8-1-2024

Redefining Spaces of Resistance Against Capitalism and Patriarchy: A Study of Females in Society and the Family as Presented by Mrs. Warren's Profession (1893) by Bernard Shaw and A Raisin in the Sun (1959) by Lorraine Hansberry

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UNVEILING EUREKA MATH: A CRITICAL PEEK BEHIND THE CURRICULAR CURTAIN

A Dissertation

by

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Submitted in Partial Fulfillment of the

Requirement for the Degree of

DOCTOR OF EDUCATION

Major Subject: Curriculum and Instruction

The University of Texas Rio Grande Valley
August 2024

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August 2024

ABSTRACT

Goldin-Weeks, Jennifer L., Unveiling Eureka Math: A Critical Peek Behind the Curricular Curtain. Doctor of Education (Ed.D.), August 2024, 260 pp., 13 tables, 42 figures, 144 references.

While contemporary mathematics education literature argues that curricular resources, such as textbooks, play a significant role guiding mathematics teachers in what and how they teach, it also shows that these resources often do not do what they say they will do, nor adhere to the standards they claim to further. When it comes to the most utilized mathematics curricular resource in the United States, Eureka Math, there is a lack of scholarly literature investigating these types of claims made by the publisher. This study aimed to explore how the Eureka Math curricular resources are positioned within the context of the Common Core State Standards of Mathematical Practice (CCSS-MP) and mathematics education more broadly and, subsequently, how the content of the resources then positions it in relation to the constructivist-aligned practices of the CCSS-MP.

Through a descriptive case study design, I first engaged in a critical discourse analysis of two webpages of the Eureka Math website to provide both a description and an interpretation of the text and images, followed by an explanation of a macro-level context of mathematical crisis that was revealed through the relevant literature. I then conducted a content analysis of tasks provided in a module of the Eureka Math curriculum using an adapted framework focused on three characteristics deemed essential for high-quality context-based tasks: type of context, type of information, and type of cognitive demand.

Findings from the critical discourse analysis revealed that Eureka Math's online platform functions to convince consumers that their products provide a superior level of rigor, real-world problem solving, and conceptual understanding through its implementation. The content analysis showed that overall, few tasks (25%) throughout the module were context-based tasks, with very few (4%) being high-quality in the type of information provided, even fewer (3%) in the type of cognitive demand required, and none (0%) in the type of context featured. While this study focuses on the Eureka Math curriculum specifically, it aims to also inform broader discussions of mathematics curriculum and, more broadly, conversations about curriculum decision-making in all content areas.

DEDICATION

I dedicate this piece to my family. While this culmination of my doctoral learnings is a true accomplishment, my greatest joy will forever be my place in your lives as daughter, sister, mama, and wife.

ACKNOWLEDGEMENT

As I have expressed many times, I could not have accomplished this goal without the unwavering support and guidance of my committee.

Dr. Laura Jewett, you were the perfect mentor for me – from celebrating my strengths to pulling me up when I had self-doubt. You will always remain one of the smartest people I have ever known, and I will forever look up to you. I was honored when you agreed to take me under your wing as a dissertation candidate, and I am humbled to look back at the work I have accomplished through your guidance. Thank you for never giving up on me.

Dr. Aguilar, your expertise in mathematics education was a rock upon which I could build my work. You worked with me as a partner when guidance from afar was not enough. Your empathetic words of encouragement that encompassed my life as a mother provided me with necessary perspective in my greatest times of need.

Dr. Badenhorst, thank you for the long hours you spent reviewing the countless iterations of my work that I have sent you. Your patience in skillfully guiding me through your area of passion of critical discourse analysis made all the difference in my success. I appreciate how you went above and beyond the call for me.

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CHAPTER I

INTRODUCTION

This study was inspired by my professional experiences both in my role as a middle school math teacher and as a district-level leader of a K-12 mathematics program at a public charter school district. During my time in these positions, I utilized and led the implementation of the Eureka Math curricular resource in my own classroom and in the classrooms that I oversaw in grades K-12. In the early years of the initial implementation of the Common Core State Standards (CCSS) in New York State, I had the unique opportunity to work alongside two teacher colleagues who were hired by the vendor charged with the creation of the EngageNY English Language Arts (ELA) curriculum. This curriculum was created and disseminated side-by-side with the Eureka Math (EngageNY Math) curriculum under the same initiative by the New York State Education Department (NYSED). This experience allowed me to get a look behind the curtain, figuratively speaking, to see the creation, revision, and distribution process of a state-produced curricular resource running parallel to that of the Eureka Math (EngageNY Math) curriculum.

The purpose of this descriptive case study is to explore how the Eureka Math curricular resources are positioned within the context of the Common Core State Standards of Mathematical Practice (CCSS-MP) and mathematics education more broadly and subsequently how the content of the resource then positions it in relation to the constructivist-aligned practices of the CCSS-MP. This study provides insight into the context in which Eureka Math's publisher, Great Minds, has positioned the curricular resource and how closely the lessons align with the constructivist-aligned approaches embedded in the CCSS-MP. Research shows that often curriculum designed for the teaching and learning of mathematics does not, in fact, do what it says it will do, nor adhere to the standards to which it claims alignment (EdReports, 2015, Polikoff, 2015; Remillard & Kim, 2020). While a study for practitioners from EdReports (2015) shows that the Eureka Math curricular resource aligns to both the Common Core State Standards of Mathematical Content (CCSS-MC) and the CCSS-MP, there is a lack of scholarly literature investigating this claim. In fact, there is often little literature investigating curricular claims regarding the alignments, content, and anticipated outcomes of specific curricula, even though such claims are influential in terms of adoption decisions. As Phillips (2014) advocates, "selecting good textbooks is important for student success" (p. 182). Similarly, the purpose of this descriptive case study was to provide a deep analysis of a specific curriculum. This study aims to explore how the Eureka Math curricular resources are positioned within the context of the CCSS-MP and mathematics education more broadly and subsequently how the content of the resource then positions it in relation to the constructivist-aligned practices of the CCSS-MP guided by the following questions:

- Research Question 1: How does Great Minds discursively position the Eureka Math curricular resources within the context of the Common Core State Standards of Mathematical Practice and more broadly, mathematics education?
- Research Question 2: How does the content of Eureka Math curricular resources produced by Great Minds position the curriculum in relation to constructivist-aligned approaches to the Common Core State Standards of Mathematical Practice?

While this study focused on the Eureka Math curriculum specifically, this study also stands to inform broader discussions of mathematics curriculum and, more broadly still, conversations about curriculum decision-making.

Toward these ends, Chapter II provides a review of the literature divided into eleven sections. First, I provide an explanation of the theoretical framework that undergirds this study. In the next two sections, I share information about the role of textbooks in mathematics education and the process by which textbooks are often chosen for use. Next, I share findings from contemporary literature regarding which mathematics curricular resources have been found to be the most utilized in the United States. Further, I introduce a framework that has been utilized in numerous research studies and which provides structure to the process that it is believed mathematical tasks go through from creation to student learning. I continue with the background of the Common Core State Standards of Mathematics (CCSS-M) and an explanation of both sets of substandards that make up the standards as a whole. I then provide an overview of research studies conducted previously to explore differing aspects of mathematics textbooks. Next, I provide an overview of information regarding Eureka Math, the most widely used mathematical curricular resource in the United States, including a background of the creation of the curriculum, the accessibility options, the structure of the resource, and details regarding a recent large-scale review that was done of its materials. I follow with a look into what contemporary literature says about the broader context of mathematics education that the Eureka Math curriculum is positioned within. I continue by presenting literature revealing the significant level of difficulty that U.S. students have with proportional reasoning. I conclude with a section explaining the significance of providing students with opportunities to engage in context-based tasks in the mathematics classroom paired with detailing the characteristics of those tasks that make them effective.

In Chapter III, I discuss how I will pursue this descriptive case study. I explain why I contemplated several different methodologies and my reasoning for ultimately choosing to employ a descriptive case study. In the first section, I describe the overarching structure of this descriptive case study which features content analysis and discourse analysis. I explain both data sources and the processes I employed to analyze the data derived. I do this by first explaining Fairclough's (2014) micro-meso-macro structure that supported my critical discourse analysis (CDA) of the publicly available materials from the Eureka Math publisher, Great Minds. I then connect this to a larger social context that this study is positioned within to explain the appropriateness of this approach to analysis. Next, I share the specifics about the content source, the Eureka Math curricular resource itself, including how the specific lessons to be analyzed were chosen. This section is followed up with an explanation of the data analysis process I utilized including details regarding the adapted instrument that framed my analysis. I then discuss how trustworthiness was ensured as well as share the personality of the researcher. The chapter closes with an appendix of a visual of the instrument in table format.

In Chapter IV, I detail my findings that resulted from engaging in a critical discourse analysis to look at two webpages of the Great Minds website in a multimodal exploration. I examined the ways that Great Minds describes the Eureka Math curriculum using Fairclough's (2014) 3D model and details the findings revealed at the micro- and meso-levels. I then present my findings from contemporary literature that describe the broader macro-context of a *crisis* of mathematics education that the curriculum is positioned within. The findings in this chapter serve as a mechanism to pull back the veil of the larger context, allowing for a more complete exploration of Eureka Math's positioning in relation to the CCSS-MP by way of the content of the curricular resource in the next chapter.

In Chapter V, I report the findings from a content analysis of the tasks within Grade 5, Module 3 of the Eureka Math curriculum. I utilize a framework adapted from the work of Wijaya et al. (2015) and Remillard and Kim (2020) to explore three characteristics that they deemed essential to high-quality context-based tasks including the *nature of the context, the type of information given in the tasks,* and *the level of cognitive demand that the task requires.* I report findings found as a whole and detailed within the Application Problem, Concept Development, and Problem Set components of the lessons.

In the final chapter, Chapter VI, I interpret the findings from the critical discourse analysis featured in Chapter IV and content analysis of Chapter V using contemporary research. I explain possible implications on further research both in the ways this study could be modified in future replications and the ways it could be expanded beyond its current scope. I conclude with the ways that this study may serve a significant role in informing discussions about Eureka Math, mathematics curriculum as a whole, and even more broadly, curriculum decision-making in content areas beyond mathematics.

Both historically and currently there exists a call from researchers for increased focus on the analysis of mathematical textbooks (Ball & Feiman-Nesmer, 1988; Baker et al., 2010; National Research Council, 2004; Reys et al., 2004; Senk & Thompson, 2003; Stylianides, 2014; Weiss et al., 2001; Woodward & Elliot, 1990). It has been recognized that textbooks play a significant role in guiding mathematics teachers in what they teach and how they teach it (Arican, 2018; Brown et al., 2016; Cai & Cirillo, 2014; Cohen et al., 2003; Fan, 2013; Fan et al., 2013; Grouws et al. 2004; Johnson, 2017; Jones & Tarr, 2006; Kastberg, 2012; and Livy &

Herbert, 2013; Peterson, et al. 2021; Stacey, et al., 2009; Stein & Kim, 2009; Schmidt et al., 2001, Stylianides, 2014, Thompson, et al., 2012). Therefore, textbooks play an important role in determining what students experience in the classroom. Additionally, it has been determined that teachers in the early stages of their careers (Drake, 2002) and teachers lacking strong foundational mathematical knowledge (Cohen et al., 2003) report feeling the least prepared and, therefore, rely heavily on the textbooks they choose or are mandated to utilize. With mathematical textbooks having such a strong influence on what mathematics teachers teach and how they teach it, it appears important to understand what is within the pages of the textbooks being utilized. According to Kaufman et al. (2018), Great Minds, 2016; Opher et al. (2017), and EngageNY (2017), Eureka Math is the most utilized curricular resource for mathematics instruction in the United States of America. While previous analyses have been conducted to explore the alignment of Eureka Math with the Common Core State Standards of Mathematical Content, only one has been conducted to explore the extent to which the curricular resource meets the requirements of the Common Core State Standards of Mathematical Practice (EdReports, 2018). The CCSS-MP are designed to detail attributes of mathematically-proficient students whom facilitators of mathematics instruction should be aiming to cultivate. In this study, I explored where I felt this analysis from EdReports stops short. Due to the calls for increased research of mathematics textbooks (Hadar, 2017; Polikoff, 2015; Remillard & Kim, 2020; Shield & Dole, 2013; Thompson et al., 2012; Wijaya et al., 2015) paired with the fact that Eureka Math is the most utilized curriculum in the U.S (EngageNY, 2017; Great Minds, 2016; Kaufman et al., 2018; Opher et al., 2017), I felt a deeper analysis of the aims, claims, and content of Eureka Math curriculum was highly warranted. I am hopeful that this study will serve as a resource to support leaders tasked with textbook selection in making well-informed and confident decisions when choosing a resource that best aligns to their districts' goals for mathematics teaching and learning. I also hope that this study sheds light on the complexity of curricular decision-making and the necessity for curriculum leaders and researchers to engage with this complexity in more nuanced ways.

Positionality of Researcher

In 2009, I took a position in the role of Middle School Mathematics teacher at an expeditionary learning charter school in New York State. During my time in this role, the New York State Department of Education (NYSED) adopted the Common Core State Standards for mandatory use in both mathematics and ELA. As I will detail in Chapter I, using Race to the Top grant funds, NYSED hired two vendors for the creation of the new mathematics and ELA CCSS-aligned curricular resources that would be available state-wide. For mathematics, NYSED hired Great Minds and for ELA, Expeditionary Learning. As a teacher at an expeditionary learning school, I got a front-row seat to the transition and the development of curricular materials for teachers state-wide. My teammate was among the teachers hired by Expeditionary Learning to produce a portion of the ELA curriculum. I bore witness to the innumerable number of hours they spent working on the project while also juggling their responsibilities as classroom teachers. I saw the collaboration that was taking place between my teammate and other teachers in different parts of the state. I also covered for their classes when they had to travel out of town to tend to components of the project that required more intense collaboration. But possibly most revealing and impactful for me as a practitioner-turned-researcher was the very real uncertainty and elements of novice that the teachers expressed in their first-time work as curriculum developers. In witnessing my teammate and friend develop a curricular resource from the ground up that would go on to be utilized by so many nation-wide, I felt such pride in them. All the while, in the back of my mind, were parallel sentiments that I was drawing about the math curriculum. I had thought that the math curriculum was most likely being created by teachers who were doubling as fledgling curriculum developers who, themselves, may have felt uncertainty as they developed in their own skills and abilities. While I held pride and appreciation for the work that was done, I simultaneously held skepticism in the reliability of the curricular resources that were being produced.

In 2018, I began a three-year journey working in a district-level role as a Mathematics Achievement Specialist at Unity Public Schools (pseudonym) in Austin, Texas. From my perspective, the school district centered its identity around the students and families that it served: the majority of whom were of backgrounds that are historically under-served. From the moment I interviewed for the role, it was clear that conversations about the purpose of the school's existence and the purpose of each individual staff member were rooted in serving students and families of ethnic backgrounds and economic status that have historically been shown to receive lower-quality educational services. Members of the district's community, including teachers, school leaders, volunteers, and families, demonstrated this focus in a variety of ways: from the trainings provided to teachers, to the thoughtful choices of literature that graced the school libraries, to the continuous almost catch-phrase question asked, *what would best serve the students and families of our community?* when debating decisions.

Through my role as Mathematics Achievement Specialist, I found overseeing the implementation of mathematics teaching and learning eye-opening for me as a professional. Working to support teachers enlightened me to the ways both small-scale and large-scale decisions were made and how plans were constructed and carried out. While I entered the role with previous experience as a mathematics educator who was always afforded autonomy to choose and implement my mathematics curricular resources, I could see that this differed from the view held by my colleagues who, instead, favored the utilization of one uniform curricular resource by all mathematics teachers in our school district. The curricular resource chosen for Unity Public Schools was Eureka Math. I learned through trainings, meetings, and casual conversations that most of my colleagues believed the utilization of a singular mathematics curricular resource that had been deemed *high-quality* in all mathematics classrooms grades pre-Kindergarten through twelve was imperative to the success of the students in our schools. It was clear that there was consensus in the belief that the utilization of the same curricular resource in all grade levels would provide what was described as vertical alignment of the mathematical strategies students were taught to use. Additionally, it was believed that the dedication to a resource of high-quality would ensure that teachers were both accurate in their content knowledge and effective in the pedagogical approaches that they enacted.

My professional experiences, like the ones just described, generated an abundance of wonderings for me. At the center of these wonderings has been the nagging question of how we can better support those who are tasked with making large-scale curricular decisions in school districts with accurate information about the materials that are available to choose from. My presiding professional experience at Unity Public Schools has led to my passion for assisting those residing over mathematics curricular decisions with thorough analysis of this popular option. I am hopeful that this study can serve as a factor of support in engaging in a well-informed selection process and ultimately a confident decision for district and school leaders tasked with choosing a resource that best aligns to their district's mathematics goals.

I have attended professional development sessions centered on Eureka Math, observed teachers utilizing the curricular materials, and I have supported the implementation of them.

After all this interaction, I have still been left with the same skepticism. While this study afforded me the opportunity to look at this curriculum critically, it was designed with a methodology through which I utilized analysis tools to help minimize my bias. The design allowed me to draw on my expertise to lend insight into my study while employing a process of analysis that helped ameliorate the bias that comes from this same expertise. While I was not seeking to be objective, I was hopeful that the analysis would help me see the curriculum in a new and different light to explore how the standards are furthered and how it works in tandem to further the approaches of quality mathematics instruction.

Definition of Terms

Alignment- the process of ensuring that the specified curriculum is consistent with enabling students to reach the milestones outlined in the standards. A curriculum that is aligned to the standards is one that ensures students have the opportunity to access the content and skills outlined in those standards (Center on Standards and Assessment Implementation (CSAI), 2018)

Application Problem- the component of the Eureka Math lessons intended to "provide students with an opportunity to apply their skills and understandings in new ways" and serve "as a springboard into the new learning of the day" (Anderson, 2017, para. 5)

Assessments- tools used to gather evidence related to student learning. These may include summative, benchmark, or interim test, or may involve more informal methods of data collection through a formative assessment process (CSAI, 2018)

Concept Development- the component of Eureka Math lessons intended to "address new content being studied" (Anderson, 2017, para. 4)

Curriculum- shapes how students will gain the knowledge, skills, and abilities as described in the standards (CSAI, 2018)

*Open Educational Resource- "*teaching, learning, and research resources that reside in the public domain or have been released under an intellectual property license that permits their free use and re-purposing by others" (Atkins et al., 2007, p. 4)

Pedagogical Approach- refers to both explicit and implicit messages about how students should interact with mathematics, one another, the teacher, the textbook, and other learning tools around mathematical ideas and teachers' roles in leading and supporting students' interactions and learning (Remillard & Kim, 2020)

Problem Set- the component of Eureka Math lessons intended to serve as "an additional set of carefully crafted problems" and is intended to be "generally about 10 minutes of additional practice" (Anderson, 2017, para. 4)

Problem Solving- engaging in a task for which the solution method is not known in advance (NCTM, 2013)

Standards- indicate what students should know and be able to do within a particular content area (CSAI, 2018)

Task- the questions proposed in the textbook through which students are expected to produce an answer utilizing operations (Amaral & Hollebrands, 2017)

Teacher's Edition- communicates with teachers and supports them in shaping lessons, monitoring student progress, and providing additional support (Remillard & Kim, 2020)

CHAPTER II

LITERATURE REVIEW

For many mathematics teachers, textbooks – both printed and online – serve as a main resource in their lesson design (Grouws et al., 2004; Peterson et al. 2021; Stacey et al., 2009; Stein & Kim 2009; Schmidt et al., 2001; Stylianides, 2014; Thompson et al., 2012). Unfortunately, much contemporary literature such as Ahl et al. (2016), Bryant et al. (2008), Jones and Tarr (2006), Shield and Doe (2013), and Wijaya et al. (2015) have asserted that popular mathematics textbooks in the United States fall short in the strength to which they further pedagogical and instructional practices. As a result of their findings, some researchers have encouraged textbook authors to include effective pedagogical practices within the covers of their textbook products and to have practitioners/leaders inspect the resources they are utilizing to determine if such features are present (Bryant et al., 2008). While the notion of teacher scrutiny of textbooks may sound promising, it is not uncommon that teachers are left out of the decision-making process yet mandated to utilize particular materials (Phillips, 2014). In addition to a lack of teacher autonomy, the same appears to be true in the case of many leaders as often it is the state level governments who hold varying degrees of control over the textbooks that school districts adopt (Phillips, 2014).

This chapter introduces literature that illuminates questions of instructional pedagogy within mathematics curricular resources which are notions that this study aims to bring insight to. I begin with a look at the role of textbooks in United States classrooms and, conversely, the schools' roles in textbook development and use. I accomplish this by beginning with the theoretical framework that undergirds this study. Following this, I explore the role that mathematics textbooks play for students and teachers as well as the ways in which these textbooks are chosen for use. Additionally, I provide insight into the Eureka Math curricular resource published by Great Minds. Then, I review a specific framework that has been utilized to describe the pathway that tasks take from their creation to student learning, followed by previous studies that have investigated mathematics textbooks. I continue with a section providing background of the Common Core State Standards of Mathematical Practice and the pedagogical practices that are embedded within them. I then provide information about the critical discourse analysis approach to analyzing text. I discuss the importance of proportional reasoning by reviewing literature that speaks to students' successes and struggles with the mathematical concepts. I conclude the chapter with a section explaining the significance of engaging students in context-based tasks and describing the characteristics that are believed to make them high-quality.

Theoretical Framework

The purpose of this section is to create a strong foundation that lends direct insight into the perspectives upon which my study is designed. As an educator and researcher, I have been deeply influenced by the constructivist theory of learning, and so it is naturally the lens through which I viewed the literature and the Eureka Math lessons themselves. This theory has shaped the way that I define what math learning is and, therefore, is at work throughout this study. While Great Minds does not explicitly state which theories of teaching and learning compose the foundation upon which their curriculum was designed – nor the pedagogical approaches it was designed to facilitate, it is clear to me that the Common Core State Standards for Mathematical

Practice (CCSS-MP), to which it asserts alignment to, are infused with tenets of constructivist theory. These tenent present in the standards are the ones I tend to notice and gravitate toward and are, therefore, the ones that formed the lens through which I adapted my instrument. Constructivism has played a major role not only in my own teaching and research but also in the greater arena of mathematics education.

The theory of constructivism has spanned decades: continually evolving and expanding into new territories thanks to the works of well-known contributors such as Dewey (1929), Bruner (1961), Vygotsky (1962), and Piaget (1980). Although its path has traversed close to a century thus far, the theory continues to hold significant value in the educational sector and is currently making a resurgence as an important part of contemporary educational reform (Bada & Olusegun, 2015; Fosnot & Perry, 2005). The tenets of constructivism are argued to have their roots in a variety of today's pedagogical trends – including discovery-based learning (Chase & Abrahamson, 2018), cooperative learning (Tran, 2013), project-based learning (Mong & Ertmer, 2013), and most applicable to this study, the Common Core State Standards of Mathematics (CCSS-M) (Hekimoglu & Sloan, 2005).

Defining the term *constructivism* can present challenges as the long-standing history and wide interest in the constructivist theory has inevitably led to many descriptions of the term (Bada & Olusegun, 2015; Simon, 1995). Several subcategories, including delineations such as *radical constructivism* (von Glasersfeld, 1987) and *social constructivism* (Vygotsky, 1962), lie under the larger umbrella term of constructivism. It is, however, imperative to first distinguish a universal and foundational belief that constructivist theory is not a theory of teaching, but rather is a theory of learning (Bada & Olusegun, 2015; Clements, 1997). Clements (1997) highlights the important distinction that the constructivist philosophy is a "perspective on how people – all people – learn, all the time" (p. 198). In a broader sense that speaks to the learning process, Blais (1988) describes constructivist theory as an individual's transformation from a novice to an expert in relation to a concept via an educational process. He describes that rather than a focus on quantity of knowledge, the end goal of expertise is centered on the quality of the individual's ability to process information and use the tools they possess and are given to navigate a situation.

Constructivist theory is founded in the notion that there is no absolute truth and no objective reality, rather knowledge can only exist inside the minds of individuals (Driscoll, 2000; Fosnot, 1996; Simon, 1995). The belief that there is no objective reality outside of an individual to which we can compare the knowledge that they have created also indicates that each individual's knowledge is constructed from their own experiences (Simon, 1995), and so as we gain new experiences, we use our own previous perceptions and understandings to construct new knowledge (Elliot et al., 2000; Simon, 1995; Wertsch, 1997). From a constructivist perspective, when our experiences do not match with our past understanding, disequilibrium occurs, and our minds begin to engage in the adaptive learning process (Simon, 1995). It is believed that we can then create new or modified concepts as we reflect on that learning process (Simon, 1995). The result of this is that a learner is continuously reworking the mental models that they hold of the world as they encounter new experiences (Bada & Olusegun, 2015). Simon (1995) eloquently summarizes these ideas in explaining, "learning is the process by which human beings adapt to their experiential world" (p. 4).

In the designing of learning experiences, many believers of constructivist theory advocate that rather than an authoritarian, all-knowing carrier and transmitter of knowledge, teachers should place themselves in the role of a facilitator who engages learners to construct their own understandings through tasks in which they are active participants rather than passive viewers who would be simply absorbing information (Garmoran et al., 2000; Gruender, 1996). Skills to perform or behaviors to adhere to are deprioritized in favor of development of deep conceptual understanding gained through engaging in authentic tasks (Fosnot, 1996).

As discussed above, constructivism is a theory of *learning*, not of *teaching.* This fundamental difference points to the reality that constructivist theory does not describe a specific way of teaching but rather, details ways in which knowledge is constructed (Simon, 1995). While this is true, practical suggestions and requirements for constructivist-aligned pedagogies in the teaching of mathematics have become increasingly visible in educational resources, frameworks, and mandates (Fosnot, 2015; Remillard & Kim, 2020). A significant embodiment of constructivist-aligned pedagogical practices is demonstrated in the creation and adoption of the Common Core State Standards for Mathematical Practice (CCSS-MP) (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA & CCSSO], 2010). This set of standards is applicable to mathematics classrooms, grades K-12 and sits in concert with a separate set of content standards for each grade level. The CCSS-MP are described as a set of standards that "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (NGA & CCSSO, 2010, p. 6). In other words, this set of standards is *not* a list of prescriptive actions for a teacher to take, rather details the ways that children proficient in mathematics should display their proficiency.

The CCSSI (CCSSI, 2013) reports the eight standards that make up the CCSS-MP were adapted from the National Council of Teachers of Mathematics' *Adding It Up* report (National Research Council, 2001) and the National Council of Teachers of Mathematics' Process Standards (NCTM, 2000), which have been recognized for their strong reflection of constructivist pedagogy throughout (Hekimoglu & Sloan, 2005). It is stated that "these practices rest on important ‘processes and proficiencies’ with longstanding importance in mathematics education” (NGA & CCSSO, 2010, p. 6). The CCSS-MP’s foundational footing upon NCTM’s *Adding It Up* report (National Research Council, 2001) and NCTM’s process standards (NCTM, 2000) was significant to this study as it demonstrates the strong grounding of the CCSS-MP in constructivist-aligned practices.

Constructivist theory tells us that individuals build knowledge from their own perceptions and experiences in their view of the world (Simon, 1995). This is echoed in mathematics researchers’ calls for the use of context-based tasks to support students with mathematical problem solving (Clements & Battista, 1990; Simon, 1995). The terms ‘real-world tasks’ and ‘authentic-tasks’ are often used interchangeably with the term context-based tasks. Constructivists believe that students learn by linking new information with what they already know (Bada & Olusegun, 2015; Phillips, 1995). In an educational setting, this new learning is affected by the context in which it is taught because students apply their existing knowledge and real-world experience to the given situation (Bada & Olusegun, 2015). In the mathematics classroom, this can be embodied by providing students experiences to problem solving through tasks positioned in real-world situations. Bada and Olusegun (2015) explain the belief that “by grounding learning in authentic, real-world contexts, constructivism stimulates and engages students. Students in constructivist classrooms learn to question things and to apply their natural curiosity to the world” (p. 68). In the eyes of a constructivist, the opportunity to engage in mathematical experiences that are situated in real-world context is an essential aspect to a learner’s development in mathematical proficiency.

