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The Effects of Using Dynamic Geometry Software While Exploring the Properties of Quadrilaterals

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THE EFFECTS OF USING DYNAMIC GEOMETRY SOFTWARE WHILE EXPLORING THE PROPERTIES OF QUADRILATERALS

A Dissertation

by

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ABSTRACT

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The purpose of my study was to determine the effectiveness of using dynamic geometry software while exploring the properties of quadrilaterals. Dynamic geometry software can be effective if used properly in the exploration process as students construct their own knowledge through constructivism. Students were given the Van Hiele Test as a pretest to determine their van Hiele level prior to taking a high school geometry course. The results determined that the majority of the students were not at the level needed for the course, which is typically taught at a van Hiele level 3. Students in the treatment group used GeoGebra while exploring the properties of quadrilaterals using a framework in which students are put at the center of their learning and the teacher acts more like a facilitator rather than being the primary source of information. Students were given a unit assessment after completing the unit and the Van Hiele Test was administered again as a posttest during the last month of the course. The results showed that students in the treatment group performed better as a whole in the unit assessment and had more students advance to the next van Hiele level, however, it is possible that they may continue to struggle with future assessments that contain a geometry section. The perceptions survey showed that students had a positive experience during this process.

DEDICATION

The completion of my doctoral studies would not have been possible without the support of my family. My mother, Raquel Jaramillo, my father, Francisco J. Jaramillo, my brothers, Mario A. Jaramillo, Josue J. Jaramillo, Francisco I. Jaramillo, and my sisters, Yvette Davis, Juliana Jaramillo, and Irene Jaramillo. Thank you for all your support and motivation.

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I would also like to thank my colleagues who have helped me throughout my courses and have given me advice and direction as I went through this process. We have been through some tough challenges but by the conversations we’ve had, it has helped me tremendously in completing the program.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

At the beginning of my teaching career, my teaching style mirrored a style based on past experiences which could be classified as *traditional.* I was accustomed to this style of teaching as I progressed through the grade levels and even up to college. A *traditional* style could be described as teacher-centered (Mthethwa et al., 2020; Sherman, 2014), students seating in straight rows (Birgin & Uzun Yazıcı, 2021), teacher lecture (Dietiker & Richman, 2021), students memorizing and recalling the information presented (Dogruer & Akyuz, 2020; Ganesan & Eu, 2020; Mthethwa et al., 2020) such as the theorems presented in geometric concepts, and students were then assessed to determine conceptual understanding through a multiple-choice and/or open-ended assessment. This type of style would often lack in providing students with opportunities to discuss the material to obtain a deeper understanding of the concepts (Ganesan & Eu, 2020).

I adjusted my teaching practices by attending various professional development sessions that included more opportunities for discussion amongst the students. To create an atmosphere that promotes student engagement through discussions, I transitioned from a *traditional* style to a student-centered (Sherman 2014;), inquiry-based environment (Dietiker & Richman, 2021; Kondratieva & Bergsten, 2021;) which allowed for more student involvement while challenging their problem-solving skills and conceptual understanding. Uwurukundo et. al. (2022) concluded in their study that students who were taught using GeoGebra, had higher conceptual understanding than those taught using traditional methods. Therefore, it would be beneficial for my students if the lessons incorporated GeoGebra as a learning tool.

As technology improved, so did my teaching strategies. My students were often engaged in student-centered group activities which allowed for collaboration and discussions amongst their classmates. My coworkers and I collaborated and shared strategies to improve the curriculum to effectively deliver content based on past experiences acquired through professional development, and best practices from past mentors or coworkers. On several occasions, strategies on how to effectively use the calculator to enhance student learning when exploring concepts, were presented by either a coworker or a specialist. The calculator was and continues to be a primary tool used for calculations, analyzing graphs, and various other functions, however, its effectiveness in developing conceptual mathematical understanding is based on the teacher's experience and knowledge and how they incorporated it into their lessons (Richardson, 2014), therefore, I collaborated with my colleagues to increase our technological skills with the calculator. We were often modifying the materials for the curriculum as we improved those skills, to include various calculator activities that promote exploration through the integration and use of the calculator.

During the pandemic, our students had limited, if any, access to calculators due to the school's supply, which meant that students and teachers had to find alternatives for students who did not have access to one. I started integrating the use of the dynamic geometry software (DGS), GeoGebra, which is a downloadable software that could easily be accessible through smartphones, tablets, and computers, and could also be used without internet access. I had learned about the existence of GeoGebra through professional development, however, because students had access to calculators, I focused mainly on the calculator's functions. Due to the pandemic, I revisited GeoGebra and it was the main source for my instruction throughout the remote instruction period. Students had access to GeoGebra's online scientific and Computer Algebra System (CAS) calculators, which could perform the calculations needed for their mathematics courses and is the reason why I incorporated it more into my lessons.

Steffen and Winsor (2021) found that teachers were able to determine that students struggled with proofs due to never truly understanding the properties of quadrilaterals and were motivated to discover new methods and strategies to help the students better understand these properties using DGS. Although the study used preconstructed dynamic shapes on GeoGebra, Steffen and Winsor's data did support the fact that their students' knowledge of quadrilaterals increased. My study focused on students having to construct the figures, providing additional insight on the use of DGS while exploring the properties of quadrilaterals. GeoGebra contains functions that help in the visualization of geometric concepts which will be further described in the proceeding sections.

Context of the Study

During the pandemic, most school districts notified their teachers that they would not be returning to in-person instruction after Spring Break 2020 due to the spread of the COVID-19 virus. It was during this time that my fellow coworkers and I started to realize some of our technological deficiencies, challenging our skills required for remote instruction. We had to adjust our curriculums due to the resources that were available during this time and had to adjust how we communicated and supported our students. With the guidance and support from our administrators, we were able to overcome some of those challenges in preparation for the 2020-

2021 school year by completing online professional development courses that guided us on how to navigate through these online applications effectively.

As we transitioned from in-person to remote learning, my colleagues and I had to enhance our technological skills to work with applications such as Google Classroom, Google Meets, Google Docs, Google Slides, Kami, Jamboard, Pear Deck, and dynamic geometry software such as Desmos and GeoGebra, used to increase student engagement and deliver quality instruction remotely to our students. Before the pandemic, it was easier to monitor student engagement through in-person instruction (Cai et al, 2020), which is why I selected applications that would be interactive while allowing multiple students participate in the activities and seeing them engage with one another during the challenges of the pandemic. For example, the breakout rooms and polls tools in Google Meets, the collaboration tools embedded in Google Docs, Google Slides, and Jamboard allowed students to work simultaneously on a shared document, and the interaction tools in Pear Deck which allowed for Questions and Answers while presenting the content material. The tools embedded in the applications provided me with alternative methods to check and assess for conceptual understanding while promoting conversations amongst the students while remote instruction was being delivered.

My students faced many challenges during remote instruction due to the pandemic. Depending on their knowledge of the different types of technologies they had available to them, they experienced different levels of challenges since many of them were not accustomed to using online applications. However, as their knowledge on the technologies being used increased, they explored other online resources even further, including those with an embedded calculating or graphing tool such as Desmos or GeoGebra. There were students who often used online resources to support their learning, while others were not aware or did not know how to properly use them (Hoyos, 2021). Another challenge my students faced was related to the internet availability. Depending on how many users were connected or the strength of the signal, they were sometimes disconnected from the Google Meets. It was important that students had access to the resources needed to ensure conceptual understanding with an offline option in case internet connection became an issue.

When exploring geometric concepts through remote instruction during the pandemic, as I was accustomed to using manipulatives during the learning process for conceptual understanding, I sought applications that would provide a similar learning experience as if the student was in the classroom. Having a physical object to observe and manipulate provided a visual aid for my students who struggled to visualize the object when one was not provided. For example, before the pandemic, I would have my students construct quadrilaterals using a protractor and graph paper, making it easier for them to measure the angles and segments to make that connection regarding the measurement of opposite angles and sides. A faster, similar process was used using DGS, helping with the time constraints we had. Although the goal was for students to have a strong abstract understanding of symbolic mathematics, manipulatives can aid those students who struggle with symbolic representations (Cooper, 2012). The use of physical manipulatives was rarely possible during the pandemic, therefore, applications that offered virtual manipulatives had to be integrated such as GeoGebra's access to virtual manipulatives that helped with visualization of the properties of quadrilaterals. Overall, GeoGebra contains many features that teachers can implement in their lessons or activities to help minimize some of the challenges we had during the pandemic and can be further used in future mathematics courses.

The Texas Education Agency (TEA), has the Texas Essential Knowledge and Skills, TEKS, for high school geometry courses. The TEKS contain the concepts need to be covered for students to receive credit for the course. Based on these TEKS, a student is expected to investigate patterns to make conjectures including those that pertain to the diagonals of quadrilaterals and the interior and exterior angles of polygons (TEA, 2012). Students are also expected to prove that a quadrilateral is a rectangle, square, or rhombus by using the opposite sides, opposite angles, or diagonals, as well as apply these relationships to solve problems (TEA, 2012). The theorems in Table 1 are presented in the Texas Edition of the Geometry textbook (Pearson, 2016).

Table 1: Unit 6 Theorems

Theorem	If a quadrilateral is a parallelogram,
6-3	then its opposite sides are congruent.
6-4	then its consecutive angles are supplementary.
6-5	then its opposite angles are congruent.
6-6	then its diagonals bisect each other.
Theorem	If a parallelogram is a rhombus,
6-13	then its diagonals are perpendicular.
6-14	then each diagonal bisects a pair of opposite angles.
Theorem 6-15	If a parallelogram is a rectangle, then its diagonals are congruent.
Theorem 6-19	If a quadrilateral is a parallelogram with perpendicular, congruent diagonals, then the quadrilateral is a square.
Theorem 6-20	If a quadrilateral is an isosceles trapezoid, then each pair of base angles is congruent.
Theorem 6-21	If a quadrilateral is an isosceles trapezoid, then its diagonals are congruent.

The Pearson (2016) geometry textbook was adopted by the school district for all high school campuses. The structure of the lessons for each unit scaffold concepts from previous sections, allowing students to build on their skills as each concept is introduced.

When the theorems were presented to the students, I would have them write them down and they would apply them to answer questions on their assignment without providing them with an opportunity to explore the properties of the quadrilaterals for themselves. I found that my students often struggled when they were assessed in applying the theorems due to them not being able to memorize the properties for the quadrilaterals, which is a similar experience shared by Steffen and Winsor (2021) and states that teachers must adjust their approach to help students with their conceptual understanding to better comprehend the properties of these quadrilaterals. It is because of this that I incorporated DGS, GeoGebra, to help with the exploration process. Well-structured DGS activities provide students with opportunities to work at their own pace and level of geometric thinking (Steffen & Winsor, 2021). Alternatives to memorizing the theorems needed to be explored, such as the implementation of DGS, to determine the best strategy for our students and their comprehension of these concepts. Even after the pandemic, few teachers incorporated DGS into their curriculums, but are now doing so after seeing the advantages of their students using it to prepare for state assessments. Although geometry concepts are not tested in the Algebra 1 End-Of-Course state assessment exam, which is what high schools in Texas are graded on, national assessments provided sufficient data to determine if students were understanding the geometric concepts taught at the high school level.

The National Assessment of Educational Progress (NAEP, 2022), assess what our nation's students know and can do in mathematics, science, reading, and writing and have been doing so since 1969. "NAEP is a congressional mandated project administered by the National

Center of Education Statistics (NCES), within the U.S. Department of Education and the Institute of Education Sciences (IES)" (NAEP, 2022). Educators can use the results to determine progress and find ways to improve education. The mathematics assessment "measures students' knowledge and skills in mathematics and their ability to solve problems in mathematical and real-world contexts" (NAEP, 2022). Grade 12 assessment content areas include:

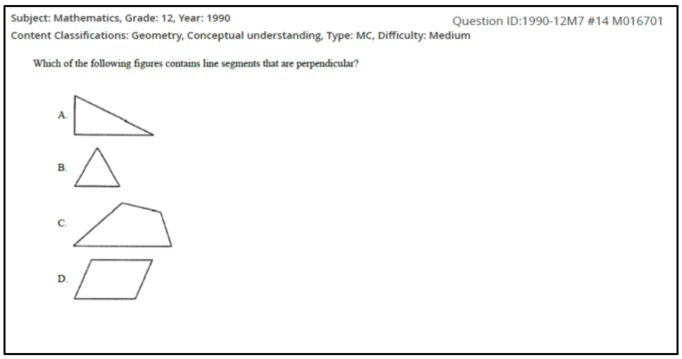
- number properties and operations - measures whether a student can represent, calculate, and estimate with numbers.
- measurement and geometry - use measurement tools to measure and apply a process to solve problems, as well as understand the relationships between two- and three-dimensional shapes.
- data analysis, statistics and probability - measures the understanding of data representation, characteristics of data sets, probability, and experiments.
- algebra - measures the understanding of patterns, using variables, algebraic representation, relationships, and functions.

Each section has questions with a different level of complexity:

- Low-complexity - questions specify what to do, which usually involves carrying out a routine mathematical procedure.
- Moderate-complexity - questions often require multiple step responses.
- High-complexity - often require abstract reasoning.

The analysis of questions provided by NAEP (2022) pertain to properties of polygons, including quadrilaterals. Figure 1 below shows that students are not performing at the level expected in the understanding of figures and their properties. To answer the question stated in Figure 1 correctly, students need to know the properties of polygons, including those of a quadrilateral. Answer choice A contains a right triangle formed by two sides perpendicular to each other. Answer choice B and D contain an altitude perpendicular to the base that can be formed when calculating the area, however, the shape itself is not formed by using line segments perpendicular to each other.

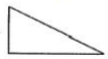
Figure 1: NAEP (2022), Mathematics, Grade: 12, Year: 1990



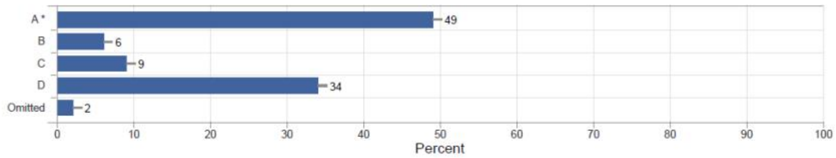
This may be the reason why the second highest choice selected was D, as seen below in Figure 2. Although 49% of the students selected the correct response, 34% selected the figure that resembles a parallelogram. Perpendicular line segments are characteristics of certain quadrilaterals such as the rectangle and square and are also discussed in the properties for a rhombus, where the intersection diagonals are perpendicular to each other. It is possible that students may have made a connection with the concept of area, in which the altitude, perpendicular to the base, is needed to correctly calculate the area of the figure.

Figure 2: Item Analysis for Figure 1

The correct answer is:

A. 

Percentages for grade 12 mathematics, by Find Perpendicular Line Segments in a Figure [M016701] and jurisdiction: 1990
1990¹, National



NOTE: Detail may not sum to totals because of rounding. Some apparent differences between estimates may not be statistically significant.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1990 Mathematics Assessment.

Figure 3 asks students to construct a parallelogram with specific properties using a ruler. Only 19% of the students were able to properly construct a parallelogram that had perpendicular diagonals, which are properties of a rhombus or square.

Figure 3: NAEP (2022), Mathematics, Grade: 12, Year: 1996

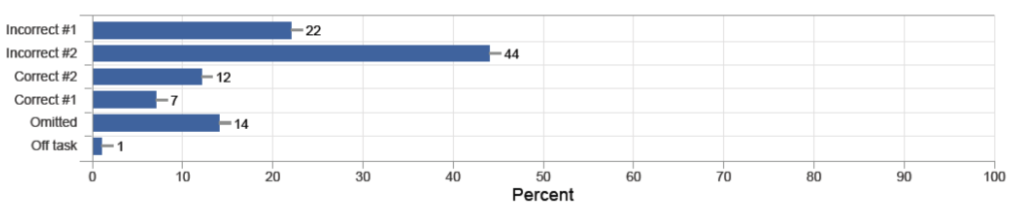
Subject: Mathematics, Grade: 12, Year: 1996

Question ID:1996-12M13 #2 M060001

Content Classifications: Geometry, Procedural knowledge, Type: SCR, Difficulty: Hard

In the space below, use your ruler to draw a parallelogram that has perpendicular diagonals. Show the diagonals in your sketch.

Percentages for grade 12 mathematics, by Draw a Parallelogram With Perpendicular Diagonals [M060001] and jurisdiction: 1996
1996¹, National



NOTE: Detail may not sum to totals because of rounding. Some apparent differences between estimates may not be statistically significant.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

Based on the data provided by NAEP (2022), students continue to struggle with questions pertaining to properties of polygons including quadrilaterals. Geometry is a subject tested in the Scholastic Aptitude Test (SAT) and the American College Testing (ACT). It is imperative that students fully understand geometric concepts including the properties of quadrilaterals, which are then used in future geometry concepts.

