?

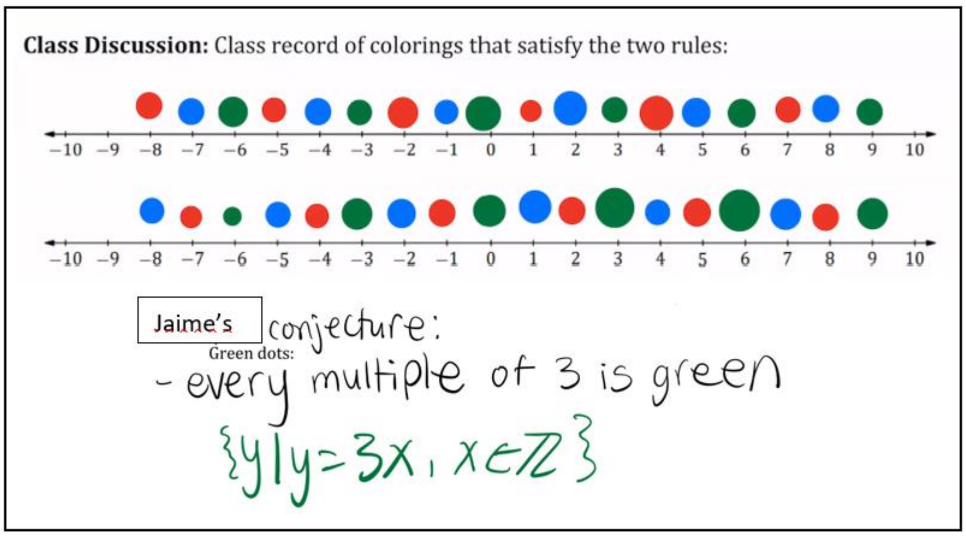

Figure 8: Jaimes' conjecture.

This pedagogical move of recognizing students' individual contributions could also be an important way to support student's identity as mathematicians and build their confidence in sharing their ideas, as we saw in the previous example. The recognition of their ideas being shared to the whole class could contribute to fostering an environment where the student's ideas are heard and valued. Thus, this pedagogical move seemed to enact the noticing for equity principle of supporting students in developing their mathematical identities by giving students recognition for their productive mathematical ideas.

Revoicing students' contributions. Another pedagogical move where the instructor seemed to attend to students' contributions in a way where students felt their ideas were heard and valued was by revoicing their contributions. Sometimes when sharing voluntarily or called-on contributions, the students would lower their voice to respond, and their idea would not be heard by everyone in the classroom. Therefore, Dr. Evans would revoice their idea and add on to it. In this particular move, I refer to the times where she revoiced the students' ideas while evaluating them as correct contributions. In the following transcript, we show how this pedagogical move was enacted.

> **Instructor**: What are the two partition properties they satisfy?
> **Monica**: (Begins to answer).
> **Instructor**: Yes, go for it.
> **Monica:** The union of subsets and the subsets, they're disjoint.
> **Instructor:** Hhmm, they are disjoint. There's no green dot that are blue, ... And then their union, putting them all together we get the whole number line. Right? So, perfect example. Thank you, Group 5. Yeah, we can clap. (Instructor and class clap).

This example shows how Dr. Evans revoices Monica's idea to show approval to her response, instead of just evaluating her response as correct. Revoicing her idea can be a way of recognizing the student's idea, giving them the credit for it. This can also be helpful when students are hesitant about their answers and express their idea with an interrogative tone. By revoicing, Dr. Evans can give reassurance that what their saying is correct and make them feel confident about their contribution. By doing this, the instructor can support the students' mathematical identity.

Attending to the productivity of a student's contribution when it is not completely correct or precise. In the case where the students made a contribution that was not correct, Dr. Evans would try to understand where that misunderstanding might be coming from by asking the student to explain their thinking. She would sometimes pose the idea to the rest of the class or right away think of a counterexample for it. In most of these instances where a contribution was not correct, the instructor would make sure to validate the students' contribution, mention that such misunderstanding was common, and/or claim that what they were talking about was hard or tricky to understand. This comment could help the student not feel bad about being wrong and not feel alone in thinking like that. Given that having such misunderstanding could be very common, the instructor would then show why something would not work that way by giving a counterexample. This way, the contribution ends up being a productive thing to talk about and helped other students avoid thinking that way. In the following transcript, the instructor was talking about the identity elements for addition and multiplication. Then, Jaime shared that the identity for division should be 1.

> **Instructor**: Okay, so Jaime is conjecturing that the division identity is one. Jaime, why is that?
>
> **Jaime**: Since identity is like something that remains it unchanged, so if 0 divide by 1 is 0 and then so on and so forth.
>
> **Instructor**: Mhmm.. I can do random numbers. What about this? [$1 \div 5 = \frac{1}{5}$] Is that true? I see Berenice shaking her head. Berenice, what are you thinking?
>
> **Berenice**: 1 divided by 5 is $\frac{1}{5}$, so that's not equal to 5.
>
> **Instructor**: Yeah, it changes it, right? So that's not equal to 5. So division identity is pretty tricky because it works as long as it comes second, but if it

comes first, it doesn't work out. Because of the way that we defined identity, it was like this, where $a * I = a = I * a$. The identity has to work on both sides. So we would say that the identity for division does not exist. But that was a really good conjecture, I'm glad it came up.

As we can see in the example, the instructor found a way to make Jaime's contribution productive, by acknowledging it and evaluating it. Working out a counterexample can also be productive for the rest of the class to be reminded about how identity is defined and how it works.

Overall, the way Dr. Evans attended to incorrect contributions was respectful and followed her class norm that "it's okay to be wrong". Fostering this norm seemed to be helpful to prevent students from feeling nervous about being wrong and not being afraid to participate even if they are not sure about their idea. By giving value to the students' contributions and fostering a growth mindset, were students can refine their thinking, the instructor can support the students in developing their mathematical identity and makes sure they are not discouraged from further participating, as they would be reminded their contributions would still be valued.

Connect with students to honor individual strengths

Below is listed the pedagogical move that was matched with the practice of connecting with students to honor their individual strengths. This particular move supported the relation the instructor had with her classmates, getting to know who they are as people. The instructor provided instances of the class where students could engage in mathematical tasks in a fun way. This move also contributed to students feeling free to express themselves in different ways such as, showing support to their classmates by clapping or snapping their fingers, or even make funny comments, fostering a relaxed atmosphere where students could engage in mathematics but also enjoy the process of learning.

Engaging in humor and play. One of the ways that Dr. Evans kept her class entertaining and engaging was by adding some humor and play. She would make jokes or use slang to make comments when talking to students. This allowed the students to feel comfortable in class, as well as allowing respectful but fun interactions between the instructor and the students.

A humorous class interaction occurred when students had to represent functions using gestures or dance moves. In the following transcript, the instructor asked the different groups to represent functions in multiple ways such as a table, a diagram, a graph and as a gesture.

> **Instructor**: (calls on group #2 where Gerardo is a member).
> **Gerardo**: (walks out of the group right as their speaker is about to begin explaining their work).
> **Instructor**: (in a teasing tone) I love how Gerardo dips out right as we… (laughs) He's like "I'm out".
> **Gerardo**: (sit down with his group) I was just trying to…
> **Instructor**: Alright, Gerardo. Tell us about your group's work.
> **Gerardo**: So, we chose $f(x) = \sin(x)$, the domain will be all real numbers and the range from -1 to 1. Our gesture is a little wavy.
> **Instructor**: Now you gotta show us (laughs).
> **Gerardo**: The gesture for this graph would be (waves his arms) just a wave.
> **Instructor**: (reenacts his gesture) Just a wave, okay. And where do you see well-defined and everywhere defined in this gesture?
> **Gerardo**: Well, I don't remember that part of it.
> **Instructor**: That's okay. Let's say my body is the y-axis and my arms are the x-axis. Then, if a have a sine wave, (does the sine gesture) everything along the x-axis has an output, so that's our everywhere defined. Where do we see well-defined?
> **Manuel**: For every one input there is only one output.
> **Instructor**: Every input has only one output… It would pass the vertical line test. Super fun stuff. Gerardo, you can now go!

The freedom Dr. Evans had to playfully tease or joke around with students made the classroom feel less tense, and this feeling could help students realize that even if the content can get difficult, they can still have fun learning it.

I connected this move to the practice of connecting with students to honor individual strengths because it seemed to allow the instructor and students to have a relaxed but respectful relationship where they could joke around to keep the class entertaining and memorable for the students.

Make system of schooling explicit

The practice of making system of schooling explicit was recognized by how Dr. Evans highlighted the way students have learned procedural methods in mathematics and how the algebraic properties and structures they learned in this class where the reasons behind why those procedures work as they do and make sure students could see how what they are learning now will inform their future teaching, which can make the content more meaningful to them.

Formalizing mathematical concepts that have previously been used by students in secondary mathematics. One of the goals the instructor had for this course was to formalize student's understanding of algebra through the reinvention of algebraic structures. For example, learning why it is okay to cancel out terms when solving equations, why we can find an inverse of a function by interchanging x and y in its equation representation, and why some operations work the way they do. In the following class episode, we see how Dr. Evans exemplifies the equal sign as an equivalence relation.

Instructor: What about equality? Where do we see equality come up?
Gerardo: That is such a broad question!
Instructor: Right? Literally, equality comes up everywhere! The equal sign is an equivalence relation.
Instructor: How can we show it satisfies the reflexive property?
Gerardo: $x = x$.
Instructor: (revoices and write down) $x = x$. How do I show symmetric?
Jaime: $x = y$.
Instructor: We say if $x = y$ then $y = x$.
Instructor: Transitive?
Rosa: $x = y, y = z$, then $x = z$.
Instructor: Thanks, Rosa. We've used these [equal signs] our entire lives never knowing what they're called. When we solve $x = y + 2$ and $y + 2 = x$ and our middle or high school teachers just tell us it's totally fine, it's just changed around, that the order of the equation didn't really matter. This is why (points to proof) See Figure 9.