Revisiting the idea of constructivism as a theory of how an individual learns, I turn to the method of critical discourse analysis, one of the two approaches of data analysis that was utilized in this study. Pederson (2009) argues that tenets of the theory of constructivism have traces of connection to the methodological foundation of critical discourse analysis. Both assume that one's own reality is unique and constructed through their own experiences in a setting of historical and cultural context that both produces and reproduces aspects of the reality through language and experience. The constructivist theory of learning asserts the idea that there is no absolute truth and no objective reality, rather knowledge is created on an individual level based on one's own experiences (Bada & Olusegun, 2015; Driscoll, 2000; Fosnot, 1996; Simon, 1995). Additionally, from a constructivist's perspective it can be said that people arrive to learning situations with knowledge and understandings from their own previous experiences (Bada & Olusegun, 2015; Driscoll, 2000; Fosnot, 1996; Simon, 1995). The knowledge from those previous experiences influences that individual's perspective on the new learning situation as well as the knowledge they will ultimately construct from it (Phillips, 1995). Being a researcher who considers herself a subscriber to constructivist beliefs, and considering it is the theory that undergirded my study, I understand the individual lens to which I arrived at this study ultimately shaped the way I interpret the text and images that I analyzed in my discourse analysis process. I am also cognizant of the fact that the decisions that I made in generating codes, applying those codes, and then categorizing the data indicated by those codes was subjective because they were created by the lens through which I viewed the images and text.

Constructivist theory of learning has had significant influence in the professional lives of mathematics education for practitioners, researchers, and curriculum developers. The tenets of the theory have had a strong presence in my own teaching and research and most certainly compose the lens I looked through when designing and conducting this study. While I will not be judging the Eureka Math curriculum to determine its adherence to the tenets of constructivist theory, its influences will be present throughout my interpretation of this prominent mathematics textbook.

Mathematics Textbooks Play an Important Role

This section discusses the significance of curricular resources, mainly textbooks, in educator's curricular and pedagogical decision-making and the degree to which they rely on such materials. As Thompson et al. (2012) claims, "textbooks are a particularly critical link between the intended and attained curriculum in school mathematics" (p. 254). Likewise, I discuss these concepts because possessing an understanding of who utilizes curricular resources, to what extent, and why provides a foundation that connects to the large picture of why the curricular textbook materials suggested or mandated for use by educators matter and more broadly why the decision-making process behind the choice should be carefully considered.

Contemporary mathematics education literature argues that curricular resources, such as textbooks, play an important role in supporting mathematics teachers in facilitating student learning of the mathematical content that they are tasked with teaching (Cai & Cirillo, 2014; Drake, 2002; Grouws et al. 2004; Hadar, 2017; Peterson, et al. 2021; Remillard & Kim, 2020; Stacey, et al., 2009; Stein & Kim 2009; Schmidt et al., 2001; Stylianides, 2014; Thompson et al., 2012). Thompson et al. (2012) suggests that textbooks, "help teachers identify content to be taught, instructional strategies appropriate for a particular age level, and possible assignments to be made for reinforcing classroom activities" (p. 254). Chang and Silalahi (2017) suggest "in mathematics education, mathematics textbooks play a particularly prominent role in guiding teachers on specific materials to teach" (p. 236). Research such as Arican (2018), Brown et al. (2016), Cohen et al. (2003) Fan (2013), Fan et al. (2013), Johnson (2017), Jones and Tarr (2006), Kastberg (2012), and Livy and Herbert (2013) all discuss the extent to which textbooks are utilized by educators and the factors that play into the way that they are utilized. Brown et al. (2016) and Kastberg (2012) both found that a teacher's lack of mathematical knowledge pertaining to the content topics they are responsible for teaching often leads to a heavy reliance on the content presented in the textbooks that they are utilizing. Consequently, according to Cohen et al. (2003), Freeman and Porter (1989), and Reys, et al. (2004) textbooks play an important role in mathematics teachers' decision-making processes, as they utilize curricular resources to guide their choices about the content of the instruction as well as the pedagogical approaches they employ. Cohen et al. (2003) discuss that while it would be unreasonable to assert that the resources used by a teacher causes learning, they state, "systems of instruction are the cause, and resources are the facilitators or inhibitors of teaching and learning" (p. 135). Textbooks are utilized by teachers to support in making decisions about what content to teach and how to teach it.

According to Drake (2002), the extent of mathematical knowledge that a teacher possesses is not the only factor that influences teachers' levels of reliance on textbook materials. In their mixed-methods study Drake (2002) explored the relationships between the stage of the career that teachers were in and their understanding and implementation of mathematics education reform. For this study, career stages were measured as "years of experience" (p. 314) and were categorized into six stages with time boundaries: 1 to 3 years, 4 to 6 years, 7 to 11 years, 12 to 20 years, 21 to 30 years, and 31-plus years. To conduct his study, Drake analyzed data from a survey previously completed by 595 elementary teachers in the state of California and examined the in-depth case studies of six elementary teachers who were implementing mathematical curriculum that met the standards of reform. The concept of mathematical education reform is described by Drake (2002) as practices grounded in core principles that “students explain their thinking, work together, and solve problems requiring higher order thinking skills” (p. 314). Drake (2002) focused on teacher’s incorporation of reform practices into their mathematics teaching and found that teachers at different stages of experience differed greatly in their understanding of reform, the level at which they perceived they were prepared to teach aligned to the principles of reform, and the mathematical pedagogy that they engaged. In the qualitative results, career stage was shown to have significant effects on both teachers’ perceived preparation to teach reform mathematics and their utilization of reform teaching practices. While teachers with fewer than three years of teaching experience held positive feelings about mathematics reform, they also held the lowest levels of understanding about reform and reported feeling the least prepared of all the groups to teach according to reform principles. Additionally, these teachers reported engaging in the lowest levels of implementation of reform practices and the highest levels of traditional teaching practices. Teachers in the three groups ranging with years of experience between four and twenty years, yielded results that were very similar to one another. They possessed high levels of understanding of mathematical reform and felt more prepared to teach in ways aligned to reform principles than the teachers who were early in their career, and they reported higher levels of actually teaching with alignment to reform principles and lower levels of traditional teaching.

Possibly most relevant to my study is that while teachers in the earliest stages of their career, measured by one to three years of experience, possessed the most positive disposition regarding mathematical reform, these same teachers reported being the least prepared to implement associated practices. Additional details released through the case study illuminated that the teachers with this same level of experience were incorporating mathematical reform practices precisely as they were written in their textbook resource. Drake (2002) reports that during much of the interviews with teachers at this experience level, they chose to center the conversation on their use of and reliance on the textbook materials. The teachers reported engaging in very little adaptation of what was presented in the textbook they were utilizing, but rather implemented the curriculum very literally. Possibly most impactfully is Drake's (2002) finding that, "much of the principled core of reform mathematics teaching was not presented by the curriculum in the ready-to-use format and, as a result, was not adopted by these early career teachers" (p. 325). So, a type of chain reaction took place. While the teachers earliest in their careers report possessing a high level of positive regard for mathematics education reform, they are the least likely to feel prepared to implement the aligned practices. Therefore, such teachers turn to implementing what is given in their textbook resources very literally, many times verbatim. However, this literal implementation falls short of the intended reform principles because much of the reform practices are not presented in the textbook as advertised by the textbook's publisher.

These findings regarding mathematics teachers' reliance on textbooks are significant to my study because they lend insight into the gravity of mathematics textbooks in some teachers' planning and facilitation of learning in their classrooms. This literature sits at the foundation of this study and supports the idea that the textbooks suggested or mandated for use by teachers possess the capability of markedly shaping what and how mathematics is taught to students in their classrooms. This information furthers the importance of examining the content of mathematics textbooks and is the frame in which both the problem and research questions are positioned.

The relationship between the level of educators' experience and the extent to which they adhere to content prescribed in mathematics textbooks was examined by Cohen et al. (2003).

Similar to the findings of Drake (2002), Cohen et al. (2003) argue that teachers possessing a strong foundation of knowledge in their content area do not adhere strictly to its content but instead tend to use textbooks as a guide for their instructional decisions. Cohen et al. (2003) state, "teachers who know a subject well, and know how to make it accessible to learners, will be more likely to make good use of a mathematics text, to use it to frame tasks productively and use students' work well, than teachers who don't know the subject or know it but not how to open it to learners" (p. 125). Cohen et al. (2003) emphasize that teachers who possess strong mathematical knowledge are able to utilize textbooks as one resource among others to guide the creation and facilitation of activities that they plan for student learning. In contrast, and most concerning to me, teachers lacking a strong foundation of mathematical knowledge (Cohen et al., 2003) or who are early in their careers (Drake, 2002) tend to utilize textbooks by engaging in literal execution of the activities presented. These realities regarding the relationship between teacher knowledge and reliance on textbooks is significant in demonstrating the strong influence that textbooks have on classroom instruction. While this research looks at both the significance of textbooks in educator's curricular and pedagogical decision-making and the degree to which they rely on such materials, my study is, in large part, a response to this literature that demonstrates the pivotal role that textbooks play in instructional decision making. Likewise, the way in which textbooks have an influence regarding how teachers facilitate lessons in their classrooms. The literature presented in this section demonstrates that teachers rely on textbooks. In the next section, I will look at how these textbooks are chosen.

How Textbooks are Chosen for Use

This section discusses the different ways textbooks are chosen for use in school districts in the United States. I detail the different levels of support or hindrance of school districts' autonomy in their textbook selection process afforded to them by state government policies. As Phillips (2014) claims, "who gets to decide what books are used in America's public-school classrooms varies by state" (p. 182). As I present in this section, literature shows that not only do state level governments hold varying degrees of control over the textbooks that school districts adopt, but also that there are factors that have influence over these levels of control. Additionally, I present findings from data examining the influence of use of the most widely utilized mathematics curriculum in the United States. I discuss these concepts because understanding the differing ways that school districts are permitted or hindered from making their own decisions regarding the curricular resources utilized within their school districts supports my desire to investigate the ways in which such textbooks further constructivist-aligned pedagogy of the CCSS-MP.

In school districts around the United States of America, there are a variety of different ways that the textbooks adopted for use are chosen (Phillips, 2014). Multiple layers often make up the process of a district's textbook adoption. Varying degrees of choice are authorized through state-level policies combined with input from a variety of stakeholders including school district administrators, classroom teachers, parents, and religious organizations.

Phillips (2014) set to be the first to examine the roles played by "decision making costs, effectiveness of voters, religious composition, power of teachers, and propensity of state governments" (p. 181) in support or hindrance of school districts' textbook selection policies. She went about this by examining textbook selection policies for biology textbooks in public schools in the United States of America. The decision-making power regarding district textbook adoption varies by state and is dictated by state-specific policies. Phillips (2014) named three predominant groups as a classification of this decision-making power: *Complete Choice States,* *Recommended List States,* and *Restricted Choice List States*. States categorized as *Complete Choice States* have policies in place that allow each school district to choose the textbook they use. Ultimately, within these states it is teachers, parents, and principals who make the final decision regarding the textbook of choice. *Recommended List States* are those that publish a list of textbooks that are recommended, not mandatory, for district adoption. In states categorized in the Restricted *Choice List*, school districts may only choose, adopt, and utilize textbooks from a list approved at the state level of government. As of 2014, twenty-nine states fit into the *Complete Choice States* category, 12 into the *Recommended List States*, and eight into the *Restricted Choice List*.

Phillips (2014) found the factors that played a role in state level textbook selection policy restrictions were state revenue share, heterogenous income levels, district size, advanced degrees, and teachers' unions. There was a strong positive correlation between the probability of a state possessing policies that classify it as a *Complete Choice State* and increased prevalence of those following fundamentalist religions. The correlation being that the fundamentalist groups assert their powers to insist autonomy within their communities. In the state of Indiana, a *Recommended List* state, fundamentalist groups played a salient role in districts' applications for waivers that would allow them to choose their own textbooks. Additionally, as hypothesized, Phillips (2014) found that states with more educated populations also have less restrictive textbook choice policies in place. This finding corroborates what previous findings from authors (Husted & Kenny, 2007; Adams & Kenny, 1986; Schmidt et al., 1996), who attribute this reasoning to the demand, expressed for more local control by more highly educated people who also are more involved in the decision-making processes in their school districts and are more confident in the decisions that they make.

The findings regarding the size of school districts yielded the opposite correlation as the probability for states to have more restrictive textbooks policies increased as the state's school districts became larger. Phillips (2014) attributes this to the belief that larger school districts foster less parental involvement and therefore, increased compliance with the implementation of state policies that are more restrictive regarding autonomy. This study also led to the conclusion that the greater the income inequality of the state, the more apt the districts were to accept restrictive policies at the state level, foregoing applying for waivers that would allow them to make their own choices in lieu of adhering to the state mandated textbook lists. Lastly, findings of this study also conclude that the greater the amount that districts receive in contributions from the state, the more restrictive the decision-making policies are regarding textbook choice. The one factor that was found to lack influence on the state level textbook selection policy restrictions was homeownership. Phillips (2014) reported that a significant theme revealed in her interview data was district leaders' desire for "high-quality, reliable, independent" information about textbooks.

Kaufman et al. (2017) conducted a mixed methods analysis designed to investigate the uses of EngageNY to better understand how it and similar curricular resources could support teachers in implementing learning aligned to the CCSS-M. To achieve this, they utilized data and information from three sources: Google Analytics that were provided by NYSED, the RAND Corporation's American Teacher Panel survey, and interviews conducted with mathematics teachers who utilized the EngageNY curricular resources. They found that when it came to adopting and implementing mathematics curricular resources in their classrooms, teachers reported that "district and state standards, guidelines, and assessments" (p. 33) were most likely to influence their choice. Likewise, data reported from the American Teacher Panel in Kaufman et al.'s (2017) study revealed that the teachers utilizing EngageNY named "school district requirements and recommendations" (p. xi) as the chief reason for choosing the curriculum. Specifically, just over one-half of the teachers in New York and California who participated in the survey reported that their district *required* that they utilize the EngageNY materials for use in their ELA and mathematics instruction. Further, between 80 and 90 percent of those participants indicated that their district either *required or recommended* their use of the EngageNY curriculum.

While recommendations to teachers on textbook choice may often come from school districts, it is also the case that state level departments of education make specific recommendations and provide avenues to easily access particular curricular materials and not others (Kaufman et al., 2017). Kaufman et al. (2017) report that "while we do not have systematic data on which states recommend EngageNY, we know that some states provide links to EngageNY or encourage its use as a resource to support standards implementation" (p. 3). Some of the states taking part in this are named as New York, Arizona, California, and Louisiana.

Phillips (2014) and Kaufman (2017) provide important insight into factors that influence school districts' choices regarding the curricula that they adopt within their school systems. Knowledge of state policies that either limit textbook options or have the potential to influence school districts' choices by providing recommended resources provided reason to the significance of my study. As was demonstrated by Kaufman (2017), multiple mathematics textbooks, including EngageNY, are both mandated and recommended by school districts for use by their teachers. The implementation of thorough analyses of these curricula's alignments – to both the content and pedagogical strategies that they are advertised in furthering – are arguably important investigations. My study aims to examine the pedagogical approaches present in the Eureka Math curricular resource to evaluate its alignment to the pedagogical standards it claims to embody.

Popular Mathematics Curricular Resources in the United States

Contemporary literature demonstrates that while there is a wide variety of curriculum utilized by mathematics teachers in the U.S., there is also little data regarding which of them is most utilized (Schwartz, 2023). In a 2016 study conducted by the RAND corporation (Opfer et al., 2018), teachers reported utilizing twelve different math curricula. These curricula were Envision Math from Pearson Scott Foresman; Go Math from Houghton Mifflin Harcourt; EngageNY; Everyday Mathematics / Everyday Learning from McGraw Hill; Investigations in Number, Data, and Space from Pearson Scott Forseman; Math Expressions from Houghton Mifflin Harcourt; Math Connects from MacMillian / McGraw Hill; Eureka Math from Great Minds; Harcourt Math or HPS Math from Houghton Mifflin Harcourt; Math in Focus / Singapore Math from Great Source-Houghton Mifflin Harcourt; Glencoe Math from McGraw Hill; and Holt McDougal Mathematics from Holt McDougal-Houghton Mifflin Harcourt.

The Center for Education Market Dynamics (2023) brings attention to the dominance of a group of three publishers that they dubbed," The Big Three": McGraw Hill, Houghton Mifflin Harcourt (HMH), and Pearson (currently Savvas Learning Company). They cite statistics that have shown that, as of 2013, 90% of curriculum sales were from these three companies. This organization has collected information regarding the math curricula that are utilized in districts across the U.S. to aggregate and provide this data publicly in an effort to expand access to high quality K-12 teaching materials for historically underserved students. As of October of 2023, the nonprofit has identified this information for 934 districts, which, by their calculations, represents

52% of all U.S. students. According to The Center for Education Market Dynamics (2023), as of 2023 the math curricula that stand beside Eureka Math as the most utilized in elementary schools include the following: EnVision Series from Savvas Learning Company; HMH Go Math! Series from Houghton Mifflin Harcourt; Illustrative Mathematics Series from Kendall Hunt / Imagine Learning / McGraw-Hill Education; i-Ready Classroom Mathematics Series from Curriculum Associates; Math Expressions Series from Houghton Mifflin Harcourt; Bridges in Mathematics Series from The Math Learning Center; McGraw Hill My Math Series from McGraw-Hill Education; Everyday Mathematics from McGraw-Hill Education; and Zearn Math Series from Zearn. Curriculum for Middle School grade levels were also investigated and, in addition to Eureka Math, include: HMH Go Math! Series from Houghton Mifflin Harcourt; Big Ideas Math Series from Big Deals Learning, LLC; Glencoe Series from McGraw-Hill Education; Illustrative Mathematics Series from Kendall Hunt / Imagine Learning / McGraw Hill Education; enVision Series from Savvas Learning Company; Connected Mathematic Project Series from Savvas learning Company; Carnegie Learning Math Solution Series from Carnegie Learning; i-Ready Classroom Mathematics Series from Curriculum Associates; and Core Connection Series from CPM Education Program. Using the data that has been collected by The Center for Education Market Dynamics (2023), the organization concludes that the Big Three Publishers still have a strong hold of the mathematics curriculum market, making up approximately 55% of both elementary and middle school use (p. 8). However, the organization acknowledges that Open Educational Resources (more on this in the next section) that are freely accessible online, such as Eureka Math, are proving to be strong competitors with The Big Three as their “growth has been rapid” (p. 8) in gaining larger shares of the market.

Of all the curricula available in the U.S., Eureka Math (formerly EngageNY) has been named by numerous scholars as the most utilized (EngageNY, 2017; Great Minds, 2016; Kaufman et al., 2018; Opher et al., 2017). While there seems to be little literature investigating the reasons why Eureka Math is, in fact, the most utilized of all the available curricula in the U.S., data discussed above from Kaufman et al.'s (2017) survey of U.S. teachers revealed that the top two reasons they reported utilizing the EngageNY curriculum was due to their *desire to help students meet state standards and prepare for assessments that aligned with state standards* and *school district requirements and recommendations* for use of the curriculum. Further, these results showed that those teaching in states that had adopted the CCSS-M were 65% more likely to report using the EngageNY curriculum than those in states that had not adopted the standards.

The information provided in this section provides a broad view of the variety of curriculum available for use in mathematics classrooms and gives shape to the larger context in which the Eureka Math curriculum exists. This knowledge begins to set the stage in an illustration of the gravity to which the Eureka Math curriculum stands apart within an abundance of other mathematics curricula, and why it was chosen as the focus of this study. The prevalent use of the Eureka Math curriculum is indicative of the influence that it has on the teaching and learning that is taking place in classrooms across the United States. It is for this reason that I have chosen to focus on this particular curriculum above all others. The next section will provide details regarding the Eureka Math curriculum from its creation to its wide-spread adoption, further painting a more comprehensive picture of the curriculum that serves as the focus of this study.

Mathematical Tasks Framework

In this section, I introduce a framework designed to break down the process that mathematical tasks go through starting with the creation of their written form through the learning that results for students. The framework was developed, adapted, and utilized in several ways within the literature. This framework was the most useful to my study as it helped me to think about the theorized process of a task through the lens of each distinct stage that can be studied throughout the larger process. While two of the researchers discussed in this section chose to utilize the framework to study the implementation and results of specific tasks, author Stylianides (2014) adapted the framework to give justification to his evaluation of the *initial* written form of tasks in textbooks. Regarding the relationship between the framework and textbooks, Stylianides states, "the framework not only highlights the dynamic interaction between students and teachers as they work on tasks during classroom work, but also points to the important role that textbooks can play in contributing to or setting the stage for this work" (p. 64). Stylianides' (2014) work most closely aligns to my own study in that we both aim to analyze the way in which tasks are presented to their users within textbooks, *before* any interpretation, adaptation, or implementation is enacted upon them.

Stylianides (2014) designed and conducted a study with a focus on evaluating how textbooks presented activities centered on reasoning-and-proving in both student and teacher textbook materials. In addition to reporting on the study that was conducted, Stylianides expanded his work to delve into the methodological challenges that encompass textbook evaluation and the subsequent design of his research. As a result, he presents a methodological approach that he encourages other researchers to utilize in analyzing textbooks *before* they are interpreted and implemented by the textbook user.

In his own analysis of mathematical textbooks, Stylianides (2014) slightly modified and utilized a framework for mathematical tasks that was developed by Stein et al. (1996). Stein et al. (1996) suggest that their framework, titled *Mathematical Tasks,* represents the process that mathematical tasks go through from their original written form to the culmination of student learning. As presented by Stein et al. (1996), this process is composed of three distinct phases and two subphases. The two subphases are strategically placed between the first and second phase and the second and third phases of the framework. Stylianides' (2014) version of this framework, as seen in Figure 1, exclude the subphases and utilize only the main three phases of Stein et al.'s (1996) original framework.

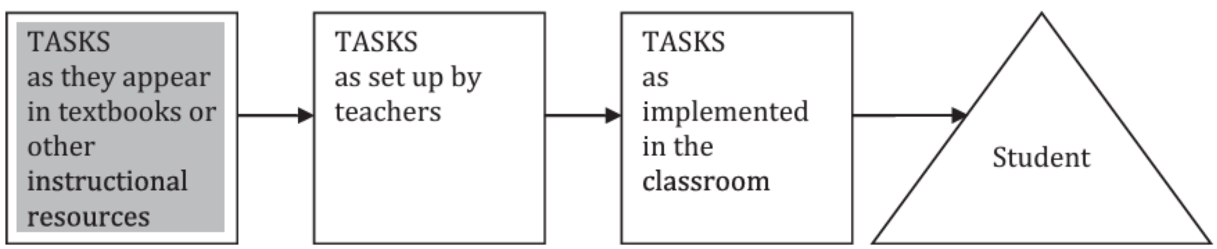


Figure 1: Mathematical Tasks Framework
Note. *Mathematical Tasks Framework in* Stylianides, G. J. (2014). Textbook analyses on reasoning-and-proving: Significance and methodological challenges. *International Journal of Educational Research (64),* p. 64.

The three phases that make up Stylianides' (2014) *Tasks' Framework*, include *TASKS as they appear in textbooks or other instructional resources TASKS as set up by teachers TASKS as implemented in the classroom.* Stylianides (2014) suggests that mathematical tasks pass through the three named phases and ultimately culminate in students' learning and that each of the phases holds the potential to have significant influence on what the students learn.

According to Stylianides (2014), the framework demonstrates "the important role that textbooks can play in contributing to or setting the stage" (p. 64) for the interactions that take place between teachers and students as they engage in each task together. He suggests that this can be seen by the origin of the tasks via the first phase, *TASKS as they appear in textbooks or other instructional resources*. One reason Stylianides expressed for choosing to center his study on this first phase of the framework is his belief that textbooks are a “less explored and insufficiently exploited, lever point for impacting change in classroom practice” (p. 64). To do so, he evaluated mathematical activities presented in 12 of the 24 units that make up the Connected Math Curriculum for grades six, seven, and eight to determine the opportunities that are provided for students to engage in proving-and-reasoning.

Stylianides’ (2014) sentiments regarding the importance of evaluating mathematical tasks directly in the curricular resource *before* interpretation and implementation by the user were echoed by Thompson et al. (2012) and paired with their choice to utilize the first phase titled, *TASKS as they appear in textbooks or other instructional resources,* of Stylianides’ modified framework. In their study, Thompson et al. (2012) evaluated contemporary high school mathematics textbooks to determine the extent to which opportunities to engage in reasoning and proving were present. Thompson et al. (2012) support their decision to focus on the contents of the textbooks rather than the set up or implementation of the tasks stating, “textbook analysis is a first, but important step in understanding students’ opportunities to learn reasoning and proof” (p. 282). Thompson et al. (2012) expand on this justification by suggesting that if opportunities are not written into the textbook, then it is less likely that they will be found in the classroom instruction which is represented in the third phase of the framework.

Stylianides (2014) emphasizes the importance of designing studies that examine textbooks separate from the interpretation, implementation, or student results they initiate. He also goes on to expand that his deliberate choice to focus on the first phase of the *Mathematical*

Tasks Framework also serves the purpose of providing imperative knowledge to researchers who conduct studies that focus on the two other phases (*TASKS as set up by teachers* and *TASKS as implemented in the classroom*) and, ultimately, the culminating result of student learning.

The structure of a mathematical task as laid out by the Mathematical Tasks Framework highlights the importance of mathematics textbooks as their own stand-alone factor of influence on the end result of student learning. This concept provides significant reason to examine mathematics textbooks in their actual form – before their interpretation by teachers – and stood at the heart of this study. Positioning the findings and implications of this study within the structure of this framework provides a clear justification for its need and how it can benefit practitioners in the future.

Common Core State Standards of Mathematics

Since the release of the Common Core State Standards (CCSS) for grades K-12 in 2010, most states of the United States of America have chosen to adopt the standards for mandatory use in their public schools. According to the CCSSI's website, which was last updated with a report from 2013, forty-one states had adopted the CCSS-M (CCSSI, 2013). These creators of the CCSS-M celebrate the standards' intended role to "deliver on the promise of common standards, the standards must address the problem of a curriculum that is 'a mile wide and an inch deep'" and claim that the standards "are a substantial answer to that challenge" (CCSSI, 2010, p. 3). Through the release of the CCSS-M, many sentiments regarding their necessity stood upon a message that mathematics education needed improvement and that the creation and adoption of these standards was a necessary solution (Schmidt, 2012). Messaging regarding the intent of the CCSS-M includes reports that their creation and widespread adoption was "an unprecedented action to promote systemic improvement in mathematics education" (Brahier et al., 2015) and meant to "raise student achievement by influencing how teachers teach and how students learn" (Coburn et al., 2016, p. 243). It's clear that a great amount of emphasis has been placed on the implementation of these specific standards as being a response to calls for improved mathematics education in the United States (Tian & Gunderson, 2020; Provasnik et al., 2016).

The CCSS-M are structured with two different sets of standards: the Standards for Mathematical Practice (CCSS-MP) and the Standards for Mathematical Content (CCSS-MC) (CCSSI, 2010). Each grade level has its own set of Standards for Mathematical Content which is titled "Grade Level Standards" in the CCSSI document. These Standards for Mathematical Content are intended to "define what students should understand and be able to do in their study of mathematics" (CCSSI, 2010, p. 4) in each specific grade level. It is intended that the CCSS-MC be paired with the CCSS-MP. The CCSS-MP are presented as one set of standards for use with students in grades K-12. The stated purpose of these standards is to "describe varieties of expertise that mathematics educators of all levels should seek to develop in their students" (CCSSI, 2010, p. 6). Mateas (2016) succinctly describes the CCSS-MP as being "focused on the way that students think about that content" (p. 93).