Students need to have a strong understanding of the properties of each concept to determine how to properly set up and solve the multi-step linear equation to determine the value of the variable, the length of the side, or the measure of the angle. Before taking the geometry course at the high school level, students must have a thorough understanding of basic concepts that are introduced throughout the curriculum based on previous classes taken during their elementary and middle school grades. My study first determined a student's current level of understanding based on the Van Hiele Test, interpreted through the van Hiele Model (Usiskin, 1982). Crowley (1987) believed that teachers could use the van Hiele Model of geometric thought to guide their instruction and how they assess the students and mentions that if students are taught appropriately based on the van Hiele Model, the students can sequentially progress through the different levels starting with the basic level, to the most rigorous level.

The van Hiele levels of thinking are not age dependent (Herbst et al., 2017). Despite their experiences of learning geometric concepts at the elementary and middle school levels, it is possible that some students will remain at lower levels of thinking (Herbst et al., 2017). Prior to attending high school, students must have completed at least up to Level 2 to understand the concepts introduced (Özçakir et al., 2020). Knowing the student's level helps with developing suitable learning activities, materials, and instructions, to provide them with learning environments that will help them advance through the levels. Students taking high school geometry courses are usually taught at a Level 3 (Crowley, 1987).

With the current advancement of technology, it is imperative that teachers increase their technological skills and implement them in their face-to-face instruction to prepare students for the post-pandemic future in the classroom especially if they need assistance on how to use technology to enhance learning (Edwards & Robichaux-Davis, 2020). Technology is changing the world and learning to use it will ensure success (Bransford et al., 2007), especially in the field of education. When teaching and learning mathematics, technology is essential because it enhances student learning and influences the mathematical content taught in the classroom (National Council of Teachers of Mathematics [NCTM], 2000).

There are certain skills students need become familiar with throughout their secondary and post-secondary experience which will prepare them for future careers such as collaboration, critical thinking, and exploration, which can be obtained through the effective use of technology in the classroom. Therefore, it is important that schools make the current technology available to the students (Darling-Hammond et al., 2007) giving them more opportunities to engage in abstract activities due to the capabilities of a computer being able to perform multiple actions so easily (Hollerbrands, 2007). Tasks, such as creating multiple versions of a polygon, that can easily be completed with the use of technology, rather than with the traditional pencil and paper.

The pandemic highlighted many challenges for our school district such as the accessibility of technology for our students. The internet provided access to a vast amount of information through a simple search through a phone, tablet, or computer. Students also obtained solutions to mathematical problems by understanding the capabilities of the calculator or applications and had access to dynamic geometry software such as GeoGebra, which allowed for more student-centered activities. If used properly, the use of technological tools such as graphing calculators and DGS can deepen a students' mathematical content knowledge and support mathematical thinking (Sherman, 2014). Based on their study, Uwurukundo et. al. (2022) concluded that the students who learned with GeoGebra succeeded more than their counterparts who did not learn with the software. Having a better understanding on how to effectively use technology in the classroom to practice skills will help students in their future mathematical courses. My study provided additional data to determine the effectiveness of GeoGebra in the context of working with the properties of quadrilaterals, a concept that is tested on standardized tests. It provided those teachers who have not used, or have rarely used, DGS in their lessons an insight of its capabilities as well as students' perceptions while working with GeoGebra.

Purpose of the Study

The purpose of the study was to determine the effectiveness of GeoGebra in the development of conceptual understanding while exploring the properties of quadrilaterals and whether the knowledge was obtained for future reference. GeoGebra's dynamic feature allows for the exploration of these properties which include the interior angles, the lengths of the sides, and diagonals which can be explored through the dragging and slider features leading to open discussions amongst the students in the class. Students can modify the quadrilateral by dragging a vertex or a side and are able to see the effects in real time, therefore making it easier for them to test and verify their conjectures, a conclusion reached based on a pattern of specific examples or past events (Pearson, 2016). For example, when students explore the interior angles of a quadrilateral, they can make a conjecture on opposite angles based on their observations, modify the quadrilateral to test their conjecture, and eventually verify their conjecture.

GeoGebra contains many functions such as *Point*, *Segment*, *Line, Polygon*, *Perpendicular Line*, *Parallel Line*, *Angle*, *Distance or Length*, *Perpendicular Bisector,* and *Angle Bisector* which students can use for visualization, measurement, and construction of multiple quadrilaterals such as a parallelogram, rhombus, square, rectangle, trapezoid, and kite faster than the traditional ruler, protractor, paper, and pencil method. Rather than giving the students the theorems associated with these shapes to be memorized, students can use GeoGebra to explore a quadrilateral's properties and make their own conjectures based on their interaction with the GeoGebra functions.

Prior to the pandemic, the calculator was the main source of technology used in my classroom. After the pandemic, GeoGebra provided a tool for my students that could be used outside of a classroom setting in case a calculator was not easily accessible. My focus is on GeoGebra and the questions that drove this study include the following:

1. How effective is GeoGebra when exploring the properties of quadrilaterals in conceptual understanding?
2. Based on the van Hiele levels of thinking, to what extent do the student's level of geometric thinking about the properties of quadrilaterals change after the implementation of GeoGebra?
3. What are the students' perception in the use and integration of GeoGebra while learning about geometric concepts, in particular the ones related to quadrilaterals?

Justification for the Study

Studies in geometry and DGS have mainly included area (Dogruer & Akyuz, 2020; Peterson, 2022), volume (Lo & White, 2020), triangle properties (Adelabu et al., 2019; Contreras, 2022; Dietiker & Richman, 2021; Quinn, 2020;), transformations (Chamberlin & Powers, 2022; Hollerbrands, 2007; Özgün-Koca & Enlow, 2020; Wasserman et al., 2020), upper-level coursework (Armstrong & McQuillan, 2020; Wasserman & McGuffey, 2021), circle properties (Ganesan & Eu, 2020; Mthethwa et al., 2020;), and functions (Bailey et al., 2021; Belnap & Parrott, 2020; Birgin & Uzun Yazıcı, 2021; Bülbül et al., 2020; Edwards et al., 2021; Fung & Poon, 2021; Kondratieva & Bergsten, 2021; McCulloch et al., 2021; White, 2020). However, there is a gap in the literature focusing on the properties of quadrilaterals such as a rhombus, square, rectangle, parallelogram, trapezoid, and kite. Steffen and Winsor (2021) focused on preconstructed quadrilaterals, however, this study had students construct their own quadrilaterals using the tools provided by DGS. My study provided insight of the effectiveness of using GeoGebra while exploring the properties of quadrilaterals, explored students' perceptions of GeoGebra, and determined their level of geometric thinking based on the van Hiele levels of thinking. Rather than just memorizing the properties for the assessment, students needed to fully understand them for future reference, including standardized testing. Having a strong foundation of the properties of various figures, including quadrilaterals, prepares students when exploring future concepts such as area and volume.

Delimitations

My study was limited to geometry courses at a specific high school, within a specific school district. The population is predominantly Hispanic, and the classes consisted of about 25 to 30 students. The students were grouped in either a College Prep (CP), or Honors course. My study was conducted using two classes labeled as Honors courses.

Summary

My journey has taken me where I am today. After 20 years of teaching, how I approach the geometric concepts has been modified throughout the years and technology has played a key role in that process. Teachers can educate students on how to properly and effectively use technology to increase their conceptual knowledge and take charge of their own learning process. The open discussions and collaboration can help them acquire those skills needed for their future, while utilizing various forms of technology that they themselves may be working with later in their careers such as the online video conferencing tools. The offline features that GeoGebra offers also benefits those students who do not have internet access, or their connection is limited.

The students of the future will be working with various forms of technology. It is important that we integrate technology to transform our classrooms into a student-centered environment which promotes discovery (Bülbül et al., 2020; Mthethwa et al., 2020). In this environment, the teacher's role is to guide the students through proper questioning, motivate them to explore (Birgin & Uzun Yazıcı, 2021; Fung & Poon, 2021), and create an environment the promotes discovery (Birgin & Uzun Yazıcı, 2021). Technology is effective when students are engaged in the activity that is authentic (Belnap & Parrott, 2020), therefore, the teacher would need to strategically embed it into the curriculum. GeoGebra is an example of this type of technology. With its many functions, teachers can create material the promotes exploration and discovery while at the same time challenging students to make and test conjectures within their classes and modifying them through open discussions. It has helped my students in their conceptual understanding of the properties of quadrilaterals.

CHAPTER II

LITERATURE REVIEW AND THEORETICAL PERSPECTIVE

This section provides information on GeoGebra and its many functions that help students with measurement, constructions, visualization, and discussions while learning geometric concepts. The literature review shows that GeoGebra can be an effective tool, however, there are few studies that focus on the exploration of the properties of quadrilaterals. Using GeoGebra to explore these properties, provides an environment in which students can construct their own knowledge based on their experiences, constructivism. Additional information is provided in the paragraphs that follow.

Technology has played a key role in mathematics especially during recent years due to the COVID-19 pandemic. Teachers sought various methods to remotely deliver the content and students were challenged with the task of becoming familiar with the different applications that were being used to receive that content from their teachers. The National Council of Teachers of Mathematics, NCTM, (2000) have emphasized the importance of technology in the teaching and learning of mathematics and how it enhances student learning. Properly utilizing technology provides students with an opportunity to focus more on decision making, reflection, reasoning, and problem solving (NCTM, 2000). Technology's ability to perform many tasks such as calculations quickly and accurately, allows for more time for conceptualizing and modeling (NCTM, 2000). Geometry, as described in Herbst et al. (2017), provides students with the means to interact with the experimental world through observation, measurement, and manipulation. I focused on the use of technology while exploring geometric concepts, specifically properties of quadrilaterals.

Online applications and the use of technology help students focus on improving their skills needed for their future success. Experiences with different forms of technology in the classroom such as a calculator and online applications like GeoGebra provide students with different experiences and different types of opportunities to enhance their learning (Karam et al., 2017). It can even change a student's attitude toward learning and motivate them to learn (Watson & Trotman, 2019). Adelabu et al. (2019) concluded from their studies that the students showed motivation, had a better understanding of the concepts, and performed higher when dynamic geometry software was used. In the study conducted by Ganesan and Eu (2020), they were able to conclude that most students who used Geometer's Sketchpad, a type of DGS, enjoyed mathematics and found that the lessons were more interesting. The students also stated that they were able to participate in classroom discussions with their classmates and teacher. The importance of effectively using technology to enhance student learning is essential in preparing students for the careers of the future. Teachers can use GeoGebra as a tool to provide students with the opportunity to focus on those skills through its dynamic features.

GeoGebra is available for offline use, making it easier for students to gain access to its many functionalities (Mthethwa et al., 2020), is free, available in multiple platforms, different languages, and supports algebra, geometry, spreadsheet, computer algebra system (CAS), graphics, and probability (Lo & White, 2020; Meadows & Caniglia, 2021). Dynamic geometry software applications such as GeoGebra, Desmos, and Sketchpad contain features that allow students to explore mathematical concepts with the use of its many functions such as dragging, construction, and measurement tools providing opportunities for visualization, which leads to collaboration and discussion amongst their classmates.

Several studies reported a statistically significant difference when utilizing DGS (Adelabu et al., 2019; Dogruer & Akyuz, 2020; Ganesan & Eu, 2020). The researchers reported a substantially higher score for the experimental group, the group that was using DGS, and/or a significant improvement between the pre- and post-test (Birgin & Uzun Yazıcı, 2021; Ganesan & Eu, 2020; Mthethwa et al., 2020), improving student achievement. The researchers showed that the participants in their control and experimental groups had similar abilities before receiving the treatment (Adelabu et al., 2019; Birgin & Uzun Yazıcı, 2021; Ganesan & Eu, 2020; Mthethwa et al., 2020). It has also been stated that DGS improves a students' understanding of abstract concepts and allows for a deeper understanding (Birgin & Uzun Yazıcı, 2021; Dogruer & Akyuz, 2020; Fung & Poon, 2021; Mthethwa et al., 2020; Zulnaidi et al., 2020). Several studies conducted used a quasi-experimental (Adelabu et al., 2019; Alkhateeb et al., 2019; Birgin and Yazici, 2021; Fung and Poon, 2021; Ganesan and Eu, 2020; Zulnaidi et al., 2020) design and used instruments that would measure a students' achievement while using DGS (Ozcakir and Cakiroglu, 2019) such as GeoGebra (Adelabu et al., 2019; Alkhateeb et al., 2019; Birgin and Yazici, 2021; Fung and Poon, 2021; Mthethwa et al., 2020; Zulnaidi et al., 2020) and Sketchpad (Ganesan and Eu, 2020).

The use of DGS also helped students make connections regarding the properties of the objects rather than memorizing the information that is normally given to them (Birgin & Uzun Yazıcı, 2021; Disbudak & Akyuz, 2019), found the lessons more interesting, improving their confidence and motivation (Dogruer & Akyuz, 2020; Ganesan & Eu, 2020; Mthethwa et al., 2020), was the preferred method from the conventional (Ganesan & Eu, 2020), and created a less stressful environment due to the discussions (Ganesan & Eu, 2020). Students who used DGS were observed to have more success when solving problems and justifying their statements, while those who did not, provided a limited justification for their answers (Mthethwa et al., 2020).

Although DGS has many useful functions as previously described, there can be small miscalculations when measuring angles. For example, while preparing the lesson in measuring complementary, the sum of two angles equals 90 degrees, and supplementary, the sum of the two angles equals 180 degrees, angles, I encountered a slight miscalculation when the sum did not equal to 90 or 180 degrees, but rather 89.9 and 179.9 respectively. I had to explain to my students that sometimes these errors do occur.

The students in the Edwards et al. (2021) study preferred physical models when exploring three-dimensional objects because it made it more meaningful. In the study conducted by Sherman (2014), it was observed that there was no evidence to suggest that most of the students were engaged in higher-level thinking even though students manipulated, measured, and constructed the figures. It was also found that although students are more independent in the learning process while they explore concepts through DGS, it provided challenges in trying to monitor and evaluate students' work (Fung and Poon, 2021). Although these studies do provide results that contradict my study, my study provided insight on how students perform when exploring the properties of quadrilaterals using DGS, which is not commonly explored.

Theoretical Framework

Based on the van Hiele Model, if students are taught appropriately, students move from the initial, or basic level, described as visualization, to the highest level, rigor (Crowley, 1987). The van Hiele Model of geometric thought can serve as a guide towards instruction as well as assess the students' abilities (Crowley, 1987). Crowley (1987) describes the model as follows:

- Level 0: Visualization – Geometric figures are recognized by their shape or physical appearance, not by their parts or properties
- Level 1: Analysis – Students begin to distinguish the characteristics of figures and the properties are used to conceptualize classes of shapes
- Level 2: Informal Deduction – Students can establish the interrelationships of properties both within figures and among figures
- Level 3: Deduction – The role of undefined terms, axioms, postulates, definitions, theorems, and proof is seen
- Level 4: Rigor – Geometry is seen in the abstract

The van Hiele levels of thinking are not age dependent (Herbst et al., 2017). Despite their experience of learning geometric shapes starting at the elementary level, it is possible that some students will stay at lower levels of thinking (Herbst et al., 2017). Prior to attending high school, students must have completed least up to Level 2 to understand the concepts being introduced (Özçakir et al., 2020). Knowing the student's level helps with developing suitable learning activities, materials, and instructions, to provide them with learning environments that will help them advance through the levels. Students taking high school Geometry courses are usually taught at a Level 3 (Crowley, 1987). By knowing a student's level based on the van Hiele Model, I was able to adjust the instruction methods to provide a proper environment for my students' conceptual understanding.

To provide students with a proper learning environment based on their level of geometric thought, I looked to an adapted version of the Van Hiele's Model of Geometric learning used by Webre et al. (2018). This model served as the foundation of my theoretical framework due to it being adjusted specifically for DGS such as GeoGebra. The model presented in Webre et al. (2018) is as follows:

- Stage 1: Teacher introduces an open-ended problem with proof as an objective and then chooses an instructional strategy that facilitates students' reasoning and problem-solving skills
- Stage 2: The student is prompted to utilize the dynamic geometry technology and investigate the present problem's situation to generate a conjecture
- Stage 3: Students are prompted to state or make a conjecture
- Stage 4: Students are encouraged to test their conjecture
- Stage 5: Students directed to prove/disprove their conjecture

NCTM (2000) emphasizes that as students transition from middle school to high school, they should learn the how to use deductive reasoning and be exposed more to formal proof techniques so that they can have opportunities to solve problems and prove their conjectures through convincing explanations, which can be accomplished by using the adapted framework by Webre et al. (2018). This framework provides opportunities for students to advance through the van Hiele Model levels.

Rather than providing my students with the theorems, the stages in this framework ask to engage students in a learning environment in which they can make conjectures based on their observations regarding, for example, the relationship between a parallelogram's opposite angles or sides, the diagonals, and the sum of consecutive angles. Students use GeoGebra to explore these properties as they create multiple representations of the same figure by dynamically modifying the dimensions while maintaining its properties and testing their conjectures. Once they have tested and verified their conjectures, the theorems are then introduced to verify their conjectures. Due to these experiences, students can construct and take charge of their learning through constructivism.