In this class transcript, we can see Dr. Evans uncovering equality as an equivalence relation. By writing out the proof for such statement, students can learn why the properties of equality hold and the importance of maintaining both sides of an equation equal. Having a reference about why things work out the way they do, can support students' understanding of mathematical procedures and concepts.

understanding of mathematical procedures and concepts.

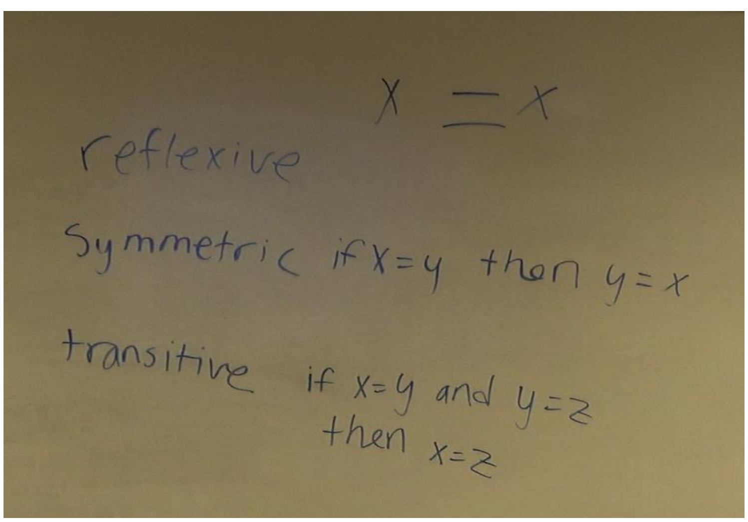

Figure 9: Proving equality is an equivalence relation.

At the end of the equivalence relation objective, the instructor wrote out the student’s ideas of where they have used equivalence relations before in middle school or high school mathematics (see Figure 9). Through this discussion, Dr. Evans can show students how the things they have learned before can be given a name, and by formalizing such concepts, students learn about why these structures can be treated as they are, by exploring their properties.

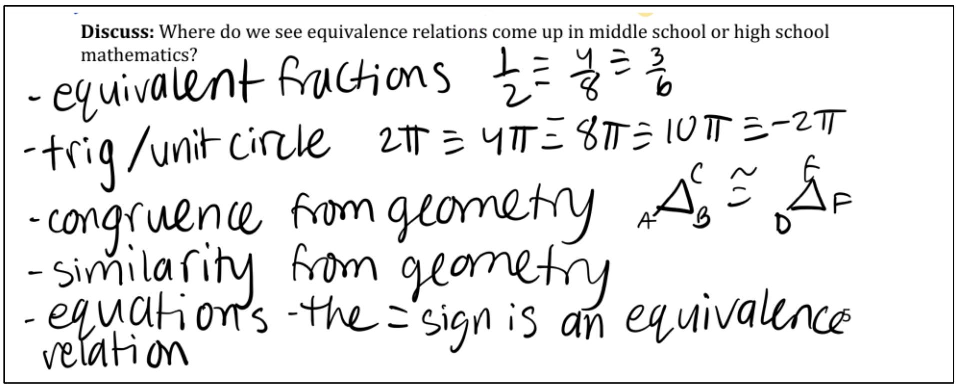

Figure 10: Equivalence relations from middle school or high school.

I associated this move with the practice of making system of schooling explicit because Dr. Evans made comparisons about how students have been previously taught about mathematics and now they were able to uncover the reasons behind what they learned, which can support students to situate what they were learning in this class and connect it to the content they will eventually be teaching.

Summary of Noticing for Equity pedagogical moves

The course was structured in a way that the students were positioned as competent mathematics learners, where the instructional tasks were sequenced in a specific way to develop their intuitive understanding towards a formal understanding of algebraic structures. Therefore, majority of the tasks seemed to were designed to *leave students to grapple with mathematical ideas*, so it was first important to *make norms explicit for doing mathematics* to make sure the students felt comfortable in having the role of conjecturing and sharing ideas. To do so, the instructor asked the students to come up with their *classroom norms that would need to be fostered during class*. Once these norms were established, the instructor *assigned group roles* for students to work collaboratively and would let them *share their ideas* with the whole class. Having students present their work every class contributed to supporting *students in developing mathematical identities*, and the instructor promoted that by *revoicing students' contributions*, *recognizing students' individual contributions,* and a*ttending to the productivity of student's contributions that were not completely correct or precise*. While doing this, she connected with student's individual strengths through *engaging with humor and play*, allowing students to feel more comfortable and motivated during class. I infer that this process allowed the instructor to make the system of schooling explicit by uncovering mathematical objects that students have used throughout their student life as important algebraic concepts through the tasks and class discussions. Overall, I found Dr. Evans used these pedagogical moves that can help enact the five noticing for equity practices from van Es et al. (2017).

EQUIP Summary

Equity Quantified In Participation (EQUIP) is an equity analytic tool that quantitively measures participation patterns in terms of distribution among the classroom's gender and/or race (Shah et al., 2016). Reinholz & Shah (2017) suggest seven dimensions in which students' participation can be measured. For this study, the distribution of participation inside Dr. Evans's classroom was evaluated within 2 dimensions of EQUIP, teacher solicitation method and explicit evaluation based on how it was distributed among genders. In the class, there were 11 women students and 9 men students. Therefore, the interactions were expected to be distributed equally between men and women. However, it was found that out of 105 observed interactions over five class sessions, 73 were from men students and only 32 were from women students, which represents 69.52% versus 30.47% of interactions, respectively. The *equity ratio* between observed and expected interactions from men, 1.55, demonstrates that the men students exceeded what would be equitably distributed participation. Whereas the equity ratio for women interactions, 0.55, shows how women students had fewer interactions than expected.

Table 4: Distribution of participation opportunities among genders.

Gender	Observed Interactions	Expected Interactions	Equity Ratio
Men	73 (69.52%)	47 (57.75%)	1.5531915
Women	32 (30.47%)	58 (47.25%)	0.5517241

Similarly, when analyzing the Teacher Solicitation Method, I found that men students were overrepresented in each of the types of solicitation methods Dr. Evans used: when calling on a group, calling on a student, calling for volunteers, and even when participation was not called on, the contributions were majority by men. For every type of solicitation method, the equity ratio for men resulted higher than 1, indicating an exceeded expected participation. On the other hand, the equity ratio for women in each of the solicitation methods was less than 1, showing how underrepresented they were when being asked to participate. An important thing to point out from this EQUIP dimension is how women did not share any contribution that was not called on, raising the question about whether they feel afraid to share their ideas without being asked.

Table 5: Teacher Solicitation Method among gender.

Gender	Called on group	Called on student	Called on volunteer	Not called on	Total
Women	13 (37.14%)	2 (28.57%)	17 (30.90%)	0 (0%)	32
Men	22 (62.85%)	5 (71.42%)	38 (69.09%)	8 (100%)	73

The second dimension that I coded is Explicit Evaluation. This dimension describes whether Dr. Evans evaluated the student's idea or left their correctness open. Based on the instructor's responses, I coded for the following methods of evaluation: Elaborate whenever the instructor used a student's idea and further explained their response to clarify and refine their thinking or to further explain aspects of the task. Evaluate was coded whenever the instructor agreed with the student's ideas by nodding or by showing excitement for a student's idea. I assigned the code Revoice when the instructor would repeat the student's idea or rephrase it as a form to evaluate the idea, most of the time to show agreement. The last code was Follow-up question. This code was assigned in the instances where a student would share an idea that was not completely precise or correct and the instructor would ask to follow up questions to guide their thinking into the correct answer.

Table 6: Explicit Evaluation among gender.

Gender	**Elaborate**	**Evaluate**	**Revoice**	**Follow-up question**	**Total**
Women	8 (25.80%)	9 (23.68%)	8 (53.33%)	7 (33.33%)	32
Men	23 (74.19%)	29 (76.31%)	7 (46.66%)	14 (66.66%)	73

Within this dimension, the results showed again favorable results towards men but an underrepresentation for women. Although there were only 15 instances of revoicing to evaluate an idea, this was the method of evaluation that resulted in an equity ratio of 1, meaning that the observed interactions matched what the expected interactions, determining an equitable and even distribution of the way Dr. Evans responded to the student's contributions.

Overall, the results showed a high level of inequity in terms of gender, as the opportunities to verbally participate in class discussions were not equally distributed among men and women. From a total of 105 observed interactions, 55 of them were volunteer contributions, and the majority of them were made by men. Even when the students had randomly assigned group roles and the speakers for each class were different, still there were more men who participated when the instructor called on groups. This indicates that careful selection for assigned roles should be made in order to manage the participation in a more equitable way, where both men and women have equal chances to share their ideas.

Student's perception about their instructor's noticing for equity

In the first part of our study, I identified the pedagogical moves Dr. Evans enacts with the goal of fostering equity and inclusion inside her classroom and I measured how the distributions of participation opportunities are managed among female and male students. Now, I am interested to learn about whether the students notice how she fosters equity and inclusion, how her students perceive these moves, and whether they feel the practices in this course have made them feel any difference regarding their sense of belonging in the mathematics community and their mathematical confidence.

To learn about this, I present the case of two students in Dr. Evans's class, Allison, and Raziel, Who responded the sense of Belonging scale by Good et al. (2012) were they were given 30 questions regarding their experience in the math community, responding on an 8-point Likert scale (1 = strongly disagree; 8 = strongly agree) and a confidence questionnaire by Parsons et al. (2019) with 5 questions about their confidence and liking of the mathematics subject and two open ended questions that explained their answers to the Likert scale questions. Both participants were interviewed about their experience in mathematics before Dr. Evans's course and after her course. The students describe the actions Dr. Evans did to promote equity and inclusion and whether they contributed towards their mathematical confidence and sense of belonging.