The eight Common Core State Standards of Mathematical Practice (CCSS-MP) were designed using resources from the National Council of Teachers of Mathematics (NCTM) as their foundation (CCSSI, 2010). Two NCTM resources were utilized: NCTM's Process Standards and NCTM's Strands of Mathematical Proficiency. NCTM's Process Standards are a part of the organization's Principles and Standards for School Mathematics (NCTM, 2000) which is an outline of suggested components of a high-quality mathematics program for schools. The Process Standards that are nested within this larger document have been recognized for their strong reflection of constructivist pedagogy (Hekimoglu & Sloan, 2005). The standards include problem solving, reasoning and proof, communication, representation, and connections. The Strands of Mathematical Proficiency are featured in NCTM's *Adding It Up* (NCTM, 2001), a publication aimed at providing guidance for mathematics curriculum, teaching, and professional development (NCTM, 2001). These Strands of Mathematical Proficiency are adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition. The rooting of the CCSS-MP within these two resources from NCTM demonstrates the strength of the connection between them and the influence of constructivism on their design.

A search in JSTOR for journal articles, book chapters, and research reports using the phrase "common core state standards" with the year parameters 2010 to 2022 resulted in 53,200 items. When narrowing criteria to focus on the content of mathematics by adding "AND mathematics", 23,541 items of the same type resulted. This search illuminates, not only the prevalence of literature regarding the CCSS, but the high proportion of content specific to mathematics with approximately 44% identified as such. The topics of the academic content available concerning the CCSS and mathematics together reveals the prevalence of materials regarding this topic. This literature spans a wide range of topics including strategies for supporting students who are members of special population groups (Koelsch et al., 2014), resources to support parents in making the transition (Opfer et al., 2016), activities for implementation of the standards (Buchheister et al., 2015; Estes et al, 2014), and CCSS's relationships to student achievement (Schmidt & Houang, 2012). Specifically, the literature reveals that there is an effort to address an ongoing issue that mathematics professionals of all types are facing: determining how to effectively implement the Common Core State Standards of Mathematics in American schools.

The transition from mathematics standards previously in place to the CCSS-M has proven to be a giant shift for those in roles of leadership and teaching (Allensworth et al., 2022; Coburn, et al., 2016; Hamilton et al., 2016) and was described by Coburn et al. (2016) as "a significant departure from current practice in most U.S. classrooms" (p. 243). While NCTM (2013) touts that the CCSS-M "offers a foundation for the development of more rigorous, focused, and coherent mathematics curricula, instruction, and assessments that promote conceptual understanding and reasoning as well as skill fluency" they also continued to acknowledge that practitioners are facing issues in implementation. A journal article from NCTM (Bleiler et al., 2014) that details the experiences of teachers and leaders in a professional development institute focused on implementing the CCSSM, headlines the article with a quote from a fourth-grade mathematics teacher who asked, "How are we going to be able to achieve the goals of the Common Core if we cannot agree on the meaning of the Standards for Mathematical Practice?" (p. 337). Differing messages from NCTM, such as these, provide a firm depiction of the issues centered on implementation that are facing mathematics educators in CCSS-M-adopted states. While the standards may be viewed as important, the evidence indicates that there is significant struggle in the pragmatic execution required for the intended benefits to come to fruition.

There may be few more poignant resources to this point than NCTM's creation of a new set of Principles to Actions in 2014, titled Principles to Actions: Ensuring Mathematical Success for All (NCTM, 2014). In a piece featured in NCTM's *Mathematics Teacher* journal, Brahier et al., (2014) explain that while the CCSS-M are intended to provide an inspiration for developing new resources and assessments, they do not provide details to support in the creation and implementation of instructional materials. It is made clear that the new set of Principles, "provides direction in filling the gap between the adoption of the CCSS-M and the enactment of policies and programs required for its widespread and successful implementation" (p. 656). The recommendations for the document are articulated in three sets of actions, "one set for leaders and policy-makers in districts and states or provinces… one for principals, coaches, specialists, and other school leaders: and one for teachers" (p. 657). Steps to support in the implementation of the CCSS-M, such as the creation of the *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014), demonstrate the ongoing needs for support and guidance that spans a wide range of stakeholders.

Perhaps one of the most significant aspects creating dissonance between the articulated expectations of the CCSSM and the actual implementation of those expectations may be the Standards of Mathematical Practice (Allensworth, 2022). This set of standards is designed to "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (CCSSI, 2010, p. 6). While they inform mathematics professionals as to what students should be able to do as a result of their teaching, the standards lack specific guidance for the pedagogical strategies teachers might use in order to produce the stated developments in their students (Allensworth et al., 2022). This lack of guidance is compounded by the reality that the implementation of such pedagogical strategies would require a large shift away from the instruction approaches to which teachers in traditional classrooms have become accustomed (Allensworth et al., 2022; Mateas, 2016). Along these lines, Allensworth et al. (2022) assert "strong implementation has proven challenging, especially in terms of the practice standards" (p. 1). A research project funded by the National Science Foundation revealed that teachers held misconceptions about the CCSS-MP that made the implementation of the standards challenging (Education Development Center, 2016).

The initial and ongoing adjustments taking place in schools have proved to be a popular focus in contemporary research. The CCSS-MP provides guidance for teachers on what students need to learn through the CCSS-CS and the skills that children proficient in mathematics should be able to demonstrate in their work with mathematics through the CCSS-MP. Unfortunately, the CCSS-MP, which provides little guidance on how teachers should get students to the place of embodying the goals of the CCSS-MP, have left teachers struggling with their implementation. Coupled with the literature regarding the significance of textbook use, it is reasonable to suggest that perhaps teachers struggling with knowing how to implement strategies that support the development of the CCSS-MP are turning to textbooks for support. This lays strong groundwork for this study as it emphasizes the need to investigate such textbooks to more thoroughly understand their alignment to the CCSS-M as well as their adherence to the claims their publishers make about the characteristics and features.

Studies that Evaluate Mathematics Textbooks

Studies focused on mathematical textbooks are plentiful and vary in purpose. Some of the most common topics include the following: mathematical content presented in the resource (Alajmi, 2012; Gabriel et al., 2013; Kar et al., 2018), the pedagogical approaches prescribed by adherence to the textbook's lessons (Otten et al., 2013a; Otten et al., 2013b; Stylianides, 2009), how teachers utilize the materials (Atanga, 2014; Brown et al., 2009; Choppin, 2011), and measuring the impact of curricular resources on student learning (Grady et al., 2012, Stein et al., 2007). Literature concerning the examination of the pedagogical approaches featured within mathematics textbooks was significant to the development of this study.

Literature focused on measuring effectiveness of mathematics textbooks by analyzing student achievement on state achievement assessments is quite common (Grady et al., 2012, Stein et al., 2007). Studies investigating this topic tend to look similar while either investigating the impact of one curriculum or comparing multiple. Grady et al. (2012) may serve as a strong example of this type of investigation. Through this study, the researchers evaluated the results of sixth-grade students' performances on the Illinois Standards Achievement Test for mathematics to compare three different mathematics curricula. Each curriculum was implemented for seven years. The students' results were utilized to judge what was deemed the overall effectiveness of the curriculum on student achievement. Like this study from Grady et al. (2012), the use of standardized assessment data to measure student achievement appears to be a fairly common practice in research of mathematics textbooks.

There is a wide span of topics concerning the examination of prescribed pedagogy in mathematics textbooks. Bieda et al., (2013), Davis (2010), Otten et al. (2013a), Otten et al. (2013b); Stylianides (2009), and Thompson et al. (2012) evaluated textbooks on the presence of opportunities provided for students to reason and prove. Stylianides (2009) designed a methodological approach for examining the opportunities provided for students to engage in reasoning and proving. Doabler et al. (2010) evaluated three math textbooks in search of the presence of eight instructional principles previously shown to support struggling students in gaining math proficiency. Bryant et al. (2008) examined mathematics textbooks in search of eleven "critical features of instruction". Jitendra et al. (2005) explored the extent to which five mathematics textbooks adhered to NCTM's Curriculum and Evaluation Standards for School Mathematics (1989) and instructional design criteria specific to teaching problem-solving. Sood and Jitendra (2007) conducted a study to examine four first-grade textbooks to determine their level of adherence to, what they called, principles of effective instruction. The pedagogical approaches prescribed within a curricular resource is a focus in many studies that have varied in both the resources being examined and pedagogical approaches being evaluated.

Previous analyses of mathematics textbooks have shown that the resources are lacking in the quality of context-based tasks (Wijaya et al., 2015), are deficient in the variety of tasks featured (Ahl, 2016; Shield & Dole, 2013), and require insufficient levels of cognitive demand (Johar & Yusniarti, 2018; Jones & Tarr, 2017) even though the textbooks claimed to meet these requirements. In a 2015 study conducted by Wijaya et al. (2015), the authors evaluated three eighth-grade mathematics textbooks to measure the type of contexts of the math activities and the type of information given by the task, the type of cognitive demand required to complete the tasks. The results revealed that only approximately 10% of the tasks analyzed were high-quality context-based tasks that required students to utilize mathematical modeling to solve rather than simply reproducing what they had done in previous problems with substitution of new numbers. This study draws heavily from the work of Wijaya et al. (2015) because of their focus on the pedagogical approaches featured in the textbooks rather than the mathematical content alignment.

In response to the adoption of the CCSS-M EdReports, an independent nonprofit released reports of their reviews of twenty K-8 math textbooks from well-known publishers (Heitin, 2015). The reviews were conducted to evaluate each curriculum's alignment to the CCSS-CS and CCSS-MP. According to a summary written by Heitin (2015), seventeen of the twenty math textbook series reviewed by EdReports were determined to be out of alignment with the Common Core State Standards, which they claimed to be aligned to. A year prior, researchers Schmidt and Houang (2014) conducted their own analysis of textbooks claiming alignment to the CCSS-M. Their analysis of 50 textbooks found that, on average, 70% of the CCSS-M grade level standards were addressed at each grade level. Additionally, the results yielded the realization that many of the publishers who had claimed their textbooks had been redesigned and edited to properly align to the CCSS-M, were in fact identical to their pre-common core versions. In an article for *Education Week*, Schmidt was quoted in the piece calling these textbook publishers, “snake oil salesmen” who “tell poor districts and teachers [their textbooks] line up to” the CCSS (Herold, 2014, para. 4).

Both scholarly and practitioner evaluations of textbooks’ fidelity to the claims of their curriculum’s features, characteristics, and alignment have produced results showing those claims to be unfounded (Ahl, 2016; Heitin, 2015; Johar & Yusniarti, 2018; Jones & Tarr, 2017; Polikoff, 2015; Shield & Dole, 2013; Wijaya et al., 2015). This pattern of unfounded claims by textbook producers served as reason for additional exploration through this study. The call is out from researchers and scholars for further evaluations of textbooks in relation to their claims of both content and pedagogical alignment (Hadar, 2017; Polikoff, 2015; Remillard & Kim, 2020; Shield & Dole, 2013; Wijaya et al., 2015) and this study responded to that call.

Eureka Math Curricular Resource

This section discusses multiple aspects regarding the Eureka Math curricular resource. I share the genesis of the Eureka Math curriculum, the ways in which the curriculum can be accessed, and claims made about the curriculum. I then detail the structure of the curriculum itself and provide a brief overview of the types of research that is available from Eureka Math’s creator and publisher, Great Minds, as well as review an influential analysis from EdReports. As I present in this section, literature and artifacts show that the creators of Eureka Math have received a substantial amount of funding to create and disseminate the curricular resource and provide evidence of claims that those efforts have successfully made this mathematics curriculum not only the most utilized in the United States, but also popular around the world. I discuss these concepts because this background information about Eureka Math provides a more holistic understanding of the creation, maintenance, and reputation of the curricular resource – helping to paint a picture that sets the stage for my exploration into the lessons itself.

Creation and Access

In 2001, a movement called The Open Educational Resources (OER) began (Remillard & Kim, 2020). OER is defined by The Hewlett Foundation as "teaching, learning, and research resources that reside in the public domain or have been released under an intellectual property license that permits their free use and re-purposing by others" (Atkins et al., 2007, p. 4) and essentially are a way to provide educational resources, including curricula, to all members of the public, free of cost. OER are often touted for their "potential as a powerful tool for reducing inequalities of educational opportunity and promoting innovative strategies to improve educational problems" (Bliss & Smith, 2015, p. 9). The Hewlett Foundation, which as of 2015 had donated over $170 million to the OER movement, explains that in 2011 their program took on a strategic focus in supporting the implementation of the newly released CCSS. This was done specifically by providing funding to EngageNY, an initiative launched by the New York State Education Department (NYSED), created to oversee the development and dissemination of openly licensed curricula aligned to the Common Core State Standards for K-12 English Language Arts and Mathematics – all free of cost. Historically, the costs associated with developing and sustaining open resource curricula has been maintained by philanthropies, nonprofits, and universities (Cavanagh, 2015). The EngageNY initiative was funded by a $700 million award from the federal government's Race to the Top grant money, with $26 million allocated to the creation of the mathematics curriculum (Tampio, 2017). Great Minds, a nonprofit publisher, was chosen by EngageNY to develop the mathematics curriculum (Cavanagh, 2015; Great Minds, 2023b). According to Great Minds (EdReports, 2023a), the curriculum was developed by "more than 120 teachers and mathematicians" (para 1). Great Minds produced the K-12 mathematics curriculum for EngageNY. In 2011, New York State launched the EngageNY website (EngageNY.org) where the curriculum was made available free of cost (Cavanagh, 2015; New York State Department of Education, 2016). According to NYSED, the website itself "was redesigned each year between 2011 – 2015 based on feedback from educators and has since become a national resource" (p. 5).

In addition to being available as an open resource on the NYSED website, the creator of EngageNY has also provided the curricular materials on their own website. The materials are renamed as Eureka Math and are available free of charge in PDF form on their website, labeled as *Eureka Basic Curriculum Files.* Great Minds explains, "The original OER curriculum is available on the EngageNY and Great Minds sites for free" (Great Minds, 2023c). This free version of Eureka Math includes student and teacher editions of the curriculum, curriculum maps, and curriculum overviews. On a page titled *EngageNY Is Eureka Math* of the Great Minds' website, the publisher explains that it has made the EngageNY curriculum available for purchase in an enhanced version. Great Minds explains that the enhanced version is displayed on their website in a different manner to provide superior navigation of the materials and is paired with added features including embedded instructional videos, printed materials, digital resources and manipulatives, and teacher and parent support materials (Cavanagh, 2015, Great Minds, 2023b; Tampio, 2017). Additionally, advertisements for the Eureka Math curriculum can be found in multiple journal publications from NCTM (National Council of Teachers of Mathematics, 2015a, b, c).

According to EngageNY (2017), Great Minds (2016), Kaufman et al. (2018), and Opher et al (2017), as of 2017, Eureka Math was the most utilized mathematics curricular resource in the United States. Estimates from New York officials tout that as of 2014, the Eureka Math curriculum had been downloaded 20 million times world-wide (Cavanagh, 2015). Further, according to statistics reported in the New York Education Department's Race to the Top Executive Summary Report (2016), from the years 2011 to 2015 the EngageNY website that housed the Eureka Math curriculum had "143 million page views, 32 million downloads and more than 10 million unique visitors worldwide" (p. 5).

The Curriculum

The Eureka Math curriculum is available for grades pre-kindergarten through twelfth grade and is broken into three main bundles: Pre-K through fifth grade, sixth through eighth grade, and ninth through twelfth grade. The prekindergarten through grade five cluster is titled *A Story of Units.* The portion of grades six through eight is titled, *A Story of Ratios,* and grades nine through twelve, *A Story of Functions.* The curriculum's lessons are clustered into modules based on related topics. The number of modules that make up a school year's curriculum vary by grade level, from as little as four modules in Algebra II to at most, eight modules in grade two. Curriculum maps provided with the Eureka Math curriculum include the suggested number of days to complete each Module. These timelines range from as little as twelve days for Module 2, *Addition and Subtraction of Length Units,* in grade two to 50 days for the Prekindergarten Module 3, *Counting to 10*.

On its website, Great Minds (2023e) provides, what they call, 36 case studies with the stated purpose "to learn how our educators have used rigorous knowledge-building content to transform their classrooms into dynamic, equitable learning environments that keep all students engaged and supported on their path to greatness" (para. 1). On a separate webpage, Great Minds (2023e) also provides two research briefs and two white paper reports.

In 2015, EdReports released a comprehensive review of twenty math curricula claiming to be aligned to the CSSS-M. Both the Common Core State Standards of Mathematical Content (CCSS-MC) and the Common Core State Standards of Mathematical Practice (CCSS-MP) were evaluated. Among those curricula reviewed was the Eureka Math curricular resource for grades K-8. On their website, EdReports' (2023b) stated goal of these reports is that they "offer evidence-rich, comprehensive information about a program's alignment to the standards and other indicators of quality" (para. 1).

According to EdReports (2023e), there are three gateways through which each curriculum has the possibility of being evaluated. The terms for the ratings are *Does not Meet Expectations, Partially Meets Expectations,* and *Meets Expectations*. The first two gateways focus on the curriculum's alignment and are titled *Focus and Coherence* and *Rigor and Mathematical Practices*. According to EdReports (2023e), the "materials must meet or partially meet expectations for the first set of indicators to move along the process" (para. 5). A curriculum moving along to *Gateway Three* is then rated and receives an overall rating of its *alignment* to the CCSS-M based on a cumulative rating of *Gateways One* and *Two*. Only those curricula that are rated as *meets expectations* for the second gateway of alignment move on to be rated for *Gateway Three*, *Usability.*

Overall, the Eureka Math curricula for grade levels K-8 were rated as "Meets Expectations" in both *Gateways One* and *Two*. EdReports (2023e) explains that this rating indicates "alignment to the fundamental design elements of the materials" (para. 4). The curricula for grades K-5 were rated as *Meets Expectations* for *Gateway 3* (*Usability*); whereas grades six through eight were rated as *Partially Meets Expectations*. EdReports (2023e) defines usability as the “degree to which materials are consistent with effective practices for use and design, teacher planning and learning, assessment, and differentiated instruction” (para. 14).

Eureka Math overall was rated as *Partially Meets Expectations* within gateway two, a gateway focused on *rigor*. Within this gateway, Eureka Math fell short in two areas regarding rigor. The first indicator is described as, “the standards for Mathematical Practice are identified and used to enrich mathematics content within and throughout each applicable grade” (para 7). The second is that the “materials carefully attend to the full meaning of each practice standard” (para 13). the area of meaningfully connecting the CCSS-MP and CCSS-MC. On their website, EdReports (2023d) explains that the reports were provided to the publishers of each curriculum and responses from the publishers were welcomed and would be paired with the reports as made available on their website. In their written response to EdReports regarding their review of the K-8 Eureka Math curriculum, Great Minds wrote that the Eureka Math “curriculum was built to embody the math practices by embedding them in what the curriculum asks students to do every day” (EdReports, 2023d, para. 7) and specified that they made the choice to provide one indicator of the CCSS-MP for each lesson. They continued to address EdReports saying, “your review has encouraged us to more carefully spell out the full meaning of each mathematical practice, helping teachers to do the same in their instruction” (para. 8).

The 2015 evaluation from EdReports has stood to be a substantially influential report. It often seems to be cited as a means of establishing credibility for Great Minds’ Eureka Math curriculum in its reportedly strong alignment to the CCSS-M. In response to the reports’ release in 2015, Heitin (2015) emphasized that out of the 20 curricula that were evaluated, “In all, just one curriculum series stood out from the pack. Eureka Math, published by Great Minds, a small

Washington-based nonprofit organization, was found to be aligned to the Common Core State Standards at all grade levels reviewed" (p. 1). Heitin (2015) quoted Polikoff's opinion about the influence of the report when he stated, "I think people really will pay attention to this, and I think it will affect [curriculum] adoption processes going forward" (p. 1). Additionally, the great level of influence that the EdReports ratings has had can also be seen in a 2018 RAND report (Opfer et al., 2018) which focused in part on exploring which instructional materials teachers reported utilizing in their mathematics classrooms and subsequently determined the proportion of teachers who were utilizing curriculum deemed *highly-aligned* with CCSS compared to the amount that were not. As Opfer et al. (2018) detail, the determination of whether a curriculum was considered highly-aligned or not was done "using a measure of alignment derived from reviews of EdReports, an independent organization that has conducted many reviews of published textbooks for their alignment with the Common Core" (p. 2). Reading through the contemporary literature regarding mathematical curriculum provides an eye-opening account as to the high level of significance that this 2015 EdReport (2015) review of the Eureka Math curricular materials holds.

Conclusion

The creation of the Eureka Math curriculum was initiated by the New York State Department of Education to provide an Open Educational Resource aligned to the CCSS-M and free of cost. The cost of its creation was funded by money given to NYSED from the U.S. federal government's Race to the Top grant program. These funds were utilized to hire the publisher Great Minds to execute the creation. Great Minds hired more than 120 teachers to create the pre-kindergarten through twelfth grade curriculum which was then made publicly available via NYSED's EngageNY website. The curricular resource gained popularity and, by

2017, became the most widely used in the United States (EngageNY, 2017; Great Minds, 2016; Kaufman et al., 2018; Opher et al., 2017). A 2018 comprehensive review of the curricular resource rated the curriculum as being aligned to both the CCSS-MC and the CCSS-MP (EdReports, 2023a).

The prevalence of use of the Eureka Math curriculum is indicative of the influence that it has on the teaching and learning that is taking place in classrooms across the United States. It is for this reason that I have chosen to focus on this particular curriculum above all others and why I believed that a closer look into its claims of alignment to the CCSS-MP was warranted by this study. Based on their survey of teachers across the U.S., Kaufman et al. (2017) provide a key insight that one factor that most influences teachers to utilize the Eureka Math curriculum is because their "districts mandated or recommended" (p. 35) its use. I believe that a curriculum that is mandated or recommended for use by school districts across the U.S. is worthy of exploration by scholars and hence, became the focus of this study. The determination of how the content of the curriculum is positioned in relation to the constructivist-aligned pedagogical practices of the CCSS-MP can provide teachers, as well as school and district leaders key information regarding the way that mathematical learning would be facilitated in classrooms utilizing Eureka Math. Additionally, it may give them ideas of where gaps in particular approaches may lie so that they can attend to them as desired.

The Discourse of Math Crisis

While reviewing the literature for this component of my study, I noticed within the literature that the need for analysis of textbooks was often reasoned by a broader concern of U.S. students' performance in mathematics. The sentiments of concern ranged from giving attention to the content areas that were of students' lowest performance, to criticisms of the performance of U.S. children when compared internationally. I noticed sentiments of what I would call an "educational crisis" consistently present in the literature concerning textbook analysis and Cobrun et al. (2016) seemed to agree. They argue that the most recent wave of crisis for higher student achievement in mathematics began in the 1980s. In 1989, National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989*)* were released – advocating for considerable changes in the content and pedagogy being implemented in U.S. schools due to low student performance. During this time, standards-based reform began to take shape as states developed their own mathematics standards and aligned them to curriculum, professional development, and assessments (Smith & O'Day, 1990). Interwoven into these curricular adjustments was what Coburn et al. (2016) terms the "accountability movement". In 1995, the Trends in International Mathematics and Science Study (TIMSS) was first implemented in the United States. By the release of the results from the third administration of the exam, panicked discussions about U.S. children's deficits took center stage again (Washington et al., 2012). In 2001, No Child Left Behind (NCLB) legislation was passed and signed into law by President George W. Bush in response to concerns that U.S. students were losing their competitive edge over their international counterparts No Child Left Behind Act, 2001). NCLB ushered in significant changes for schools by requiring state-level education departments to administer annual assessments in reading and mathematics for students in grades 3-8. These measures were utilized to evaluate schools and teachers and report subsequent ratings publicly. Schools failing to meet the proper level of progress were taken over by the state (Coburn et al., 2016). As Coburn et al. (2016) explains, "states began to more aggressively publish results from testing (Adequate Yearly Progress reports) on the theory that public pressure would serve as an important lever for accountability and published information on achievement gaps within schools and districts in hopes of decreasing those gaps" (p. 245). The implementation of this high-stakes accountability seemed to generate a sense of panic as schools scrambled to keep their doors open and staff intact. Several years later, in 2009, President Barack Obama enacted the Race to the Top grant program that incentivized states to, yet again, work to improve student performance. In a speech to introduce to the Race of the Top program, Obama (2009) stated,

> If you set and enforce rigorous and challenging standards and assessments; if you put outstanding teachers at the front of the classroom; if you turn around failing schools – your state can win a Race to the Top grant that will not only help students outcompete workers around the world, but let them fulfill their God-given potential. (para. 11)

His words echo a sense of competition and urgency in improving student performance.

One of the ways that states could earn money through this program was to create new learning standards for English and math or, alternatively, they could more simply adopt a new set of standards that would be made by the Department of Education. This legislation created an environment of competition as states across the country began to compete for their share of the funds and *beat* others for their place at the top. Soon after this, 40 states had adopted the new CCSS for English and Mathematics, a set of standards touted "to deliver on the promise" of achieving "great focus and coherence" (NGA & CCSSO, 2010, p. 6). In the introduction to the CCSS-M official document it is stated, "for over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country” (NGA & CCSSO, 2010, p. 3).

The more I read about mathematics textbooks, the more the crisis took shape. Years upon years of an iterative process of strategizing, enacting, measuring, and then criticizing emerged. Remillard and Kim (2020) and Washington et al. (2012) posit that the cycle of crisis has a long history dating as far back as WWII when fears of our international inferiority fell on the shoulders of math and science educators in the name of preserving the existence of the United States of America as a country. Interlaced as an even finer thread, appears to be a continual discussion about the curricular materials and their individual relationships to solving or worsening the crisis of mathematics education.

A sense of transfer of epistemic authority from teachers to textbook publishers seems to veil these discussions. At a time when teachers are feeling a loss of authority (Caronia, 2023), a sentiment takes shape that creators of the textbooks hold knowledge that others do not. In conjunction with the emergence of this mathematical crisis in the literature, a study from Pini (2011) utilized critical discourse analysis to examine the text and images displayed on the websites of prominent education management organizations; organizations who manage schools for a profit. She examined these through the lens of corporate authority and explains,

> Corporate discourses are persuasive because they influence people’s perceptions of reality. Private companies present themselves as being able to offer that which education has seemingly lost: 1) in managerial terms, high quality and efficiency; and 2) in political terms credibility and legitimacy. (Pini, 2011, p. 269)

For me, a question emerged from this fusion of ideas: to what extent have the larger social conditions of mathematics education shifted from textbook publishers being providers of instructional resources for reliable, high-quality, supportive teachers to publishers advancing unreliable, unverified curricular products that override teacher expertise?

Through a prevalence of literature on the topic, the emergence of a crisis of mathematics education formed a broader context for exploration in this study. The surrounding discourse of the *crisis* was significant not only to understanding how Great Minds positions the Eureka Math curriculum discursively within the context of mathematics education, but also to more fully understand how the publisher positions the curriculum in relation to the CCSS-MP..

U.S. Students' Struggle with Proportional Reasoning

In deciding which lessons of the Eureka Math curricular resource to explore for this study, I took the work of Stylianides (2014) into account. As I explain in further detail in Chapter III, Stylianides (2014) evaluated numerous textbooks and expressed his belief that continued examination of mathematical textbooks is an imperative aspect to understanding the larger picture regarding what they provide as a starting point for the teachers who utilize them. He advocates for careful consideration when choosing which parts of the textbooks to conduct detailed analysis. Stylianides (2014) posits that investigations are best conducted on topics that "are important from a disciplinary perspective and with which students and teachers tend to face significant difficulties" (p. 68). In alignment with this approach, I reviewed the literature to identify which subsection of mathematics has been shown to be the most challenging for U.S. students. I found that the concern of US students' inferior performance to their international counterparts continues to be a topic of contemporary literature (Tian & Gunderson, 2020; Provasnik et al., 2016). In 2015, the Programme of International Student Assessment (PISA), an assessment measuring children's mathematical literacy in 47 countries across the globe, was administered (Organisation for Economic Co-operation and Development, 2021). The results showed that students of the United States ranked 39th of 47 in mathematics performance (Organisation for Economic Co-operation and Development, 2021), bolstering concerns of U.S. students' inferior performance. Students in the United States tend to struggle with understanding concepts involving proportional reasoning, making it one of the most difficult mathematical concepts for students to learn (Gabriel et al., 2013; Siegler et al., 2011). Within the umbrella of proportional reasoning lies a variety of concepts, among which fractions stand out. U.S. students' challenges with fractions have been recognized for over four decades – indicated by their performance on national assessments (Carpenter et al., 1981; Namkung & Fuchs, 2019). For example, data from a 2017 National Assessment of Educational Progress (NAEP) showed that when asked to determine if a given fraction was less than, greater than, or equal to one-half, approximately 32% of grade four students were correctly able to do so.