Constructivism

The subject of geometry works well if the manipulation of objects is being used for exploration of the concepts. Young students learn best when they have opportunities to actively manipulate objects through hands-on learning (Goodman, 2014) and it helps with visualization. Piaget is credited by many educational psychologists with the development of the notion that children create meaning and knowledge on their own (Goodman, 2014), also known as constructivism. When students try to make sense of new information, they first try to associate it with information they already know, their existing knowledge. When faced with experiences or situations that challenge their way of thinking causing an imbalance, they must then alter their way of thinking based on the new information to restore that balance (Amineh & Asl, 2015). Through his observations, Piaget describes four distinct periods of cognitive growth (Goodman, 2014):

- First period (birth – approximately 2 years old) - sensori-motor stage of development
- Second period (approximately 2 – 7 years old) – pre-operational period of development
- Third period (approximately 7 – 11 years old) – concrete operational stage of development
- Fourth period (approximately 12 – 15 years old) – formal operational stage

Goodman (2014) states that in the sensori-motor stage, children explore the world through their five senses. In the pre-operational, children use various forms of play to develop language and other life skills. In the concrete operational stage, children begin to learn how to add, subtract, spell, and perform other mental tasks. Students learn by actively participating in group work and use manipulatives. Constructivism is important in this stage due to students' interaction with the concepts, using prior knowledge acquired from the beginning stages. In the last stage, formal operational, students' logical thinking skills, developed in the prior period, are enhanced by thought processes that can include abstract and hypothetical ideas. The four period are highly criticized due to the population that Piaget studied, which failed to account for diversity of children's experiences, globaly.

Vygotsky believed that a child's cognitive development is influenced by family, community, and culture and that a child's growth occurs through scaffolding within a space he called the Zone of Proximal Development (ZPD) (Goodman, 2014). As we scaffold through the properties of quadrilaterals using meaningful activities, we can deliver lessons in a way to help students understand these concepts through ZPD as they advance through the scaffolding process. Vygotsky's belief that the process of knowing is affected by other people and the community and culture help shape that knowledge (Amineh & Asl, 2015), is the approach that closely resembles what happens in my classroom. Students are constantly collaborating with one another, questioning contradictions to their belief, therefore, developing new knowledge based on those experiences. I typically use a mixture of the two when first introducing a concept.

Using DGS, creates an environment that places students at the center of the learning. Based on past personal experiences, my students worked collaboratively on exploring the properties of quadrilaterals while using DGS, created and tested their conjectures, and reflected on their experiences as we had open discussions while working with the properties of quadrilaterals. These are characteristics consistent with a constructivist approach to mathematics (Davis & Maher, 1990, as cited in Francisco, 2005). In a study by Francisco (2005), the students preferred to build or “discover” their own knowledge rather than have teachers tell them through a lecture. Prior to taking the high school geometry course, students were introduced to basic geometric concepts at earlier grade levels. Constructivism involves students obtaining new knowledge and accommodating it with existing knowledge (Barrett & Long, 2012). Students obtain the new knowledge based on their personal experiences with the activities. Due to technology being an important tool in their learning process, students can take control of their independent learning due to the vast amount of data and information students have readably available. With the information currently available through the internet, students can easily search a specific geometry concept, become familiarized with the information, and comprehend the material with little teacher instruction. The teacher will serve as a guide as opposed to an information dispenser.

Technology allows for members of a group to collaborate and help one another as they explore through the material. These social relations and interactions help shape a child’s cognitive functioning and is something that may be useful for teachers to consider (Goodman, 2014). As the students work through the stages of the framework presented by Webre et al.

(2018), they will have opportunities to construct the quadrilaterals, observe the changes as they explore the different properties, collaborate, and discuss their findings with their classmates, thus taking charge of their learning process. Having the ability to dynamically make changes to the polygons provide an environment where students can construct their knowledge.

Dynamic Geometry Software

Studies have shown that the use DGS in mathematics teaching has increased the students' achievement levels (Adelabu et al., 2019; Alkhateeb & Al-Duwairi, 2019; Birgin & Uzun Yazıcı, 2021; Dogruer & Akyuz, 2020; Disbudak & Akyuz, 2019; Fung & Poon, 2021; Ganensan & Eu, 2020; Mthethwa et al., 2020). The DGS environment provides ways of representing and manipulating geometric objects instantly (Adelabu et al., 2019; Belnap & Parrott, 2020; NCTM, 2000; Herbst et al., 2017; Meadows & Caniglia, 2021; Ozakir & Cakiroglu, 2019), which would be time-consuming if traditional materials were used such as a compass, ruler, paper, and pencil.

Dynamic geometry software gives students a sense of control due to its ability to easily undo any mistakes (Adelabu et al., 2019, p. 54) reducing the level of anxiety and negative attitude towards mathematics, however, to enhance the learning benefits, students would need to work on tasks with little to no guidance (Olsson & Granberg, 2019) to allow for exploration of the concept, in this case the properties of quadrilaterals.

Dragging

Dragging involves selecting a quadrilateral's vertex or side and moving it to a different location, therefore modifying its characteristics. The GeoGebra functions can be described as follows:

Point – plots a point on the screen

Segment – connects two points to create a segment

Polygon – creates a polygon by connecting a series of points

Perpendicular Line – draws a line perpendicular to a selected line/segment

Parallel Line – draws a line parallel to a selected line/segment

Angle – calculates and displays the angle measure by selecting three points

Distance or Length – calculates and displays the length of a segment

When students explore quadrilaterals, they create them by using the *Point* and *Segment*, or the *Polygon* function. However, when quadrilaterals are created using either of these two methods, the properties are not maintained. If a student created an isosceles trapezoid and were to drag one of the vertices, it loses some of the properties. If the *Polygon* function were used, they can drag the entire polygon without modifying the properties but defeats the purpose when trying to maintain the attributes when a vertex is dragged. The goal is to be able to drag a vertex, modifying the lengths of the sides and the interior angle measures, but maintaining the properties of, for example, an isosceles trapezoid which has two identical side lengths and parallel bases.

To properly construct a parallelogram, rhombus, square, rectangle, or trapezoid, the *Perpendicular Line* and/or *Parallel Line* functions will need to be used in addition to the *Point* and *Segment* functions. Regardless of the method used to construct the quadrilateral, they can then manipulate the object by *dragging* a vertex, receiving instant feedback on the object's characteristics while creating multiple versions instantaneously providing opportunities for students to observe those changes and make their own conjectures (Bülbül et al., 2020; Kondratieva & Bergsten, 2021; Hollerbrands, 2007; Özçakir et al., 2020; Sherman, 2014). The *dragging* capability is seen as an important feature when exploring geometric objects because it engages a students' understanding of geometry (Adelabu et al., 2019; Bailey et al., 2020,) and provides a learning environment to foster geometric thinking (Özçakir et al., 2020). The *slider* functionality can also be manipulated by *dragging* which causes a change in the object resulting in an immediate change (Fung & Poon, 2021). Examples of the *slider* function include modifying the number of sides of a polygon and observing the effects of the interior and exterior angles.

Measurement

In a geometry classroom, students have access rulers and protractors which they can use to measure the sides and angle of a quadrilateral. Once students have a strong foundation in measuring objects using these tools, they can then explore the measurement functions in GeoGebra. The *Angle*, and *Distance or Length* functions provide measurements for the angle measures and the length of a segment respectively. The measurement capabilities had a vital role in this study due to students exploring the characteristics of quadrilaterals as stated in the theorems. When exploring a quadrilateral's characteristics, students can view the dynamic changes in the measurements of the sides and angles and make their own conjectures without knowing the theorems beforehand. They can use the measurement functions to check for accuracy (Adelabu et al., 2019) and verify their conclusions (Contreras, 2022; Ganesan & Eu, 2020; Hollerbrands, 2007).

Construction

According to the geometry TEKS (2012), students are expected to use constructions, drawing geometric figures using compass and a straightedge (Pearson, 2014), to validate conjectures, a conclusion reached through inductive reasoning (Pearson, 2014), about geometric figures. The tools typically used during the construction process include a straightedge such as a ruler and/or compass, but the geometric objects tend to be static. Students would need to construct several different versions to explore its characteristics, which can be time-consuming. While using GeoGebra, students can construct an object using functions such as *Parallel* and *Perpendicular Lines* when working with the properties of parallelograms. There is a process that students must follow to ensure that the object is constructed properly to avoid distortion by *dragging* a vertex. The process (See Appendix A) is as follows:

Step 1: Plot three points in the Geometry section of GeoGebra using the *Point* function.

Step 2: Connect Points A and B using the *Segment* function. Do the same with Points B and D.

Step 3: Use the *Parallel Line* function to draw the line parallel to Line BD by clicking on Line BD and Point A. Repeat the step with Line AB and Point D.

Step 4: Use the *Intersect* function to plot the point where the two lines intersect.

Step 5: Select the *Show / Hide Object* and click on the two intersecting lines. Then click on the *Move* function.

Step 6: Select the *Segment* function to connect Points A and E, and E and D. Select a vertex and modify the figure by *dragging*.

Step 7: To display the angle measures, select the *Angle* function, then select three points with the middle point being the vertex. Do this to all four angles.

The *Polygon* function can create objects which can be moved anywhere in the environment, however, *dragging* a vertex distorts the figure, making it difficult to observe the characteristics. With proper planning, teachers can build on a student's knowledge on GeoGebra's functions to effectively construct objects which allows for resizing while maintaining its properties. The GeoGebra functions allow for multiple constructions in a shorter period by *dragging*, giving students more time to focus on exploring, making conjectures, and validation (Adelabu et al., 2019; Meadows & Caniglia, 2021; Mthethwa et al., 2020). Constructions aid in visualization, increases student engagement (Meadows & Caniglia, 2021) and promotes discussions (Conner, 2020, Quinn, 2020).

Visualization and Exploration

Having the ability to *drag* different parts of well-constructed object and precisely measure the lengths of sides or angles creates an environment where students can observe the object's many characteristics as they dynamically change so that they can then make and test conjectures based on the visual (Bülbül et al., 2020; Contreras, 2022). Visualization is important for the conceptual understanding of geometric concepts (Birgin & Uzun Yazıcı, 2021; Fung &

Poon, 2021; Ganesan & Eu, 2020; Lo & White, 2020 (sliders); Meadows & Caniglia, 2021; Wasserman & McGuffey, 2021). With the various objects introduced in geometry, sketching a diagram helps in the visualization of abstract geometric ideas (Adelabu et al., 2019; Hollerbrands, 2007) and can aid with arithmetical, algebraic, and statistical concepts (Ganesan & Eu, 2020, p. 58).

Dynamic geometry software can be described as a dynamic exploration environment (Bailey et al., 2020; Dogruer & Akyuz, 2020; Kondratieva & Bergsten, 2021; Özçakir et al, 2020) as opposed to paper and pencil, which is more static. Exploring is important when working with the mathematical process (Armstrong & McQuillan, 2020; Birgin & Uzun Yazıcı, 2021; Dogruer & Akyuz, 2020; Fung & Poon, 2021; Ganesan & Eu, 2020; Hollerbrands, 2007; Sherman, 2014) such as writing definitions (Lovett et al., 2020), formulating and testing conjectures (Bailey et al., 2021; Contreras, 2022; Dietiker & Richman, 2021; McCulloch et al., 2021; Özçakir et al., 2020; Wasserman et al., 2020), or summarizing observations (Hoyos, 2021) that can be used for reflection (Hollerbrands, 2007). Students can work with ease knowing that if they make mistakes, they can be easily fixed, building up their confidence and motivation to become problem solvers (Birgin & Uzun Yazıcı, 2021; Quinn, 2020).

Collaboration and Discussion

Having students explore by *dragging*, measuring angles and segments with the measurement functions, and construction functions embedded in GeoGebra, provides opportunities for collaboration and open discussions amongst their classmates. Goodman (2014) mentions that teachers should consider Vygotsky's thinking as to how culture and community help in a student's development process. Although there are many different types of learners, open discussions can benefit all students due to the conversations. Studies have shown that participants who then collaborated with their group members, had discussions based on each of their conjectures and had an opportunity to defend it using DGS (Dogruer & Akyuz, 2020; Lovett et al., 2020; McCulloch et al., 2021). A learning environment which promotes exploration and construction of their own knowledge through discussions with their peers supports conceptual understanding (Birgin & Uzun Yazıcı, 2021). Students may change or modify their conjectures based on the discussions due to something being overlooked. Through discussion, students may discover new information that they themselves may have missed, creating an environment for conceptual learning (Dogruer & Akyuz, 2020; Lovett et al., 2020; Meadows & Caniglia, 2021). It benefits the student when teachers create tasks that promote collaboration in which students can then reflect on when problem-solving using technology (Adelabu et al., 2019; Baker et al, 2021; Belnap & Parrott, 2020; Hoyos, 2021).

Summary

GeoGebra has many useful embedded functions which are useful in the exploration of geometric concepts. Students can properly construct figures based on their characteristics and manipulate the object to make conjectures about the different attributes and what they observe. They can see the changes in the length of the sides and the measures of the angles of multiple figures as they occur in real-time while providing an environment that is open to collaboration and in-depth discussions of the geometric concepts. Rather than giving students the theorems, they can construct the figures and create multiple versions while *dragging* a vertex. GeoGebra provides teachers with an opportunity to create a more student-centered environment, allowing them to construct their own knowledge through constructivism.

CHAPTER III

METHODOLOGY

In this chapter, I discuss the methodology used to conduct my research and analysis. I describe the design of my study, the participants, and the methods used in collecting and analyzing the data with the main goal of answering the research questions:

1. How effective is GeoGebra when exploring the properties of quadrilaterals in conceptual understanding?
2. Based on the van Hiele levels of thinking, to what extent do the student's level of geometric thinking about the properties of quadrilaterals change after the implementation of GeoGebra?
3. What are the students' perception in the use and integration of GeoGebra while learning about geometric concepts, in particular the ones related to quadrilaterals?

The purpose of my study is to determine the effectiveness of using GeoGebra when exploring the properties of quadrilaterals based on the levels of the van Hiele Model amongst freshman and sophomore students in a geometry course while providing them with the opportunity to practice on enhancing their technological skills. Based on the TEKS (2012), students should have a basic understanding of two-dimensional geometric concepts prior to taking the high school level course. I first determined a student's current level through the van Hiele Model using the Van Hiele Test, which served as a guide on the materials that were used for instruction. GeoGebra was used for the tasks in exploring the properties of a quadrilateral, Topic 6 Polygons and Quadrilaterals, and once the unit was completed, the students were given the Van Hiele Test a second time to determine their new level of understanding. Uwurukundo et. al. (2022) states that GeoGebra contributed to their students' understanding of mathematical concepts, therefore, testing it on the properties of quadrilaterals provided teachers with a resource in case they were hesitant in using it in future lessons.

Research Design

A quantitative approach, following a quasi-experimental design, was used for this study due to the *treatment* variable (i.e., GeoGebra) being observed to determine the effectiveness when studying the properties of quadrilaterals. A quasi-experimental design was appropriate for my study because students who participated in this study were randomly placed in each group by the school administrators (Mills & Gay, 2019). The sampling could be classified as a stratified sampling, as described in Mills and Gay (2019), due to a certain number of Honors students being selected from a population of students that includes those in an Honors course, and those that are not. One group was exposed to GeoGebra, while the other was taught by a teacher-developed lesson plan when teaching the properties of quadrilaterals. According to Mills and Gay (2019), experimental research produces the soundest evidence when cause-effect relations are being observed. This coincides with the purpose of the study, which was to determine if GeoGebra provided an environment that assisted students in the conceptual understanding of the properties of quadrilaterals.

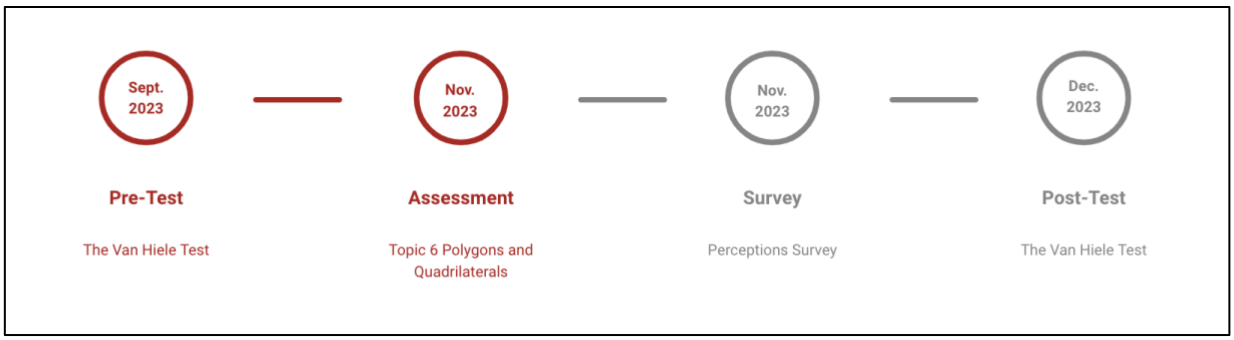
A nonequivalent control group design was implemented due to the pre-test, treatment, and post-test being administered to intact classes, and not individuals which are the characteristics of a nonequivalent control group design (Mills & Gay, 2019). This design added validity threats such as regression and interactions between selection, maturation, history, and testing (Mills & Gay, 2019). To reduce the validity threats and to help strengthen the study, two geometry honors courses at similar levels were selected. One group was randomly selected as the treatment group and the other as the comparison group. The honors geometry course was purposely selected to be the target of the study because of its feasibility.