Participant A: Allison

Before Dr. Evans's class. Allison is working towards a Mathematics major with a middle school teacher certification, having recently switched majors from Special Education. She found active learning impactful for her learning, specifically inquiry-oriented courses as she thinks finding information by herself gives her a rewarding moment and that information is easier to remember. Prior to switching to Mathematics, she says she did not enjoy working in groups, as people would not talk and would not know what to do. Regarding equity and inclusion during group work, she mentioned having previous experiences where one or two members of the group would dominate the discussion, preventing the rest of the group from sharing their opinions. When asked about her participation during class discussion, she said it would depend on how confident she felt about her idea but the majority of the time she would hold her thoughts to herself if she was not sure if her answer was right or wrong: “I’m so used to traditional types of lessons where, If I’m wrong, they’re like ‘that’s incorrect,’ and move on”. If one of her previous professors would call her out, Allison said she would “go into panic mode”, and preferred when the instructor would give a heads up about asking questions.

Allison said her mathematical confidence will depend on the content of the class, given that higher level courses of math have brought her confidence down, but she does find having confidence impactful towards learning mathematics:

> I feel like math is foundational…When you work on one concept and you build on to another, you’re building that foundation…If I feel confidence in my first level, then I’ll feel comfortable moving on to the next one.

The change in major made Allison feel she did not quite belong at first because her classmates would have a lot more experience with mathematics education courses than her: “It feels like everyone in that room knew more than I did”. Working in group work was a way where Allison felt more belonging: “it made me feel more part of this math community”.

Allison recognized the importance of having strong mathematical confidence and a sense of belonging in math as she mentioned,

> It's important to be confident now, to have a sense of belonging now, because it's going to show when you are the one in front of the classroom, and you're the facilitator… If you, the teacher, don't have confidence yourself, then maybe the students might not have that confidence as well.

Finally, when asked about whether she thought having a strong mathematical confidence and sense of belonging could determine the way she performed in class, she said a lowered mathematical confidence would make her doubt yourself and feel discouraged from going on.

After Dr. Evans's class. Allison said that Dr. Evans's class has been interesting and fun, going from simple things to complex concepts: "At first I was intimidated by it because it looked really complex…but little by little when we're exploring it as like in our groups and with Dr. Evans, it was really interesting to see this more in depth*"*. When asked about the classroom social norms, she said the biggest one for her was how everyone's ideas were valued and evaluated.

> Any ideas are appreciated, any input is accepted, and especially by Dr. Evans…Maybe we find like misconceptions, and she'll always be like, "Oh, that's a good point to make, that is something that we should look out for" and then we keep in mind.

Regarding active versus traditional learning, Allison mentioned that she likes inquiry-oriented based instruction because she can build knowledge from her own and group's ideas but mentioned that a tricky part from it would be whenever they have misconceptions because she says she doesn't want to be taught the wrong thigs, whereas to her, traditional learning is more definitive because she would get the answer from her teacher. However, she finds learning through an active setting more memorable because she is involved in the process of coming up with the ideas.

I asked Allison what moves she noticed Dr. Evans did to promote participation in class. She mentioned she liked the group roles Dr. Evans assigned, recognizing one of the observed pedagogical moves of *assigning group roles for collaborative work*,

> It's good to see my name, and then like oh, I'm speaker of the day. So, then I'm ready, mentally preparing what to say…But me personally, as someone that can get overwhelmed in a social setting, if I'm told in advance that I'm going to be the one presenting I feel a little bit more calm because I already know it's coming.

Even though she finds learning about what her role in the group is in every class helpful, she thinks that sometimes students who are not the speaker might not put the same effort into coming up with ideas and solving the task. Allison also talked about how sometimes Dr. Evans would call students on the spot and thinks that at the end of the day, everyone can be the speaker. She said:

> I like that every now and then she'll keep us on our toes, and she'll just call on us. I know her, as always that any input is accepted. Regardless of how right or wrong it is because in the end of the day we're all learning and our idea is part of this like, it's complementary to our learning. I don't know. That's the best way I could say, it's productive.

In this interview transcript, Allison noticed how Dr. Evans *attends to the productivity of a student's contribution when it is not completely correct or precise.* She *ex*plained that knowing that the instructor would react like this to a wrong answer, made her feel less anxious about being called out to participate during class discussions and made her be attentive to what was going on in class, as she could be called out any time. Throughout the post-interview, she mentioned how this move positively impacted the way she participated in class, because she would not feel afraid of being wrong or getting laughed at. She also talked about an instance where she had an idea during group work and Dr. Evans asked her to present it to the class, when she was not the speaker.

> She'll let us know like, "Oh, by the way, Allison, I really like the idea. I would actually like for you to share that idea with you or with the class…and maybe I wasn't like speaker of the day. I was probably you know, any other role. But yeah, everyone is a speaker in the end.

Allison recognized the pedagogical move of *recognizing student's individual contributions.* I asked her to elaborate on how this action from her instructor made her feel and she responded, *"*I feel like I really contributed something... It's a good feeling when you get a sort of praise by the teacher...It's still nice that everyone is still going to be spotlighted or highlighted.*"* Having this recognition by Dr. Evans gave Allison a boost in her confidence, which is always a good thing to have and might have encouraged her to contribute with more ideas. She felt her contributions were valued as Dr. Evans would tell them "little statements of praise" like, "Oh that's a really great point to make*", "*Oh, did everyone catch onto that?" or "I want everyone to hear what Allison has to say."

Allison thought the participation among the students in the classroom was well distributed. She made this conclusion by saying that she heard everyone's voice in the class throughout the time spent on the course, which is something that had not happened in her previous courses. She noticed that even if there was not enough time for a group to present their work during class, Dr. Evans would still include a picture of the group's whiteboard in the notes, so that everyone could still see their work. This made her feel that everyone's ideas were important.

Allison said Dr. Evans and working with her classmates improved her mathematical confidence. In this course, she felt supported by her instructor and her classmates as they would be encouraging about coming up with new ideas and building up from them, which made her feel more comfortable in sharing and felt more competent in what they were learning and eventually what she will teach.

This course made Allison realize the importance of sense of belonging. She said if she did not feel she belonged, she would be discouraged from learning, but during this course, she felt she belonged because of how welcomed and accepted her ideas felt by Dr. Evans. Allison thinks her sense of belonging benefited the way she participated in class, as she thinks if she was not comfortable, she would just sit in class and be quiet. Allison compared her previous experience in a mathematics classroom versus this course and realized that the main difference was the task of setting the classroom norms, identifying Dr. Evans's pedagogical move of *fostering classroom norms.*

> One difference immediately that I thought of was Dr. Evans from the very first week that we joined the class was asking us to provide like, what would we want of each other to feel respected, what would we want of each other to feel like, everyone matters, and for everyone to be included and I think when we had that discussion, we just empathized with one another in the class...

Allison also thinks the structure of the class has allowed her to identify herself as a mathematician and throughout the tasks she has found her confidence as an aspiring math educator.

Finally, she was asked about the role of an instructor in building up students' sense of belonging and mathematical confidence, recognizing that a teacher plays a big role. Allison thinks Dr. Evans did a really good job in improving her sense of belonging and confidence.

> I walk away each class every like Monday, Wednesday, feeling like I learned. I learned well that day, and I know I'm gonna learn even more like the next session... I think the connection that she's created with us it makes us more comfortable to ask questions when we have questions? So that we never doubt on ourselves.

Allison concluded that her sense of belonging and mathematical confidence increased after taking Dr. Evan's class.

Participant B: Raziel

Before Dr. Evans's class. Raziel is majoring in Interdisciplinary Studies, going for his middle school mathematics certification, and a Kinesiology minor. Raziel was originally aiming for an engineering degree but realized he was not passionate enough for it, so he decided to step back. Raziel has always liked math and said his grades and performance have always been at a high standard. When asked about whether he prefers traditional versus active learning, he said some topics are easier to learn traditionally, but the majority of the time, he leans towards a more active approach.

Raziel said he had enjoyed working collaboratively but had encountered himself in negative experiences where some members of the group would not work, and his grade would be brought down by someone else's actions. He usually takes up the role of asking other members of the group questions and helping them out if they do not understand. He feels confident about doing so mostly because of his experience because he claimed he has always understood most things pretty clearly. Raziel mentioned he tends to voluntarily answer questions during class discussions, but his participation would depend on the confidence level he has in his answer. If he felt his answer was wrong, then he would prefer to stay quiet, and he would mentally double check his answer when answering a question from an instructor, to make sure he was not making a mistake. When asked about whether he had noticed inequitable distribution of participation opportunities in his mathematics courses, he mentioned how he noticed the distribution changed depending on the class he is in:

> I did notice that it was more females talking in the education classes than males. But in like the math classes that I've had that, it's more males talking than females, and I'm not sure if that's just cause there is more males in the class than females,

> or because just the males felt more confident in speaking...I don't recall if there was like, how split the classes were in terms of gender. But I did notice that in math, men would talk more than women and in my education classes, the women would talk more than men.

Regarding his mathematical confidence, Raziel thinks he has a good level of confidence because of the way he understands concepts, so the confidence is determined by the level of understanding he has on a certain topic. He believes that having a strong mathematical confidence helps him enjoy more the content.

> Having a high level of confidence kinda makes me like it more cause I'm more confident in that, kinda like, I'm like relating it to. I guess like sports, if you're confident in your abilities to shoot a basketball, then you're gonna be more inclined to shoot the basketball. So, the same thing with Math, if you're confident with your skills in math, you're gonna be more inclined to do math. So, you're obviously gonna enjoy it more.