There is a large body of literature concerning proportional reasoning in mathematics education spanning a wide range of foci including: student achievement in proportional reasoning (Torbeyns et al., 2025), the causes of the challenges that students face (Boyer et al., 2008; Gabriel et al., 2013; Siegler et al., 2011), the evaluation of interventions to support students' increased mastery (Scheilbling-Seve et al., 2022), the relationship between understandings in proportional reasoning and future successes (Torbeyns et al., 2015), and strategies to support students in mastering proportional reasoning concepts (Tian & Gunderson, 2020). As the literature shows, students in the United States tend to struggle with mathematics as a whole and have particular difficulty within the domain of proportional reasoning thus, making the skills and concepts within its boundaries some of the most difficult for U.S. students to learn (Gabriel et al., 2013; Siegler et al., 2011).

Proportional reasoning is a topic in mathematics education that encompasses a variety of concepts and skills. Boyer et al., (2008) defines proportional reasoning as "understanding the multiplicative relationships between rational quantities (a/b = c/d)". Within students' educational careers, the successful understanding of proportionality is vital for success in middle school (Tan & Gunderson, 2020), foundational to success in both algebra and geometry (Boyer et al., 2008; Fazio & Siegler, 2010), and imperative in the future studies of mathematics and sciences (Fazio & Siegler, 2010; Gabriel et al, 2013). Additionally, challenges with proportional reasoning can lead to mathematics anxiety in students (Gabriel et al., 2013). In addition to serving as a critical foundational component of mathematics education, it is also imperative to the success in students' future professional and personal lives in practical ways: making economic decisions, interpreting temperatures, scaling a recipe up or down, using a map, calculating the supplies needed for home improvement projects, determining unit price, and comparing sizes among objects (Ahl et al., 1992; Cramer & Post 1993; Fazio & Siegler 2010; Gabriel et al., 2013; Karplus et al., 1983; Lesh et al., 1988; Mousley, 2021).

Within the body of literature dedicated to proportional reasoning, an inquiry emerges to seek out understanding regarding specific causes for students' struggles. Challenges and misconceptions that students face range from difficulty distinguishing between proportional vs. non-proportional situations (De Bock et al., 2007; Fernandez et al., 2008; Van Dooren et al., 2005, 2008), to theories of deep misconceptions about characteristics of fractions (Gabriel et al., 2013; Siegler et al., 2011).

Siegler et al. (2011) argue that U.S. students' difficulties with proportional reasoning ranges not only in fraction arithmetic but with less complex concepts such as fraction placement on a number line or determining fraction magnitude. He believes that the major misconception elementary students hold about fractions is that they treat them like whole numbers (Siegler, 2010). Both Fazio and Sielger (2010) and Siegler et al. (2011) explain that one of the primary roadblocks for students' successful understanding of fractions is the tendency to mistakenly apply whole number concepts to rational numbers. This concept is believed to have been first termed *whole number bias* by Ni & Zhou in 2005 (Ni & Zhou, 2005). Whole number bias is described as a belief that the properties of whole numbers apply to all numbers (Siegler et al., 2011).

In a study conducted and discussed by Gabriel et al. (2013), fourth-, fifth-, and sixth-grade students were assessed to determine the specific difficulties they were facing when learning fractional concepts. The researchers administered a test to students in grades four, five, and six. The researchers found strong patterns in students' mastery of the concept. It was determined that proportion, part-whole through partitioning, and arithmetical operations with simplification of fractions proved challenging to students. Additionally, students struggled with placing or locating fractions on a number line, ordering fractions in ascending or descending order, identifying equivalent fractions, addition and subtraction of fractions with different denominators, graphically representing improper fractions, or including improper fractions in the ordering of fractions. Like Siegler et al. (2011), Gabriel et al. (2013) attribute much of the lower performance to *whole number bias*. Through whole number bias, many students utilize properties of whole numbers when attempting to solve problems with fractions because they lack a deeper conceptual understanding of fractions (Fazio & Siegler, 2010; Gabriel et al., 2013;

Siegler et al., 2011, Stafylidou & Vosniadou, 2004). Gabriel et al. (2013) uses an example to demonstrate Ni and Zhou's (2005) concept of whole number bias: when asked 3/4 + 2/5 = ?, the majority of students in this study answered 5/9. Whole number bias leads students to add the numerators together and use that value as the sum's numerator and add the denominators to be used as the sum's denominator. This foundational concept is imperative for success with proportional reasoning, emphasizing the importance of attention to building conceptual understanding in the domain of proportional reasoning (Fazio & Siegler, 2010; Siegler et al., 2011).

Literature shows consensus among researchers in their belief that students struggle to perform well with proportional reasoning due to a lack of conceptual understanding (Fazio & Siegler, 2010; Siegler, 2010). Fazio and Siegler (2010) define conceptual understanding as "knowledge of what fractions mean" (p. 6) and give the example, "their magnitudes and relations to physical quantities, an understanding of why arithmetic procedures with fractions are mathematically justified and why they yield the answers they do" (p. 6). In this sense, conceptual knowledge in mathematics is distinguished from procedural knowledge, which can be described as "the ability to execute a series of steps to solve a problem" (Fazio & Siegler, 2010, p. 6). Fazio and Siegler (2010) go on to explain, "a student might have the procedural knowledge to solve fraction division problems through inverting the divisor and multiplying the inverted divisor by the dividend but might lack conceptual knowledge regarding why this procedure is mathematically justified and why it yields the answers it does" (p. 6). Researchers have dedicated time and effort in search of the most successful approaches to the incorporation of both procedures and conceptual knowledge (Hecht et al., 2003). Overall, there appears to be consensus among many researchers that conceptual knowledge may support the acquisition of procedural knowledge (Hecht et al., 2003). When it comes to recommendations of how to increase students' success with fractions, there is much concord in the focus of building conceptual understanding.

Mathematics teachers play a crucial role in students' development of conceptual knowledge (Kazemi, 1998). A variety of approaches are recommended for teachers looking to support students in building conceptual knowledge of fractions (Gabriel et al., 2013; Kazemi, 1998; Stafylidou & Vosniadou, 2004). These strategies may fall into two different categories: content-based and pedagogical-based approaches. Some of the content-based approaches include the use of manipulatives and visual representations (Fazio & Siegler, 2010), while pedagogical approaches include engaging students in academic discourse designed to help students build on their own thinking (Ghousseini et al., 2017; Kazemi, 1998; Kazemi & Hintz, 2014) – requiring justification for answers and strategies used (Kazemi, 1998) and presenting fraction problems in meaningful, real-world contexts (Fazio & Siegler, 2010). The pedagogical approaches named here align to several standards featured in the Common Core State Mathematics Content Standards and are discussed in further detail in the section below.

Stylianides (2014) was highly influential in helping me decide which modules of Eureka Math to examine for this study. As discussed in this section, Stylianides (2014) advocates for careful consideration when choosing which mathematical topics to focus on when conducting detailed analysis of curriculum and advocates that researchers engage with topics that are both significant to children's mathematical development and cause difficulty for students and teachers. As discussed in this section, contemporary literature (Tian & Gunderson, 2020; Provasnik et al., 2016) demonstrates that areas of proportional reasoning meet both of these criteria: particularly in the topic of fractions. This literature informed the choice to center the exploration of the Eureka Math curriculum on modules that support teachers in working with students on concepts of fractions. I discuss details of these modules and their focus further in Chapter III.

Context-Based Tasks

Researchers express that one of the primary goals of mathematics education is children's ability to utilize mathematics in a variety of contexts in their daily lives (Boaler, 1993; De Lange, 2003; Graumann, 2011; Muller & Burkhardt, 2007; Niss et al., 2007; Remillard & Kim, 2020; Wijaya et al., 2015). A pedagogical best practice advocated as a support in accomplishing this goal is the opportunity for students to engage with mathematics situated within real-world contexts (Clements & Battista, 1990; Karagiorgi & Symeou, 2005; NGA & CCSSO, 2010; Remillard & Kim, 2020; Wijaya et al., 2015). The utilization of context-based tasks is considered a pedagogical strategy that fosters this type of experience (deLange, 1995; Remillard & Kim, 2020; Wijaya at al., 2015). The pedagogical strategy of engaging students in context-based tasks was central to this study as I explored ways in which the CCSS-MP are aligned in the Eureka Math curricular resource through the exploration of the tasks provided in the textbook.

The framework that I utilized as the instrument for my content analysis is adapted from the work of Wijaya at al. (2015) and Remillard & Kim (2020). Wijaya et al.'s (2015) framework reflects their belief that there are four essential characteristics of context-based tasks. These characteristics are *the nature of the context, the purpose of the context-based task, the type of information given in the tasks,* and *the type of cognitive demand that the task requires*. As I describe in Chapter III, I utilized an adapted version of the framework for this study and incorporated a framework adapted by Remillard & Kim (2020) which supported the evaluation of the levels of cognitive demand of tasks. To support a deeper understanding of my instrument, this section explains what context-based tasks are and details the essential characteristics relevant to this study.

The context of a mathematical task allows students to give meaning to a problem and supports students in developing their ability to problem-solve (Cooper & Harries, 2022; Heuvel-Panhuizen, 1996; Wijaya et al, 2015). While the terms real-world tasks, authentic-tasks, and context-based tasks are often used interchangeably, I will be consistently utilizing the term *context-based tasks,* to maintain uniformity with both Wijaya et al. (2015) and Remillard and Kim (2020).

Wijaya et al. (2015) proclaim that, "the primary requirement for students' learning to solve context-based tasks is that students should be offered experiences to deal with essential characteristics of context-based tasks and should be given the necessary practice in handling these characteristics" (p. 44). The characteristics utilized by Wijaya et al. (2015) were inspired by de Lange (1995) in which he advocated that the context in which the problems are situations is the most essential. In their own study, from which I have drawn from for the methodology of this one, the researchers investigated mathematical textbooks to determine the extent to which they offered opportunities to engage in solving context-based tasks and the connection between those opportunities and the difficulties that students were facing in solving them. They concluded that a lack of opportunities to learn to solve context-based tasks may cause students to have difficulty solving these types of tasks independently.

The Modeling Process

De Lange (1995) posits that a key characteristic of what may be identified as a high-quality context-based task is dependent on the type of context in which the problem is situated. He believes that three categorizations for the type of context are, *no context, camouflage context,* and *essential and relevant context,* with essential and relevant being the type of context necessary for yielding the desired benefits for students. The term, no context is used to describe a task that is absent of a context, often consisting of only numbers and operations. De Lange (1995) and Wijaya et al. (2015) also use the term *bare tasks* to describe tasks written absent of a context. Tasks described as displaying camouflage contexts may look as though they are positioned within a real-world context using words but are tasks that do not require modeling because the process required to solve the task is obvious (de Lange, 1995; Wijaya et al., 2015). In contrast, tasks considered essential and relevant are written within a real-world situation. Wijaya et al. (2015) explains that the setting of the situation can be of the real world or of fantasy which can be imagined by students, and can include personal, scientific, public, or occupational information.

One significant benefit of engaging students in essential and relevant context-based tasks is that these tasks require students to use the process of mathematical modeling to solve them. Blum & Leiss (2007), who are proponents of mathematical tasks that require students to engage in the modeling process, define modeling as a process to "simplify and structure the given real situation, to translate it into mathematics and to interpret mathematical results obtained" (p. 7). Additionally, Niss and Blum (2020) explain,

> Mathematical modelling consists of representing the main elements of a context with mathematical entities and the questions pertaining to the context with mathematical question. The whole world enterprise then consists of seeking answers to mathematical questions and interpreting these answers in terms of the context. (p. 2)

Pollack (2011) provides an explanation of the modeling process grounded in the idea of real-world context by asking the question, "what's different about reasoning in the outside world from reasoning in mathematics?". He responds with a play-by-play breakdown of the process that a learner engaging in mathematical modeling goes through by detailing:

> The real situation usually has so many "angles" to it that you can't take everything into account, so you decide which aspects are most important and you keep those. At this point you have an idealized version of the real-world situation which you translate into mathematical terms. Now you have a *mathematical model* of the idealized question. Then you apply your mathematical instincts and knowledge to the model and get interesting insights, examples, approximations, theorems, and algorithms. You translate all this back into the real-world situation, and you hope to have a theory for the idealized question. But you have to check back: are the results practical, the answers reasonable, the consequences acceptable? If so, great! If not, take another look at the choices you made at the beginning and try again. (Pollack, 2011, p. 64)

This description from Pollack (2011) highlights another chief component of context-based tasks: the amount of information included in the task. Here, Pollack illustrates the common sense point that real-world situations require a person to decipher which information is relevant to solving a problem and which is unnecessary. Through his description, he mirrors this with context-based problems – advocating that this structure be reflected in them. While the presence of superfluous information provides students with the important opportunity to select relevant information, intentionally providing less information than is needed is equally as beneficial. Niss and Blum (2020) concur as they also emphasize the practical need for students to determine which information is important in solving a task. Structuring a context-based task with missing information requires students to use their knowledge of the situation's context to derive and add the necessary missing information (Wijaya et al., 2015). For example, a context-based task requiring students to convert hours to minutes may leave out the essential information that 1 hour = 60 minutes with the intention of requiring students to derive that necessary information from the context of the situation and their own knowledge. Importantly, the amount of information given within a task is a chief component of deriving the full benefits from context-based tasks.

The process of mathematical modeling that is made possible for students by their engagement in *relevant and essential* context-based tasks aligns with aspects of the characteristics that the CCSS-MP seek to develop in mathematically proficient students (NGA & CCSSO, 2010). CCSS-MP1, *Make sense of problems and persevere in solving them* explains, "mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals" (p. 6) and "younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem" (p. 6). CCSS-MP1 emphasizes the importance for students to be able to make decisions about the information given in tasks, in order to determine which to utilize, as well as, making sense of the problem through the utilization of the modeling process. Additionally, there is overlap found in CCSS-MP2, *Reason abstractly and quantitatively*. This standard of mathematical practice states that mathematically proficient students should display "the ability to decontextualize- to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents” (NGA & CCSSO, 2010, p. 6). This process is similar to the one described by Pollack (2011) in the play-by-play type of explanation above. The description of CCSS-MP2 goes on to detail that mathematically proficient students should be able to “create a coherent representation of the problem at hand: considering the units involved; attending to the meaning of the quantities, not just how to compute them; and flexibly using different properties of operations and objects” (p. 6). Both of these components echo the sentiments regarding making sense of the meaning of quantities rather than simply applying operations to a problem.

The process of mathematical modeling has significant benefits for students learning and doing mathematics (De Lange, 1995; Niss & Blum, 2020; Pollack, 2011; Remillard & Kim, 2020; Wijaya et al., 2015). If the central goal of mathematics education is to prepare mathematically literate citizens for the world outside of the classroom (Boaler, 1993; de Lange, 2003; Gruamann, 2011; Muller & Burkhardt, 2007; Blum et al., 2007; Wijaya et al., 2015), then it can be argued that the greatest benefit of any component of mathematics education would be to directly serve this objective. Contemporary researchers (e.g., Pollack, 2011; Niss & Blum, 2020; Remillard & Kim, 2020; Riyanto et al., 2017) are in consensus that learners who engage in mathematical modeling improve their ability to problem-solve in the real-world situations they encounter outside of the classroom. Pollack (2011) exemplifies this notion in an example, proposing that when deciding on the necessary time to depart a location to arrive at the airport in time to make a flight, one is utilizing mathematical modeling. The individual is faced with a real-life problem that features an endless amount of information in which they need to sift through to determine what is relevant and needs to be used versus what is irrelevant to solving this particular problem. Interestingly, the information used may differ based on the individual, the location, or other needs. The process to be used is also flexible. Options of mathematical knowledge may enter their mind, and they must choose a strategy to utilize in determining the time to depart for the airport. While not entirely congruent, situations such as these are supported by the experience of engaging in mathematical modeling in the structured mathematics classroom setting, leading to its benefits outside.

Proponents of mathematical modeling via context-based tasks (de Lange 1995; Pollack, 2011; Remillard & Kim, 2020; Wijaya et al., 2015) propose that a task's ability to provide students with the opportunity to model given problems is reliant on the task being *essential and relevant* and intentionally providing too much or too little information necessary for the solving of the task. The presence of these types of tasks within the Eureka Math curriculum will support understanding of the extent to which the curricular resource furthers the constructivist-aligned pedagogical practices related to modeling through context-based tasks.

Cognitive Demand

It has been long been claimed that engaging primary students in mathematical experiences centered on reasoning and sensemaking increases their ability to solve complex mathematical problems and helps them develop deeper levels of understanding than experiences focused on memorization and use of procedures (Fennema et al., 1993; Hiebert & Carpenter, 1992; Stein & Smith, 1998). Stein and Smith (1998) propose that mathematical tasks that require students to reenact a procedure they have memorized provides a much different learning experience than tasks that require students to think conceptually and make connections. These levels of cognition required to complete tasks are often discussed in mathematics education and have been termed *cognitive demand.* Cognitive demand can be described as "the kind of thinking processes entailed in solving a task" (Stein & Lane, 1996, p. 57). Some actions requiring high levels of cognition include constructing, synthesizing, applying, and explaining or justifying one's reasoning. In contrast low levels of cognition can include reproducing information, memorizing, recalling, applying algorithms, routines, or procedures (Remillard & Kim, 2020).

Within mathematics research, Doyle (1983, 1988) and Stein (Henningsen & Stein, 1997; Stein & Lane, 1996; Stein et al., 2000, 2009; Silver & Stein, 1996) have been highly influential in their development of frameworks incorporating cognitive demand with context-based mathematical tasks. Doyle (1983, 1988) categorized instructional tasks into three categories: *memory tasks, procedural / routine tasks, and comprehension / understanding tasks*. Henningsen and Stein (1997), on the other hand, identified four types of cognitive demands: *doing mathematics; the use of formulas, algorithms, or procedures with connection to concepts, understanding or meaning; the use of formulas algorithms, or procedures without connections to concepts, understanding, or meaning;* and *memorization*. The type, doing mathematics, was considered as requiring the highest cognitive level, and the levels moved down in level of demand respectively in the list.

As Amaral and Hollebrands (2017) declare, "just because a problem includes context does not mean that the problem will be of high cognitive demand" (p. 1169). In fact, it is possible for tasks positioned in real-world contexts to require very low levels of cognitive demand, and – depending on the curricular resource being utilized – it may also be extremely common (Remillard & Kim, 2020). Designing context-based tasks that require a high level of cognitive demand of students is essential in order for them to yield the benefits of context-based tasks. Tasks requiring high levels of cognitive demand promote connections between mathematical concepts and mathematical procedures while also promoting better learning of concepts. The level of cognitive demand required by a mathematical task is a factor that directly impacts the type of problem solving a learner is engaged in and level of understanding a learner takes away from their time engaging in the task (Amaral & Hollebrands, 2017; Doyle, 1983; Fennema et al., 1993; Henningsen & Stein, 1997; Hiebert & Carpenter, 1992; Remillard and Kim, 2020; Stein & Smith, 1998).

There is a heavy emphasis on context-based tasks within contemporary literature of mathematics education (Blum & Leiss, 2007; Clements & Battista, 1990; Karagiorgi & Symeou, 2005; NGA & CCSSO, 2010; Niss and Blum, 2020; Remillard & Kim, 2020; Wijaya et al., 2015). Scholars are clear that the benefits of well-designed context-based tasks are abundant (Amaral & Hollebrands, 2017; Blum & Leiss, 2007; Cooper & Harries, 2022; de Lange, Niss, and Blum, 2020; 1995; Riyanto et al., 2017; Remillard & Kim, 2020; Heuvel-Panhuizen, 1996; Wijaya et al., 2015). Aspects of context-based tasks as tasks that would yield such benefits include its position within a real-world context (de Lange, 1995; Wijaya et al., 2015), the amount of information given in the task (Pollack, 2011; Wijaya et al., 2015), and the level of cognitive-demand it requires of students to successfully solve it (Doyle, 1983, 1988; Henningsen & Stein, 1997; Niss and Blum, 2020; Stein & Lane, 1996; Stein & Smith, 1998; Stein et al., 2000, 2009; Silver & Stein, 1996). These three components of high-quality, context-based tasks sit at the foundation of this study as they compose the content of the instrument I utilized to engage in the content analysis of the mathematical tasks within the Eureka Math lessons. As discussed above, the instrument that I employed is directly adapted from the work of Wijaya et al. (2015) and Remillard and Kim (2020) and stands as a crucial part of this study. I will discuss the instrument and its usage further in Chapter III.

Conclusion

In this chapter I have explained the theoretical framework focused on the constructivist theory of learning that undergirds this study. I have introduced literature that brings to the forefront the strong influence that mathematics curriculum has on the teaching and learning that takes place in U.S. classrooms, while providing insight into the ways in which these curricula are decided upon by school districts. This influential nature of curriculum lays the foundation for the importance of this study. I have also detailed information from contemporary literature that provides names of the curricular resources most often utilized in mathematics education. I followed with insight into a framework utilized to provide a structure to the process that mathematical tasks travel through from their creation to student learning. While an overview of the CCSS-M provides a broad context as to the standards to which the Eureka Math curriculum says it is aligned, additional details of the pedagogical practices seen through a constructivist lens provide the structure through which the study is positioned. Next, I provided an overview of information regarding Eureka Math, the most widely used mathematical curricular resource in the United States. I explained some of the background regarding the creation of the curriculum and the way in which it can be accessed, the structure of the resource, and details regarding a large-scale study that was conducted on the materials. I followed with a look into contemporary literature that helps paint a picture of the surrounding context of mathematics education that the Eureka Math curriculum is positioned within. I continued by presenting literature that revealed the significant level of difficulty that U.S. students have with proportional reasoning. This section laid out the literature that influenced my decision to focus my exploration of Eureka Math modules on those of the proportional reasoning sub-concept of fractions.

I then concluded the chapter with details regarding context-based tasks – including aspects that have shown to make them high-quality and yield the most important benefits for students learning mathematics. The components of a high-quality, context-based task compose the content of the instrument I utilized to engage in the content analysis of the mathematical tasks within the Eureka Math lessons, and, therefore, sit at the foundation of this study.

The next chapter describes the process of conducting this case study as informed by literature. Specifically, I will present details of both the critical discourse analysis and content analysis processes that were utilized to explore the Great Minds website and Eureka Math lessons, respectively.

CHAPTER III

METHODOLOGY

The purpose of this descriptive case study was to explore how the Eureka Math curricular resources are positioned within the context of the CCSS-MP and mathematics education more broadly and subsequently how the content of the resource then positions it in relation to the constructivist-aligned practices of the CCSS-MP. Examining materials provided by Great Minds to users and potential users of the Eureka Math curricular resource provided me detailed information as to the claims the publisher makes about how it furthers constructivist-aligned approaches to the CCSS-MP. Examining teacher materials of specific topics in the Eureka Math curricular resource provided an inventory of the alignment to the claims made by the publisher. To these ends, the study focused on the following research questions:

1. Research Question 1: How does Great Minds discursively position the Eureka Math curricular resources within the context of the Common Core State Standards of Mathematical Practice and more broadly, mathematics education?
2. Research Question 2: How does the content of Eureka Math curricular resources produced by Great Minds position the curriculum in relation to constructivist-aligned approaches to the Common Core State Standards of Mathematical Practice?

This chapter discusses the process of exploring these questions. First, I describe my research design and the rationale for my approach to exploring the Eureka Math curriculum.

Next, I will introduce details regarding methodology including the participants, data collection, and data analysis. I conclude with ethical consideration of my study and issues of trustworthiness.

Research Design

Case Study

When weighing the options of design types, I found that, unlike comparative studies through which the researchers could investigate a curriculum's effectiveness, a case study allows the researcher to examine the mechanism that led to the effects identified and studied in a comparative study (National Academies of Sciences, Engineering and Medicine, 2007). According to the National Academies of Sciences, Engineering, and Medicine (2007), a case study aims to "provide insight into mechanisms at play that are hidden from a comparison of student achievement" (p. 167). Like the work of both Thompson et al. (2012) and Stylianides (2014) as represented by the *Framework for Mathematical Tasks* discussed in Chapter I, I looked into such mechanisms since I aimed to investigate pedagogical decisions that occur *before* the curricular materials are implemented, *before* teachers make instructional decisions to personalize the curricula's implementation, and *before* student assessment results are produced. I sought to examine the curriculum that teachers are using to investigate how those materials address specific pedagogical approaches.

Hays (2004) explains that case studies are also utilized to examine contemporary issues over a short period of time (Hays, 2004). This claim by Hays (2004) encapsulates the second reason that I chose the case study design to examine my research questions since the curricular resource is currently being utilized and claims to meet the CCSS-MP that are currently in place in much of the United States.

Additionally, the National Academies of Sciences, Engineering, and Medicine (2004) claim that “case studies may provide additional specificity that is necessary and helpful to practitioners in assessing the probability of successful use in their settings” (p. 168). In alignment to this claim, utilization of the case study design bolstered the ability to utilize the findings of my study to support teachers and those in influential decision-making roles to make well-informed decisions when seeking out curricular resources aligned to the Common Core State Standards of Mathematical Practice (CCSS-MP).

Descriptive Case Study

Under the umbrella of case study, I made the decision to use the descriptive case study design. According to Mills et al. (2012), descriptive case studies can be thought of as intensive or focused case studies. My study was highly focused on one specific mathematics curricular resource, rather than examining multiple different curricula. I believe that these two characteristics of a descriptive case study effectively illustrate the nature of my study and subsequently led me to my choice of this design type for my study.

Secondly, Hays (2004) asserts that descriptive case study design also holds the purpose of examining contemporary issues over a short period of time. My interest in analyzing textbooks currently utilized and claiming to meet to further the CCSS-MP directly aligns to this design approach because it is a concern currently relevant in schools. Additionally, the National Academies of Sciences, Engineering, and Medicine (2004) claim that “case studies may provide additional specificity that is necessary and helpful to practitioners in assessing the probability of successful use in their settings” (p.168). The descriptive case study design bolstered the ability to utilize my findings of this study to support teachers and those in influential decision-making roles to make informed decisions when seeking out curricular resources aligned to the CCSS-MP.

Data Sources and Analysis

In this section, I explain both the data sources and processes utilized to analyze the data derived from the sources. These are described together for each data source rather than in separate sections to capitalize on their interconnectedness and to better explain the flow of the processes. As Hays (2005) details, case studies in educational research often rely on a wide variety of data collection including interviews, focus groups, classroom observations, and content analysis of school documents, students' records, letters, memoranda, meeting minutes, action plans, curriculum plans, lesson plans, and budgets. In my use of a descriptive case study design, I utilized two types of data analysis approaches: discourse analysis and content analysis. The discourse analysis centered on the promotional materials that Great Minds provides on their website. The content analysis focused on the teacher guides that are a part of the Eureka Math curricular resource.