Originally, both groups were set to be taught by the same teacher, however, due to scheduling and teacher assignments by the school's administration, two different teachers with similar levels of expertise were assigned to teach the Honors courses. As I understand the constrain of the above, this represents an uncontrollable limitation. To mitigate the effects of not being able to control for this variable, I controlled the assessment that was implemented, and the lesson plan used to teach the topics.

The two teachers and myself met frequently and planned the lessons to ensure the consistency of instruction to both the comparison and treatment groups. The treatment group followed the theoretical framework described in Webre et al. (2018) while the comparison group used a similar process while using teacher developed activities. We (i.e., the teachers and I) ensured that both groups received quality instruction to prepare for the unit assessment.

The timeline in Figure 4 shows the date for each assessment. Students first took the pretest to determine their van Hiele Level prior to taking the course. High school geometry concepts are typically taught at a van Hiele Level 3 (Crowley, 1987), therefore, students should be at van Hiele Level 2 coming into the course. Knowing a student's van Hiele level prior to taking the course, provided the teacher with information on how approach the curriculum while using GeoGebra. After completing Unit 6, Polygons and Quadrilaterals, students took a unit assessment to determine their comprehension of the material. Additional conversations with the teacher in the treatment group were held to determine the date for the perception survey, to be completed after taking the unit assessment. The students were then given Van Hiele Test again as a posttest to determine their new van Hiele Level and was used as part of the data to determine GeoGebra's effectiveness.

Figure 4: Research Study Timeline



The pre- and post-test, the unit assessment, and perceptions survey were all used to find the answers to research questions. The unit assessment was primarily used to determine the effectiveness GeoGebra while exploring the properties of quadrilaterals, question 1. The unit assessment was selected in a collaborative effort between the teachers who participated in the study and myself as researcher. We looked at the unit assessments given in the past (i.e., in previous years) and decided that the one used for the study best tested student's comprehension of the concepts introduced in this unit. The pre- and post-test results were used to determine if GeoGebra had an effect in the van Hiele levels of thinking, and if so, to what extent, my research question 2 was successfully answered. Knowing the van Hiele student's level prior to taking the course, gives teachers an idea of where students are and how to approach the material being introduced in the unit. The perceptions survey answered my research question 3, the students' perception in the use and integration of GeoGebra while exploring the properties of quadrilaterals. The results are presented in Chapter 4.

Population and Sample

The Geometry Honors course groups were selected from a high school in the district in which the comparison group received teacher developed instruction and the treatment group worked with GeoGebra activities. Both groups (i.e., comparison and treatment) were exposed to the same lesson plan. Each group consisted of approximately between 25-30 high school sophomore to senior students, ages ranging from 14 to 16, predominantly Hispanic with almost an even mixture of female and male students. The teachers in each group were experienced geometry teachers with 3+ years teaching experience in geometry and about 28 years of combined teaching experience. One is female and the other is male. Teacher A in the comparison group, had a total of 26 students, 81% Hispanic. In total, 19 students out of the 26 assented to participate in the study, from which 67% were female and 33% male. Teacher B, the treatment group, had a total of 30 students, from which approximately 87% were Hispanic, 22 assented to participate in the study, but only 17 had all parts of the study completed, 53% were male and 47% female. To maintain consistency only students who had completed both the pre- and post-test, and the unit assessment were considered for the data analysis.

Data Analysis and Instruments

As mentioned above, two sections of the honors geometry course (i.e., treatment and comparison group) were considered for the study, from which the effectiveness of using GeoGebra to explore quadrilaterals was analyzed. The Van Hiele Test (See Appendix C) as presented by Usiskin (1982), was administered prior to instruction to determine the current level of the student based on their middle school experiences. This test was developed by Usiskin (1982) and was used in his study which consisted of 2699 students enrolled in a one-year geometry course in 13 schools. The low-level questions required some sort of analysis and overall the questions were generally more conceptual than those found in a standard test (Usiskin, 1982). Because of the quality and content focused of Usiskin's test, I decided in consultation with the teachers participating in the study, to use the same test to determine the student's van Hiele level.

The Van Hiele Test consists of a total of 25 questions, 5 for every level of the van Hiele Model. Students were given 35 minutes to complete the test. The same test was administered to both the treatment and comparison group. The treatment group explored the properties of quadrilaterals using GeoGebra while the comparison group used a teacher developed lesson with the calculator as a primary source of technology.

The Van Hiele Test was then administered again (i.e., posttest) after the completion of the geometry curriculum, including the 6 sections in Unit 6, and was used to determine the effectiveness of GeoGebra. Enough time needed to pass to avoid the memorization of answers when it was first administered as the pretest. In terms of data analysis, I used the Statistical Package for the Social Sciences (SPSS) to analyze the data. The quantitative variables *Score1* and *Score2* represented the weighted score based on the responses given by the participants for the pre- and post-test, respectively. A *t*-test for non-independent samples was used due to the data being collected and analyzed coming from a single group's performance on the pre-test and post-test (Mills & Gay, 2019, p. 515). A special *t*-test for non-independent samples was needed because the scores were non-independent (Mills and Gay, 2019).

As mentioned in the research questions, I also evaluated the student's perceptions of using GeoGebra to have a better understanding of the students' attitude and opinions while using GeoGebra. Students in previous courses have stated the benefit of integrating GeoGebra as a pedagogical strategy to explore the properties of quadrilaterals, and I wanted to find evidence that supports the hypothesis that other students potentially would have a similar experience. To evaluate student's perceptions, I used a slightly adapted version of the survey (See Appendix D) developed by Ganesan and Eu (2020). Their study used Geometer's Sketchpad rather than GeoGebra. In my study, I used GeoGebra. The adaptations were needed to meet the needs of the study in terms of student's grade level and focus of the questions related to the topic of my study. The survey was implemented with the treatment group (i.e., the group exposed to GeoGebra) to capture their perception in the following categories:

i. on the GeoGebra usage (Items 1, 11, and 14)

ii. on how GeoGebra helps for understanding (Items 2, 5, 6, 8, 10, and 13)

iii. on their abilities to communicate when using GeoGebra (Items 3 and 9)

iv. on their attitudes towards geometry when using GeoGebra (Items 4, 7 and 12)

The scale used a five-point Likert scale format to assess students' responses for each related section. (1 – Strongly Disagree, 2 – Disagree, 3 – Neutral, 4 – Agree, 5 – Strongly Agree).

The results were analyzed based on the categories and summarized in frequencies and percentages.

Students were also given a Topic 6 Polygons and Quadrilaterals assessment (See Appendix E). The topic assessment is made up of 23 open-ended questions testing the students conceptual understanding of the geometric concepts introduced in this topic. Questions 10 and 14 of the unit assessment were omitted due to treatment group not being able to complete the lesson prior to the unit assessment. The students' scores were graded on agreed point-scale system. The students' scores were analyzed based on the points acquired compared to the total points possible. The most commonly missed and answered individual questions were also analyzed to determine concepts that students struggled with the most and least, respectively.

Lesson Structure

The lessons were structured based on the school adopted textbook, Pearson Geometry – Texas Edition. Each section builds on the previous section through scaffolding. It was during this time that the treatment group became better familiarized with key GeoGebra functions that would help them guide through the sections that followed. By scaffolding the sections and building on prior knowledge and experiences through constructivism, students were able to gain knowledge through Vygotsky's Zone of proximal development as described in Goodman (2014).

The unit of focus was Topic 6: Polygons and Quadrilaterals and was implemented in both groups (i.e., treatment and comparison) of the honor geometry course that participated in the study. The first section of the topic served as practice for students to become familiarized to some of the functions embedded in GeoGebra. Following the adapted framework as stated by

Webre et al. (2018), the teacher introduced the exploration of the interior and exterior angles of a polygon and explained the instructions of the activity as stated in the worksheet in Appendix B. The students first selected a vertex and then connected that vertex to the remaining vertices, thus creating separate triangles. They then added all the triangles created and made a conjecture about the sum of the interior angles of the polygon. The information of the triangle was given, the sum of the interior angles, and students used that information to determine the sum of the interior angles of a quadrilateral, thus completing Stage 1 of the adapted framework. Stage 2 prompted students to predict the sum of the interior angles for the remaining polygons while using GeoGebra to verify their prediction. The worksheet prompted the students to work from a quadrilateral up to a decagon while filling out the information regarding the number of sides, number of triangles created, and the sum of the interior angles and recorded their answers on the worksheet. In Stage 3, the students then were challenged to create an equation for an *n*-sided polygon. In Stage 4, students were given the opportunity to test their conjecture or formula, and Stage 5 asked them to prove or disprove their conjecture. The use of GeoGebra allowed for this process to be completed in a timely manner, giving students an opportunity to discuss their findings with other groups.

The remaining sections of Topic 6 include properties of parallelograms, rhombi, rectangles, squares, trapezoids, and kites. A similar process from the first section was followed. Stage 1 consisted of students analyzing a theorem as presented for each quadrilateral, for example comparing the lengths of the diagonals, the angles formed by the diagonals, and lengths of the sides. Stage 2 consisted of the proper construction of the quadrilateral using DGS. This process is important because it challenged the students to use parallel and perpendicular lines, along with other functions provided by the DGS environment to properly construct the quadrilaterals to ensure they were observing accurate data. Stage 3 asked them to make a conjecture and test it in Stage 4. They finally proved or disproved their conjecture in Stage 5. Prior to this process, students took the pre-test to determine the level of their knowledge based on the van Hiele Model and took the post-test after completing the given tasks. This process took between 5 to 7 days due to the 90-minute blocks.

Reliability and Validity

To better understand any misconceptions students may have about quadrilaterals, teachers can assess their students to determine their prior knowledge. This provides teachers with an opportunity to intervene and ensure comprehension of the concept before assessing again to determine if the intervention was successful. There are a few geometry concepts that have been covered at the middle school level, so it is beneficial to the teacher to determine any prior knowledge or misconceptions. The assessment used to determine a student's skill level was designed according to the methods of estimating reliability and types of validity as described in Reynolds et al. (2009).

The test-retest approach does have significant limitations due to the same test being administered, which include carryover effects (Reynolds et al., 2009), making it suitable for this method of estimating reliability, however, it was administered with considerable time between the two administrations to avoid the reliability estimate from being inflated due to memorization of the questions from when it was first administered (Reynolds et al., 2009). After calculating the results of the pretest, students participated in the study using GeoGebra to determine the effectiveness when exploring quadrilaterals. A posttest determined if students advanced to the next level of the van Hiele Model.

Advantages to an assessment include having a better understanding of a student's skill level based on their responses. After their skill level is determined, proper intervention was provided to help strengthen the understanding of the concept. A limitation regarding the assessment includes that lack of determining if a student truly understands the properties of quadrilaterals, or if they are simply following a process, such as memorizing a formula.

Summary

Technology has modified how teachers deliver instruction. Classrooms have the capability of changing from a traditional setting into something more student-centered. Teachers should take advantage of the technology's graphing, visualizing, and computing features to enhance a student's learning opportunity by creating mathematical tasks that highlight these features (NCTM, 2000). Dynamic geometry software is a useful application with many tools that can be used for various geometric concepts. The ability students have in being able to modify a polygon's dimensions dynamically, allows students to view different lengths and angles for the shape, which is something that cannot be so easily replicated through paper and pencil. The functions embedded into GeoGebra provide opportunities to have an interactive experience with a variety of two-dimensional shapes (NCTM, 2000).

The Van Hiele Test was given as a pretest to determine if the student is at the proper level of understanding prior to taking the high school geometry course. Due to some topics being covered at the middle school level, it is important to know the level of the student prior to working with GeoGebra to determine what activities were needed to be used to evaluate its effectiveness. The framework adapted by Webre et al. (2018), based on the van Hiele Model, was used to evaluate the level of the student, and determine if students' level on understanding increased after working with GeoGebra. The Van Hiele Test was given as a posttest to evaluate their new level of understanding. The results from the pre- and post-test, along with the unit assessment, were compared to determine the effectiveness of using GeoGebra.

Dynamic geometry software can be used with a coordinate plane, giving students the opportunity to incorporate algebra into their exploration, which will be useful as they solve a wider array of problems in geometry and algebra (NCTM, 2000, p. 42). NCTM (2000) also state that in secondary grades, the coordinate plane can be helpful as students work on discovering and analyzing properties of shapes. They can be represented analytically, making that fundamental connection between geometry and algebra (NCTM, 2000). Future studies will be conducted on the effects of DGS when combining geometric and algebraic concepts.

CHAPTER IV

RESULTS AND DATA ANALYSIS

The assessments were given based on the timeline described in Figure 4, depicted in chapter three above. The data collected from the pre- and post-test, unit assessment, and perceptions survey were used to answer the questions that drove my study. The sections that follow show the results for each assessment and the questions that drove my study were answered.

The Van Hiele Test

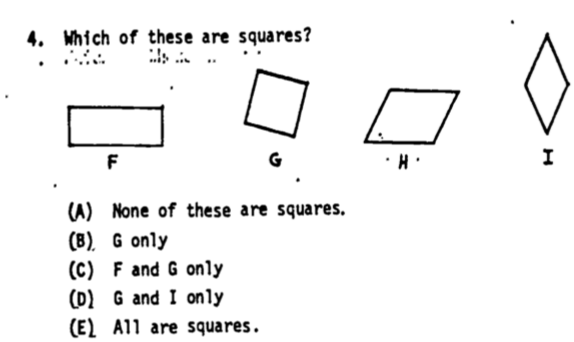
The participants in both the treatment group (N = 17) and comparison group (N = 19) were given The Van Hiele Test (See Appendix C) as the pre- and post-test which consisted of 25 questions, with a 35-minute time limit. Although the treatment group had 30 students registered for the course and the comparison group had 27, those that assented to participate in the study, answered both the pre- and post-test, and took the unit assessment, were considered as part of the study.

The multiple-choice assessment was scored based on the criteria as stated in Usiskin (1982) and each student was given an initial level based on the pretest results and an updated level after the posttest. The levels start with Level 0 – Visualization and continue to Level 4 – Rigor. For data analysis purposes, a level of 0 was used for students who were not yet at Level 0, therefore each level was increased by one unit (i.e., Level 0 = 1, Level 1 = 2, Level 2 = 3, Level 3 = 4, and Level 4 = 5).

As previously mentioned, students in a high school geometry course are typically taught at a van Hiele Level 3. As we progressed through the concepts, students eventually encountered material considered to be at a van Hiele Level 4. These sections typically consisted of writing proofs while using the theorems and postulates for their reasoning as introduced in those sections. Based on my experience as a geometry teacher for over 10 years, students typically struggled with questions considered to be a van Hiele Level 4.

A sample question from the first three levels, presented below, depict the most commonly missed question from that specific level. At a van Hiele Level 0, students should be able to recognize shapes. Although geometry concepts are scaffolded throughout the grade levels, other mathematics courses are taken prior to the high school geometry course. This may be a reason as to why students tend to forget some of the basic properties of geometric shapes. Figure 5 depicts a question where students have to determine which of the shapes are squares based on prior knowledge about the properties of squares.

Figure 5: The Van Hiele Test Question 4

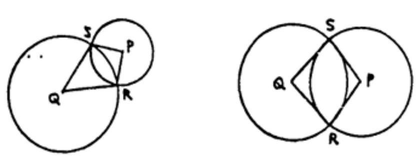


A square and rhombus do have four congruent sides, option I, however, students must know that a square contains four vertices that measure 90-degrees. In both the pre- and post-test, this question was answered in incorrectly 37% of time by the comparison group and 47% of the time by the treatment group, option E and C being the most popular choices, respectively. The fact that squares can be considered to be rectangles, but not the other way around, may be the reason why students selected option C. The reason for option E needs to be further explored in future studies.

Through scaffolding, like Vygotsky's ZPD (Goodman, 2014), students then become familiar with the properties of figures and how to properly identify them using labels. At a Level 1, students should be able to identify specific properties of figures using proper notation. Students must know the properties of a kite and rhombus to be able to answer the question in Figure 6 such as that a kite has one pair of opposite congruent angles, a rhombus has two pairs of congruent opposite angles, and that the diagonals are perpendicular to each other.

Figure 6: The Van Hiele Test Question 10

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.



Which of (A)-(D) is not always true?

(A) PRQS will have two pairs of sides of equal length.

(B) PRQS will have at least two angles of equal measure.

(C) The lines $\overline{PQ}$ and $\overline{RS}$ will be perpendicular.

(D) Angles P and Q will have the same measure.

(E) All of (A)-(D) are true.