In our first interview, Raziel said he had not thought about sense of belonging related to mathematics, but once he related his idea of belonging with math, he concluded that,

> I don't feel like I don't belong in a mathematics field, because it's something that I've only ever had, like a positive experience with. So having that positive experience with math for my confidence, for my enjoyment, my grades.

Raziel specifically recalled one of his teachers from high school that had a positive impact on his sense of belonging because the professor would recognize his potential and create specific class content for him to advance even more. He then also related his decision of leaving engineering because he felt he did not belong there. Therefore, he recognized the importance of having a sense of belonging in what you are doing. Finally, he said that he sees how mathematical confidence has an impact on the way he performs in class but his sense of belonging in math does not necessarily affect it.

> There is like a connection between the two. Because obviously, if you're more confident, you're gonna feel like you belong there. But it's also like your confidence is a factor in your like, in your sense of belonging, but it's not like the main thing.

After Dr. Evans's class. During our second interview I asked Raziel how he felt about about Dr. Evans class, and he mentioned he felt good about the content, mentioning the tasks were about "dealing with the nature of how of why things are the way they are*"*. For example, he mentioned how he has always dealt with binary operations but never knew what they were called. Raziel noticed an important part of Dr. Evans's class structure and pedagogical move, *defining mathematical concepts in a formal way*.

Other things Raziel liked about the structure of Dr. Evans's class was that it was very inquiry-based oriented, interactive and, there was lots of group work where everyone has a particular role in their group, so "everybody's held accountable for doing something*"*. Raziel noticed the pedagogical move of the instructor of *assigning group roles for collaborative work.* However, when asked about whether this move promotes equity, he said*:*

> it does promote it, cause obviously everybody has a role. But at the same time, it doesn't guarantee that everybody's gonna be doing what they're supposed to be doing. I would say it promotes it, but it doesn't guarantee it.

Raziel also noticed how Dr. Evans would *share group work's ideas to the whole class*. I asked him to elaborate whether he felt Dr. Evans attended well to their ideas.

> She takes pictures so everybody can see what like, just in case you can't directly see what they're writing. But I mean, yeah, she goes to every group, and she asks about what we're thinking. And sometimes, if we get like an idea that's different, or an idea that has been brought, she always expands upon it.

Throughout his interview, Raziel mentioned a few examples of times he participated in class by sharing his ideas to the class and how Dr. Evans would elaborate on them. Raziel said he felt confident in participating because he knew the instructor and his classmates were fostering the social norms they set at the beginning of class. He knew no one would make fun of him or say something negatively if he was wrong. He also mentioned that having Dr. Evans recognize and talk about his ideas would make him feel more confident,

> It's not necessarily the way she talks about my ideas, just the fact that she is talking about the ideas. It does have like an effect, and it does improve my confidence, and it like kinda leads me to be more inclined to share more throughout the class.

Raziel also pointed out how Dr. Evans attends to ideas that are incorrect, perceiving the pedagogical move of *attending to the productivity of student's contribution when it is not completely correct or precise.* He said,

> The way she handles incorrect answers without like, directly, like just telling you like flat out "You're wrong", it's always with positive reinforcement and kind of like leading you towards the correct answer.

When asked about how Dr. Evans elicits student contributions, Raziel mentioned the "*moment of private think time*":

> She has us do private think time. So when she asked us questions, she'd say, "Take a moment of private think time", and then we'll talk about it with our groups or with our partners. So that kind of gives everybody a time to think about it first before someone just blurts out the answer.

Raziel recalled the time when he shared the conjecture about identifying the identity from a Cayley table by looking at whether an element operated with itself resulted in that same element (recall to Figure 7.) and Dr. Evans made a proof about this conjecture:

> She did on the board, and proved that what I said was correct with like actual mathematical proofs and everything. It obviously like, you feel acknowledged, and you feel accepted.

. Within this example, Raziel perceives Dr. Evans's way of *recognizing students' individual contributions*, as well as how she *engages students in the mathematical practices that mathematicians use.*

> She'll always mention people's ideas, and she'll sometimes, she'll like write it on the board. She's like kind of illustrating everybody's ideas, and how she always in the filled in notes on Blackboard it always has pictures of everybody's ideas so you can see what other people are thinking.

Overall, Raziel said he feels confident about the content they have learned in class, recognizing that the instructor does “a lot of stuff” that makes it easier to understand, such as having well structured notes, color coding activities, and inquiring towards formalizing a definition. He also mentioned how two of his classmates have made him feel more confident because they would always compliment the way in which he understands things, and how they are supportive of his ideas.

Just talking about the ideas, establishing norms, making sure that you feel included with like the different roles, and elaborating when you have an answer that isn't completely correct were some of the things Raziel noticed Dr. Evans did to foster the students’ sense of belonging.

> It's not the case for me but it could have been the case for somebody else where they didn't feel like they belonged in like a math classroom, and then they had Dr. Evans, and then all the stuff that she's doing, establishing the norms, making sure everybody's ideas are heard, giving you positive reinforcement when you have a right answer or a wrong answer is all the stuff that she does to make you feel like you're included and that you belong there. It can like, kind of revitalize someone's passion.

Although he recognizes that he did not need much more than his individual experience with mathematics and how easily it is for him to understand the concepts, he was able to notice how fostering these practices could benefit others who may not experience math the way he does.

Raziel concluded that his sense of belonging stayed the same because he already felt a good sense of belonging towards math, but he noticed improvement in his confidence.

Summary of students’ perceptions of Dr. Evans’s noticing for equity

Based on how the participants talked about the perceived pedagogical moves, I inductively coded the interview data segments with the constructs of mathematical confidence (Parson et al., 2009) and sense of belonging (Good et al., 2012) to make explicit connections about the way the students felt the pedagogical moves impacted them. From the ten pedagogical moves that I found in the classroom observations, the students were able to also identify eight of them; Raziel identified all eight moves, and Allison identified six of them. As seen on Table 7, some of the pedagogical moves were connected with both of the constructs of students' confidence and sense of belonging. For example, *fostering classroom rooms* helped the students feel comfortable in sharing their ideas in front of the whole class, but at the same time, knowing that their idea would be valued by their peers and Dr. Evans, regardless of the correctness of the idea, also helped them build up their confidence to participate. Similarly, Dr. Evans *recognizing students’ individual contributions* boosted the students’ confidence, and in consequence, having this “moment of praise”, as Allison called them, fostered their sense of belonging.

Table 7: Participants’ connections between pedagogical moves to mathematical confidence and sense of belonging.

Construct	Perceived Pedagogical Moves	Student(s)
Mathematical confidence	Fostering classroom norms	Allison, Raziel
	Recognizes students’ individual contributions	Allison, Raziel
	Attending to the productivity of student’s contribution when it is not completely correct or precise	Allison, Raziel
	Sharing group work’s ideas with whiteboards // Gallery walks	Allison, Raziel
	Defining and understanding mathematical concepts in a formal way	Raziel

Table 7: cont.

	Assigning group roles for collaborative work	Allison, Raziel
	Give students a "moment of private think time"	Raziel
	Engaging students in the mathematical practices that mathematicians use	Raziel
Sense of Belonging	Fostering classroom norms	Allison, Raziel
	Attending to the productivity of student's contribution when it is not completely correct or precise	Allison, Raziel
	Sharing group work's ideas with whiteboards // Gallery walks	Allison, Raziel
	Assigning group roles for collaborative work	Allison, Raziel
	Recognizes students' individual contributions	Allison, Raziel
	Engaging students in the mathematical practices that mathematicians use	

These inferences about an improved sense of belonging and mathematical confidence align with the results from the students' pre and post questionnaires. Shown in Table 8 are the results for both Allison and Raziel. As it can be observed, Allison had an improvement in both her confidence in both sense of belonging and confidence, whereas Raziel maintained his sense of belonging and showed improvement in his mathematical confidence.

Table 8: Mean Scores of Pre and Post questionnaires from Allison and Raziel.

Student	**Sense of Belonging**	**Mathematical Confidence**
Allison	**Before:** 5 **After:** 5.5	**Before:** 4 **After:** 5
Raziel	**Before:** 4.5 **After:** 4.5	**Before:** 4 **After:** 5

CHAPTER VI

DISCUSSION

Given that noticing for equity can support the instructor in noticing aspects of mathematical activity that make students feel more or less empowered (van Es et al., 2017), it is important to understand how these practices can be managed inside the classroom. The purpose of the present study was to understand how the commitments of an instructor to notice for equity can influence aspects of the students' experience in doing mathematics, particularly how their sense of belonging and mathematical confidence is fostered through their instructor's pedagogical moves. Furthermore, I wanted to make explicit how these pedagogical moves for noticing for equity were enacted and prove whether they contributed to an equitable management of participation opportunities during class discussions, particularly how these opportunities are distributed among men and women.

One of the main findings of this study were the ten pedagogical moves that were identified from observing Dr. Evans's class and their connection with van Es et al.'s (2017) five noticing for equity practices. I infer these practices were enacted by the following pedagogical moves:

1. Make norms explicit for doing mathematics was identified by Dr. Evans's move of *fostering of classroom norms* because these norms provided students with the guidelines and behavior they should follow when working collaboratively and sharing mathematical ideas.

2. Leave students to grapple with mathematical ideas was seen when Dr. Evans *gave students a "moment of private think time"* and *assigned group roles for collaborative work* because in both of these moves, the instructor intended to let students take up space in the classroom and position them as competent mathematics learners.
3. Support students in developing mathematical identities was recognized when Dr Evans *shared group work's ideas, engaged students in practices mathematicians use, recognized student's individual contributions, revoiced student's contributions,* and *attended to the productivity of student's contribution when it was not completely correct or precise*. In all of these moves, Dr. Evans contributed to fostering students' identity as mathematicians by giving them authority over their knowledge, allowing them to conjecture about algebraic structures, and evaluating their ideas in a productive way.
4. Connect with students to honor individual strengths was shown by Dr. Evans's *engaging in humor and play* as it allowed her to create a connection with the students that made them feel more comfortable to share their thoughts and ideas.
5. Make system of schooling explicit was achieved by Dr. Evans when she *formalized mathematical concepts that had previously been used by students in secondary mathematics* since she would compare how students were taught some things in high school without knowing why the procedures worked. Through these comparisons, the students could make meaningful connections between what they already knew and the new formalized understanding of that.