Discourse Source and Analysis (Great Minds)

Fairclough (2014) explains that by analyzing texts, a person is analyzing the relationship between texts, processes of interpretation, and the social conditions of interpretation that surround. This can also be thought of as an analysis between t*exts, interactions,* and *the contexts in which they sit* – each of these three representing a level of an overarching structure in which the text is positioned. A visual of this relationship can be seen in Figure 2. Fairclough (1989) emphasizes that a text is just a small part of discourse, often referred to as the *micro-level*. He posits that written text is both a product and a resource and that text is a product of a process of text production and a resource for the process of text interpretation. While the discourse can be thought of as “the whole process of social interaction” (p. 57), in this study I approach the text as a product of this larger process of social interaction. Text is essentially one of the most visible parts of the social interaction process that we can grasp onto as an entry point into a greater understanding. Therefore, the text itself is only one part of discourse analysis. While it may be the most visible part, there are the less visible processes of production and interpretation at play. Fairclough (1989) explains that multiple factors influence the way that an individual interprets and produces text. These factors include “their knowledge of language, representations of the natural and social worlds they inhabit, values, beliefs, assumptions” (p. 57). He argues that these factors are socially generated and iteratively shaped while continuously being internalized by the individual. This is all to say that while people may interpret and produce text on an individual level from the ideas in their own minds, those ideas have been shaped by the society that the person lives and functions within. This prominence is the reason that the analysis of text is only one component of critical discourse analysis (*micro)* and why, through a critical perspective, it is believed that the influences from society must also be analyzed through the interactions and the larger context. The interactions that are analyzed, often referred to as the meso-level, include the relationships between individuals’ cognition and the text produced, which Fairclough (1989) explains as “transitory social events” (p. 59). Additionally, the broadest level of analysis, often termed the *macro-level*, describes the more “durable social structures” (p. 59).

As advocated by Fairclough (1989), the analysis at each of these three levels – t*exts, interactions,* and *the contexts in which they sit* – is conducted in different ways due to the object of focus at each. He describes three stages of critical discourse analysis:

- Description: the stage which is concerned with formal properties of the text (utilized for the *text* / micro-level)
- Interpretation: the stage concerned with the relationship between text and interaction – with seeing the text as the product of a process of production, and as a resource in the process of interpretation (utilized for the *interactions / meso-level*)
- Explanation: the stage concerned with the relationship between interaction and social context – with the social determination of the processes of production and interpretation and their social effects. (p. 58) (utilized for the *contexts in which they sit* / macro-level)

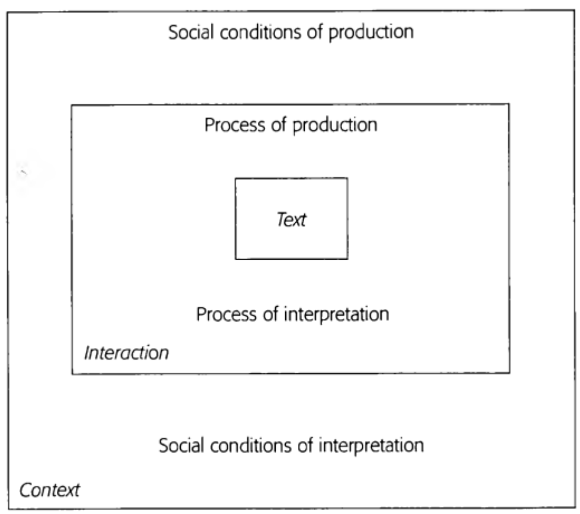


Figure 2: Relationship Between Texts, Interactions, and Contexts
Note. Reprinted from Language and Power (p. 58) by N. Fairclough, 1989, Routledge.

I conducted my investigation utilizing the structure of micro-macro-meso levels (Fairclough, 2002) to analyze the discourse embedded in the Great Minds website in examination of the claims the publisher makes about its Eureka Math curricular resource as well as how it relates to the larger context of education in the U.S. Because Eureka Math is the most widely utilized curricular resource in the U.S., I was interested in understanding how their claims about the curriculum align with the reality of their products and, more broadly, to education in the U.S. I did this by focusing on the following questions:

- Micro: What are the claims that Great Minds makes about their Eureka Math curricular resource?
    - What are its features?
    - Who does the curriculum support?
    - What should school and teachers expect to result from using this curriculum?
    - What are its aims?
    - What are the discursive and linguistic features?
- Meso: Who is Great Minds?
    - Why is this curriculum offered for free from this publisher?
    - What types of modalities are available from Great Minds and what utility do they serve?
- Macro: How do the claims made by Great Minds relate to the larger context of education within the U.S.?
    - What are Great Minds' positions on the current state of education in the U.S.?
    - How does this curriculum fit into the current state of "crisis" of mathematics education in the U.S.?

I began my CDA by briefly examining information at each of the micro-, meso-, and macro-levels to give me a sense of the availability of materials and literature. This brief examination helped me to get a sense of the direction that I needed to travel in my deeper analysis in the next steps. For example, as discussed above, my preliminary engagement with literature involving these topics culminated in my recognition of a theme of discourse related to a *crisis* of underperformance in mathematics among students in the U.S. I began to build the foundation of this background knowledge regarding the larger macro-level that the smaller investigation into Great Minds is positioned within. This same scenario held true for the launch of my examination of materials related to the micro- and meso-levels. Beginning by building the foundations of knowledge at each level will support my investigation.

After my initial investigation of each level, I moved into strategically examining materials for each of the three levels. As discussed above, according to Fairclough (2014), the analysis at each of the three levels of his framework are conducted differently. Beginning at the micro-level, I engaged in the description stage which was concerned with the formal properties of the text itself. I did this by analyzing the materials publicly available on Great Minds' website. I utilized Fairclough's (1989) guide to analysis for this stage. Fairclough's guide is composed of three categories of focus for the text analysis: *vocabulary, grammar, and textual structures*. These three categories house ten questions that I employed to direct my analysis of the Great Mind's website. As Fairclough (1989) explains, there is a value in the analysis of both text and visuals and advocates that the investigation of images is especially revealing in the discourse of advertising. As I conducted my analysis, I found that I had correctly anticipated that much of my analysis would be conducted by viewing the informational materials as advertisements that were created and placed with the purpose of persuading those making curricular decisions to utilize their product. On the Great Minds website there were a variety of different types of visuals utilized, which I subsequently analyzed in alignment to Fairclough's (1989) recommendations regarding images in advertising.

Fairclough (2013) believes that there are three ways that advertisements work ideologically. These include *building relations, building images,* and *building the consumer.* In the case of *building relations,* advertisers utilize an approach to discourse that goes one-way: from the advertisers as the producer of the discourse and the audience of the advertisement as the interpreters. In Fairclough's (2013) opinion, the relationship that is built between the two is a personalized relationship in which a *synthetic personalization* of the audience members takes place. This synthetic personalization can be done by the utilization of the term *you*, for example, or through the practice of using familiar advertising components such as intriguing the audience, highlighting the benefits of the commodity, or an appeal for the audience to take action as a result of the ad. *Building images* is another way that advertisers employ ideology. This is usually done through both verbal and visual modalities which create an ideological construct which as Fairclough (2013) explains, "is both used as a vehicle for the generation of the product image and produced and reproduced in its own right in the process" (p. 206). A type of ideological frame, in which the product sits, is created or reinforced. The goal of the advertiser is to convince the consumer that they, too, should desire to fit into this frame. Lastly, is the ideological work of the advertisement in *building the consumer.* In this sense, the advertiser creates an ideological frame which the consumer utilizes to interpret the advertisement and realize their belonging to a larger community of consumers whose "needs and values and tastes are those embedded in this frame" (p. 207). Exploring the materials from Great Minds through the lens of advertising discourse was an approach that yielded interesting findings via the critical discourse analysis I employed.

Throughout this study, two separate terms were employed regarding the access of the Eureka Math curricular resource: *utilize* and *purchase*. This was because, according to my initial viewings of the Great Minds website, the materials are offered free of charge in what is termed "basic PDF form" and are accessible by completing an online form to request access and create an account. There are also options to purchase what are termed enhanced versions of the Eureka Math curricular resource as well as professional development support. These options are accessible for use after completing an online form to begin initial contact with a representative.

At the completion of my analysis at the micro-level, I engaged in analysis at the meso-level by engaging in the description stage. This stage was concerned with the relationship between text and interaction – with seeing the text as the product of a process of production, and as a resource in the process of interpretation. Fairclough (1989) explains that the relationship between the written text, as examined in the first stage, and social structure is indirect. He posits that the text takes on meaning due to its embedment in social interactions, where it is produced and interpreted. This indirect relationship that is formed by the production and interpretation of the text is based on the background of *common-sense assumptions* that the text is positioned upon. Fairclough (2013) utilizes the term "cues" to describe features of the written text, utilized by the producers, that trigger the common-sense assumptions that are present in the mind of the interpreter. These assumptions are utilized by the interpreter as they make sense of the text from that particular point of view. The process of analysis in this stage of what Fairclough (1989) terms, *description*, is represented in a diagram that can be seen in Figure 3. The process features six main domains of interpretation: two relating to the interpretation of the context, as seen at the top of the diagram, and four to the interpretation of the text, as seen at the bottom. Essentially, in their analysis, a researcher would like to understand exactly what is going on in a participant's mind. Since this is not actually possible, the researcher must go through the same actions of reading the text as the participant interpreter with the added duty of explicating what he or she is doing.

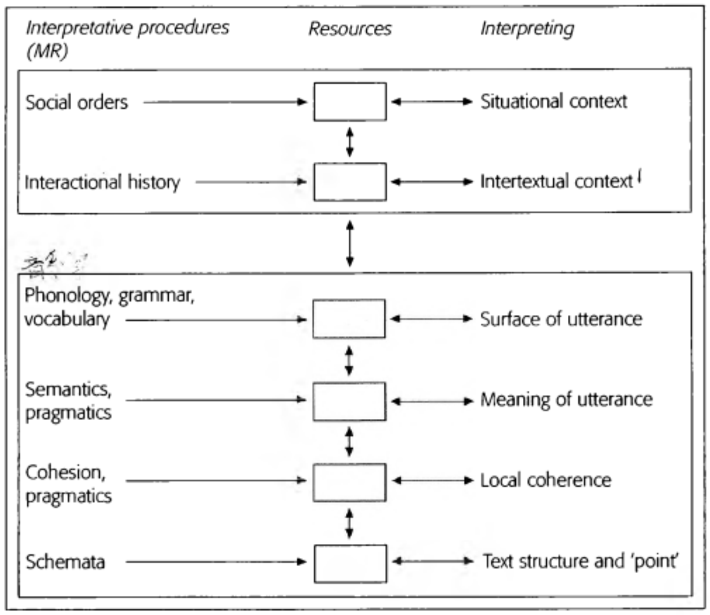


Figure 3: Process of Analysis in Description Stage
Note. Reprinted from Language and Power (p. 156) by N. Fairclough, 2013, Routledge.

Finally, I broadened out another level to gain an understanding of the state of mathematics education in the United States through the macro-level, which is comprised of the social context. The analysis at this level is conducted as the stage of *explanation*. In this stage, the focus was on the relationship between interaction and social context – with the social determination of the processes of production and interpretation and their social effects. (Fairclough, 1989). The relationships discussed at the second, meso-level, are only functional in concert with the institutional and societal contexts in which they are positioned, which are represented by the macro-level. The assumptions at play in the meso-level are of such common-sense due to the larger macro ideologies that create the conditions for the ongoing societal discourse that previously and continuously shapes those beliefs. These beliefs, and the commonsense assumptions that result, are embedded within social struggles and relationships of power that the discourse participants are largely unaware of. These ideologies are explored at this broader macro-level in what Fairclough (1989) describes as a type of "unveiling or demystification" (p. 155). Fairclough (1989) details that the process of reproduction is what connects the two stages of interpretation and explanation of the meso- and macro-levels. In essence, the reproduction connects the components in a way that is cyclical: the social structures shape one's ideas in the mind which are expressed through discourse, those discourses either reinforce or change what is in an individual's mind – which effectively change or reinforce the social structures more broadly. He explains that the stage of explanation is "to portray a discourse as part of a social process, as a social practice, showing how it is determined by social structures, and what reproductive effects discourse can cumulatively have on those structures, sustaining them or changing them" (Fairclough, 2013, p. 172).

It is recommended to utilize three sets of key questions to engage in analysis for the *explanation* stage. Fairclough (2013) writes these questions:

- Social determinants: what power relations at situational, institutional, and societal levels help shape this discourse?
- Ideologies: what elements of MR (members' resources) which are drawn upon have an ideological character?
- Effects: how is this discourse positioned in relation to struggles at the situational, institutional, and societal levels? Are these struggles overt or covert? Is the discourse normative with respect to MR (members' resources) or creative? Does it contribute to sustaining existing power relationships, or transforming them? (p. 175)

During this stage, it was imperative I acknowledged that I arrived with my own assumptions and rationalizations about society and therefore had to continuously remember that I was working with the purpose to bridge the gap between myself and participants through understanding and theory of the society that surrounds the text discourse.

It took me some time to arrive at the decision to utilize critical discourse analysis rather than other methods. I found the body of research related directly to my purpose of exploring materials from a textbook's publisher to be merely non-existent and the body of research of topics similar to be sparse. Most of the literature concerning critical discourse analysis of mathematics textbooks or their related materials center on the analysis of the contents through critical lenses such as gender (Parker, 1999), cultural traditions (Pepin & Haggarty, 2001), and voice (Herbel-Eisenmann, 2007). The purpose of this analysis appears to be a unique investigation for which I was unable to find closely-aligned literature. This led me to reframe my thinking and examine my thoughts regarding the motivations that undergird the textbook analysis. Underneath the investigation of the publishers' claims seems to be what I will describe as a corporate drive to persuade and ultimately sell their product to a consumer. This led me to believe that approaching my analysis through the lens of corporate goals turned Great Minds' publicly-available informational materials into advertisements created and placed with the purpose of persuading. Adjusting the lens through which I was viewing my investigation opened new pathways to my approach and bolstered me in creatively constructing an approach that afforded this meaningful analysis to take effect.

Content Source and Analysis (Eureka Math Curricular Resource)

Choosing Eureka Math

Remillard and Kim (2020) advocate that a researcher's rationale for the selection of a particular curriculum as the focus of an analysis shapes the study being conducted. The Eureka Math curricular resource was chosen according to two important criteria: its claims to adherence to the CCSS-MP and its prevalence of use in the U.S. According to EngageNY (2017); Great Minds, 2016; Kaufman et al. (2018), and Opher et al. (2017), Eureka Math is the most utilized curricular resource for mathematics curriculum in the United States of America. This was a major factor in solidifying my choice to examine the Eureka Math curricular resource. As I have previously explained in Chapters I and II and will continue to detail later in this chapter, while I had already had an interest in examining this specific curriculum, the prevalence of its utilization in the United States confirmed and added weight to my decision.

Content Area of Focus

Stylianides (2014) argues that it is best to focus on topics that are "important from a disciplinary perspective and with which students and teachers tend to face significant difficulties" (p. 68). The lessons that were sampled were identified as engaging students in the CCSS Domain of "Number and Operations- Fractions". This domain contains content focused on proportional reasoning and runs through a portion of the curriculum in grades three through five. I chose from this domain of the CCSS because of the prominent concerns around students' poor performance with skills and concepts related to proportional reasoning as I summarized from the literature in Chapter I. The concept of fractions is described in the literature as an essential foundation for other math skills in later years and possibly most heavily in Algebra (Hannich, 2009). I chose these grade levels due to their focus on fractions. Stylianides (2014) advocates that detailed investigations of tasks are best conducted on topics that are two things: important to the discipline that they are within and one with which are challenging to students. Students in the United States tend to struggle with understanding concepts involving proportional reasoning, making it one of the most difficult mathematical concepts for students to learn (Gabriel et al., 2013; Siegler et al., 2011). Among the many skills and concepts, proportional reasoning is arguably the most challenging for U.S. students.

Fraction concepts align with specific standards within the CCSS-M. The number of standards focused on fractions is highest in grades 3, 4, and 5, featuring three, seven, and seven standards respectively. These standards are the focus of specific modules of the Eureka Math curricular resource.

In grade three, the three CCSS-M of focus are 3.NF.A1, 3.NF.A2, and 3.NF.A3. These standards sit at the foundation of grade 3's Module 5: *Fractions as Numbers on the Number Line*. The Module features thirty lessons. In grade four, the seven CCSS-M of focus are 4.NF.A1, 4.NF.A2, 4.NF.B3, 4.NF.B4, 4.NF.C5, 4.NF.C6, and 4.NF.C7. These standards can be found as the focus of two modules: Module 5: *Fraction Equivalence, Ordering, and Operations,* which features 45 lessons, and Module 6: *Decimals and Fractions,* featuring 20 lessons. Grade five also features seven CCSS-M which are: NF.A1, 5.NF.A2, 5.NF.B3, 5.NF.B4, 5.NF.B5, 5.NF.B6, 5.NF.B7. They can be found within three consecutive modules of the Eureka Math resource. The first module, Module 3: *Addition and Subtraction of Fraction,* features 22 lessons. Module 4 is titled *Multiplication and Division of Fractions and Decimal Fractions* and has 32 lessons. Lastly, Module 5: *Addition and Multiplication with Volume and Area* wraps up the concept of fractions in the K-5 learning trajectory that Eureka Math has termed, The Story of Units. This module features 25 lessons. This information is synthesized in Table 1.

Table 1: Alignment of Eureka Math Modules and CCSS-M

Grade	Standards	Module	# of Lessons
3	3.NF.A1, 3.NF.A2, 3.NF.A3	Module 5: Fractions as Numbers on the Number Line	30
4	4.NF.1; 4.NF.2; 4.NF.3; 4.NF.4	Module 5: Fraction Equivalance, Ordering and Operations	45
	4.NF.1; 4.NF.5; 4.NF.6, 4.NF.7	Module 6: Decimals and Fractions	20
5	5.NF.1; 5.NF.2	Module 3: Addition and Subtraction of Fractions	22
	5.NF.3; 5.NF.4; 5.NF.5; 5.NF.6; 5.NF.7	Module 4: Multiplication and Division of Fractions and Decimal Fractions	32
	5.NF.4; 5.NF.6	Module 5: Addition and Multiplication with Volume and Area	25

I then narrowed down the module that I analyzed by examining the topics of focus for each module. Module 3 in Grade 5 is the first time within the curricular resource that students learn adding and subtracting fractions featuring unlike denominators. In keeping with advice from Stylianides (2014) to choose a topic that is challenging to students, I knew from my own teaching experience that this topic met this description.

Materials Analyzed

The Eureka Math curricular resource features Teacher Editions, Student Editions, and a Supplemental Materials packet. In choosing which set of materials to focus my content analysis on, I again found parallel in the work of Remillard and Kim (2020) who faced the same decision in their own analysis of five popular mathematics curricular resources. They chose to analyze the Teacher's Editions because their analysis was focused on investigating the opportunities for pedagogical strategies provided by the curricular resource which they felt could best be accessed via the materials that the teachers had access to and would use regularly: the Teacher's Editions. I mirror these strategic goals in this study. The Teacher Editions are the only component of Eureka Math materials that provide exemplar vignettes, standard-alignment indicators, solutions to problems and tasks, and sections intended to provide teachers with additional insights including advice for the teacher regarding implementation. For this reason, I chose to analyze the Teacher Edition of this curriculum. Within the teachers' guides I examined tasks, teacher instructions, exemplar vignettes, group activities, independent practice problems. I looked at lessons in the Teacher Editions for Module 3 in Grade 5.

Content Analysis

Content analysis can be defined as "a systematic, replicable technique for compressing many words of text into fewer content categories based on explicit rules of coding" (Stemler, 2001, p. 1). I conducted an extensive review of literature related to the analysis of textbooks to determine alignment to the CCSS-MP. In the analysis conducted by EdReports (2023c), the raters took each of the CCSS-MP and described the ways in which the textbook demonstrated that standard in a comprehensive way. Most of these descriptions also quoted one task from the unit that demonstrated that standard. Here I will provide an example of this rating for CCSS-MP1. EdReports (2023c) states:

> In Module 4, Lesson 3 students independently demonstrate conceptual understanding of place value. Students interpret two-digit numbers as either tens and some ones or as all ones. Problem Set Question 1 states, 'Count as many tens as you can'. Complete each statement. Say the numbers and sentences. ___ ten ___ ones is the same as ___ ones. (para. 12)

This process used by EdReports to search the curricular materials for ways that an evaluator could find that the CCSS-MP were demonstrated did not seem sufficient for my work. I wanted the approach to my content analysis of the materials to provide readers with a more detailed idea of alignment. In contrast to EdReports' process, I sought to examine every task present in each lesson and for each of those tasks, explore how the CCSS-MP were put into practice and to what extent.

Overall, the method for coding each math activity was adapted from the work of Wijaya et al. (2015). I adapted the instrument in three different ways. The authors utilized the framework to analyze students' opportunities to learn to solve context-based tasks in three mathematics textbooks. They define context-based tasks as "problems presented within a 'situation' which can refer to a real world or fantasy setting, can be imagined by students, and can include personal, occupational, scientific, and public information" (p. 44). Wijaya et al. (2015) analyzed the math activities featured within three grade-eight mathematics textbook lessons. The textbooks were chosen for the analysis because of their similar structures: both at the textbook and the lesson levels. The criteria that made up their framework were four characteristics that the authors believe make up a strong context-based task. In examining the textbooks, the tasks in the three textbooks were coded by one author using a combination of three frameworks. The second author then coded a random selection of 15% of the tasks to check for reliability of coding. The results revealed that only approximately 10% of the tasks analyzed were context-based tasks.

The framework used by Wijaya et al. (2015), titled *Context-Based Task Analysis Framework,* features four task characteristics: type of context, purpose of the context-based task, type of information, and type of cognitive demand. The authors explain that while they utilized the work of de Lange (2003) as the foundation for naming and defining the characteristic of type of context, they also chose to extend this piece of analysis to incorporate the second characteristic, purpose of the context-based task.

The first characteristic of the framework, *type of context,* is based on three categories previously established by de Lange (1995): *no context*, *camouflage context*, and *relevant and essential context*. According to Wijaya et al. (2015), tasks deemed as having *relevant and essential* context meet the definition above and require mathematical modeling to solve. As discussed in Chapter I, the process of mathematical modeling ultimately requires students to employ the process of mathematization which is explained by Wijaya et al. (2015) as the requirement to "transform the context situation into a mathematical form" (p. 45). Tasks considered having a camouflage context are "merely dressed-up bare problems" (p. 45) whose mathematical operations necessary to solve the task are obvious and therefore do not require mathematical modeling. The category, no context, identifies a task that "refers only to mathematical objects, symbols, or structures" (p. 52). These tasks can also be referred to by Wijaya et al. (2015) as *bare tasks*.

This second characteristic, purpose of the context-based task, was intended to "distinguish whether a context-based task is used for *mathematical modeling or applying mathematics*" (p. 49) based on whether it comes before or after the explanation section. I chose to remove this characteristic as I initially believed it was not conducive to my work with the Eureka Math curriculum due to the structure of the lessons in grade 5. Specifically, the parts of the Eureka Math lessons being explored are named as the following: Application Problem, Concept Development, and Problem Set. I believed that the Eureka Math lesson structure was incompatible with the two indicators of before or after the explanation section because the Concept Development component of the lesson itself is where the explanation takes place. It, in fact, is named as an entire section of explanation. This meant that the entire Concept Development section would not have fit into either of the indicators of the purpose of the task characteristic. At first, I attempted to modify the sub-categories of this characteristic to include a third that indicated tasks of the Concept Development as neither before nor after the explanation, and rather served as the actual explanation. I decided against this approach because I wanted to ensure accuracy with identifying the purpose of the task determined from the indicator. Since Wijaya et al. (2015) had not created an indicator like this, I didn't want to make assumptions and risk being inaccurate. I landed instead on an alternative approach to utilizing the characteristic of purpose of the task that seemed more compatible with the specific structure of the Eureka Math lessons. I chose to explore each lesson in three different chunks which matched the three different chunks. I first began by coding the tasks within the Application Problem component of a lesson. I then recorded all the codes in my spreadsheet in a tab dedicated to only the Application Problem tasks for the lessons of the module. I then repeated this process with the tasks in the Concept Development by exploring the tasks to code them and then recording the codes onto a separate tab dedicated to the Concept Development tasks. I concluded each lesson by doing the same with the tasks of the Problem Set. This supported me in being able to analyze the data and to recognize patterns of each lesson component and ideally would have somewhat mitigated the tensions between the task characteristics of purpose of context-based tasks from Wijaya et al.'s (2015) framework and the Eureka Math lesson structure. This delineation allowed me to easily separate my analysis to examine patterns within and between the components of the lessons. In Chapter VI, I share my thoughts on how I would have gone about this process differently if I had known then what I know since conducting the study.

The second adaptation that I have made to this instrument is to section for the types of cognitive demand characteristic. As discussed in Chapter I, there are several frameworks and categorizations that organize different outlooks of the level's cognitive demand. While Wijaya et al. (2015) feature three levels of cognitive demand, Remillard and Kim (2020) posit five. The additional two levels included in the latter referenced framework are the result of more specificity to the cognitive demand levels in the middle of the hierarchical structure. This additional specificity seemed to necessitate an increased level of details to differentiate the levels in the middle. In their quest to analyze the tasks featured in five popular mathematics textbooks, Remillard and Kim (2020) constructed a framework adapted from that of Stein et al. (2000). While Stein et al. (2000) utilized four levels of cognitive demand (memorization, procedures without connections, procedures with connections, doing mathematics) for the purpose of analyzing mathematical tasks, Remillard and Kim (2020) made some modifications including adding a level of cognitive demand and expanding details to the descriptions of each level. Added as a level was *procedure with superficial connections.* The team explained that this level was added to reflect tasks in which they believe a curriculum developer was attempting to create a task requiring a meaningful procedure, but they did not design it in a way that would require a higher level of cognitive demand. In their framework, Remillard and Kim (2020) also expanded the descriptions for each of the five levels by adding details of the interpretation of the ultimate goals that a task seems to be seeking to accomplish. An example of these additions of the goal of a task is the addition of the language, "master the procedures having connections as possible scaffolding" (p. 35) for the level of procedure without connection. Remillard and Kim (2020) also explain that they integrated ideas from the work of other researchers who had utilized the framework themselves (Otten & Soria, 2014; Smith & Stein, 1998; Boston & Candela, 2018). In choosing to adapt Wijaya et al.'s framework by substituting Remillard and Kim's (2020) framework in place of Wijaya et al.'s characteristic of type of cognitive demand, more depth and specificity was added to the instrument. I found that the additional specificity was of great support for me in contextualizing the data.

The final adaptation that I made to Wijaya et al.'s (2015) framework was the development and display of codes for each sub-category in the table. These codes can be seen in the table in Appendix 1. The purpose of these codes was to allow me to more succinctly code directly in the Teacher's Edition.

To summarize, the first adaptation that I made to Wijaya et al.'s (2015) *Context-Based Task Analysis Framework* is in the removal of the characteristic of applying mathematics in exchange for coding and recording the codes separately for the three different components of the lesson. My second adaptation was the utilization of Remillard and Kim's (2020) *Analytic Framework for Cognitive Demand of Tasks* in lieu of the original included in Wijaya et al.'s (2015) framework. Lastly, I modified the framework by adding a column to the table where I listed the codes for my coding process.

As I interpret the data through the discussion of my findings in Chapter VI, I chose to assign descriptive terms for the ranges of percentages to promote consistency in my explanations. I developed this scale myself and found great utility in it. This scale descriptors were intended to be utilized when discussing a particular percentage of the tasks that met a characteristic: 0 – 20%- very few / very little; 21 – 40%- few / little; 41 – 60%- approximately half; 61 – 80%- many; 81 – 99%- majority; 100%- all.

Trustworthiness

In a case study approach to research, trustworthiness is essential to ensuring that the study is "worth paying attention to" (Lincoln & Guba, 1985; Shenton, 2004). There are multiple perspectives regarding the components that should be in place to increase the trustworthiness of a qualitative research study – each featuring corresponding approaches that are advocated for being employed (Altheide & Johnson, 1994; Creswell & Miller, 2000; Guba, 1981; Lather, 1993; Lincoln & Guba, 1985; Maxwell, 1992; Maxwell, 1996; Merriam, 1998; Schwandt et al., 2007). Due to their comprehensive nature, the work of Guba (1981) and Schwandt et al. (2017) were used in guiding my integration of practices to increase the trustworthiness of this study. The components of dependability, confirmability, credibility, and transferability comprise the concept of trustworthiness I worked to attain. Each component is coupled with a variety of approaches that I employed to strengthen the level of each one (Guba, 1981; Schwandt et al., 2007). A lack of any one, or combination of the four components serves as a threat to the trustworthiness of the study. In this study, I took deliberate steps to address each of the four components.