They must also know that the bar on top of the two variables is used to represent a segment and that perpendicular segments form a 90-degree angle. In both the pre- and post-test, this question was answered in incorrectly 79% of time in the comparison group and 59% of the time in the treatment group. Options B, C, and E were selected about the same number of times. It is possible that students may not have read the question correctly, overlooking the word “not”.

At a Level 2, students should be able to identify relationships between quadrilaterals. In Figure 7 below, students must know the properties of specific parallelograms and be able to compare them to each other by identifying key attributes. Rectangles and squares contain vertices that measure 90-degree angles, therefore, eliminating options C and D. This question determines whether a student can classify a square as a rectangle, however, a rectangle can not be classified as a square due to a square having four congruent sides. Option B was the most common answer selected by both groups, meaning that students did have a general idea, but were not able to distinguish that one characteristic that makes the statement false.

Figure 7: The Van Hiele Test Question 14

14. Which is true?

(A) All properties of rectangles are properties of all squares.
(B) All properties of squares are properties of all rectangles.
(C) All properties of rectangles are properties of all parallelograms.
(D) All properties of squares are properties of all parallelograms.
(E) None of (A)-(D) is true.

Due to high school geometry typically being taught at van Hiele Level 3, students should at least be at van Hiele Level 2 coming into the course. Based on the pretest results shown in Table 2 below, it was determined that 26% of the students in the comparison group and 35% in the treatment group, were at a van Hiele Level of 2 or above. Although the treatment group had more students prepared to take on the challenge of learning high school geometry, 29% of the students did not score high enough to register a van Hiele Level compared to 11% in the comparison group. It seemed that the comparison group had results like one another, whereas the treatment group was more diverse.

Table 2: Pretest Van Hiele Test Results

Comparison Group	**Pretest Level**	**Treatment Group**	**Pretest Level**
C1	1	E1	2
C2	1	E2	0
C3	3	E3	3
C4	2	E4	1
C5	2	E5	1
C6	1	E6	0
C7	1	E7	0
C8	2	E8	0
C9	1	E9	3
C10	2	E10	3
C11	3	E11	3
C12	2	E12	0
C13	3	E13	3
C14	1	E14	1
C15	0	E15	1
C16	1	E16	2
C17	5	E17	3
C18	3		
C19	0		

Note. Pre- and post-test level 1 is a van Hiele Level 0.

The posttest was given after completing the course to determine their new van Hiele level of understanding. The results are listed in Table 3 below. The data shows that only 2 students from each group were not able to register a van Hiele Level after completing the course. It is interesting that 3 out of the 4 students regressed from their previous level.

Table 3: Posttest Van Hiele Test Results

Comparison Group	Posttest Level	Treatment Group	Posttest Level
C1	0	E1	0
C2	1	E2	1
C3	5	E3	4
C4	2	E4	1
C5	2	E5	3
C6	2	E6	3
C7	0	E7	1
C8	3	E8	1
C9	1	E9	3
C10	3	E10	2
C11	2	E11	3
C12	1	E12	0
C13	3	E13	3
C14	2	E14	1
C15	1	E15	3
C16	1	E16	2
C17	5	E17	2
C18	1		
C19	1		

Note. Pre- and post-test level 1 is a van Hiele Level 0.

Overall, the average van Hiele level of the students in the treatment group were (M = 1.53, SD = 1.28) for the pretest, and (M = 1.94, SD = 1.20) for the posttest. The results of the paired t-test showed that (t = -1.383, p = .186), and therefore must accept the null hypothesis, that DGS did not have a significant effect when exploring the properties of quadrilaterals, however, the data does show a slight increase in the mean score and that 41% of the students increased to the next van Hiele Level, and 2 of those students increased more than one level. There could be other factors that resulted in a decrease for three students, such as test fatigue due to state assessments and course final exams, but that aspect was not observed. The average van

Hiele level of the students in the comparison group were (M = 1.79, SD = 1.23) for the pretest, and (M = 1.89, SD = 1.41) for the posttest. The overall posttest mean was slightly higher on the treatment group even though the comparison group had a higher mean on the pretest. High school geometry is typically taught at a van Hiele Level of 2 and based on the pretest data, only 26% of the students in the comparison group and 35% of the treatment group were on level or above, and 11% in the comparison group and 29% in the treatment were not even at a Level 0, which may be the reason why our students often struggled with geometry. The results show that GeoGebra did have an impact, even if it was a slight impact, on the overall improvement in van Hiele levels for the students in the treatment group.

The Unit Assessment

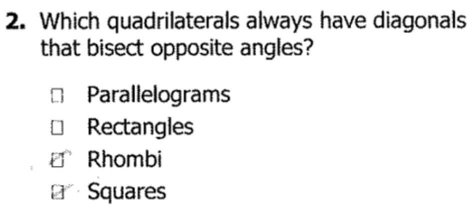
Students were then assessed on Unit 6: Polygons and Quadrilaterals from the Pearson (2016) textbook. While using GeoGebra, the students in the treatment group were given an opportunity to explore the properties of quadrilaterals using the theoretical framework as introduced in Webre et al. (2018). Table 4 below shows the results of that assessment using an agreed upon scoring rubric developed in a combined collaborative effort between the teachers participating in the study and myself. The assessments were scored twice, once by the teacher and once by me. The unit assessment results show that 53% of the students in the comparison group and 59% in the treatment group received a grade of 70 or above, which is a passing score based on our school grading system. Overall, the mean of the comparison group was 66.79%, while the treatment group mean was 67.82%. This data shows that our students continue to struggle with understanding the properties of quadrilaterals. Even though the treatment group used GeoGebra to explore the concepts, they did perform slightly better than the comparison group, showing that using GeoGebra does benefit the students when exploring the properties of quadrilaterals.

Table 4: Unit 6 Assessment Scores

Comparison Group	Score	Treatment Group	Score
C1	98	E1	79
C2	72	E2	44
C3	100	E3	52
C4	37	E4	42
C5	42	E5	77
C6	91	E6	86
C7	50	E7	31
C8	45	E8	17
C9	95	E9	95
C10	97	E10	93
C11	79	E11	72
C12	58	E12	49
C13	93	E13	98
C14	59	E14	47
C15	13	E15	93
C16	44	E16	98
C17	99	E17	80
C18	100		
C19	47		

The data for the unit assessment was analyzed individually for the overall score, while each question was analyzed overall including both groups. Figure 8 depicts a task in which students were asked to check the box of the quadrilaterals whose diagonals always bisects its opposite angles. Students must know the properties for each quadrilateral and be able to distinguish those properties that are unique to a specific shape. Students not being familiarized with those properties may be the reason as to why this question incorrectly answered by 62% of the students from both groups. Question 2 can be classified as Level 2, informal deduction, where students can establish the interrelationships of properties both within figures and among figures as described in Crowley (1987). The two quadrilaterals whose diagonals bisect its opposite angles include both the rhombi and squares.

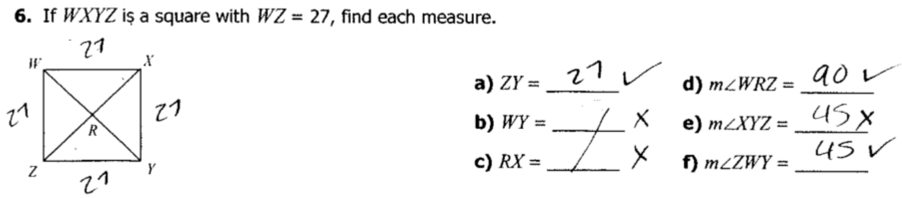
Figure 8: Unit 6 Assessment – Question #2


2. Which quadrilaterals always have diagonals that bisect opposite angles?

- ☐ Parallelograms
- ☐ Rectangles
- ☑ Rhombi
- ☑ Squares

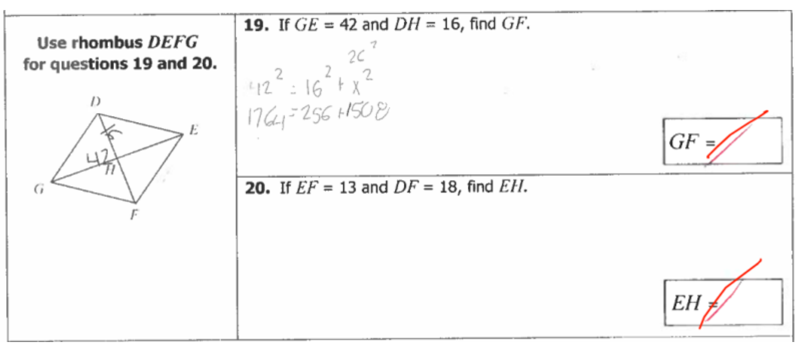
The second most incorrectly answered question, shown in Figure 9, also focuses on the properties of a rhombus. Students must first understand that the diagonals bisect each other and form 90-degree angles in the center. They must then use the Pythagorean theorem to find the length of the third side to determine the length of the second diagonal, which may have been the reason as to why most of the students missed this question. It is possible that students were not aware that the central angles form 90-degree angles, therefore the Pythagorean Theorem could be used. The third and fourth most missed questions were 6b and 5e, shown in Figure 9 and Figure 11 respectively, also included the properties of a square and rhombus, which based on these results, it seems that students struggle with the properties of a rhombus and square in general.

Figure 9: Unit 6 Assessment – Question 6e



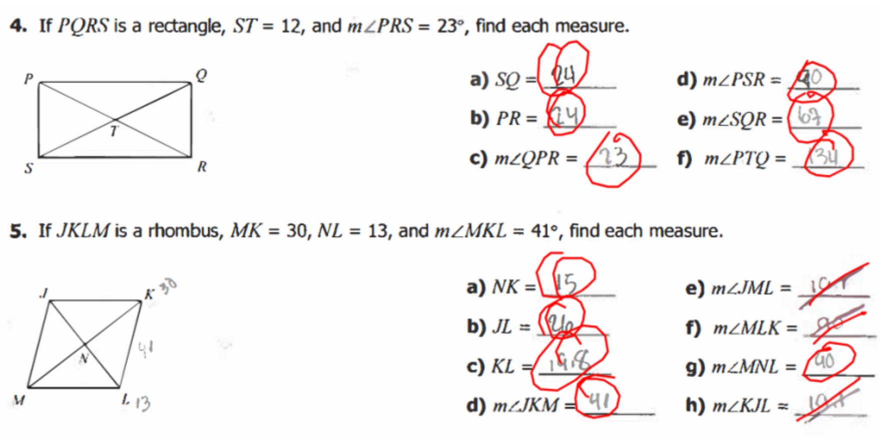
Having to solve for the missing segment using the Pythagorean Theorem may have been the reason why students missed these questions, like question 19 and 20 in Figure 10, however, that aspect of the question was not observed in this study due to the Pythagorean Theorem being explored extensively in the prior unit. Figure 10 does show that the student was aware that they needed to use the Pythagorean Theorem but were not able to do so successfully. The actual reason as to why it was not answered properly needs to further exploration.

Figure 10: Unit 6 Assessment – Question 20



On the contrary, the two most accurately answered questions are shown in Figure 11 below, included 4a, and 5a. These two questions would fall under the van Hiele Level 1, analysis, where students begin to distinguish the characteristics of figures and properties and are used to conceptualize classes of shapes as described in Crowley (1987). Based on the results, 89% of the students from both groups were able to accurately use the information given to determine that lengths of the diagonals of a parallelogram. In question 4a, students knew that ST represented half of SQ, so they simply had to double the value of ST to determine SQ. In question 5a, students were given the length of NK given that MK was 30.

Figure 11: Unit 6 Assessment – Question 4 and 5



The Perceptions Survey

Students in the treatment group were also given a perceptions survey. The studies presented in the literature review, lacked in this area. There were comments given by the teachers as to what they saw, however, not many actually surveyed their students to fully understand how they felt about working with DGS, such as GeoGebra. Table 5 shows the results of the Perceptions Survey given to the students after completing the unit assessment.

Table 5: Perception Survey - Results

Perception on Usage	1-2	3	4-5
1. GeoGebra is easy to use.	13%	39%	48%
11. GeoGebra guides me to explore the properties of quadrilaterals.	8%	35%	57%
14. I acquired a good experience from the opportunity to use GeoGebra.	4%	31%	65%
Perception on Understanding			
2. GeoGebra can illustrate the diagram more clearly.	13%	22%	65%
5. I get to learn the topic in greater depth.	13%	39%	48%
6. I can draw and solve problems with GeoGebra.	9%	4%	87%
8. I understand the lesson better when using GeoGebra compared to using textbooks.	17%	22%	61%
10. I am more confident at solving problems.	13%	48%	39%
13. Using GeoGebra is a good way for me to learn about the properties of quadrilaterals.	4%	22%	74%
Perception of Communication			
3. GeoGebra helps me in my discussion in the classroom.	13%	39%	48%
9. I get to interact with both my teacher and classmates in the Classroom when I use GeoGebra.	13%	22%	65%
Perception on Attitude Towards Geometry			
4. Learning mathematics is more fun now.	35%	35%	30%
7. Learning mathematics is easier now.	35%	35%	30%
12. Now I enjoy mathematics.	48%	30%	22%

Note. 1 (Strongly Disagree), 2 (Disagree), 3 (Neutral), 4 (Agree), 5 (Strongly Agree).

The data shows that only 26% of the students in this category agree that mathematics is more fun, easier to learn, and enjoy mathematics. Approximately 57% agree or strongly agree on the perception of usage on that GeoGebra is easy to use, guides them through the exploration activities, and had a good experience. Approximately 62% agreed or strongly agreed that GeoGebra did help on the perception of understanding. Students were able to illustrate the diagrams clearly, learned the topic in greater depth, prefer GeoGebra over the textbook, and improved their confidence. Approximately 57% agreed or strongly agreed that GeoGebra helped in classroom discussion with classmates and the teacher.

Students' perceptions regarding the use of GeoGebra were analyzed through an adapted version of a survey developed by Ganesan and Eu (2020), as mentioned before. The survey was implemented to determine a students' perception on the usage of GeoGebra, how it helps for understanding, on communication, and attitude towards geometry (Table 6).

Table 6: Item Analysis for Students' Perception Survey

Item	Mean	Standard Deviation
1	3.43	0.84
2	3.70	0.93
3	3.35	0.88
4	2.91	1.04
5	3.35	0.88
6	3.96	0.77
7	2.91	1.04
8	3.57	0.95
9	3.65	1.15
10	3.26	0.86
11	3.57	0.95
12	2.57	1.04
13	3.91	0.79
14	3.78	0.80

Overall, students seem to have a favorable perception while working with GeoGebra except for items 4, learning is more fun, (M = 2.91, SD = 1.04), 7 (M = 2.91, SD = 1.04), and 12, now I enjoy mathematics, (M= 2.57, SD = 1.04) as shown in Table 6 above, which represents the students' perception on their attitudes towards geometry. The items with the highest mean, 6, I can draw and solve problems with GeoGebra, (M = 3.96, SD = 0.77) and 13, using GeoGebra is a good way for me to learn about the properties of quadrilaterals, (M = 3.91, SD = 0.79) as seen in Table 4 above, show that students had a positive experience while exploring the properties of quadrilaterals using GeoGebra. The data in the perceptions survey gives teachers an idea of what students thought as they worked through the properties of quadrilaterals, data that is typically not included in the exploration of working with DGS. It is important that we take into consideration what students think about working with DGS and how it can benefit them in future math courses.

Results

Due to the data that was collected from the assessments, I was able to answer the research questions that drove my study. The unit assessment was used to determine conceptual understanding, the pre- and post-test were used to determine if students advanced in their van Hiele Level of understanding, and the perceptions survey to determine how students felt after exploring the properties of quadrilaterals using GeoGebra. The results are as follows.

How effective is GeoGebra when exploring the properties of quadrilaterals in conceptual understanding? The most missed questions, by more than 50% of the students combined from both groups, dealt with the properties of a rhombus and square, mainly that when the diagonals intersect, they form 90-degree angles in the center. Students were often required to use the Pythagorean theorem for these questions to calculate for the missing side. Whether or not this caused incorrect answers, was not explored in this study. The remaining questions were successfully answered by more than 50% of the total students. The unit assessment resulted in 53% of the comparison group and 59% of the treatment group passing with a grade of 70 or above. This shows that students working with DGS did perform better than the comparison group.

Based on the van Hiele levels of thinking, to what extent do the student's level of geometric thinking about the properties of quadrilaterals change after the implementation of GeoGebra? Although the mean of the pretest for the treatment group (M = 1.53) and posttest (M = 1.94) showed only a slight increase, the increase was slightly more than the comparison group pretest (M = 1.79), and posttest (M = 1.89). Most of the students in the treatment group, 82%, and 74% in the comparison group remained at the same level or showed improvement. Approximately 41% of the students in the treatment group and 37% in the comparison group did advance to the next van Hiele level, which does show improvement. This shows that the student's level of geometric thinking about the properties of quadrilaterals did slightly improve more in the treatment group. There were only two situations where the students went from a van Hiele level of 2, to a level 1, and one instance where the student went from a van Hiele level of 1 to a van Hiele level 0 in the treatment group. The comparison group had four students decrease one level, and one decreased two levels. This may have been caused by factors that were not focused by this study such as test fatigue. Around this time, some of the students went through state assessments as well as course final examinations. Overall, the treatment group did perform better overall than the comparison group.