These pedagogical moves showed to be good for managing respectful collaborative work and class discussions, establishing a good relationship between the instructor and her students, eliciting students' thinking, and having students share their ideas without fear of being wrong.

Another important thing to highlight is the context in which these pedagogical moves were enacted. In my results I talked individually about the ten moves. However, most of the time, Dr. Evans enacted multiple moves in one interaction with a single student. I mention this to point out the difficulty of attending not only to the students' mathematical understanding, but also to make sure their understanding is attended to in the most equitable way possible. For instance, when students gave erroneous answers, the easiest thing a teacher could do would be evaluate the answer as incorrect and point them to the correct answer. However, as Dr. Evans showed during her class, attending to erroneous contributions can be done in an equitable way, where first and foremost the students feel their idea is considered and valued by their instructor. Then, the instructor could come up with a counterexample or show why the idea is not completely correct, and then scaffold their thinking into a correct solution. It could also be the case that the student is given reassurance that talking about their mistake is a productive thing to do to solve any misconception. Noticing student thinking in-the-moment, attending to their thinking and trying to understand where their idea is coming from, coming up with a counterexample, explaining how their thought can be a common misconception, and recognizing the student for their contribution, can occur all in a small short of time and can represent only one of the multiple interactions between instructor and student per class day. Noticing equitably requires an instructor to be present and ready for occurrences as this one. In no way is noticing for equity easy. However, the importance of doing it can be a determining factor for students to feel included and valued.

Active learning has been promoted as an instructional method that can benefit various aspects of student's learning but can contribute to inequities inside the classroom such as participation disparities (Flores, 2007; Serbin & Mullins, 2019; Aguillon et al., 2020), gender biases (Reinholz et al., 2022), and cultural and social differences (Theobald et al., 2020). Researchers continue to explore ways in which teachers can promote fair and inclusive learning opportunities in mathematics active learning courses and investigating what can be done to mitigate potential inequities within active learning instruction. One of my wonderings with this study was whether the students could recognize the commitment and efforts their instructor has towards fostering equity and inclusion, in this case, how Dr. Evans promoted this in the classroom.

A major finding was learning that students *do notice* it and can sympathize with the importance of everyone having an equitable experience when learning mathematics. I analyzed data from interviews I had with two of Dr. Evans's students, a woman and a man, Allison and Raziel, and matched what they described with my list of pedagogical moves. Out of the ten pedagogical moves I identified, only the moves of revoicing students' contributions and engaging in humor and play were not explicitly described by the students. I cannot conclude whether this was because they forgot about it during the interview setting or maybe they did not consider it as something that contributed to equity and inclusion. By inferring about the pedagogical moves used by the instructor and the way students described their experiences with such moves, I associated each move to the constructs under investigation: *sense of belonging* and *mathematical confidence* and obtained the following results:

1. Fostering classroom norms was a move that seemed to foster students' *sense of belonging* since it created a judgement-free environment inside class, which also allowed students to feel comfortable in participating, knowing that their ideas would be considered regardless of being right or wrong, supporting their *mathematical confidence*.

2. Giving students a “moment of private think time” seemed to allow students into finding out how to deal with a task by themselves, eliciting their thinking in a position where students needed to use their previous knowledge, apply definitions, or make conjectures based on their intuitive understanding, which all guided them to work with their cognitive skills, contributing to their *mathematical confidence*.
3. Assigning group roles for collaborative work was a role identified with the students that I infer fostered their *sense of belonging* by having them work together. Sharing their ideas and coming up to agreements about their final answers allowed them to connect with each other, which at the same time fostered their *mathematical confidence*, as having shared ideas gave them assurance to share in front of the class.
4. Sharing group work’s ideas with a whiteboard and Gallery Walks seemed to foster students’ *mathematical confidence*, as their ideas were driving Dr. Evans’s task sequence, taking up space in the classroom. Having their opinion be considered and valued by their instructor and peers could foster their *sense of belonging*, as they could perceive themselves as the contributors to advancing the mathematical agenda of class.
5. Engaging students in the mathematical practices mathematicians use appeared to contribute both towards their *mathematical confidence* and *sense of belonging*. The students had authority over their mathematical activity, which helped them experience mathematics in a different way, where their ideas can be negotiated and constructed towards a formalized understanding.
6. Recognizing students’ individual contributions could foster students’ *mathematical confidence* and *sense of belonging*. To have their idea recognized by the instructor and choosing to talk about it to the rest of the class made students feel they were making a

good point, regardless of the correctness of the idea, and made them feel their contribution was being valued.

7. Attending to the productivity of a student's contribution when it is not completely correct or precise seemed to give students *mathematical confidence* to share their understanding even if they were not sure if it was right or wrong. Fostering a classroom where making mistakes is productive helped students feel comfortable in participating, as this reassured them about how they can make mistakes and still *belong* in the mathematics community.
8. Defining and understanding mathematical concepts in a formal way appeared to foster *mathematical confidence*, as students were able to understand why high school procedures worked the way they do and in a future they could use this knowledge to respond to their own students.

Although these last two moves were not explicitly talked about by the students during their interviews, I can infer that both can foster students' *sense of belonging*.

9. Revoicing students' contributions can let students know that their idea is being considered by their instructor. This can also be a sign where the instructor tries to understand or make clear to others what the student is trying to say, which could also make the student feel understood.
10. Engaging in humor and play was a big part of Dr. Evans's class. I assume this move might not have come up in the interviews as students may only perceive this as the instructor's personality, but by the observations I conducted during her classes, I could see that this move contributed a lot to the students being comfortable and relaxed but at the same time paying attention to the class.

The intentions the instructor had for enacting the observed pedagogical moves may have been focused towards noticing for equity, but in consequence of the constructs I investigated in this study, they also resulted in being good ways that could foster students' sense of belonging in mathematics and build their mathematical confidence. Although this could be concluded by relating the pedagogical moves to the existing literature, our findings become meaningful when the students who experienced this noticing for equity from their instructor are the ones making the connections.

Based on the sense of belonging and confidence questionnaire the students filled out before and after Dr. Evans's class, Raziel showed an increase in his mathematical confidence, whereas his sense of belonging remained the same. On the other hand, Allison results showed an increase in both her sense of belonging and confidence. These results match with my findings of Dr. Evans's pedagogical moves fostering equity and inclusion and our hypothesis that such moves can foster students' sense of belonging and mathematical confidence.

Finally, when analyzing Dr. Evans's work through the EQUIP analytic the results showed there was an inequitable distribution of participation opportunities among the gender demographic of the classroom, being that men had more participation interactions with the instructor than women in different types of solicitation methods that elicit student thinking: calling on a group, calling on a student, calling for volunteer and the interactions that were not called on. Although the difference in men and women demographic is not that significant, 9 men students and 11 girl students, the EQUIP analytic showed a major difference among the equity.

Discussion of findings

Equitable distribution for participation opportunities

Despite the evident commitment from Dr. Evans to notice for equity, her management of distributing participation opportunities between women and men resulted in an overrepresentation among men students. But what exactly does this imply about her teaching? Dealing with power dynamics inside the classroom is complex. Mathematics education research has shown the evident gender gaps when implementing active learning due to the social narratives that have been historically incentivizing the favoring of male students. The use of active learning can be in most cases be favorable towards students who are men (Maries et al., 2020; Reinholz et al., 2022) and thus, attention needs to be paid towards reducing the possibility of such disparity. For instance, the assignment of group roles seemed to be a productive way to get students to work collaboratively and hypothetically have everyone in the classroom participating. However, when analyzing the class discussion with EQUIP, I found that even when group roles were assigned and the speakers were rotated each class, calling on groups to participate still resulted in an inequitable distribution of participation opportunities. To help mitigate such disparities in students' verbal participation, the instructor should keep track of who has been the speaker before and making manual selection of groups to make sure the roles are equally distributed for each person in class. The instructor must also be attentive for other inequities that may arise in small groups, such as the prevailing of power dynamics.

Given the nature of active learning, inquiry-based instruction requires students' engagement and participation. Dr. Evans's most common solicitation method for students' participation was calling on volunteers. By societal structures and power dynamics, there is a higher likelihood that men volunteer to participate, given they are generally more confident in sharing idea compared to women (Serbin & Mullins, 2019). Thereby, promoting volunteerism is an inequitable way of soliciting students' contributions. One way I recommend to address and reduce the gender gap in classroom participation is soliciting students' contributions by calling out each student to participate by name. This method would allow the teacher to control who has participated, ensuring an equal opportunity for every student to be called upon and respond to their questions.

Nevertheless, women might continue to have a lack of confidence and fear in participating and sharing their ideas due to these social dynamics that society unfortunately conforms with (Ganley & Lubienski, 2016). I also recognize how calling on students by name can be an anxiety-inducing method for participation. Therefore, it is crucial to establish and foster social norms that contribute towards building student's confidence, especially women's, who are systemically disadvantaged by their gender.

Pedagogical moves to notice for equity and fostering students' sense of belonging and mathematical confidence

Despite the documented inequities in Dr. Evans's solicitation of men and women students' verbal contributions in class discussion, Dr. Evans enacted her commitment to equity through her pedagogical moves used to guide, notice, and respond to student thinking. Ten of these pedagogical moves were found that can support noticing for equity practices: fostering classroom norms, give students a "moment of private think time", assigning group roles for collaborative work, sharing group work's ideas with whiteboards, engaging students in the mathematical practices that mathematicians use, recognizing students' individual contributions, revoicing students' contributions, engaging in humor and play, and formalizing mathematical concepts that have previously been used by students in secondary mathematics.