According to Shenton (2004) and Yilmaz (2013), credibility has been described as the qualitative researcher's equivalent to internal validity for the quantitative researcher. Credibility is an aspect that aims to ensure a study measures what was intended to be measured (Shenton, 2004). This aspect of credibility is arguably believed to stand as one of the most important factors in trustworthiness (Lincoln & Sage, 1985). To bolster confidence of credibility, it is recommended that the researcher employ specific, recommended methods including adoption of research methods that are well established, frequent debriefing of sessions, peer scrutiny of the research project, use of researcher's reflective commentary, and thick description of the phenomenon under scrutiny (Shenton, 2004; Lincoln & Guba, 1985). Shenton (2004) suggests that qualitative researchers ensure that the "methods of data analysis should be derived, where possible, from those that have been successfully utilized in previous comparable projects" (p. 64). The instrument I utilized in this study, adapted from Wijaya et al. (2015) and Remillard and Kim (2020), was thoughtfully chosen to adhere to the recommendation of adopting well-established methods used in comparable projects. Additionally, to reduce the threats and strengthen credibility, frequent debriefing sessions between me and my dissertation chair took place. Creswell & Miller (2000) and Shenton (2004) advocate that recurring involvement of individuals in supervisory capacity often illuminates flaws and provides an opening for conversations that promote the development of ideas and discovery of previously unknown biases and preferences. Additionally, reflective commentary by the researcher to record initial thoughts and opinions when collecting and analyzing the data provides further strength to dependability (Lincoln & Guba, 1985; Shenton, 2004). Lincoln and Guba (1989) assert that this continuous monitoring by the researcher bolsters their ability to develop sound conclusions from emerging patterns and theories as well as support thorough analysis methods detailed in the report. To meet these standards, I engaged in reflective commentary by recording my impressions at the close of each data collection session. Details included in the commentary focused on the patterns that I found to be emerging as well as wonderings that came up regarding what would be found moving forward. These ongoing reflections were the main contributor to my thoughts regarding ways I would adjust my study if conducting it over again and additional avenues through which I could extend this study in the future. The thoughts can be seen in more sophisticated form in the *Implications for Future Research* section in Chapter VI.

Transferability in a qualitative study is, in some ways, like the idea of external validity or generalizability in a quantitative one in that the readers may be concerned or interested in determining the extent to which the findings and conclusions are applicable to other situations outside of the specific one featured (Merriam, 1998; Yilmaz, 2013). Firestone (1993) and Lincoln and Guba (1985) advocate that the researcher include abundant contextual information about the conditions in the research report. They posit that an ample amount of information –

that does not exclude chosen details – is important due to the researcher's inability to predict what would be meaningful to the readers of their report. Providing full descriptions gives opportunity for readers to develop their own connections and judge for themselves whether they find transferability from the researcher's study to their own situations (Guba, 1981; Shenton, 2004). To increase the level of transferability for this study, I included detailed descriptions regarding the following: the curricular resource itself, the data analysis methods that were employed, and the data that was collected.

According to Yilmaz (2013), it can be thought that a quantitative researcher's reliability is to the qualitative researchers' dependably. In fostering high levels of dependability, Shenton (2004) suggests that "the processes within the study should be reported in detail" (p. 71). The level of details should be so much so that it would allow other researchers to both reproduce the study through the same procedures and to evaluate the practices for adherence to sound research practices. To accomplish this, Shenton (2004) recommends the inclusion of dedicated sections in the research report that address: the research design and its implementation on a strategic level, the operational details of data gathering, and reflective appraisal of the project. While much of the remedy for increasing levels of dependability seems to overlap with those of credibility, I continued to focus on the recording of details, regardless of how insignificant they may seem.

According to Shenton (2004) and Yilmaz (2013), the aspect of confirmability can be likened to the concept of objectivity in qualitative studies. An essential component to increasing confirmability is the researcher's reflection and reporting of their own biases and tendencies (Miles & Huberman, 1994). To tackle this, Shenton (2004) recommends that qualitative researchers share admissions of their beliefs and assumptions in relation to the study as well as recognize their shortcomings in the study's methods. In Chapter I, I share my own beliefs and assumptions in relation to the Eureka Math curriculum through my own experiences with it as a teacher, coach, and district leader. Here, I also share where my beliefs align to the constructivist theory of learning, which serves as the theoretical framework of this study. In Chapter V, I explain some shortcomings of the methods that I enacted for my study. These admissions emerged from my reflective actions during my engagement with the study and serve to strengthen the confirmability of this qualitative study.

Trustworthiness plays a critical role in communicating that a study is worthwhile for an audience to read and consider (Lincoln & Guba, 1985). There are a variety of different approaches that are utilized by qualitative researchers to increase the level of trustworthiness of their studies. The work of Guba (1981) and Schwandt et al. (2017) were used to guide the actions that were taken in this study to demonstrate worthiness. Credibility, transferability, dependability, and confirmability will serve as the four components of focus. Each was addressed in unique ways, much of which centered on the recording of detailed descriptions of both processes and intra- and inter- personal reflections of the researcher. The documentation of such processes will serve as information for readers to utilize to construct their own judgements regarding the credibility, transferability, dependability, and confirmability of the study in relation to their own practice.

CHAPTER IV

FINDINGS: CRITICAL DISCOURSE ANALYSIS

In this chapter, I detail the findings that are a result of my utilization of critical discourse analysis as a tool to engage in a multimodal exploration of two web pages that I have determined to be of significance to the overall Eureka Math section of the Great Minds website. I first provide a report of my findings of the Overview page through the micro- and meso-levels of Fairclough's (2014) 3D model to provide both a description and interpretation of the text and images here. I then engage in the same process for the Professional Development page that is linked through the Overview page. Through my examination of the Overview page, I provide my description (micro) and interpretation (meso) of both the text and images utilized by the producers of the page. The structure of my writing features the description and interpretation of major components of each website in a paired fashion, by which I mean I provide a description of the formal properties of the text and images immediately followed by an interpretation of that same component. I chose to structure this section in this way to provide a fluid thought that provides clear evidence for readers about the connections being made. I found that separating the micro- and meso- levels into separate sections was not conducive to providing the most intelligible connections for the reader. For each of the two web pages, I detailed the formal properties of the text and images to help the reader to visualize the website's components while also providing screenshots from the page itself to lay a foundation for the micro-level of my analysis. These descriptions are then immediately followed by the interpretations that I have made which are aimed at examining the text as a product of a process of production by the producers of this website and a resource of interpretation for the viewers: assumed to be primarily teachers and leaders. My interpretations rely heavily on the writings of Machin and Mayr (2023) whose work provides guidance on determining the specific components of text and images worthy of attention and positions these elements in the context of western society to determine the relationship between the text / images and the interactions of both the production of the producers and the interpretations of the viewers. After providing the reader with my writings on my examination of the micro- and meso-levels of both the Overview and Professional Development pages, I progress to my findings of the macro-level of explanation utilizing the work of Coburn et al. (2016), Gutstein (2009), Klees (2020), Klein (2007), Rampal, (2019), and Washington et al. (2012) to develop an explication of the cultural context that this discourse is situated within. Here, I focus on the relationship between the producers' and viewers' interactions with the text / images as detailed through the meso-level and the broader social context in which these are positioned. As discussed earlier, the assumptions at play at the meso-level are of common-sense to the producers and viewers because of larger *macro*-ideologies engrained by ongoing societal discourse that is continuously shaping our beliefs.

Eureka Math Overview Page

Individualized Joyful Learning

I begin my analysis on the overview webpage of the Eureka Math section of the Great Minds website (Great Minds, 2023d). The top of the page displays a section featuring a photo of a student to the right and words on the left and bottom. See Figure 4. In the photo, a single student is foregrounded in front of a watermarked-type of background that features large windows with an outdoor view of trees. The student looks to be of upper-elementary or lower-middle school age. The student is Caucasian, is well-dressed in a clean and modern-style long-sleeve shirt, has brown-hair that is combed over to one side, is holding a mechanical pencil in the upside-down position, and is looking upwards, smiling. Due to the positioning of the frame of the photo only a small part of the desk or table he sits at is visible. On this desk there is no visible paper or math manipulatives.

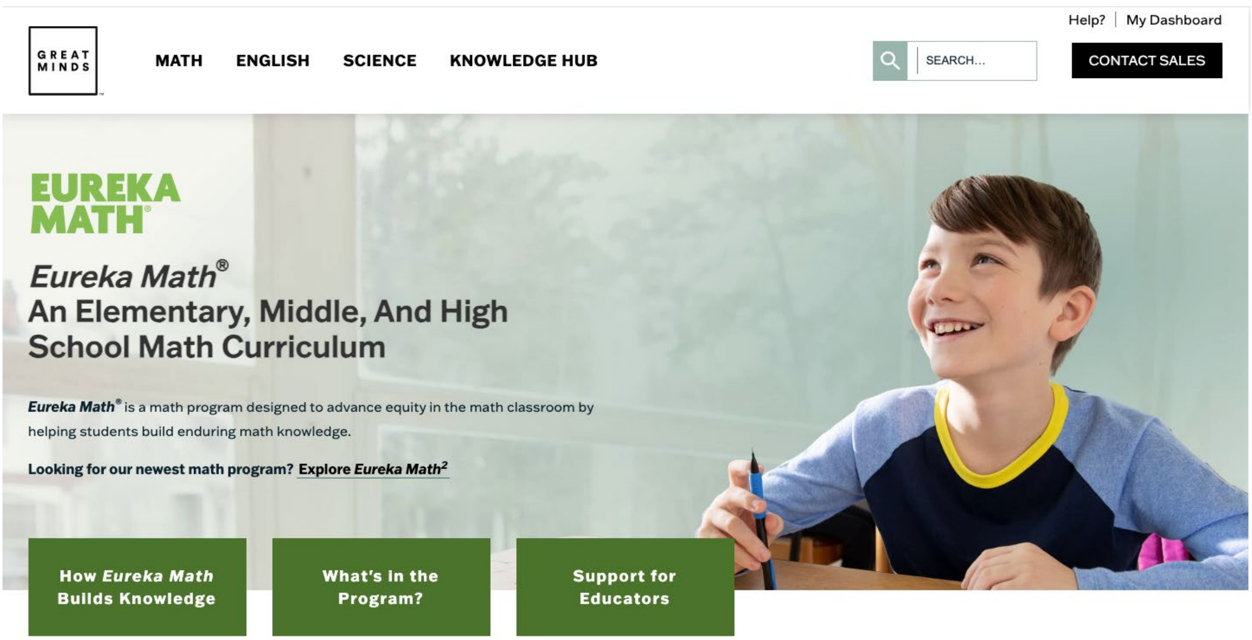


Figure 4: Eureka Math Overview Page- Top
Note. From Great Minds, (2023). Overview. Eureka Math.
https://greatminds.org/math/eurekamath

As Machin and Mayr (2023) inform, "in pictures and in film clips the viewer is always positioned in regard to the subject" (p. 130). The direction that the student is looking in contains important qualities. Kress and Leeuwen (2006) term the two types of gazes a subject can display in relation to the viewer as a *demand* and an *offer*. A *demand* is made when the subject of an image is looking directly into the camera, as if they are demanding something of the viewer in a direct communication. In contrast, when a subject is looking somewhere other than directly into the camera, it is as if an offer is being made to the viewer. Kress and Leeuwen (2006) explain, “here a real or imaginary barrier is erected between the represented participants and the viewers, a sense of disengagement, in which the viewer must have the illusion that the represented participants do not know they are being looked at, and in which the represented participants must pretend that they are not being watched” (p. 120). It is as if, as viewers, we are offered the opportunity to get a sneak-peek into the actions and emotions of the subject of an image. Here, this image of the single student who is gazing away from the camera is utilized by the producers to give the viewer the sense that they are getting a secret look into the student’s engagement with Eureka Math curriculum without him knowing. Part of this sneak peak view set up by the producers serves to portray an essence surrounding the use of the Eureka Math curriculum through the subject’s pose, the direction of the gaze, and the facial expressions.

Machin and Mayr (2023) explain that the pose of a subject in an advertisement is often intentionally utilized by the producers to communicate a particular attitude about the product being advertised. Here the student-subject is sitting straight-up, communicating attention and energy. Even more specifically, the direction of the student’s gaze is also of importance to the pose as it represents a strong metaphorical association. Machin and Mayr (2023) suggest that in western culture an upwards gaze tends to represent feelings of positivity and a downward gaze that of negativity. The student in this photo is gazing upward, communicating a sense of positivity from the producers. Paralleling this sentiment, his facial expression displays a smile, which is similarly utilized by the producers to create a sense of positivity (Machin & Mayr, 2023). Created by the producer, these characteristics of the photo’s subject may serve as a visual metaphor of the enjoyment that is experienced by students using Eureka Math.

In further exploration of the image, according to Kress and Leeuwen (2006), the distance that a viewer is from the subject of the image suggests a level of intimacy between the two. The frame size of this photo indicates that it is considered by Kress and Leewen's (2006) standards to be a *close shot* (p. 124) because the subject is viewable from approximately the height of the shoulders. This frame size puts the distance of the viewer of the image at a *close personal distance* (p. 124) to the subject, which can be described as "the distance at which one can hold or grasp the other person and therefore also the distance between people who have an intimate relationship with each other" (p.124). Personal distances contrast with *social distances* (close and far) at which "impersonal business occurs" (p. 124), creating a much less personal perception for the viewer . Kress and Leewen (2006) suggest that the size of the frame creates a perceived distance between the viewer and the subject, which, just like in everyday social interactions, is often an indication of the type of social relationship had with another person because the distance we keep from others in a real-life interactions is determined by the social relationship we hold with them (Kres & Leeuwen, 2006). When interacting with strangers, humans tend to stay at a farther distance than when engaging with a person one holds a closer personal relationship with. In this image, the producers' use of a *far personal distance* to the student subject indicates a degree of relationship that is close enough to witness to his emotions in a personal way. The combination of the intentional frame size (far personal distance), positionality (gazing upwards), and facial expression (smiling) utilized by the producer combine to communicate a comprehensive message that, as viewers, they are secretly witnessing the joy that the curricular materials can bring to students of their own.

The setting of the image is also of interest when exploring visuals through CDA. Machin and Mayr (2023) tell that advertisers use settings to "communicate ideas and values" (p. 69). A closer examination of the photo reveals that it looks as though the individual student was cropped from a different photo and superimposed onto the soft-green colored background of windows and trees. An additional students' legs hanging off the front of a classroom chair appear present under the left-arm of the featured student. Additionally, there is a pink object, possibly another student's clothing under the right-arm. The producers' intentional cropping of the image to include only one student is done to convey a focus on individualism. Machin and Mayr (2023) explain that there are several approaches that producers may employ to highlight a feeling of individualism including featuring one subject for a product meant for a large number of people. While the curriculum is created to be used with students en masse, this visual of one student provides a much different feeling about Eureka Math than an image that might picture rows upon rows of students sitting in desks in the exact same position. The concept of designing a text or image to address many people while putting the focus on a generic individual is termed *synthetic personalization* by Fairclough (1992). Machin and Mayr (2023) add that synthetic personalization "suggests that there is something personal about the social or commercial interaction taking place" (p. 62). In similar fashion, the motto in a banner at the very bottom of this web page reads, *Every child is capable of greatness.* See Figure 5. Machin and Mayr (2023) are clear that in advertisements, lexical choices are made to convince a viewer that the product was created in a *personalized manner*. Here, this motto is utilized by the producer to persuade the viewer that the Eureka Math curriculum was created with the personal needs of the users in mind and therefore utilizing the curriculum will allow teachers to meet those personalized needs of their students.

A final exploration of the image concerns the background upon which the cropped image was superimposed upon the background of windows with a view of trees. Machin and Mayr (2023) posit that advertisers often utilize aspects of nature to communicate feelings and values of *serenity* and *simplicity*. In this context, the producers utilized the background featuring aspects of nature to provide a sense of calmness and clarity in this classroom – a feeling which a viewer would most likely find favorable when facilitating mathematical learning with students. The producers intentionally use the natural aspects of trees to create a calm and controlled mood in the visual to help the viewer associate the Eureka Math curriculum with a classroom of similar characteristics.

Figure 5: Eureka Math Overview Page- Bottom Banner
Note. From Great Minds, (2023). Overview. Eureka Math.
https://greatminds.org/math/eurekamath

Support and Collaboration

On this site there are four more images featured across the webpage. I found one in particular to be most interesting to examine due to the stark differences that one might immediately notice. This image is featured in a section that comes about three-quarters of the way down the page. See Figure 6. The title to the section, written in all upper-case letters, reads, *PROFESSIONAL DEVELOPMENT,* followed by the subtitle, *Supporting Teachers Before, During, and After Implementation.* Below these titles are two distinct sections, each contained in a square shape made up of a white background. The square to the right features an image of a student written on a white board with an adult standing beside them watching on, smiling. The text below the photo features both a title and caption. In bold lettering the title reads, *Professional Learning from a Trusted Colleague* followed by the caption, *Great Minds is the exclusive provider of professional development written and delivered by the creators of Eureka Math.* Below the text is a button that can be clicked that in all upper-case lettering says, *VIEW OFFERINGS.* This button takes the viewer to the Professional Development page that I explore in the next section.

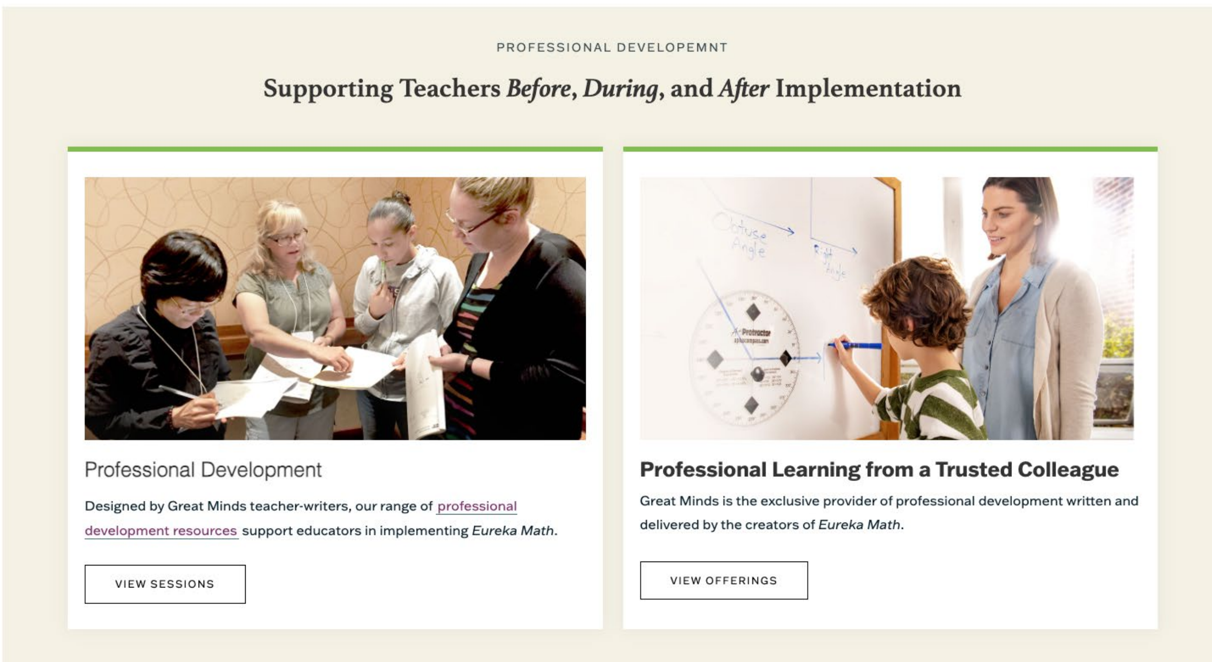


Figure 6: Eureka Math Overview Page- Professional Development
Note. From Great Minds, (2023). Overview. Eureka Math.
https://greatminds.org/math/eurekamath

The square to the left is set up in similar fashion featuring an image with text beneath. The essence of this image is quite different from the first image of the student subject found at the top of the page. The first image portrays the joy of learning in a calm atmosphere with attention on the individual. This image of the four adult subjects gives the feeling of seriousness, concentration, with a focus on prioritizing collaboration. The image features four adults who are standing together, each holding a packet of papers. Here, I present both a description of the noteworthy components of the image along with an interpretation of text / images as a product of a process of production by the producers.

The subjects look to all be women, three of whom are Caucasian and one who appears to be of Asian descent. Based on the text, it can be assumed that these subjects are meant to be teachers who are attending the professional development from Great Minds. I will refer to these subjects as *subject 1, 2, 3,* and *4* moving from left to right in the photo for clarity of communication to the reader. The three Caucasian subjects, 2, 3, and 4, are all looking down at the same piece of paper which is being held by subject 3 who has a pen held to her closed mouth as if she is deep in thought. Subject two is pointing to the paper that they are looking at together and has her mouth agape as if she is speaking. Subject 4 looks on with her head tilted to the side as if she is considering the words of subject 2 and / or the contents of the page. While this is happening, subject 1 has her head tilted downward looking at her own paper being held in her left hand at chest level. She is writing on the paper with a pen. The pose of subject 4 portrays an intense focus on the work she is completing. As a whole, the facial expressions of the four subjects portray an essence of seriousness and focus on the work they are doing.

The frame of the photo creates varying views of the subjects. It is positioned far enough away to capture all four subjects in the frame which displays subject 1 from stomach area up, subjects 2 and 3 from knee height up, and subject 4 from hip height up with a portion of her head out of the top of the frame. According to Kres & van Leeuwen (2006), this frame size is considered a *medium close shot* due to the cut off being at the subjects' waists. As discussed earlier, the size of the frame provides the viewer with a sense of the type of relationship that they have with the subject because the perceived distance it places the viewer at from the subject.

Here, the producers' use of a medium close shot is to convey the feeling that the viewer is at a *far personal distance* (Kres & Leeuwen, 2006) with the subject. A far personal distance can be described as, "the instance at which subjects of personal interests and involvements are discussed" (Kres & van Leeuwen, 2006, p. 124). This functions to create the sense that the viewer is in a social relationship close enough that viewing the work the subjects are engaging in is acceptable and welcomed. The gaze of all four participants away from the camera bolsters this feeling of welcome as it invites the viewer to take a look into their work together. Similar to the image of the single student discussed above, the subjects are making an offer to the viewer by carrying on with their work as the photo is taken rather than looking into the lens of the camera. The producers use this photo to portray a feeling that it is as if the participants of professional development are so focused and engaged in their learning and collaboration that they do not even care to glance at the camera, or they are so happily engrossed in mathematical development they forget that the camera is even there.

The happy collaborator text can be seen to reinforce explicit assertions that the curriculum is *designed by Great Minds teacher-writers* and implicit messages suggesting that this is *professional learning from a trusted colleague.* The producers' eye-level angle for the positioning of the shot is used to create the feeling of equality or symmetry of power among peers (Kres & Leeuwen, 2006). This eye-level positioning contrasts with a positioning above or below the subject, which would communicate feelings of superiority and inferiority respectively. Taking the shot from an eye-level angle serves the idea that the viewer is seeing the work of a peer.

The setting of the image is important to the goals of the producer. As discussed in the exploration of the image of the single student above, Machin and Mayr (2023) advocate that the setting of an image used in advertising is an important component in the producers' attempt to communicate a particular meaning about a product or service. The subjects of this photo are standing in what appears to be the corner of a room donning wallpaper on the walls. There are two generic-type wallpapers covering the wall, separated by a chunky wood molding placed at hip height. The wallpaper on top of this molding is tan colored and features swirls of abstract shapes made in a darker-tan / brown color. There is less shown of the bottom wallpaper which appears to be a textured tan color. Pulling from my own experiences of attending professional developments at conferences, it is easy to assume that the physical setting of this image is a hotel conference room. The details of the subjects standing in a corner of an assumed conference room could further imply that these adults are in a small working group and outside of the frame are other participants who are also positioned in similar fashion, working in small groups of their own. I believe that, paired together, the generic hotel-like conference room wallpaper and the standing poses of the individuals provide subconscious clues to a viewer who has attended a professional development conference that this image is shot at a similar type of event. The producers utilize an image with this setting to demonstrate the professional nature of their services. A sense of formality can be associated with professional learning that is held at a location outside of one's place of employment. It can be assumed that much planning and organization goes into arranging for teacher and leader participants to engage in an offsite training and that the training must be worthwhile enough to put in this amount of effort.

This text works in tandem with other images from their website aimed at giving the viewer (potential customer) a behind-the-scenes look into teachers' and leaders' actual engagement in the professional development offered by Great Minds. This sets the stage for what they want viewers to think about the service they offer. While the picture of the single student featured at the top of the webpage communicates joy and calmness of learning math through a curriculum focused on the individual student, this image functions to convince the viewer of the producers' ability to provide teachers and leaders support and collaboration through their professional development offerings. Through frame size, shot angle, gaze, and pose, the producers aim to create a sense that the viewer can picture themselves as a part of such a professional development as they see their peers engaging in – a common strategy in advertisements (Machin and Mayr, 2023).

The notion of support and collaboration is extended within the next web page that I analyzed and provided details on later in this paper.

Change is Necessary

In the last section of this page, the title *Transforming Classrooms* heads a section with a quote from an individual named Quentina T. See Figure 7. The only additional information given about the producer of the quote is a vague label of *East Baton Rouge Parish School System, LA*. The quote from Quentina T. reads, *The greatest joy was to see children discussing why they got a certain math answer and explaining different problem-solving strategies*. The producers' use of the verb, *transforming,* to describe classrooms in this title would presumably encapsulate the quote that follows. Therefore, in relationship together, the title and quote would imply that, without the Eureka Math curriculum, classrooms are void of Quentina T.'s description of *children discussing why* and *explaining different problem-solving strategies* because the classrooms have not yet been *transformed* as those that are utilizing Eureka Math. Further, it is suggested that the Eureka Math curriculum is the vehicle that produces a change from classrooms without these characteristics to those that match the description given by Quentina T.

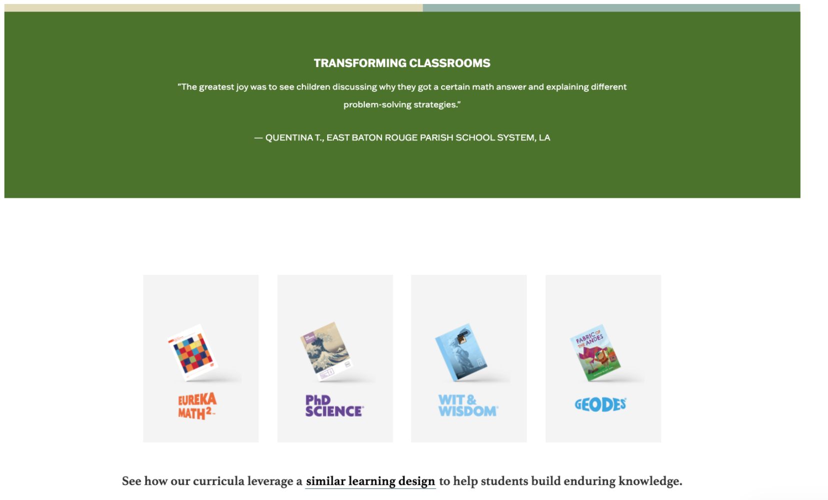


Figure 7: Eureka Math Overview Page- Transforming Classrooms
Note. From Great Minds, (2023). Overview. Eureka Math.
https://greatminds.org/math/eurekamath

The use of the word *transform* is worthy of analysis. Defined as a verb meaning, *make a thorough or dramatic change in the form, appearance, or character of* (Oxford Languages, 2023) implies that the curriculum was the cause of *transform*ation in Quintina's classroom from an unspecified previous state, to one in which students are *discussing why they got a certain math answer* and *explaining different problem-solving strategies*. The message makes the assumption that the viewer, as a teacher or leader of teachers, is in direct need of large-scale changes to the way their students are interacting through their mathematics learning, a manner that here presumably does not include the *discussion of why they get certain answers* nor *explaining different problem solving-strategies*. Additionally, this is structured by the producer to imply that the Eureka Math curriculum is uniquely and superiorly capable of almost magically producing these changes.