What were the students' perception in the use and integration of GeoGebra while learning about geometric concepts, in particular the ones related to quadrilaterals? As far as perception, most students had a favorable perception while exploring the properties of quadrilaterals using GeoGebra. In the four categories presented in Table 2, three had a favorable outcome by most of the students, approximately 57% on the perception of usage, 62% on the perception of understanding, and 57% on the idea that they were given an opportunity to have open discussions with their classmates and teacher. The one category in which students scored it low was on their perception on their attitudes towards geometry, or mathematics in general. It seems that they do not believe mathematics is more fun, easier, or do they enjoy mathematics after using DGS. Although students' perceptions are not favorable towards mathematics, DGS did help in confidence and other areas needed when exploring mathematics concepts.

In conclusion, the data showed that the majority of the students who took the high school geometry course were not at the van Hiele level they should be coming into the course. Based on the pre- and post-test, the treatment group had more students advance to the next van Hiele level and the growth was slightly better than that of the comparison group. Although most of the students from both the comparison and treatment group were successful in the unit assessment, it is possible that they will continue to struggle in future assessments that contain geometry concepts, which is consistent with the results presented by NAEP (2022). The instruments used for my study provided the data used to answer my questions that drove the study, which is consistent with the results presented in the literature review while working with DGS.

GeoGebra provided opportunities for students to explore and have discussions on the properties of quadrilaterals. Students had positive experiences as they constructed and explored these properties, so teachers should consider using DGS early in their curriculums to ensure students become experienced with its many features and use it to its full potential. The data does conclude that students who work with DGS do perform better when exploring the properties of quadrilaterals.

Limitations

At the beginning of my study, I had originally selected one teacher to teach both the treatment and comparison group. Due to scheduling, two different teachers who taught the Honors courses were selected due to each only having one section each. Although we frequently met to discuss lesson delivery, I was not able to personally monitor the classes as the selected teachers delivered instruction. Also, due to the number of Honors courses scheduled for the semester, there were a limited number of participants who assented to be a part of the study.

CHAPTER V

DISCUSSION AND CONCLUSION

In this research project, I was able to collect evidence of the effectiveness of using GeoGebra while exploring the properties of quadrilaterals. My study focused on the effectiveness of GeoGebra while exploring the properties of quadrilaterals, the extent of a student's level of geometric thinking when exploring the properties of quadrilaterals, and a student's perception while working with GeoGebra. Although the data shows a slight advantage of using GeoGebra, it is effective when trying to provide an environment where students can construct multiple polygons in a fraction of the time it would take to use a compass or ruler and paper. My study also focuses on students' perceptions while using GeoGebra in their exploration process. A brief overview, conclusions of the data, and its implications are presented in the sections that follow.

Based on the literature, studies have focused on area, perimeter, volume, transformations, and functions. There is little research focusing on how DGS affects conceptual understanding while exploring the properties of quadrilaterals, which is why this was the area of focus for my study. GeoGebra contains many functions that help with measurement, constructions, and visualization all while promoting open discussions amongst students. Doing so can challenge a student's prior knowledge if it contradicts their current understanding, having them adjust that understanding based on those discussions as they reflect on the activities. As students constructed, manipulated, and explored the properties of quadrilaterals, they constructed new knowledge based on their findings. The students were at the center of their learning whereas the teacher in the study acted more as a facilitator, rather than being the main source of information.

Dynamic geometry software is a tool that students can use on multiple platforms, in many locations. It is an alternative to calculators, which are what teachers typically use when exploring geometric concepts, however, our school does not have the resources to issue out one per student. GeoGebra is a source that students can use without having to rely on the calculator, that contains many features that help with the exploration of geometric concepts. Based on the literature, the studies that utilized DGS, such as GeoGebra, had positive results due to its many features that helped in creating dynamic shapes easily and quickly allowing for classroom discussions as they manipulated the figure in real time.

GeoGebra provides an environment where students can make conjectures based on their experiences, therefore, constructing their own knowledge through constructivism. Through Vygotsky’s ZPD (Goodman, 2014), students can learn new concepts through scaffolding as they build on their prior knowledge of geometric concepts through collaboration. The theoretical framework defined by Webre et al. (2018) contains the five stages teachers can use to guide students through the geometric concepts as they act more like facilitator rather than the center of the knowledge as students improve their van Hiele level of understanding while working with DGS.

The pretest determined that most of the students in both the comparison and treatment group were not at a van Hiele Level 2, meaning that they would more than likely struggle with the high school geometry course, typically taught at a Level 3. The mean van Hiele Level for the comparison group and the treatment group showed that the students were in fact near a van Hiele

Level 1 but were classified as a van Hiele Level 0. This means that most students were able to recognize a shape by its appearance, not by their parts or properties, but struggled when analyzing a quadrilateral based on its properties and interrelationships of those properties amongst similar figures. This brought many challenges as the teachers prepared their lessons to accommodate the students to deliver quality instruction to ensure conceptual understanding. The teacher in the comparison group used teacher developed activities, while the teacher in the treatment group used GeoGebra.

The mean score for both the comparison and treatment groups in the posttest showed that most students were still not prepared for a high school geometry course as the mean score was closer to a van Hiele Level 1 for both groups. Although 41% of the students advanced a level, the data does show that they lack conceptual understanding of geometric concepts. This is consistent with the national assessment data presented by NAEP (2022) which shows that students continue to struggle with geometric concepts in their future mathematics courses. It is important that students understand these concepts due to them appearing in multiple national assessments after students complete the course.

The unit assessment showed that only 53% of the comparison group and 59% of the treatment group scored above a 70, meaning that students continued to struggle with the properties of quadrilaterals. Although the treatment group did perform slightly better than the comparison group in both the posttest and unit assessment, it does indicate that these concepts need to be further analyzed to determine how we can better prepare our students when exploring geometric concepts. The most missed questions dealt with the properties of squares and rhombi. Students seem to understand that a square and rectangle both have four 90-degree vertices but seemed to struggle with the central angles formed by the intersecting diagonals of a square and rhombus. The Pythagorean Theorem was needed to determine the length of a segment, which may be the reason why students struggled with these questions. Further research needs to be done to determine if this was a factor. The Pythagorean Theorem is covered extensively in the prior unit, so it may be possible that students were not able to make the connection that the central angles formed create four 90-degree angles, therefore not realizing that the Pythagorean Theorem needed to be used to answer the questions correctly.

The few studies that explored students' perceptions showed that students were motivated and stated that they understood the concepts better and found the lessons more interesting. Based on the data collected in this study, it seems that students had a positive experience working with GeoGebra, but still do not consider mathematics to be fun. The data also showed that students found GeoGebra challenging to use but the majority stated that it was a good way for them to learn about the properties of quadrilaterals. Approximately 65% of the students agreed that they were able to communicate with their classmates and teacher, therefore creating an environment where students can share ideas. For students to have a better experience, I do believe that GeoGebra would need to be used from the very beginning of the geometry course and should provide students with opportunities to explore through discussions. That way students will be better familiarized with its functions and have better experiences as students explore future geometric concepts.

GeoGebra contains many features that provides students with opportunities to explore the properties of quadrilaterals in a way that students can make their own conjectures. If used properly, teachers can utilize this tool to promote open discussions in a way that students can learn from these discussions as they take charge of their learning process. For my study, GeoGebra was used to explore the properties of quadrilaterals. Teachers may want to start working with GeoGebra from the very beginning so that students can become better familiarized with the many features as they progress through the concepts. Students will have a better understanding of geometry and DGS so that they may be able to fully use it to their advantage. The ability to create multiple versions of a shape helps with the exploration process, something that is very useful in geometry, especially if students are visual learners.

Overall, the data does seem to support that GeoGebra was effective based on the pre- and post-test, the unit assessment, and perceptions survey. The increase from the pretest to the posttest and the mean score for the unit assessment were both greater in the treatment group. The perceptions survey showed mainly positive responses to working with GeoGebra meaning that students had positive experiences with DGS.

I typically embed GeoGebra in several of the geometry units as the year progresses so that my students become familiar with all of the functions available through the exploring process and have used it with both my honors students, as well as my college-prep (CP) students. Both have expressed positive experiences, but were not formally observed. Due to my study focusing on unit 6 and a specific group of students, future studies should include both honors and CP students, an interview section to help determine the exact cause of why students incorrectly answered similar questions, such as those that included the Pythagorean Theorem, and possibly embedding GeoGebra throughout the curriculum as opposed to a specific unit. I believe teachers will benefit from this study due to the students' continued struggle with geometric concepts. My study is consistent with the studies in the literature review which included area, perimeter, volume, transformations, and circles whereas teachers reported positive results while using DGS. Based on the results of my study, it is recommended that teachers incorporate DGS throughout their curriculums and focus on lessons that can possibly change students' attitudes towards mathematics.

In the future, I would like to work with teachers to design a curriculum that benefits all students coming into a high school geometry course. My study showed that the majority of the students were coming without having the proper skills and knowledge needed to be successful. It also showed that students may struggle with future assessments based on the posttest results. I would like to conduct a study as mentioned above to fully understand how we can better prepare our students, through the perspective of the student, for future assessments that include a geometry portion.

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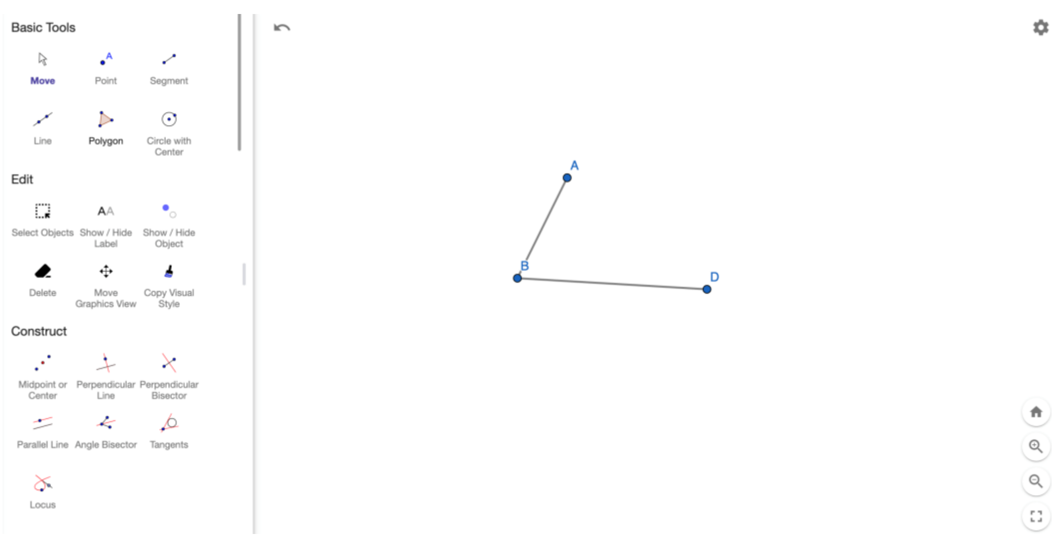
APPENDIX A

APPENDIX A

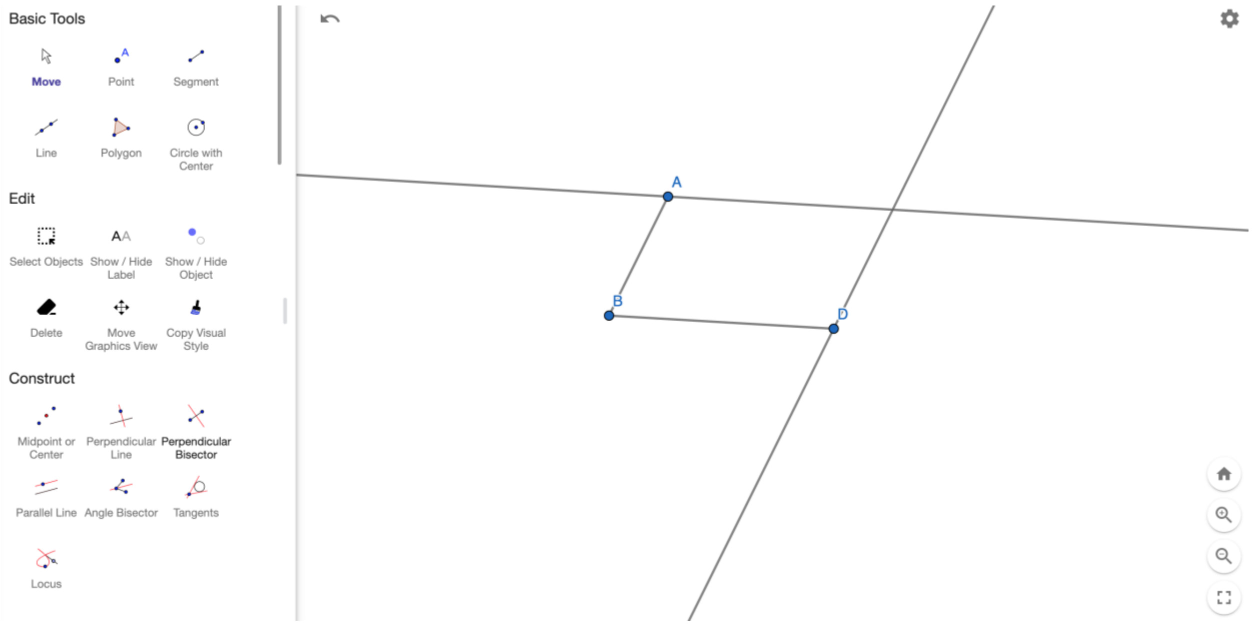
CONSTRUCTING A PARALLELOGRAM

Step 1: Plot three points in the Geometry section of GeoGebra using the *Point* function.

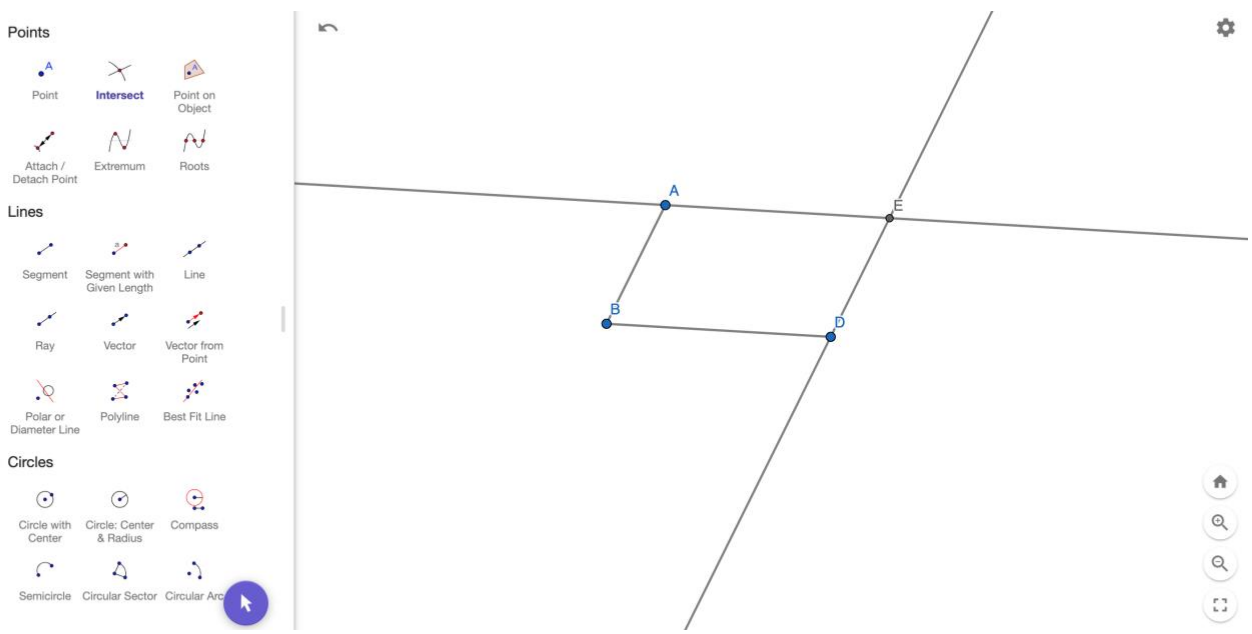
Step 2: Connect Points A and B using the *Segment* function. Do the same with Points B and D.