Fostering classrooms that allow students to take up space and interact with their peers, by working collaboratively can help build students' sense of belonging (Lahdenperä & Nieminen, 2020) and confidence (Laursen et al., 2014) because students get the opportunity to share and teach their mathematical ideas to the rest of the class. Another good way to improve students' mathematical confidence is through the encouragement of growth mindset learning (Boaler, 2013), where students are not discouraged from making mistakes and instead are invited to refine their thinking. In consequence, this growth mindset can cultivate students' sense of belonging by having their contributions being evaluated as productive.

These suggestions from the literature align with the pedagogical moves Dr. Evans enacted during her class and are good indicators that her enacted efforts were contributing to the students' sense of belonging and confidence. I leverage the idea that sense of belonging and mathematical confidence can be key factors that determine a student's performance in the classroom (Lahdenperä & Nieminen, 2020), their development of mathematical identity (Darragh, 2013), and their persistence in mathematics (Good et al., 2012). In their multidimensional noticing for equity study, van Es et al. (2022) suggested that future studies should "include student voices and perspectives to understand whether and how students experience their classrooms as equitable and affirming spaces" (p. 127). The contribution the current study adds to the literature is the inclusion of two students' voices and perceptions of how their instructor attended to equity and inclusion during the classroom. The results from the conducted interviews showed that students are noticing what is being done by the instructor to include and value every student's contributions and have a positive reaction towards such contributions. Both students, Allison and Raziel, demonstrated having an improved mathematical confidence, and Allison self-reflected on her sense of belonging being higher than before taking

Dr. Evans's class, whereas Raziel said his sense of belonging was already at a high level, which he maintained after this course. Although these results cannot be generalized to the total of students in the class, they are still good indicators to keep on promoting equity and inclusion inside the classroom, as it is fostering students' sense of belonging and their confidence to do mathematics.

Overall, we show how the efforts an instructor makes to foster equity and inclusion in the classroom can be perceived by the students and can foster their sense of belonging in mathematics and develop their mathematical confidence. I recognize the complexity of adding another aspect of class to notice, but I encourage instructors to implement this practice of noticing for equity, for it proved to be a beneficial practice to keep the class engaged and make the students feel included.

Even if noticing for equity does not guarantee an equitable experience for all, as shown in this study, continuing efforts should be done to mitigate gender gaps in participation. Instructors should not be discouraged from continuing working towards more equitable mathematics classrooms. The results of this study highlight the importance of noticing for equity and the need to add student voices to research studies to learn what they think and how they feel about the management of equity inside the classroom, as they are the ones who can actually guide educators towards making them feel valued and included.

Limitations of the study

One of the limitations I recognize for this study is the sample size of students who were interviewed. It was not my aim to generalize the perception students have towards their instructor's noticing for equity. In the end, all students have their own personal experience with mathematics and will perceive things differently from their classmates according to multiple factors (e.g., their beliefs, their gender, their knowledge). It would be interesting to interview a student who is not performing well in class or did not participate as often, to learn about their experience with the class. It is possible that some of the moves Dr. Evans did for including everyone might not be accepted by everyone in class or they might not like the idea of working collaboratively and sharing ideas.

Another limitation was the modality of the course. The class met in person every Monday and met online on Wednesdays. The analyzed class observations were only collected during five days of class in a face-to-face modality. Thus, the online interactions were not included in this study but it would be interesting to check whether there is any difference in the ways women participate in class and check whether through an online setting women students feel more comfortable sharing their ideas.

Another factor that could determine the participation of students is the level of difficulty of the class's content. This course is an advanced class in the mathematics with teacher certification major, and thus, it may be different from classes they have had before. As our interviewed students talked about, they felt comfortable sharing their ideas when they felt confident about the way they understood the content. This could also be a determining factor that can discourage students from raising their hand or answering the instructor's questions.

Lastly, the genders for which EQUIP was analyzed were limited to my assumptions of the students' gender, instead of students' self-identified gender, so it could be possible that I did not consider students who do not identify as cisgender. If this were the case, then the distribution of participation among the two EQUIP dimensions would be different as I would have another gender to account for.

Directions for future research

Further research can be done to analyze how the distribution of participation opportunities was managed in the online classes compared to the already analyzed in-person classes to learn whether there were any changes amongst the participation of women and check if the participation was better managed in the online setting or if women felt more comfortable sharing their ideas in the online chat rather than sharing them verbally. It would also be important to ask every students their self-identified gender to make sure everyone is included and get accurate results on the EQUIP measurements. Also, more instructors could be observed to analyze their pedagogical moves and match them with the moves found in this study, and if there are additional moves that could help instructors to notice for equity. Furthermore, recruiting more students who are willing to share their experience could add meaning to the results found to check whether these pedagogical moves can foster changes in the students' confidence and sense of belonging and how is it that these moves encourage them to participate during class discussions.

Implications for instructors

An imperative change that can be done by instructors to possibly reduce the gender gap in participation opportunities would be to ask less for voluntary contributions, as most likely men students will be the ones dominating the participation. Instead, instructors can start calling on students by name. Although, calling on students on the spot can cause them to have anxiety, it would allow the instructor to keep track on who has shared ideas and who has not. By doing this, the instructors can distribute evenly the participation among the gender of the students in the course. To reduce the possibility of anxiety in students who are called on to participate, it is important to foster classrooms norms and attend to the productivity if the students' contributions to remind students that their classroom is a judgement-free space and making mistakes are learning opportunities for everyone. It is also recommended that the random selection for assigning group roles can be replaced with a manual selection of groups, tracking who has been the speaker to make sure everyone gets the chance to speak up when soliciting to share the group's ideas. We further suggest that instructors can implement Dr. Evans's pedagogical moves as ways to practice noticing for equity inside their mathematics classrooms.

REFERENCES

Adiredja, A. P., & Andrews-Larson, C. (2017). Taking the sociopolitical turn in postsecondary mathematics education research. *International Journal of Research in Undergraduate Mathematics Education*, *3*, 444–465.

Adiredja, A. P. (2019). Anti-deficit narratives: Engaging the politics of research on mathematical sense making. *Journal for Research in Mathematics Education*, *50*(4), 401-435.

Aguillon, S. M., Siegmund, G. F., Petipas, R. H., Drake, A. G., Cotner, S., & Ballen, C. J. (2020). Gender differences in student participation in an active-learning classroom. *CBE—Life Sciences Education*, *19*(2), ar12.

Ames, C. (1992). Classrooms: Goals, structures, and student motivation. *Journal of educational psychology*, *84(3),* 261.

Bartell, T., Wager, A., Edwards, A., Battey, D., Foote, M., & Spencer, J. (2017). Toward a framework for research linking equitable teaching with the standards for mathematical practice. *Journal for Research in Mathematics Education, 48*(1), 7-21.

Boaler, J. (2008). Promoting 'relational equity'and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal*, *34*(2), 167-194.

Boaler, J. (2013, March). Ability and mathematics: The mindset revolution that is reshaping education. Forum.

Burton, L. (2004). Confidence is everything"–Perspectives of teachers and students on learning mathematics. *Journal of Mathematics Teacher Education*, *7*, 357-381

Chao, T., Murray, E., & Gutiérrez, R. (2014). What are classroom practices that support equity-based mathematics teaching. *Research Brief*.

Copur-Gencturk, Y., Cimpian, J. R., Lubienski, S. T., & Thacker, I. (2020). Teachers' bias against the mathematical ability of female, Black, and Hispanic students. *Educational Researcher*, *49*(1), 30-43.

Darragh, L. (2013). Constructing confidence and identities of belonging in mathematics at the transition to secondary school. *Research in Mathematics Education*, *15*(3), 215-229.

Dunleavy, T. K. (2015). Delegating mathematical authority as a means to strive toward equity. *Journal of Urban Mathematics Education*, *8*(1).

Dweck, C. S. (2006). *Mindset: The new psychology of success*. Random house.

Ellis, J., Fosdick, B. K., & Rasmussen, C. (2016). Women 1.5 times more likely to leave STEM pipeline after calculus compared to men: Lack of mathematical confidence a potential culprit. *PloS one*, *11*(7), e0157447.

Esmonde, I. (2009). Ideas and identities: Supporting equity in cooperative mathematics learning. *Review of Educational Research*, *79*(2), 1008-1043.

Flores, A. (2007). Examining disparities in mathematics education: Achievement gap or opportunity gap?. *The High School Journal*, *91*(1), 29-42.

Floro, B., & Bostic, J. D. (2017). A case study of middle school teachers' noticing during modeling with mathematics tasks. *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks*, 73-89.

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the national academy of sciences*, *111*(23), 8410-8415.

Furner, J. M. (2017). Teachers and Counselors: Building Math Confidence in Schools. *European Journal of STEM Education*, *2*(2), 3.

Ganley, C. M., & Lubienski, S. T. (2016). Mathematics confidence, interest, and performance: Examining gender patterns and reciprocal relations. *Learning and Individual Differences*, *47*, 182-193.

Gehrtz, J., Brantner, M., & Andrews, T. C. (2022). How are undergraduate STEM instructors leveraging student thinking?. *International Journal of STEM Education*, *9*(1), 1-20

Good, C., Rattan, A., & Dweck, C. S. (2012). Why do women opt out? Sense of belonging and women's representation in mathematics. *Journal of personality and social psychology*, *102*(4), 700.

Gutbezahl, J. (1995). How Negative Expectancies and Attitudes Undermine Females' math confidence and Performance: A review of the Literature.

.Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: Toward a new equity research agenda. *Mathematical Thinking and Learning*, *4*(2-3), 145-187.