This idea of transformation is, in fact, a metaphor. Machin and Mayr (2023) explain that metaphors are often utilized to replace actual concrete processes with abstractions that can be used to make arguments seem more credible. This can be done by highlighting one aspect while hiding other aspects of the process. In this situation, the producers highlight the presumably desired student actions of discussing and explaining as described above while using the metaphor to hide the complexities of the process that may include components such as the implementation of new pedagogical strategies or the establishment of conducive classroom norms to produce the transformation from one state to the next. Point being, the producers' use of the metaphor, *transform*, portrays what could be considered by many to be an arduous undertaking as a simple, overnight, and magical fix.

Differentiating Eureka Math from Others

Suppression of Data: Most Utilized Curriculum. Further down the overview page, a banner spans across the page reading, *The most widely used K-5 math curriculum in the U.S.* See Figure 8. Relevant to this statement, both Fairclough (2003) and Machin and Mayr (2023) emphasize that what is missing from a text is just as important as what is in a text. In her own examination of the websites of five prominent private schools, Pini (2011) revealed the common advertising technique used by producers to "make affirmations based on simplistic or absent evidence" (p. 285) which she attributes to their intention to veil particular characteristics in favor of others.

Similarly, on the Great Minds' website, there are several missing pieces of evidence that are missing from claims or affirmations. One of the most significant being the producers' claim that Eureka Math is *The most widely used K-5 math curriculum in the U.S.* At the most fundamental level, the actual quantity that supports this claim is absent. Approximately how many schools or teachers utilize the Eureka Math curriculum? Also important is the overall quantity of total math curriculum users that were surveyed to declare that of those Eureka Math is the “most widely used”. One might also be curious as to the process of how this data was obtained. Or additionally, the dates to which this statement applies- is this true currently, in the past, or both? Any additional details to support this statement would support the reader in making their own judgment as to the degree of truth it holds. Here, there appears to be a disregard of providing readers with evidence to support such a bold and possibly exciting claim. Machin and Mayr (2023) explain that through implied quantifications, such as this producers' claim that the curriculum is the most widely used, producers can “give the impression of objective research and scientific credibility, when in fact not specific figures are provided” (Machin & Mayr, 2023, p. 84). By intentionally withholding such important information, the producers of this webpage give the impression that there is credibility to their statement while portraying a sense of epistemic authority. Including this claim is a tactic intentionally utilized by the producers to persuade the viewer by creating a sense that the Eureka Math curriculum is tried and true because so many others are already utilizing it. Additionally, including the idea that this is the *most widely used K-5 curriculum* can function to convince the viewer that they are missing out on something that other teachers and leaders are already engaging in.

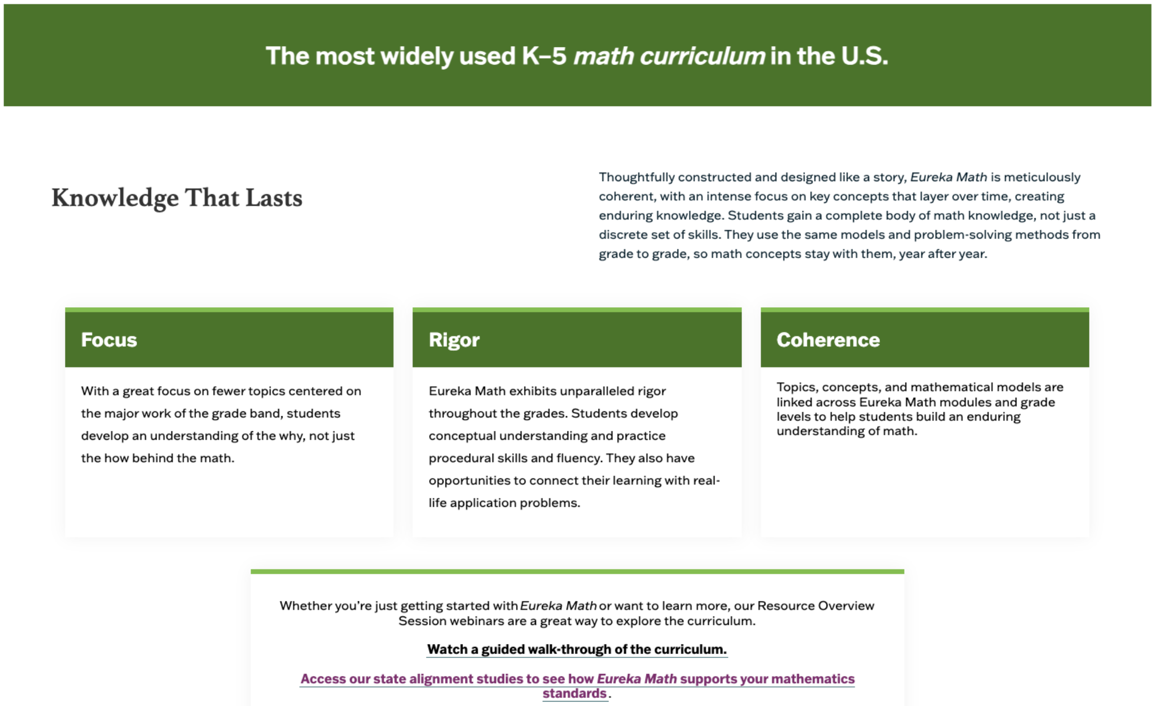


Figure 8: Eureka Math Overview Page- The Most Widely Used K-5 Math Curriculum
Note. From Great Minds, (2023). Overview. Eureka Math.
https://greatminds.org/math/eurekamath

Differentiation Through Structural Opposition. Pini (2011) points out that discursive strategies of advertising are commonly utilized to serve two related purposes: creating an attractive image of a company while producing negative definitions of competitors. This differentiation can be accomplished in a variety of ways including subtly implying differences using intentional language. A foundational aspect of Halliday's (1985) theory of Social Semiotics is that words are a part of a network of meanings. Subsequently when a single word is used in text, it can imply a sense of opposition without explicitly stating a term that directly communicates such opposition. An example given from Machin and Mayr (2023) illustrates this opposition: "when a school is described as 'safe', we may take it that a contrast is being made between safe schools and those which are not" (p. 63). Davies (2012) believes that this concept, *structural opposition,* is useful in analyzing ideology in language and is often structured by a writer's use of only one of the opposing terms. While only one term is utilized, Machin and Mayr (2023) explain that this inclusion "merely implies the differences from its opposite without these being overly stated". On this website, there exists an ongoing implied opposition between the Eureka Math curriculum and other curricular materials divorced from their actual names or any specific identifying features. This structural opposition is utilized to communicate a sense of superiority of Eureka Math to other curricula. The sentiments of opposition implied throughout the webpage focus on Eureka Math being new and innovative rather than outdated and obsolete, rigorous rather than lenient, producing enduring knowledge rather than temporary, and enjoyable rather than boring.

Superior in the Level and Type of Knowledge and Learning. Throughout the Eureka Math overview webpage, the use of structural opposition creates a sense of differentiation claiming Eureka Math's ability to produce a superior level of knowledge and skills in students than other unnamed curricula. In the section titled, *Knowledge That Lasts*, a portion of the descriptive texts reads, *Students gain a complete body of math knowledge, not just a discrete set of skills.* See Figure 9. The adjective, *complete,* is used by the producer to describe the Eureka Math curriculum, and it implies that other curricula may be incomplete. Additionally, the structure of the phrase, *not just a discrete set of skills,* implies that, in contrast to the Eureka Math curriculum, other curricula provide students with *just a discrete set of skills*.

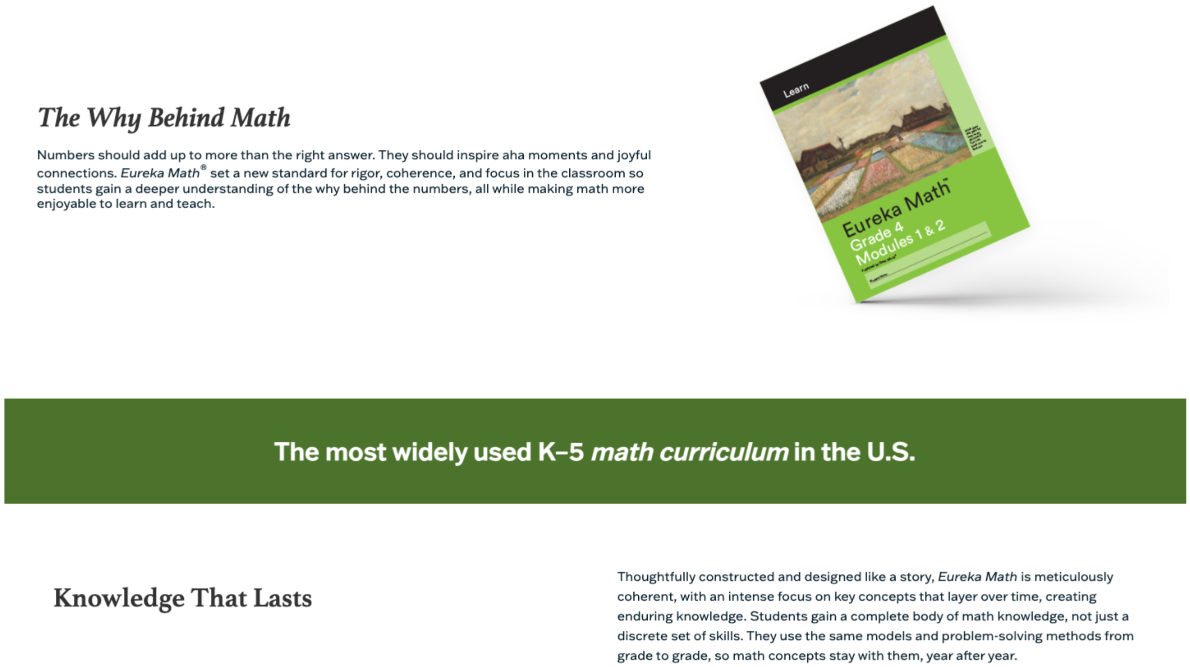


Figure 9: Eureka Math Overview Page- Knowledge That Lasts
Note. From Great Minds, (2023). Overview. Eureka Math.
https://greatminds.org/math/eurekamath

Also on this page, it reads, *Eureka Math set a new standard for rigor, coherence, and focus in the classroom so students gain a deeper understanding of the why behind the numbers, all while making math more enjoyable to learn and teach*. See Figure 9. This statement by the producer is used to imply that, prior to the availability of Eureka Math, an inferior level of *rigor, coherence, and focus* were the standard of other math curricula.

Additionally, the producer's decision to structure the adjectives of deep as *deeper* and enjoyable as *more enjoyable* implies a superiority to an unnamed *other*. This intentionally phrasing functions in communicating the producers' claim that Eureka Math provides students with a deeper understanding than other math curricula and is more enjoyable than other math curricula. One might ask, *deeper* than what? Or *more enjoyable* than what? Without specificity of naming an *other*, it is difficult for the reader to examine the accuracy of these claims or even, more simply, to make their own judgment regarding the accuracy of these claims.

In a separate section titled, *Rigor*, it is expressed that *Eureka Math exhibits unparalleled rigor throughout the grades*. See again Figure 8. The two sentences that complete this section could be assumed to define for the reader what the characteristics of *rigor* entail: *students develop conceptual understanding and practice procedural skills and fluency. They also have opportunities to connect their learning with real-life application problems*. Again, as before, void of any named opposing curricula, the assumption can again be made that Eureka Math is being compared to *other* curricula. The use of the adjective *unparalleled* implies a comparison to the unnamed *other*. It may be assumed here that other curricula are lenient in comparison to Eureka Math. Moreover, the producers' use of the adjective *unparalleled,* paired with the two sentences that follow, creates a structure of opposition promulgating that idea that in comparison to other curricula, Eureka Math is exceptional in producing students who *develop conceptual understanding and practice procedural skills and fluency* and, in contrast to other curricula, provide students *opportunities to connect their learning with real-life application problems* which presumably other curricula do not.

Conclusion of Overview Page

Through the means of multimodal Critical Discourse Analysis, I examined the overview page of the Eureka Math section of the Great Minds website to create a description (micro) and interpretation (meso) of both the text and images utilized by the producers of the page. Through the micro-level of analysis, I detailed varying formal properties of the text and images – such as the use of metaphorical terms like transforming – and the image of the single student on the main page contrasted by the visual of a group of teachers. Along with the description of each significant component, I provided interpretations of the text / images as a product of a process of production by the producers and a resource of interpretation for the viewers. This analysis revealed several connections: including the use of strategies of individualization, use of conceptual metaphor, and an overarching strategy of differentiation through structural opposition and suppression.

Professional Learning for *Eureka Math* Webpage

I chose to continue my analysis on the linked web page titled, *Professional Learning for Eureka Math* (Great Minds, 2023e). From the Great Minds' Eureka Math main page, this second page is accessed by clicking the button, *View Offerings,* in the subsection titled, *Professional Learning from a Trusted Colleague* of the section *Professional Development: Supporting Teachers Before, During, and After Implementation*. My reason for choosing to further my analysis in this section is twofold. The word *teacher* appears eleven times on the main webpage, making it the second most utilized word of substance (excluding a, the, etc.). This leads me to believe that the focus on teachers is of importance to the publisher, and this section of the main page focuses on support for teachers. Secondly, while the Eureka Math curriculum is a product offered free of charge by Great Minds, the professional development is a service that accompanies the free product but is of cost to the consumer.

You Have a Need and Great Minds Has the Solution

Establishing a Sense of Trust. The first section of the Professional Learning page provides us with some important insights. This section consists of an image with the subject set off to the right and two phrases which are positioned to the left. I will first discuss the image itself and then the text that is overlaid upon the section. The photo is of a middle-aged man sitting with an open textbook to his right, a piece of paper directly in front of him, and an open laptop in front of the paper and angled a bit to the left of him. He is holding a blue pen which he appears to be writing with. He is dressed in professional attire, wearing a brown collared-shirt donned with white-colored pinstripes. Over the shirt he is wearing a gray sweater-vest. He is looking down at the paper he is writing on with a slight smile on his face. He appears to be sitting in a classroom as a green chalkboard is visible in the background. The chalkboard is blurred, making the white writing on it illegible to the viewer. The image is like that of the student on the overview page in that it is of one individual, at approximately the same distance, and at approximately the same level. In this image, the frame is a bit smaller than that of the student on the main page. See Figure 10. Here the shot is closer, and the viewer is able to get more of an idea of what the subject is working on. As discussed earlier, Machin and Mayr (2023) provide us with the fundamental concept that the viewer of both films and pictures "is always positioned in regard to the subject" (p. 130). This positioning determines the degree of closeness to the subject that the frame provides and the height of the viewer in relation to the subject. According to Kres & van Leeuwen (2006), the photo is taken as a *medium close shot* (p. 124) and provides a feeling of closeness as the top of the subject's head is out of the frame of the picture, providing a sense that we, as the viewer, are close to him. Like the image of the four subjects engaging in professional development, this type of shot puts the viewer at a *far personal distance* (p. 124), giving one a sense of having a relationship of closeness with the subject that makes it acceptable to look at the work that he is engaging in. The angle of the shot positions the viewer at the same level as the subject – suggesting equality in status, such as one of peers rather than one of inferiority or superiority that would be obtained if positioned below or above the subject. The producers utilize this frame size of medium close shot and the angle of being at the same level to create a visual that represents equality of status between the viewer and the individual in the photo, a sentiment that is consistent throughout this webpage.

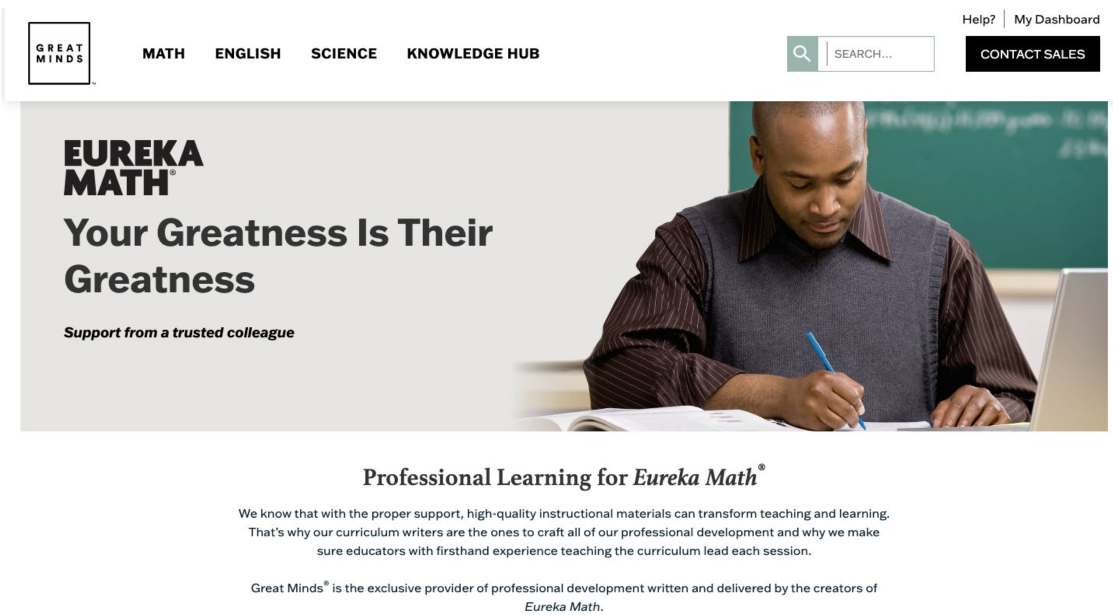


Figure 10: Eureka Math Professional Learning Page- Top
Note. From Great Minds, (2023). Professional Learning. Eureka Math.
https://greatminds.org/math/eurekamath/professional-learning

The visual, discussed above, demonstrates consistency with the text of this section. From top to bottom, the first is the Eureka Math logo in the boldest and largest font. Below, in a lighter-colored shade of black and about the same size as the Eureka Math logo but written in both upper and lowercase letters are the words, *Your Greatness Is Their Greatness*. The last word *Greatness* is written on its own line below the other four words. Lastly, in the smallest font, italicized and written in both upper- and lower-case letters, are the words *Support from a trusted colleague*, labeling the facilitators of the offered professional development as *colleagues*. The first example of the congruence of equality of status is found in this section of the page where the term colleague is utilized in the sentence, *Support from a trusted colleague*. The use of the word *colleague* rather than trainer, instructor, or similar term implies that the reader will receive support from a coworker who is of similar rank or status. The producers' intentional use of this term functions to create a relationship of closeness and trust between themselves and the organization, a feeling as if Great Minds has their best interest at the forefront.

Building a Brand Image

Turhan and Okan (2017) remind us of the most fundamental idea regarding advertising: its primary aim is to "persuade people to buy something or to accept an idea" (p. 213). They delineate that between product and non-product advertising, the former being the advertising of an actual product "to buy" and the latter being "to accept an idea". In Lischinsky's (2024) discussions of branding, he explains that, for business organizations, the "primary concern is to create a distinctive brand identity that will differentiate their offering from functionally-equivalent alternatives, as well as anchor consumer loyalty as products are quickly replaced by updated or redesigned models" (p. 2). Through this website, the producers are working to build a brand for Great Minds as a designer and provider of professional development that supports teachers and leaders in their implementation of the Eureka Math curriculum. The branding of Great Minds in relation to Eureka Math is of an organization of experts in mathematics education who can be trusted to support school personnel to accomplish their own metamorphosis from a school in need of change to one full of students attaining success.

Establishing a Sense of Epistemic Authority. The phrase *we know* is utilized twice in this web page in different two statements:

- *We know that with the properly support, high-quality instructional materials can transform teaching and learning*

- *Because we know that when teachers feel prepared and well-versed in the curriculum, students thrive.*

This phrase, *we know,* is one worth examining. Here, it is not explicitly stated who is included in the pronoun *we*, however, it is implied that it does not include the reader, but rather speaks about the unnamed individuals that comprise the Great Minds organization. Fairclough (2000) and Machin and Mayr (2023) are in agreement that the concept of *we* can be utilized to disguise power relations. When stating *we know,* text producers are speaking from a place of authority. An epistemic modality (Machin & Mayr, 2006), such as the statement made here, is used by the producer to convey a sense of trust through their extensive level of knowledge. Utilizing this modality to express a high degree of certainty can be used to convince the audience or viewer that what is being said need not be supported and that the producer can indeed be trusted (Machin & Mayr, 2006).

Fairclough (2000) and Machin and Mayr (2023) also point out that the use of *we* can be used by producers to make vague, unsupported statements. In situations of this kind, producers employ an *epistemic modality* through which they show their certainty "that something will happen or is the case" (Machin, 2023, p. 261). In this case, the producers of this webpage forego the inclusion of additional details to explain their certainty that, *with the proper support, high-quality instructional materials can transform teaching and learning* and *that when teachers feel prepared and well-versed in the curriculum, students thrive.* The producers' lack of explanation exudes a sense of *epistemic authority* over the viewers – a perception that the Great Minds organization is made up of "more epistemically competent individuals, namely the experts" (Croce, 2020). In a piece from Croce (2020), he details Hardwig's (1985) beliefs regarding the reasoning that those projecting a sense of epistemic authority (*epistemic subjects*) are able to do so with little to no empirical evidence. Croce (2020) explains that often "epistemic subjects are justified in possessing beliefs that lack evidential support to the extent that those beliefs arise out of the epistemic dependence that epistemic subjects have" (p. 5). In alignment to these ideas, the producers of this site may hold an identity that they are justified in omitting evidential support for their assertions that they *know* because the viewers, acting as *epistemic subjects,* possess a dependence on them for guidance in this epistemic domain.

In congruence with this essence of epistemic authority is the use of the adjective *proper* to describe the *support* necessary to transform the teaching and learning. As to what exactly *proper support* entails, the statement is vague. It seems to be assumed, therefore, that whatever designates support as *proper*, the professional development being offered here meets those requirements. Again, the lack of explanation provided by the producers as to what *proper support* is helps to establish Great Minds in a position of authority, possessing the knowledge necessary to deem support as proper or not.

This same statement bolsters the idea that Eureka Math is a *high-quality* curriculum that will transform teaching and learning. Being that the professional learning services advertised on this page are to support the utilization of Eureka Math, this statement seems to make the assumption that the Eureka Math curriculum consists of *high-quality instructional materials*. There is no explanation of the criteria that constitutes curricular materials to be deemed as *high-quality* or how the producers know it to be true that it is high-quality. Again, a sense of epistemic authority is exuded through the inclusion of this unsupported label as if it is information that is so obvious that it need not be explained or supported with evidence.

A sense of epistemic authority is communicated through the combined use of this statement, *we know* and the claim that the Eureka Math curriculum is of *high-quality*, both lacking additional explanations. Acting with a sense of authority in this epistemic domain may often function as a way for the producer to increase the confidence that a viewer feels regarding their product or company.

Legitimacy Through Functionalization. The follow up to the statement, *we know that with the proper support, high-quality instructional materials can transform teaching and learning,* as discussed above bares some additional meaning as designed by the producers. They state, *that's why our curriculum writers are the ones to craft all of our professional development and why we make sure educators with firsthand experience teaching the curriculum lead each session*. Here, the individuals who are credited with designing the professional development offered by Great Minds are anonymized through the use of functionalization – which, according to Machin and Mayr (2023), is depicting an individual or group "in terms of what they do" (p. 117). From their own research, Machin and Mayr (2007) have found that functionalization that identifies people by their occupation can suggest legitimacy or illegitimacy depending on the context and purpose. The producers' application of functionalization is consistent in three additional areas of this webpage, as the producer informs that the professional development is *created by the team of teacher-writers behind Eureka Math*, that it is *written and delivered by the creators of Eureka Math,* and that participants will get *support from a trusted colleague.* In these cases, the producers seem to have functionalized the designers of the professional development to establish confidence in the accuracy and reliability of their design and implementation of the professional development service, and subsequently to again build a sense of trust in the brand.

Cultivating Trust Using Overlexicalization. Returning to the subtitle that is displayed within the image and in conjunction with the statement about greatness, it reads *Support from a trusted colleague*. The adjective, *trusted* – utilized by the producer to describe the anonymous colleague who is providing support – is important as it functions to communicate the message to the reader that they can have faith in the person who would provide support from Great Minds and, more broadly, in Great Minds as an organization. In this title, there is no elaboration by the producers of why or how the support would come from a colleague or why the unnamed colleague should indeed be trusted.

While convincing the viewer that there is a need for change for their students to succeed, the producers take care to connote a sense of authenticity and trust. To do so, terms and phrases conveying this sense of authenticity and trustworthiness are extensively utilized: *support, colleague, trusted, why we make sure, ensure, coaches, help, understand, we know, set educators up for success, personalized coaching, feel confident,* and *because we know.* Often termed *overlexicalization*, this strategy may be best defined by Teo (2000) in reference to its use in media coverage as "a surfeit of repetitious, quasi-synonymous terms is woven into the fabric of news discourse, giving rise to a sense of over completeness" (p. 20). As Machin and Mayr (2023) explain, this technique often gives the impression of over-persuasion and is most often "evidence that something is problematic or of ideological contention" (p. 54). The producers' implementation of overlexicalization is logical in the social context where consumers of educational materials, mainly teachers and leaders, are being asked to put their trust in large corporations that are known to have their own financial objectives – separate from that of one's community. The producers' intentional stance of possessing epistemic authority, paired with an overemphasis on language associated with trust, function in tandem to manufacture a feeling of confidence in Eureka Math and Great Minds as a whole.

You Need Change

Persuasion Through Metaphor. Like on the Eureka Math overview page, the metaphor of transformation is utilized again on the professional development webpage. See figure 11. We discussed earlier the sentence from this web page, *we know that with the proper support, high-quality instructional materials can transform teaching and learning.* It is worth revisiting that the word *transform* is defined as a verb meaning *make a thorough or dramatic change in the form, appearance, or character of* (Oxford Languages, 2023). The claim that the curriculum will *transform* the teaching and learning happening in a classroom is a message that reinforces an overarching essence of the main Eureka Math web page. Through this message, the producer makes the assumption that the reader, as a teacher or leader of teachers, either already believes or is open to the belief that they are in direct need of large-scale changes to the mathematics teaching and learning that they are associated with – so much so that they can build on that belief.

Figure 11: Eureka Math Professional Learning Page- Middle
Note. From Great Minds, (2023). Professional Learning. Eureka Math.
https://greatminds.org/math/eurekamath/professional-learning

As discussed earlier, Machin and Mayr (2023) explain that metaphors are often utilized by producers to replace concrete processes with abstractions. The producer utilizes this metaphor of transformation to communicate that with the *proper support* from Great Minds through their professional development, the viewer's classroom will undergo a needed *dramatic change* simply by utilizing the Eureka Math curricular materials. Abstracting the concrete process that teachers, leaders, and even students would have to go through to get to the end goal is purposely concealed through the use of metaphor. The idea of making a classroom *transform* can certainly feel more alluring in comparison to synonymous verbs such as change, modify, reshape, or convert. The notion of transformation has a certain magical essence to it, portraying the idea as easy and possibly comfortable or nostalgic for viewers as associations with the transformation of caterpillars to butterflies, robots into motor vehicles via cartoons, or in the tale of Cinderella into a beautiful woman ready to go to the ball.