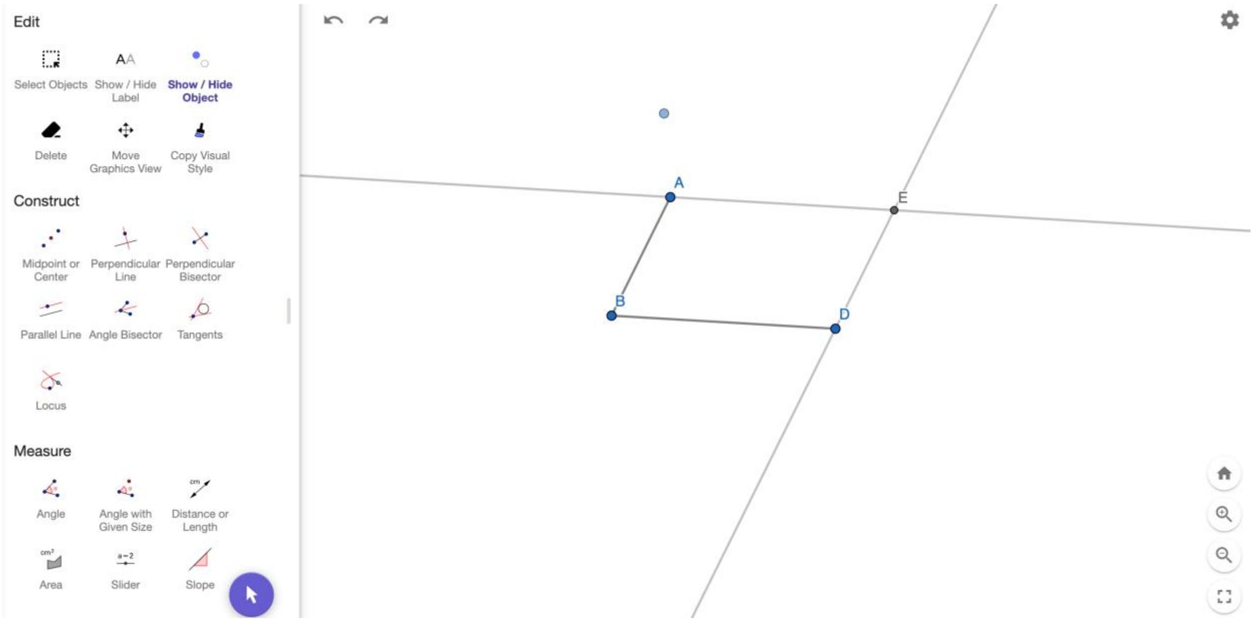
Step 3: Use the *Parallel Line* function to draw the line parallel to Line BD by clicking on Line BD and Point A. Repeat the step with Line AB and Point D.



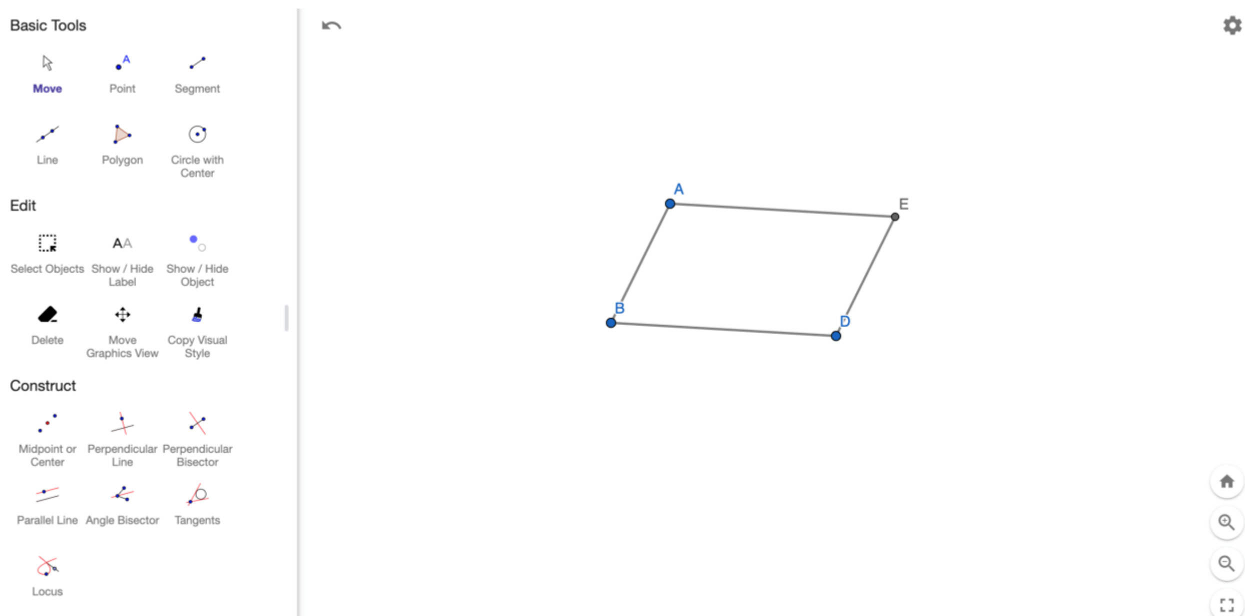
Step 4: Use the *Intersect* function to plot the point where the two lines intersect.



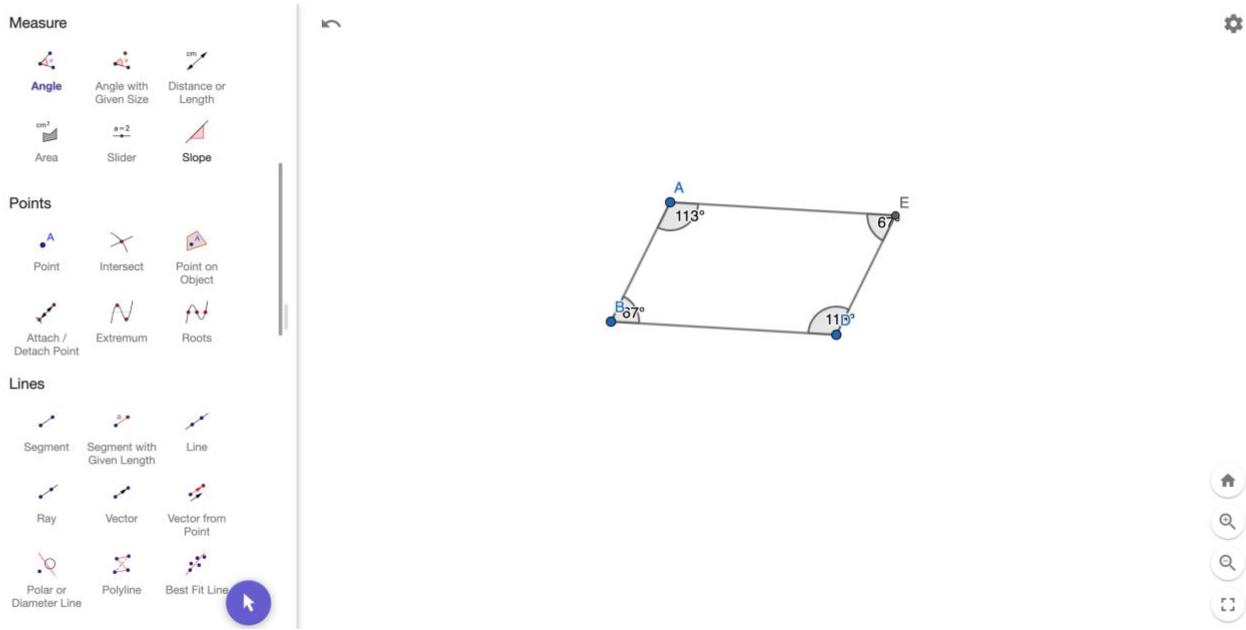
Step 5: Select the *Show / Hide Object* and click on the two intersecting lines. Then click on the *Move* function.



Step 6: Select the *Segment* function to connect Points A and E, and E and D. Select a vertex and modify the figure by *dragging*.



Step 7: To display the angle measures, select the *Angle* function, then select three points with the middle point being the vertex. Do this to all four angles.



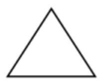
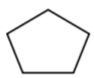
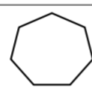
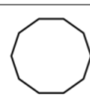
APPENDIX B

APPENDIX B

ACTIVITY 1: SUM OF THE INTERIOR AND EXTERIOR ANGLES OF A POLYGON

Directions: The sum of the interior angles of a triangle is 180°.

- Fix one vertex and make triangles by connecting to vertices of the polygon.
- Generate a rule to find the sum of interior angles of any polygon.

Name of Polygon	Figure	Number of Sides	Number of Triangles	Sum of Interior Angles
Triangle		3		180°
Quadrilateral				
Pentagon				
Hexagon				
Heptagon				
Octagon				
Nonagon				
Decagon				
n-sided	XXXXXX			

Write a rule to find the sum of interior angles for any polygon: ___________________________________

Exercise: Using the rule above, find the sum of interior angles of the given polygon.

1. Quadrilateral

2. Pentagon

3. Octagon

4. A polygon with 11 sides

5. A polygon with 15 sides

APPENDIX C

APPENDIX C

THE VAN HIELE TEST

 Test Number ____________

VAN HIELE GEOMETRY TEST*

Directions

Do not open this test booklet until you are told to do so.

This test contains 25 questions. It is not expected that you know everything on this test.

There is a test number in the top right hand corner of this page. Write this number in the corresponding place on your answer sheet.

When you are told to begin:

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.
3. Use the space provided on the answer sheet for figuring or drawing. Do not mark on this test booklet.
4. If you want to change an answer, completely erase the first answer.
5. If you need another pencil, raise your hand.
6. You will have 35 minutes for this test.

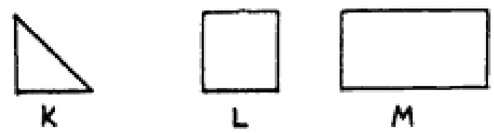
Wait until your teacher says that you may begin.

*This test is based on the work of P.M. van Hiele.

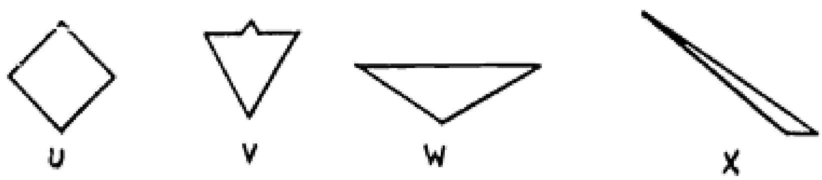
VAN HIELE GEOMETRY TEST

1. Which of these are squares?

   (A) K only
   (B) L only
   (C) M only
   (D) L and M only
   (E) All are squares.

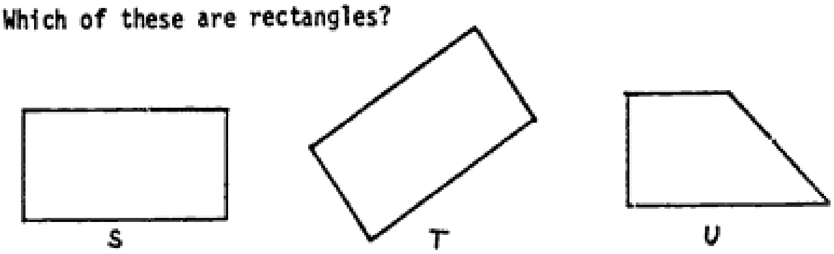


2. Which of these are triangles?



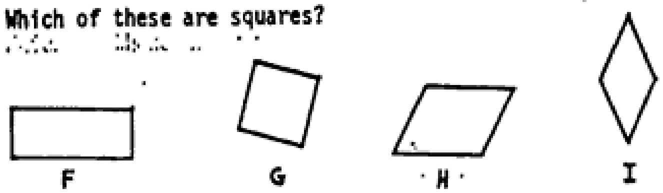
   (A) None of these are triangles.
   (B) V only
   (C) W only
   (D) W and X only
   (E) V and W only

3. Which of these are rectangles?



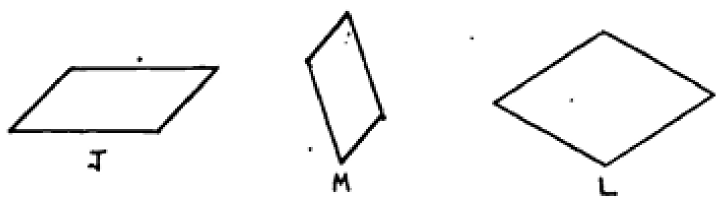
   (A) S only
   (B) T only
   (C) S and T only
   (D) S and U only
   (E) All are rectangles.

4. Which of these are squares?



(A) None of these are squares.
(B) G only
(C) F and G only
(D) G and I only
(E) All are squares.

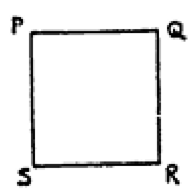
5. Which of these are parallelograms?



(A) J only
(B) L only
(C) J and M only
(D) None of these are parallelograms.
(E) All are parallelograms.

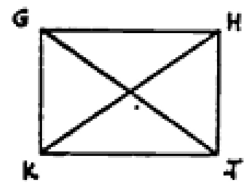
6. PQRS is a square.

Which relationship is true in all squares?



(A) $\overline{PR}$ and $\overline{RS}$ have the same length.
(B) $\overline{QS}$ and $\overline{PR}$ are perpendicular.
(C) $\overline{PS}$ and $\overline{QR}$ are perpendicular.
(D) $\overline{PS}$ and $\overline{QS}$ have the same length.
(E) Angle Q is larger than angle R.

7. In a rectangle GHJK, $\overline{GJ}$ and $\overline{HK}$ are the diagonals.



Which of (A)-(D) is not true in every rectangle?

(A) There are four right angles.

(B) There are four sides.

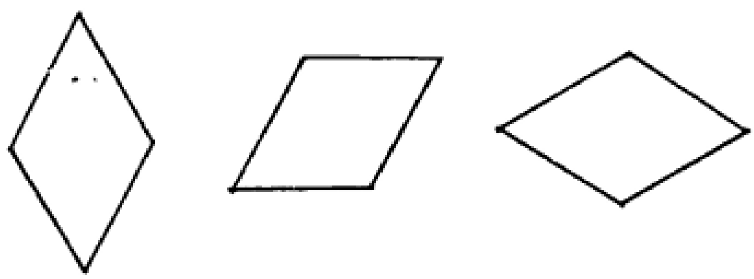
(C) The diagonals have the same length.

(D) The opposite sides have the same length.

(E) All of (A)-(D) are true in every rectangle.

8. A rhombus is a 4-sided figure with all sides of the same length.

Here are three examples.



Which of (A)-(D) is not true in every rhombus?

(A) The two diagonals have the same length.

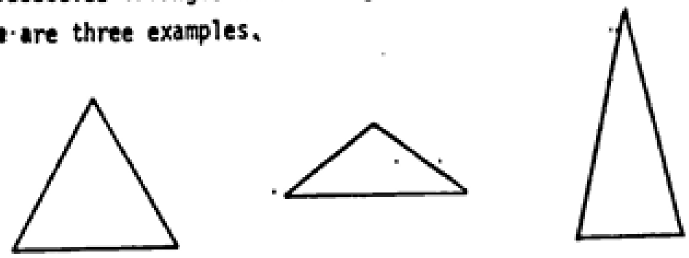
(B) Each diagonal bisects two angles of the rhombus.

(C) The two diagonals are perpendicular.

(D) The opposite angles have the same measure.

(E) All of (A)-(D) are true in every rhombus.

9. An isosceles triangle is a triangle with two sides of equal length.
Here are three examples.



Which of (A)-(D) is true in every isosceles triangle?

(A) The three sides must have the same length.

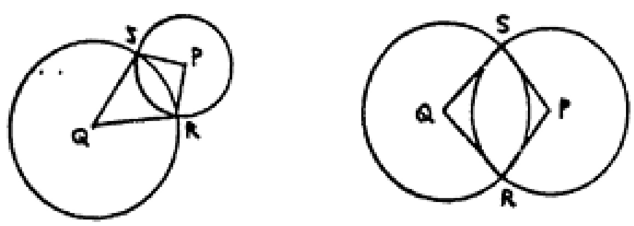
(B) One side must have twice the length of another side.

(C) There must be at least two angles with the same measure.

(D) The three angles must have the same measure.

(E) None of (A)-(D) is true in every isosceles triangle.

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.



Which of (A)-(D) is <u>not</u> always true?

(A) PRQS will have two pairs of sides of equal length.

(B) PRQS will have at least two angles of equal measure.

(C) The lines $\overline{PQ}$ and $\overline{RS}$ will be perpendicular.

(D) Angles P and Q will have the same measure.

(E) All of (A)-(D) are true.

11. Here are two statements.

    Statement 1: Figure F is a rectangle.
    Statement 2: Figure F is a triangle.

Which is correct?

(A) If 1 is true, then 2 is true.

(B) If 1 is false, then 2 is true.

(C) 1 and 2 cannot both be true.

(D) 1 and 2 cannot both be false.

(E) None of (A)-(D) is correct.

12. Here are two statements.

    Statement S: $\triangle ABC$ has three sides of the same length.
    Statement T: In $\triangle ABC$, $\angle B$ and $\angle C$ have the same measure.

Which is correct?

(A) Statements S and T cannot both be true.