Gutiérrez, R. (2011). Context matters: How should we conceptualize equity in mathematics education?. In *Equity in discourse for mathematics education: Theories, practices, and policies* (pp. 17-33). Dordrecht: Springer Netherlands.

Hamm, J. V., & Faircloth, B. S. (2005). Peer context of mathematics classroom belonging in early adolescence. *The Journal of Early Adolescence, 25*(3), 345-366.

Hand, V. (2012). Seeing culture and power in mathematical learning: Toward a model of equitable instruction. *Educational studies in mathematics, 80*(1-2), 233-247.

Hand, V., Kirtley, K., & Matassa, M. (2015). Narrowing participation gaps. *The Mathematics Teacher, 109*(4), 262-268.

Hardy, T. (2008, March). Subjectivity and confidence in mathematics education. In *Symposium on the Occasion of the 100th Anniversary of ICMI, Rome, Italy. Anais... Rome. Disponible En: Http://Www. Unige. Ch/Math/Ensmath/Rome2008/Wg3/Wg3. Html.*

Harel, G. (2013). Intellectual need. In *Vital directions for mathematics education research* (pp. 119-151). New York, NY: Springer New York.

Herzig, A. H. (2010). Women belonging in the social worlds of graduate mathematics. The Mathematics Enthusiast, 7(2), 177-208.

Hunter, R., & Hunter, J. (2022). Developing Equitable Participation Structures. *Mathematics Education Research Group of Australasia*

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. Journal for research in mathematics education, 41(2), 169-202.

Jansen, A. (2020). Rough-draft thinking and revising in mathematics. *Mathematics Teacher: Learning and Teaching PK-12, 113*(12), e107-e110.

Johnson, E., Andrews-Larson, C., Keene, K., Melhuish, K., Keller, R., & Fortune, N. (2020). Inquiry and gender inequity in the undergraduate mathematics classroom. *Journal for Research in Mathematics Education*, *51*(4), 504-516.

Jong, C. (2017). Extending equitable practices in teacher noticing: Commentary. *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks*, 207-214.

Kallio, H., Pietilä, A. M., Johnson, M., & Kangasniemi, M. (2016). Systematic methodological review: developing a framework for a qualitative semi-structured interview guide. *Journal of advanced nursing*, *72*(12), 2954-2965.

Kilic, H. (2018). Pre-service mathematics teachers’ noticing skills and scaffolding practices. *International Journal of Science and Mathematics Education*, *16*, 377-400.

Krupa, E. E., Huey, M., Lesseig, K., Casey, S., & Monson, D. (2017). Investigating secondary preservice teacher noticing of students' mathematical thinking. *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks*, 49-72.

Lahdenperä, J., & Nieminen, J. H. (2020). How does a mathematician fit in? A mixed-methods analysis of university students' sense of belonging in mathematics. *International Journal of Research in Undergraduate Mathematics Education*, *6*(3), 475-494.

Laursen, S. L., Hassi, M. L., Kogan, M., & Weston, T. J. (2014). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study. Journal for Research in Mathematics Education, 45(4), 406-418.

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education*, *46*(1), 88-124.

Lee, M. Y., & Choy, B. H. (2017). Mathematical teacher noticing: The key to learning from lesson study. *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks*, 121-140.

Leedy, M. G., LaLonde, D., & Runk, K. (2003). Gender equity in mathematics: Beliefs of students, parents, and teachers. *School science and mathematics*, *103*(6), 285-292.

Lewis, K. L., Stout, J. G., Finkelstein, N. D., Pollock, S. J., Miyake, A., Cohen, G. L., & Ito, T. A. (2017). Fitting in to move forward: Belonging, gender, and persistence in the physical sciences, technology, engineering, and mathematics (pSTEM). *Psychology of Women Quarterly*, *41*(4), 420-436.

Leyva, L. A. (2017). Unpacking the male superiority myth and masculinization of mathematics at the intersections: A review of research on gender in mathematics education. *Journal for Research in Mathematics Education*, *48*(4), 397-433.

Louie, N. L. (2018). Culture and ideology in mathematics teacher noticing. *Educational studies in mathematics*, *97*, 55-69.

Lugosi, E., & Uribe, G. (2022). Active learning strategies with positive effects on students' achievements in undergraduate mathematics education. *International Journal of Mathematical Education in Science and Technology*, *53*(2), 403-424.

Lubienski, S. T. (2002). Research, reform, and equity in US mathematics education. *Mathematical thinking and learning*, *4*(2-3), 103-125.

Maries, A., Karim, N. I., & Singh, C. (2020). Active learning in an inequitable learning environment can increase the gender performance gap: The negative impact of stereotype threat. *The Physics Teacher*, *58*(6), 430-433.

Matthews, J. S. (2020). Formative learning experiences of urban mathematics teachers' and their role in classroom care practices and student belonging. *Urban Education, 55*(4), 507-541.

Roth McDuffie, A., Foote, M. Q., Bolson, C., Turner, E. E., Aguirre, J. M., Bartell, T. G., ... & Land, T. (2014). Using video analysis to support prospective K-8 teachers' noticing of students' multiple mathematical knowledge bases. *Journal of Mathematics Teacher Education*, *17*, 245-270.

Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). Qualitative data analysis: A methods sourcebook. 3rd.

Minner, D. D., Levy, A. J., & Century, J. (2010). Inquiry‐based science instruction—what is it and does it matter? Results from a research synthesis years 1984 to 2002. Journal of Research in Science Teaching: The Official Journal of the National Association for Research in Science Teaching, 47(4), 474-496.

Moschkovich, J. N. (2011). How equity concerns lead to attention to mathematical discourse. In *Equity in discourse for mathematics education: Theories, practices, and policies* (pp. 89-105). Dordrecht: Springer Netherlands.

Parks, A. N., & Schmeichel, M. (2012). Obstacles to addressing race and ethnicity in the mathematics education literature. *Journal for Research in Mathematics Education, 43*(3), 238-252.

Patterson Williams, A. D., Athanases, S. Z., Higgs, J., & Martinez, D. C. (2020). Developing an inner witness to notice for equity in the fleeting moments of talk for content learning. *Equity & Excellence in Education*, *53*(4), 504-517.

Peterson, B. E., Van Zoest, L. R., Rougée, A. O., Freeburn, B., Stockero, S. L., & Leatham, K. R. (2017). Beyond the "move": A scheme for coding teachers' responses to student mathematical thinking. In *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 17-24). PME.

Reinholz, D., Johnson, E., Andrews-Larson, C., Stone-Johnstone, A., Smith, J., Mullins, B., ... & Shah, N. (2022). When active learning is inequitable: Women's participation predicts gender inequities in mathematical performance. *Journal for Research in Mathematics Education*, *53*(3), 204-226.

Reinholz, D. L., & Shah, N. (2018). Equity analytics: A methodological approach for quantifying participation patterns in mathematics classroom discourse. *Journal for Research in Mathematics Education*, *49*(2), 140-177.

Reinholz, D. L., & Wilhelm, A. G. (2022). Race-gender D/discourses in mathematics education:(Re)-producing inequitable participation patterns across a diverse, instructionally-advanced urban district. *Urban Education*, 00420859221107614.

Rubel, L. H. (2017). Equity-directed instructional practices: Beyond the dominant perspective. *Journal of Urban Mathematics Education, 10*(2).

Schoenfeld, A. H. (2011). Toward professional development for teachers grounded in a theory of decision making. *Zdm*, *43*, 457-469.

Serbin, K. S., & Mullins, S. B. (2019). A gendered comparison of abstract algebra instructors' inquiry-oriented instruction. In A. Weinberg, D. Moore-Russo, H. Soto, & M.

Wawro (Eds.), *Proceedings of the 22nd Annual Conference on Research in Undergraduate Mathematics Education* (pp. 533-540). Oklahoma City, OK.

Serbin, K. S., Sara Brooke, M., Marilin Annie, K., & Johnson, E. (2020, December 23). Social norms conducive to women's learning in inquiry-oriented abstract algebra. Mathematics Education Across Cultures: *Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 42, 568-572.

Shah, N., & Lewis, C. M. (2019). Amplifying and attenuating inequity in collaborative learning: Toward an analytical framework. *Cognition and instruction*, *37*(4), 423-452.

Shah, N., Reinholz, D., Guzman, L., Bradfield, K., Beaudine, G., & Low, S. (2016). Equitable Participation in a Mathematics Classroom from a Quantitative Perspective. *North American Chapter of the International Group for the Psychology of Mathematics Education*.

Sherin, M. G. (2002). A balancing act: Developing a discourse community in a mathematics classroom. *Journal of mathematics teacher education*, *5*, 205-233.

Sherin, M. G., Russ, R. S., & Colestock, A. A. (2011). Accessing mathematics teachers' in-the-moment noticing. *Mathematics teacher noticing: Seeing through teachers' eyes*, *79*.

Sparks, D. M., & Pole, K. (2019). "Do we teach subjects or students?" Analyzing science and mathematics teacher conversations about issues of equity in the classroom. *School Science and Mathematics*, *119(*7), 405-416.

Stahnke, R., Schueler, S., & Roesken-Winter, B. (2016). Teachers' perception, interpretation, and decision-making: a systematic review of empirical mathematics education research. *Zdm*, *48*, 1-27.

Sun, K. L. (2018). Brief report: The role of mathematics teaching in fostering student growth mindset. *Journal for Research in Mathematics Education*, *49*(3), 330-335.

Teuscher, D., Leatham, K. R., & Peterson, B. E. (2017). From a framework to a lens: Learning to notice student mathematical thinking. *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks*, 31-48.

Theobald, E. J., Hill, M. J., Tran, E., Agrawal, S., Arroyo, E. N., Behling, S., ... & Freeman, S. (2020). Active learning narrows achievement gaps for underrepresented students in undergraduate science, technology, engineering, and math. *Proceedings of the National Academy of Sciences*, *117*(12), 6476-6483.

van Es, E. A., Hand, V., & Mercado, J. (2017). Making visible the relationship between teachers' noticing for equity and equitable teaching practice. *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks*, 251-270.

van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers'"learning to notice" in the context of a video club. *Teaching and teacher education*, *24*(2), 244-276.