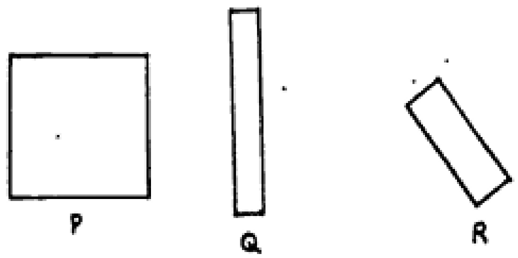
(B) If S is true, then T is true.

(C) If T is true, then S is true.

(D) If S is false, then T is false.

(E) None of (A)-(D) is correct.

13. Which of these can be called rectangles?

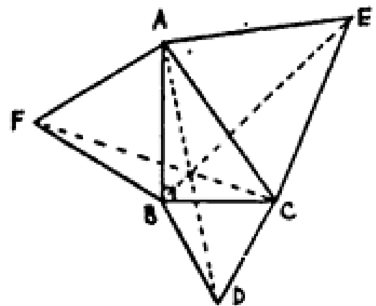


(A) All can.
(B) Q only
(C) R only
(D) P and Q only
(E) Q and R only

14. Which is true?
(A) All properties of rectangles are properties of all squares.
(B) All properties of squares are properties of all rectangles.
(C) All properties of rectangles are properties of all parallelograms.
(D) All properties of squares are properties of all parallelograms.
(E) None of (A)-(D) is true.

15. What do all rectangles have that some parallelograms do not have?
(A) opposite sides equal
(B) diagonals equal
(C) opposite sides parallel
(D) opposite angles equal
(E) none of (A)-(D)

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common. What would this proof tell you?

(A) Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(B) In some but not all right triangles, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(C) In any right triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(D) In any triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(E) In any equilateral triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.
Property S: It is a square.
Property R: It is a rectangle.

Which is true?

(A) D implies S which implies R.
(B) D implies R which implies S.
(C) S implies R which implies D.
(D) R implies D which implies S.
(E) R implies S which implies D.

18. Here are two statements.

    I: If a figure is a rectangle, then its diagonals bisect each other.
    II. If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

(A) To prove I is true, it is enough to prove that II is true.

(B) To prove II is true, it is enough to prove that I is true.

(C) To prove II is true, it is enough to find one rectangle whose diagonals bisect each other.

(D) To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.

(E) None of (A)-(D) is correct.

19. In geometry:

(A) Every term can be defined and every true statement can be proved true.

(B) Every term can be defined but it is necessary to assume that certain statements are true.

(C) Some terms must be left undefined but every true statement can be proved true.

(D) Some terms must be left undefined and it is necessary to have some statements which are assumed true.

(E) None of (A)-(D) is correct.

20. Examine these three sentences.

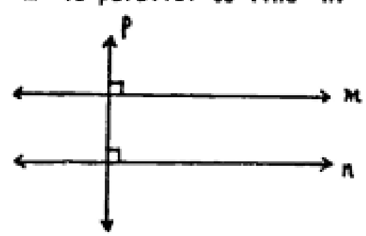
(1) Two lines perpendicular to the same line are parallel.

(2) A line that is perpendicular to one of two parallel lines is perpendicular to the other.

(3) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?

(A) (1) only
(B) (2) only
(C) (3) only
(D) Either (1) or (2)
(E) Either (2) or (3)



21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R, and S, the lines are {P,Q} , {P;R} ; {P,S} , {Q,R} , {Q,S} , and {R,S}

Here are how the words "intersect" and "parallel" are used in F-geometry. The lines {P,Q} and {P,R} intersect at P because {P,Q} and {P,R} have P in common.

The lines {P,Q} and {R,S} are parallel because they have no points in common.

From this information, which is correct?

(A) {P,R} and {Q,S} intersect.
(B) {P,R} and {Q,S} are parallel.
(C) {Q,R} and {R,S} are parallel.
(D) {P,S} and {Q,R} intersect.
(E) None of (A)-(D) is correct.

22. To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?

(A) In general, it is impossible to bisect angles using only a compass and an unmarked ruler.

(B) In general, it is impossible to trisect angles using only a compass and a marked ruler.

(C) In general, it is impossible to trisect angles using any drawing instruments.

(D) It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.

(E) No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

23. There is a geometry invented by a mathematician J in which the following is true:

The sum of the measures of the angles of a triangle is less than 180°.

Which is correct?

(A) J made a mistake in measuring the angles of the triangle.

(B) J made a mistake in logical reasoning.

(C) J has a wrong idea of what is meant by "true."

(D) J started with different assumptions than those in the usual geometry.

(E) None of (A)-(D) is correct.

24. Two geometry books define the word rectangle in different ways. Which is true?

(A) One of the books has an error.

(B) One of the definitions is wrong. There cannot be two different definitions for rectangle.

(C) The rectangles in one of the books must have different properties from those in the other book.

(D) The rectangles in one of the books must have the same properties as those in the other book.

(E) The properties of rectangles in the two books might be different.

25. Suppose you have proved statements I and II.

I. If p, then q.
II. If s, then not q.

Which statement follows from statements I and II?

(A) If p, then s.

(B) If not p, then not q.

(C) If p or q, then s.

(D) If s, then not p.

(E) If not s, then p.

Test Number ____________

VAN HIELE GEOMETRY TEST

ANSWER SHEET

Project Use Only J

ID_____ _______________

Please print

Name_____________ _________________________________ Class period___________

Last First Middle

Math Teacher ________________________ School ____________________________

Grade in School (circle): 8 9 10 11 12 Sex (circle): M F

Birth date ______ ______ ______ Test date ______ ______ ______

Month Day Year Month Day Year

Space for drawing or figuring
(You may also use the other side)

Cross out the correct answer

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E
16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E
21. A B C D E
22. A B C D E
23. A B C D E
24. A B C D E
25. A B C D E

APPENDIX D

APPENDIX D

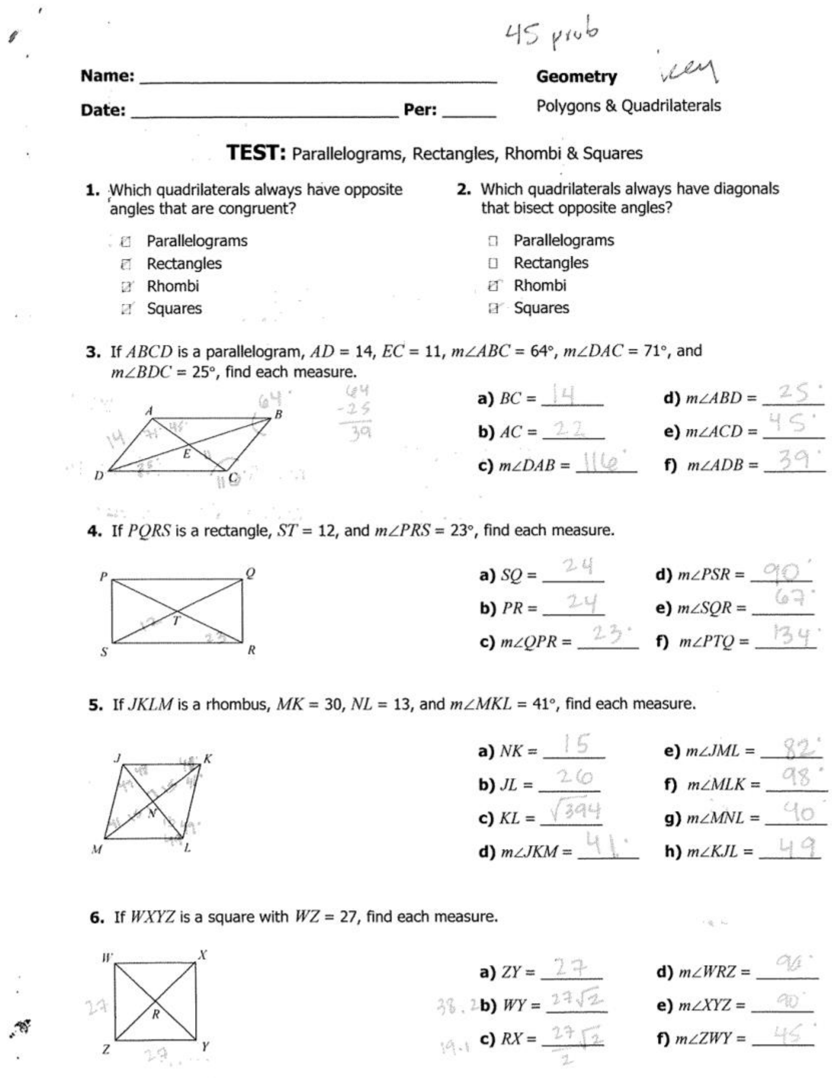
STUDENTS' PERCEPTION SURVEY

Item	1 Strongly Disagree	2 Disagree	3 Neutral	4 Agree	5 Strongly Agree
1. GeoGebra is easy to use.					
2. GeoGebra can illustrate the diagram more clearly.					
3. GeoGebra helps me in my discussion in the classroom.					
4. Learning mathematics is more fun now.					
5. I get to learn the topic in greater depth.					
6. I can draw and solve problems with GeoGebra.					
7. Learning mathematics is easier now.					
8. I understand the lesson better when using GeoGebra compared to using textbooks.					
9. I get to interact with both my teacher and classmates in the classroom when I use GeoGebra.					
10. I am more confident at solving problems.					
11. GeoGebra guides me to explore the properties of quadrilaterals.					
12. Now I enjoy mathematics.					
13. Using GeoGebra is a good way for me to learn about the properties of quadrilaterals.					
14. I acquired a good experience from the opportunity to use GeoGebra.					

APPENDIX E

APPENDIX E

UNIT 6 ASSESSMENT – POLYGONS AND QUADRILATERALS



45 prob

Name: ______________________ **Geometry** key

Date: ______________ **Per:** ____ Polygons & Quadrilaterals

TEST: Parallelograms, Rectangles, Rhombi & Squares

1. Which quadrilaterals always have opposite angles that are congruent?

- ☑ Parallelograms
- ☑ Rectangles
- ☑ Rhombi
- ☑ Squares

2. Which quadrilaterals always have diagonals that bisect opposite angles?

- ☐ Parallelograms
- ☐ Rectangles
- ☑ Rhombi
- ☑ Squares

3. If $ABCD$ is a parallelogram, $AD = 14$, $EC = 11$, $m\angle ABC = 64°$, $m\angle DAC = 71°$, and $m\angle BDC = 25°$, find each measure.

a) $BC =$ 14 **d)** $m\angle ABD =$ 25°

b) $AC =$ 22 **e)** $m\angle ACD =$ 45°

c) $m\angle DAB =$ 116° **f)** $m\angle ADB =$ 39°

4. If $PQRS$ is a rectangle, $ST = 12$, and $m\angle PRS = 23°$, find each measure.

a) $SQ =$ 24 **d)** $m\angle PSR =$ 90°

b) $PR =$ 24 **e)** $m\angle SQR =$ 67°

c) $m\angle QPR =$ 23° **f)** $m\angle PTQ =$ 134°

5. If $JKLM$ is a rhombus, $MK = 30$, $NL = 13$, and $m\angle MKL = 41°$, find each measure.

a) $NK =$ 15 **e)** $m\angle JML =$ 82°

b) $JL =$ 26 **f)** $m\angle MLK =$ 98°

c) $KL =$ $\sqrt{394}$ **g)** $m\angle MNL =$ 90

d) $m\angle JKM =$ 41° **h)** $m\angle KJL =$ 49

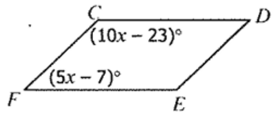
6. If $WXYZ$ is a square with $WZ = 27$, find each measure.

a) $ZY =$ 27 **d)** $m\angle WRZ =$ 90°

b) $WY =$ $27\sqrt{2}$ **e)** $m\angle XYZ =$ 90°

c) $RX =$ $\frac{27\sqrt{2}}{2}$ **f)** $m\angle ZWY =$ 45°

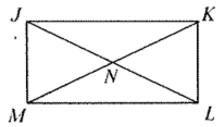
7. If $CDEF$ is a parallelogram, find $m\angle FCD$. **7.** 117°



10x − 23 + 5x − 7 = 180
15x − 30 = 180
x = 14

10(14) − 23
117

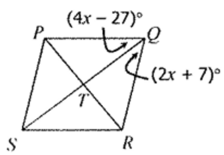
8. If $JKLM$ is a rectangle, $JL = 2x + 5$, and $MK = 7x - 40$, find MK. **8.** 23



7x − 40 = 2x + 5
x = 9

7(9) − 40
63 − 40
23

9. If $PQRS$ is a rhombus, find $m\angle PQR$. **9.** 82°



4x − 27 = 2x + 7
2x = 34
x = 17

m∠QRS = 2(17) + 7 = 41
m∠PQR = 2(41) = 82

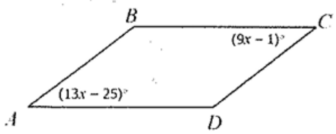
10. Quadrilateral $BCDE$ has vertices $B(-1, -1)$, $C(6, -2)$, $D(5, -9)$, and $E(-2, -8)$. Determine the most precise classification of $BCDE$: a parallelogram, rectangle, rhombus, or square. Use the distance formula to justify your answer.

BC
DE
CD
BE

dig BD
CE

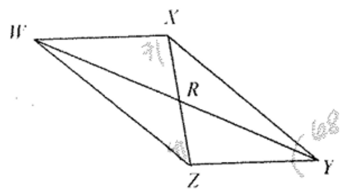
10. $BCDE$ is a square

11. If $ABCD$ is a parallelogram, find $m\angle D$.



$m\angle D$ = 127°

Use parallelogram $WXYZ$ for questions 12 and 13.



12. If $m\angle XYZ = 68°$ and $m\angle WXZ = 71°$, find $m\angle WZX$.

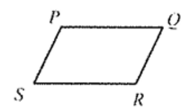
$m\angle WZX$ = 41°

13. If $XZ = 8x - 18$ and $RZ = 2x + 5$, find XR.

XR = 19

14. The vertices of quadrilateral $PQRS$ are given below. Use the distance and/or slope formulas to determine is $PQRS$ is a parallelogram. Use the diagram a a guide.

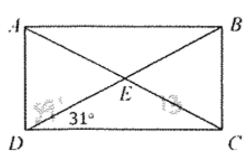
$P(-6, 4)$, $Q(-2, 7)$, $R(-1, 0)$, $S(-5, -3)$



☑ $PQRS$ is a parallelogram

☐ $PQRS$ is not a parallelogram

Use rectangle $ABCD$ for questions 14-16.



15. If $EC = 13$, find BD.

BD = 26

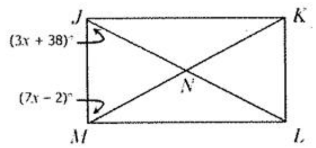
16. Find $m\angle ADB$.

$m\angle ADB$ = 59°

17. Find $m\angle DEC$.

$m\angle DEC$ = 118°

18. If *JKLM* is a rectangle, find $m\angle NML$.

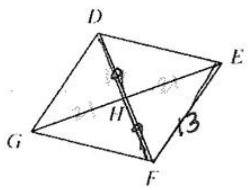


7x − 2 = 3x + 38
4x = 40
x = 10

7(10) − 2 = 68°
90
−68
22

$m\angle NML$ = 22°

Use rhombus *DEFG* for questions 19 and 20.



19. If *GE* = 42 and *DH* = 16, find *GF*.

$21^2 + 16^2 = c^2$
$\sqrt{697} = c$

26.4

GF = $\sqrt{697}$

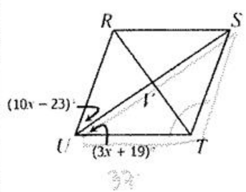
20. If *EF* = 13 and *DF* = 18, find *EH*.

$13^2 - 9^2 = b^2$
$\sqrt{88} = b$
$2^3 \cdot 11$ $2\sqrt{22}$

9.4

EH = $2\sqrt{22}$

21. If *RSTU* is a rhombus, find $m\angle UTS$.

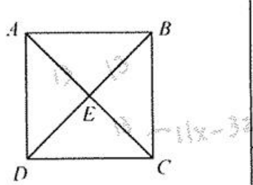


10x − 23 = 3x + 19
7x = 42
x = 6

3(6) + 19 = 37°
180 − 37 − 37

$m\angle UTS$ = 106°

Use square *ABCD* for questions 21 and 22.



22. If *AC* = 26, find *BC*.

~~$a^2 + a^2 = 26^2$~~ $13^2 + 13^2 = c^2$
~~$2a^2 = 26^2$~~ $\sqrt{338} = c$

18.4

BC = $13\sqrt{2}$

23. If $m\angle ACB = (11x - 32)°$, find the value of *x*.

11x − 32 = 45
11x = 77
x = 7

x = 7

VITA

Jose J. Jaramillo earned their degree in Doctor of Education – Curriculum and Instruction Specializing in Mathematics from the University of Texas Rio Grande Valley in July 2024. He also received a Master of Science in Computer Science Engineering in 2017 and a Bachelor of Science in Computer Science Engineering in 2003. Jose can be reached at josejjaramillo79@gmail.com.