Yolcu, A. (2019). Research on equitable mathematics teaching practices: Insights into its divergences and convergences. Review of Education, 7(3), 701-730.

Zander L, Höhne E, Harms S,Pfost M and Hornsey MJ (2020)When Grades Are High butSelf-Efficacy Is Low: Unpackingthe Confidence Gap Between Girls and Boys in Mathematics.Front. *Psychol*. 11:552355.doi: 10.3389/fpsyg.2020.552355.

APPENDIX

Table 9: Semi-structured Interview Protocol for First Interview with the student participants.

Semi-structured Interview Protocol - First interview

Previous experience – *This interview will take place at the beginning of the semester.*

- Tell me a little bit about yourself (what semester you're in, concentration, teaching experience?)
- How has been your experience as a mathematics student?
- Why did you choose to get a master's degree in mathematics?, *etcetera.*

Active learning

- Regarding teaching methods, do you prefer traditional lecturing or an active learning classroom?
- What do you think about active learning? Have you been in classes where there is more collaboration and class discussions?

Collaborative working

- Do you like working collaboratively? Why or why not?
- When you've worked in groups, do you feel your contributions are valued by your peers? Why or why not?
- Have you experienced online collaborative working? Do you think collaboration comes easier in an online modality or do you prefer being in-person?
- What is the usual dynamic you have experienced when working in groups? What role do you typically take?
- Do you think this experience has changed from being an undergraduate to now being a graduate student? Why do you think that happens?

Inequities during collaborative working

- Have you experienced or seen any gender inequity during group work? (i.e. men are the ones doing the math and women writes it down or women's opinion not being considered)

Class discourse/discussions

- Do you voluntarily participate in class discussions and group work or do you prefer not to participate?
- How do you feel about participating in class discussions?
- What are your usual thoughts when an instructor calls on you to participate?
- What is it that makes you nervous/comfortable about participating?
- What do you think could contribute to you feeling comfortable if your instructor calls on you to participate?

Inequities during classroom discourse

- Have you ever felt there were inequities during classroom participation and group work during your previous classes? (i.e. the teacher favored certain students when calling on for contributions or a person who dominated the discussions when working in groups)
- When sharing your ideas with your previous instructors, have you ever felt you were misunderstood or ignored?
- What are your thoughts on mathematics being considered a field for white men?
- Do you think men often dominate class discourses?
- From your experience, do you think male instructors favor male students during participation?
- Question about women in mathematics

Math confidence

- What do you think determines your confidence in doing mathematics? (i.e. knowledge, experience, nervousness, anxiety.)
- How confident do you feel about your contributions?
- Do you think your teacher has a role in making you feel confident about your math?
- How do you think having strong mathematical confidence would benefit the way you learn mathematics?

Sense of belonging

- In your Sense of Belonging scale, you answered you felt/did not feel part of the mathematics community. How do you think your previous class experiences have determined the way you responded to the scale?
- Do you think your teacher has a role in making you feel a sense of belonging in mathematics?
- How important it is for you to have this sense that you belong in math and your contributions are accepted and valued?
- Do you think your sense of belonging could determine the way to perform in your class?

Table 10: Semi-structured Interview Protocol of Second Interview with the student participants.

Semi-structured interview protocol – Second Interview

Course experience – This interview will take place at the end of the semester.

- How has been your experience during this course?
- What was something you liked about the structure of the class?
- How did the social norms that were set at the beginning of class help to make the classroom more equitable and inclusive?
- Do you think the instructor did a good job in reinforcing/reminding students about those social norms?

Active learning

- Do you think you learn better during active learning courses than traditional lecturing?
- Do you think working collaboratively and having class discussions benefit you in getting a better understanding of the class content? Could you explain why or why not?

Working collaboratively

- Did you feel comfortable participating in group work?
- What was the usual dynamic when you were put into groups

Inequities during group work

- Did someone take over the leader role or was the work distributed equally among participants?
- If there was a person that would lead the group, what determined this leadership?
- Do you think the instructor attended well group work interactions in a way that everyone was elicited to contribute?

Class discourse/discussions

- Did you feel comfortable participating during class discussions?
- Did the way in which your instructor solicited your participation made you feel comfortable o did it make you anxious?
- What do you think you could have done better to feel comfortable when partipating in class?
- How did the way the instructor managed class discussions help you ….
- What do you think the instructor could improve about class discussions?

Equity / Inequities during discussions

- Do you think your teacher did a good job in attending to your mathematical thinking/contributions?
- Did the way the instructor responded to your contributions made you feel your contributions were valued and accepted? Why is that?
- Do you think the opportunities to participate during class discussion were equitably distributed among the students / everyone got the same opportunity to participate?

Math Confidence

- Your Math Confidence questionnaire shows you improved/kept/lowered your mathematical confidence across this semester, why do you think that happened?
- How confident did you feel about the contributions you made during class discussions and group work?
- What do you think determines this confidence?
- How confident do you feel about the content you learned in class?
- How do you think the instructor helped in developing / increasing your mathematical confidence?
- Did your peers also had some impact in you feeling more confident? Why or why not?
- If you compare traditional lecture course with active learning course, which one do you think fosters more students' mathematical confidence?

Sense of Belonging

- Your Sense of Belonging scale, shows you improved/kept/lowered your sense of belonging in math across this semester, why do you think that happened?
- What do you think your instructor did to help improve your sense of belonging in math?
- How important it is for you to have this sense that you belong in math and feeling your contributions are accepted and valued?
- What actions did your instructor do that helped you develop your sense of belonging in math?
- Do you think your sense of belonging contributed in the way to performed in this class?
- Did the interaction with your peers helped you to increase your sense of belonging in math?
- Do you think traditional lecturing fosters students' sense of belonging?
- Do you think active learning fosters students' sense of belonging?
- What factors in an active learning course would make you feel you do not belong?

Wrap up questions

- Do you think your teacher has a role in making you confident and feeling you belong in the mathematics community?
- What recommendations would you give your instructor about her attending to equity for future courses?

Key Questions

The following questions are a subset of the questions in the student questionnaires, which corresponds to the results being presented in this paper. The original questionnaires contained approximately 50 questions in total.

1. Given a choice would you have chosen to study this mathematics module?

Yes ☐ No ☐

How confident would you describe yourself overall?
Please tick one box per question

	Very confident				Not confident
2. in mathematics?	☐	☐	☐	☐	☐
3. in statistics?	☐	☐	☐	☐	☐
4. in life in general?	☐	☐	☐	☐	☐

5. For how long have you held this opinion of your self-confidence in mathematics?

6. How do you think that your experiences of mathematics *before* coming to university have affected your confidence or liking of the subject?
(Please describe your experiences if possible)

7. Has this module helped you to feel more confident than previously?

More confident				Less confident
☐	☐	☐	☐	☐

Do you like the subject?

	Really Like				Detest
8. Like Mathematics?	☐	☐	☐	☐	☐
9. Like Statistics?	☐	☐	☐	☐	☐

10. Has this module helped you to *like* the subject more?

More				Less
☐	☐	☐	☐	☐

11. How would you describe your *attitude* to learning mathematics?

12. How would you rate your *motivation* in this area?

Really motivated				Not motivated
☐	☐	☐	☐	☐

13. Is this more or less motivation than for your other modules overall?

More ☐ Less ☐ The same ☐

14. How much time have you spent outside lectures working on this module on average in hours per week? (tick one box)

0 ☐ 1 hour ☐ 2 hours ☐ 3 hours ☐ 4+ hours ☐

Figure 11: Parsons et al. (2016) confidence questionnaire.

Math Sense of Belonging Scale

Today we have some questions we would like you to answer about your experience with math courses and in the math academic community. When we mention the math academic community, we are referring to the broad group of people involved in that field, including the students in a math course.

We would like you to consider your membership in the math community. By virtue of having taken many math courses, both in high school and/or at _________, you could consider yourself a member of the mathematics community. Given this broad definition of belonging to the math community, please respond to the following statements based on how you feel about that group and your membership in it.

There are no right or wrong answers to any of these statements; we are interested in your honest reactions and opinions. Please read each statement carefully, and indicate the number that reflects your degree of agreement.

Strongly Disagree					Strongly Agree		
1	2	3	4	5	6	7	8

When I am in a math setting,

1. I feel that I belong to the math community.
2. I consider myself a member of the math world.
3. I feel like I am part of the math community.
4. I feel a connection with the math community.
5. I feel like an outsider.
6. I feel accepted.
7. I feel respected.
8. I feel disregarded.
9. I feel valued.
10. I feel neglected.
11. I feel appreciated.
12. I feel excluded.
13. I feel like I fit in.
14. I feel insignificant.
15. I feel at ease.
16. I feel anxious.
17. I feel comfortable.
18. I feel tense.
19. I feel nervous.
20. I feel content.
21. I feel calm.
22. I feel inadequate.
23. I wish I could fade into the background and not be noticed.
24. I try to say as little as possible.
25. I enjoy being an active participant.
26. I wish I were invisible.
27. I trust the testing materials to be unbiased.
28. I have trust that I do not have to constantly prove myself.
29. I trust my instructors to be committed to helping me learn.
30. Even when I do poorly, I trust my instructors to have faith in my potential.

Figure 12: Sense of belonging scale by Good et al. (2012).

BIOGRAPHICAL SKETCH

Sthefania Espinosa was born in Tamaulipas, Mexico. She earned her Bachelor's Electrical Engineering degree in December 2021 and her Master's degree in December, 2023; both from the University of Texas Rio Grande Valley. You can contact her here: sthefaniaesp@gmail.com.