Further, the producers make the claim that engaging in the advertised professional development to implement the Eureka Math curriculum will transform the teaching and learning in their customers' classrooms without any specificity as to what exactly a so-called *transformation* would result in. What would the classroom look like? What would instruction be like? How would such a transformation be measured: with assessments, with observations? What would be assessed or observed? A significant point that Machin and Mayr (2023) make can be easily seen through these wonderings about the end results of going through the professional development. They advocate that when an individual or company uses metaphor to avoid specifying what will be the result, it creates a situation in which *no one can in fact be held accountable* (p. 226). The producers' promise to transform a classroom's teaching and learning creates such a vague and subjective picture that holding the organization accountable would be impossible as there is no solid declaration of what meeting vs. not meeting accountability standards would even look like.

Persuasion Using Moral Obligation. Next, I look again at the statement displayed at the top of the banner of the page, but this time I hone in on the statement, *Your Greatness is Their Greatness*. In this statement, the adjective choice stands out. Machin and Mayr (2023) articulate that adjectives used in advertising are often utilized to "enhance a sense of experience associated with a product or service" (p. 55). They add that rather than telling us about the functionality of the product, adjectives create an emotional evaluation of the product or service. Additionally, Machin and Mayr (2023) advocate that, "adjectives tell us about the qualities of an entity or person and are also associated with comparisons" (p. 55). In this particular statement, the term *greatness* is written twice- in reference to both the reader and an individual or group of individuals referred to as *their*. A logical assumption for the antecedent of the term *their* would be students – communicating a more complete idea that the reader's greatness is the individual student's greatness. But even further, because this page is focused on professional learning for teachers and leaders utilizing Eureka Math, it may also make sense that the meaning of the title is to communicate the belief that the level of greatness of a teacher utilizing the Eureka Math curriculum is indicative of their students' greatness. A sense of ownership and responsibility for having *greatness* is portrayed through this title. The implication here could be that students' greatness relies on the greatness of their teacher. Further, in light of the *crisis* of mathematics education, it is also implied that students' failures are a result of their own teachers' failures. This implied responsibility is part of the producers' use of *moralizing discursive strategies* to play upon the emotional bonds that are a prevalent part of the discourse in education and creating an obligation to the viewer's students. Machin and Mayr (2023) assert that this is a common strategy among the producers of advertising in the educational sector where the neoliberal discourse is regarding teachers' moral obligations to take personal ownership for their students by striving to create children of a particular type. If successful, through this language, the producers may be attempting to tap into the viewer's sense of obligation to create students of greatness and therefore persuading them to take on the responsibility of engaging in the professional development offered here to ensure this comes to fruition. Of course, in conjunction with what has previously been discussed, the producers are communicating the message that the ability for this greatness to be attained is centered on the professional development engendering a change in that individual's classroom, from what it currently is to its transformed state.

The adjective, *greatness* is a part of the Eureka Math product branding and the Eureka Math Professional Learning service branding. The producers' intentional use of word *greatness* with its root being *great* displays continuity with the publisher's name Great Minds. In this same sense, at the very bottom of every web page is a smaller banner that reads, *every child is capable of greatness*. Taken together, the producer is communicating the message to teachers and leaders that all children are capable of greatness, but for them to achieve such greatness one must first be great themselves. The further implication is that this greatness is not already the current status, but rather change is needed to achieve it and the professional development being offered here will provide that change.

Along the same lines, the producers utilize moralizing discursive strategies to reinforce their message of teacher and leader commitment to students in their statement, "*because we know that when teachers feel prepared and well-versed in the curriculum, students thrive*". Playing into the assumption of a viewer's feelings of moral obligation as is pervasive within the culture of education, this statement makes the same type of implication as discussed above: your students' successes rest on your own willingness to prepare yourself, and you are not yet prepared.

The notion communicated by positioning these two sentences on this web page is that the professional development provided by Great Minds is necessary for a teacher to attain a level of greatness or preparedness that only then will permit students to be great or thrive. The underlying assumption being conveyed by the producer is that the viewer is not currently at this level of greatness or preparedness and subsequently requires change from their current status quo. This culminates in the idea that this necessary change can be delivered by participating in the featured professional development from Great Minds to support in the implementation of the Eureka Math curricular resources.

Great Minds will Fulfill Your Need for Change

When looking through the lens of CDA, there is even more meaning behind the phrase *we know* which is utilized twice on this web page alone. As a reminder from the discussion above, the phrase *we know* is utilized in two statements. In the second example, I have included the preceding sentence which is necessary for understanding the context of the focus sentence.

- *We know that with the proper support, high-quality instructional materials can transform teaching and learning.*
- *To set educators up for success with Eureka Math, our team of teacher-writers have created the three-year plan below integrating both professional development and personalized coaching so all teachers can feel confident with the curriculum. Because we know that when teachers feel prepared and well-versed in the curriculum, students thrive.*

Machin and Mayr (2023) posit that, while consumers take on the role of those who *desire, need,* and *want,* advertisers use such words as *think, understand,* and *know* to portray themselves as possessing the unique ability to "fulfill the needs and wants through their knowing and understanding" (p. 152). The producers of this web page reinforce these roles in their use of the phrase *we know,* which functions to communicate to the viewer the assumption that they have a need and Great Minds possesses the solution. The specific need that is communicated through these sentences is related. As discussed earlier, in the first statement, the producers are communicating that the viewer needs change in the teaching and learning of their current classroom. In the second statement, they are communicating that the viewer needs to understand the Eureka Math curriculum well for their students to flourish. While a portion of the implication of these sentences is that the Eureka Math curriculum will provide support toward fulfilling these needs, the producers make it clear that simply using the curriculum is not enough. In the first, the producers indicate that the need for change can be fulfilled *with proper support* from Great Minds. In the second, it is indicated that *the three-year plan that the teacher-writers have created* that *integrates both professional development and personalized coaching* will fulfill the need for understanding the curriculum.

Through these statements, the producers are attempting to position Great Minds as the professional development and coaching provider who will be able to fulfill the needs for change that the viewer desires. The producers go on to lay out a plan of professional development and coaching for the viewers which I analyze in the next section.

The Plan for Your Change

The largest section on this webpage begins with the title, *RECOMMENDED LEARNING PLAN* written in all uppercase letters. See Figure 12. In this section a second title, written directly below in a larger, bolded font, in upper- and lower-case text read, *Implementing Eureka Math with Fidelity*. A two-sentence description we have most recently discussed reads, *To set educators up for success with Eureka Math, our team of teacher-writers have created the three-year plan below integrating both professional development and personalized coaching so all teachers can feel confident with the curriculum. Because we know that when teachers feel prepared and well-versed in the curriculum, students thrive*.

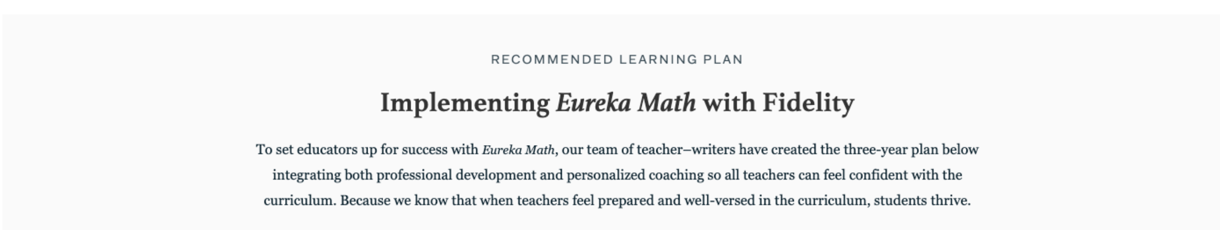


Figure 12: Eureka Math Professional Learning Page- Recommended Learning Plan
Note. From Great Minds, (2023). Professional Learning. Eureka Math.
https://greatminds.org/math/eurekamath/professional-learning

Under this banner, within this section, a three-year recommended learning plan for professional development is detailed. See figure 13. The plan is displayed in three tables featuring one for each year. Each of the three tables features two columns: colored light-green on the left and dark-green on the right. The light-green column reads, *Professional Development Workout Sequence* and the dark-green, *Personalized Coaching Sequence*. Below these column titles are six rows, three gray-colored rows span the horizontal-length of the table, each with one row of text below them. The gray-colored rows serve as headers for the one-row section below. These headers read, *SUMMER*, *FALL*, and *SPRING*. The first header in the Year 1 table contains an added detail with the title "SUMMER", reading "SUMMER (prior to implementation)". The rows with more text beneath these header rows are split in half by color difference, the section on the left being a darker-beige color and the section on the right being a lighter-beige color.

Professional Learning for *Eureka Math*®

We know that with the proper support, high-quality instructional materials can transform teaching and learning. That's why our curriculum writers are the ones to craft all of our professional development and why we make sure educators with firsthand experience teaching the curriculum lead each session.

Great Minds® is the exclusive provider of professional development written and delivered by the creators of *Eureka Math*.

PROFESSIONAL DEVELOPMENT

Created by the team of teacher–writers behind Eureka Math, our in-person and virtual professional development sessions include sessions for teachers and leaders. These sessions ensure strong initial implementation as well as sustained success.

PROFESSIONAL DEVELOPMENT OPTIONS

PERSONALIZED COACHING

Through modeling and observation, Great Minds coaches help teachers improve implementation and help leaders develop effective ways to support, understand, and evaluate day-to-day classroom practices.

COACHING OPTIONS

RECOMMENDED LEARNING PLAN

Implementing *Eureka Math* with Fidelity

To set educators up for success with *Eureka Math*, our team of teacher–writers have created the three-year plan below integrating both professional development and personalized coaching so all teachers can feel confident with the curriculum. Because we know that when teachers feel prepared and well-versed in the curriculum, students thrive.

Year 1: Building a Foundation

Professional Development Workshop Sequence	Personalized Coaching Sequence
SUMMER (prior to implementation)	
- Lead *Eureka Math* - Launch Eureka Math - Fluency in Action Resource-specific sessions: - *Eureka Math Equip*™ - *Eureka Math in Sync*™	Strong Start for Leaders: - Strategic Planning - Crafting Effective Feedback - Effective Feedback Conversations
FALL	
Preparation and Customization of a Lesson	Personalized coaching package based on school or district goals
SPRING	
90-minute professional development topic sessions (available upon request)	Personalized coaching package based on school or district goals

Year 2: Sustaining Growth

Professional Development Workshop Sequence	Personalized Coaching Sequence
SUMMER	
Any of the following sessions: - Solving Word Problems - Understanding Major Work of the Grade Band	
FALL	
Additional 90-minute topics available upon request.	Personalized coaching package based on school or district goals.
SPRING	
Additional 90-minute topics available upon request.	Personalized coaching package based on school or district goals.

Year 3: Leading with Confidence

Professional Development Workshop Sequence	Personalized Coaching Sequence
SUMMER	
Offer all new teachers the foundational professional development sessions: - Launch Eureka Math - Fluency in Action 90-minute professional development topic sessions (available upon request)	
FALL	
Offer all new teachers the final foundational professional development session: - Preparation and Customization of a *Eureka Math* Lesson 90-minute professional development topic sessions (available upon request)	Personalized coaching package based on school or district goals
SPRING	
90-minute professional development topic sessions (available upon request)	Personalized coaching package based on school or district goals

Figure 13: Eureka Math Professional Learning Page- 3-Year Plan
Note. From Great Minds, (2023). Professional Learning. Eureka Math.
https://greatminds.org/math/eurekamath/professional-learning

An aspect that stands out in the three-year plan is the producers' ambiguous nature of eight of the nine time periods of the *Personalized Coaching and Sequence*. Of these eight periods, two are blank with no assigned plan and six read, *personalized coaching package based on school or district goals*. This description of the coaching lacks details about the process or the anticipated focus or results. As discussed multiple times throughout this chapter thus far, the producers' use of a vague title with no detailed description can function to allow the viewer to come up with their own picture of what would be taking place and what would be accomplished (Machin & Mayr, 2023). This lack of specificity by the producers can also result in a lack of accountability as there are no clear or specific goals outlined for the coaching work. It can also be argued that emphasizing personalized nature of the package leaves the reader reliant on obtaining more details in an alternative way, such as contacting the company utilizing the *Contact Sales* button in the top banner of the page. While this may feel inconvenient to some, it echoes the sentiments of individualism communicated throughout the pages. In providing only a limited idea of what the personalized coaching would entail, the producers inadvertently encourage the viewer to make a personal connection with the organization, thereby opening up the communication lines between them.

Among the nine time periods of the Personalized Coaching section of the table, one period that is neither blank nor described as personalized coaching is the very first period designated in the Summer of Year 1. This section reads, *Strong Start for Leaders* and contains three bullet points below – seeming to specify the topics of *strategic planning*, *crafting effective feedback*, and *effective feedback conversations*. The titles of the topics here lack specificity to the Eureka Math curriculum or to math instruction in general. Rather, these titles seem to center on popular buzzwords utilized in the contemporary education sector such as *strategic, planning,* *effective,* and *feedback*. The producers may utilize these buzzwords to tap into the current climate of accountability that currently surrounds teachers and leaders (Washington et al., 2012).

Overall, the information provided in the personalized coaching sequence of the three-year plan is both vague and non-existent, with a small amount of trendy use of buzzwords. Through this design, the producers portray a sense of individualism in their offerings while avoiding any commitment to a particular approach.

In turning to the portion of the 3-Year Plan focused on the *Professional Development Workshop Sequence*, six of the nine periods contain the statement *90-minute professional development session that is available upon request*. Similar to what was just discussed regarding the personalized coaching sequence details, this simple statement in each lacks any specified topic or goal. Again, the producers' use of vague language with a lack of specificity functions to allow the viewer to create ideas and expectations through their own lens. The idea that the viewer holds of what the professional development could look like may work as an advantage to the producers, as the viewers imagine it meeting the needs they desire. Here, the producers seem also to be communicating the sense that the sessions will be part of a personalized plan, which aligns to the message of personalization reiterated throughout the web page.

When zooming out to examine this section of the web page as a whole, there appears to be a fundamental contrast between the title of this section and the focus of professional development and some of the details listed in the plan itself. In the title, *Implementing Eureka Math with Fidelity*, the term *fidelity* stands out. Oxford Languages (2023) defines fidelity as *the degree of exactness with which something is copied or reproduced*. With this definition in mind within the context of this section title, one might ask the question, when utilizing a scripted curriculum with *fidelity* for the purpose of copying or reproducing what is in the written lesson plan, why would a user of Eureka Math require three years of guidance from the curriculum's publisher regarding how to do so? Further, one might ask, isn't the purpose of the scripted lesson plans for the curriculum to provide guidance in written form that can be implemented by the user? And additionally, if the lesson plan does contain the guidance, shouldn't a user simply be able to read the lesson and execute it as written to *implement it with fidelity*?

In contrast to this idea of fidelity to the Eureka Math curriculum, professional development listed by the producers in the three-year plan for the Fall of Year 1 and for new teachers in the Fall of Year 3 is titled *Preparation and Customization of a Lesson*. The verb *customization* is defined by Oxford Languages (2023) as *the action of modifying something to suit a particular individual or task*. The focus on customization as a *foundational professional development session* seems counterintuitive to the overarching goal of fidelity as communicated through the title of *Implementing Eureka Math with Fidelity*. On one hand, it seems that implementing the curriculum with precision is an overarching goal of three-years of workshops and coaching, while on the other hand, the details of those services as available for purchase from Great Minds communicates a focus on how to modify the exact materials that are intended to be implemented with fidelity.

Also, in contrast to the notion of *fidelity,* is the consistent use of the term *personalized* as is utilized in the description of *Personalized coaching package based on school or district goals*, comprising six of the nine time frames of the *Personalized Coaching Sequence* of the three-year timeline. Oxford Languages (2023) defines *personalized* as *designed or produced to meet someone's individual requirements*. While this personalization refers to the coaching package that a school or district will engage in, there remains a discord in the messages being communicated by the producers as the adjectives convey such different meanings. Overall, with the combined frictions between *fidelity* and *customization* / *personalized*, the message being communicated by the producers to viewers may be interpreted as: You will be successful in the implementation of Eureka Math if you do not deviate from the lesson plan as it is written, but also modify it to meet your own ideas while also adhering to the specialized requirements determined by your school or district.

Conflicting messages on this web page may lead the reader to feel confusion and seek support in deciphering the purpose of the three-year plan while attempting to gather more information regarding the ambiguity interwoven throughout. As the reader continues to scroll down in search of additional information or clarity, they will encounter the small banner featuring the three scrolling quotes along with an online form to complete for more information about Professional Learning. Making this online form available is yet another way that the producers on the web page make it convenient to get in contact with viewers in hopes of converting them into customers.

At this point, I conclude my examination of the Great Minds webpages and move into a macro-analysis of the societal conditions that affect the textual and imagery production. While I am closing my analysis at the micro- and meso- levels, I do believe that there could be further analysis worth completing in both the web pages that I explored and the other web pages that are linked via features of the Eureka Math overview page. For example, I believe there could be value resulting from the engagement of an interested researcher in examining the additional professional development page to determine seminaries, differences, and overlaps between it and the page that I analyzed above.

Macro Analysis

In Chapter III, I shared the overarching question I aim to answer through my analysis of the macro-level of Fairclough's (1989) 3-D model. The question articulated was, *How do the claims made by Great Minds relate to the larger context of education in the U.S.?* I then laid out two supporting questions meant to guide me in my pursuit of answering that overarching question:

- What are Great Minds' positions on the current state of education in the U.S.?
- How does this curriculum fit into the current state of "crisis" of mathematics education in the U.S.?

In this section, I answer these questions through the process of explanation (Fairclough, 1989). The stage of explanation is defined by Fairclough (1989) as being "concerned with the relationship between interaction and social context – with the social determination of the processes of production and interpretation and their social effects" (p. 58). Here, we will explore the "durable social structures" (p. 59) that make up the surrounding contexts that allow for the production and reproduction of the discourse found at the meso-level of the two Great Minds' web pages as detailed above. Through the data collected from this CDA, the findings reveal the producers' and viewers' positionings within a discourse of the crisis of mathematics education situated upon a neoliberal ideology that promotes competition, decentralization, privatization, and emphasis on increased testing (Klees, 2020).

A Discourse of Crisis of Mathematics Education

In Chapter I, I shared a brief summary of the history of mathematics education in the U.S. from World War II to the recent adoption of the Common Core State Standards of Mathematics across the majority of the U.S. A significant thread woven through the history of the past six decades of mathematics education is a discourse of the presence of a U.S. national security crisis blamed by what has been deemed poor performance of America's youth in mathematics. Washington et al. (2012) explains, "the prevailing discourse in narrative in mainstream mathematics education is plagued with crisis talk" (p. 54). As we saw, these crises came in the form of national security concerns tracing back at least as far as World War II when military recruits were said to be lacking in the mathematical skills considered necessary for U.S. victory (Washington et al., 2012). These national security concerns have continued into our present-day experience in what some have termed the *accountability movement* (Coburn et al., 2016) ushered in by both President George W. Bush's No Child Left Behind federal law and President Barack Obama's Race to the Top federal grant program. Proponents of this belief claim that America is economically, politically, and technologically inferior and so vulnerable to other countries due to low mathematical performance by U.S. students, and consequently, their lack of preparation for the technical jobs needed to keep the country ahead of others in the international community.

This discourse also includes continuous calls for the mitigation of this so-called poor level of performance through reforms to mathematics education (Coburn et al., 2016; Gutstein, 2009; Klees, 2020; Washington et al., 2012). As we have seen from the response to each new iteration of this crisis narrative, each newly-perceived catastrophe is responded to with a change in mathematics education (Washington et al., 2012) that is often packaged as a need for reform (Krees, 2020). Regarding this ongoing cycle of crisis, Washington et al. (2012) provides salient context: "the invocation of crisis stands at the root of each effort, ranging from new math, to back to basics, to progressive movements, and culminating in the confluence of NCTM Standards, No Child Left Behind, and Race to the Top" (p. 58). Klees (2020), who is an avid critic of neoliberal tenets, believes such practices have been taking root in our educational system by way of, or with the support of, U.S. government entities since the start of the crisis talk. He argues that basic needs of public schools have been ignored in favor of neoliberal reform-focused solutions, asserting that "almost every reform was focused on governance-reorganization, restricting, re-engineering, knowledge management, merit pay, reform civil service laws, community involvement, decentralization, increase testing, vouchers, privatization, output-based aid, results-based finance, etc." (p. 12). A continuous narrative of crisis has bred an ongoing narrative for the need for change that is accepted and reinforced throughout society.

The narrative of mathematical inferiority of U.S. children is such a normalized part of the culture that it is readily accepted as a societal truth and rarely questioned or probed for explanation (Gutstein, 2009). Klees (2020) details that a well-known aspect of the neoliberal ideology that is widely accepted is the mindset that, "education leads to skills, skills lead to employment, employment leads to economic growth, economic growth creates jobs and is the way out of poverty and inequality" (p. 12). He attributes the ongoing acceptance of this narrative that both critiques public schools and places the blame on teachers to be a typical characteristic of the neoliberal ideology and emphasizes the prevailing prevalent message as "schools are failures and teachers are to blame" (p. 10). While the depth to which this narrative is ingrained within U.S. culture cannot be accurately measured, it is clear that the influence it has on educational policies and the way in which members of the society talk and act in accordance to it is wide spread (Coburn et al., 2016; Gutstein, 2009; Klees, 2020; Washington et al., 2012). This crisis discourse continues to reverberate through our society today as the solution to the latest iteration of the crisis has taken effect through the implementation of the Common Core State Standards Initiative.

Is The Crisis a Myth? The term *shock doctrine* was coined by Klein (2007) to describe a tactic utilized by corporate and political elites to capitalize on a period immediately following a large-scale event that causes collective shock to members of a society and during which those elites force through the implementation of radical free-market, pro-corporate measures. This process of shock doctrine is described as being designed and deployed by the School of Economics at the University of Chicago and is possibly most well-known through the efforts of Friedman, a well-known proponent and leader of the neoliberal ideology. Through her research, Klein (2007) cites both natural disasters and human-caused events after which members of society both seek and support fast and pivotal responses to correct the situation which are exploited. Klein calls the state in which people find themselves after a disaster, *disorientation*, and explains in relation to it that, "the theory is that if painful economic 'adjustments' are brought in rapidly and in the aftermath of a seismic social disruption like a war, a coup, or a government collapse, the population will be so stunned, and so preoccupied with the daily pressures of survival, that it too will go into suspended animation, unable to resist" (p. 44). Friedman (1982) himself was transparent in his support of the advantageous seizure of this critical period of opportunity, writing, "only a crisis- actually or perceived- produces real change. When that crisis occurs, the actions that are taken depend on the ideas that are lying around. That, I believe, is our basic function: to develop alternatives to existing policies, to keep them alive and available until the politically impossible becomes politically inevitable" (p. xiii-xiv).

Klein (2007) bases this hypothesis on her own research that reveals a distinct pattern of pro-corporate policies previously discussed and debated by members of societies, being suddenly thrust into implementation in the aftermath of these types of societal *shocks* world-wide. She argues that free-market ideology has been utilized to exploit these moments of shock to advance radical privatization. Klein (2007) attributes much of the exploitation of moments of shock that occurred across the globe since the 1970s to Friedman and his followers from the Chicago School due to their influential nature among political figures and the public at large.

The terror attacks of 9/11, Hurricane Katrina, and the California Wildfires are examples she provides of U.S.-specific *shocks* that were followed by the implementation of pro-corporate radical policies and the privatization of disaster response. The terror attacks of September 11 paint a vivid picture of the way in which the U.S. government utilized fear tactics in the wake of the disaster – not only to declare a *War on Terror*, but to rapidly implement policy supporting the privatization of security, warfare, and reconstruction within the U.S. Klein explains that the War on Terror is a unique war as it is being fought, for the most part, by private companies. Former President Bush outsourced functions previously conducted by the government such as healthcare of soldiers, interrogation of prisoners, and investigations of U.S. citizens to private companies. While this outsourcing was funded by the government, it also established the government as the largest consumer of the services being provided – acting as both an investor of the business and the customer providing guaranteed demand. The magnitude of this spending can be seen in the numbers: in the first five years after the events of 9/1,1 the Department of Homeland Security paid $130 billion to private contractors and, in 2003 alone, spent $327 billion on contracts with private companies. The title of *War on Terror* coupled with the nature of an objective to eliminate terrorism world-wide lacks specificity regarding a time-boundary, physical-boundary, or target to the goal. According to Klein (2007), the ambiguous and limitless nature of this government-established end goal seems to guarantee that the demand for the products and services of the privatized offshoots of the government would continue indefinitely.

Beyond the borders of the United States, fear tactics were implemented on the Iraqi people using the military strategy of *shock and awe*. This military doctrine is designed to spread "mass fear" within an enemy's society at large. In 2003, between March 20 and May 2, U.S. forces bombarded the people of Iraq by deploying more than thirty-thousand bombs and twenty-thousand precision-guided cruise missiles (Gordon & Trainor, 2006), establishing disorientation and regression within the public. This deliberate effort to squash the public's will to resist is a key psychological component of the *shock and awe* strategy. Further reaching, the broadcasting of these actions via the news media is an additional intentional effort to amplify fear world-wide by both exposing a perception of the enemy's weaknesses and the aggressor's power while also serving as a warning to others who may be considering the same actions or contemplating entering the conflict. These examples demonstrate the larger scope of an operation to design and manipulate social and economic factors to create a market engrossed further into capitalism with each passing crisis, a phenomenon that Klein calls, *disaster capitalism.*

In a later example of disaster capitalism, Klein (2007) explains how Hurricane Katrina in 2005 provides clear insight to the ways in which the government can play a part in the enabling of a disaster and then the privatization of rescue efforts and reconstruction of communities in the aftermath. In New Orleans, the government not only allowed poor levels of infrastructure to exist, putting the city in a vulnerable state before the hurricane occurred, but also failed in its response in the aftermath of the storm – both factors playing a major role in the creation of the disaster. Klein (2008) places much blame on the state government, who they hold responsible for insufficient attention and funding to basic infrastructure meant to prevent a catastrophe in the event of a storm such as a hurricane. These failures included levees meant to protect the city from flooding, having not been properly repaired from previous damage and an underfunded and disorganized public transportation system that was ill-prepared for such an emergency despite the advocacy for a well-designed plan in preparation. As Hurricane Katrina approached the city, nearly 120,000 residents who relied on New Orleans's public transportation system were unable to successfully utilize it to evacuate, finding themselves waiting on state-funded emergency response teams who never arrived. In the immediate aftermath of the disaster, the Bush administration began their implementation of neoliberal-aligned processes as the underfunded Federal Emergency Management Agency (FEMA) proved ineffective in providing the proper support, federal funds were utilized to hire private companies and private contractors to deliver the needed response. These government-run contracts with companies previously hired in the reconstruction of Iraq gained new contracts with the U.S. government in response to Hurricane Katrina totaling $3.4 billion. Among the beneficiaries were Halliburton, hired to reconstruct military bases affected by the storm; Blackwater, to provide security for FEMA employees; Parsons, to reconstruct bridges; and Fluor, Shaw, Bechtel, and CH2M Hill, to provide mobile homes to evacuees. Possibly most shocking was the contracting of work to Service Corporation International to retrieve the bodies of victims from homes and streets throughout New Orleans while forbidding local morticians to volunteer in these efforts, claiming their support would be considered as encroachment on the company's commercial territory. Service Corporation International charged the government $12,500 for the retrieval of each victim. The federal government played a central role in the disaster caused by Hurricane Katrina, both in their preparation and their response, and they utilized the opportunity of disorientation created by the shock to further the privatization of services previously rendered by the U.S. government.

Klein (Klein & Smith, 2008) cites the California Wildfires of 2007 as a poignant example of a government's exasperation of a disaster coupled with the privatization of disaster response.

Klein (2007) explains that the California government's response under Governor Schwarzenegger was inadequate and led to worse spread of the fires as they were occurring. Concurrently, as the disaster was playing out, privately-funded rescue workers from the AIG Insurance Corporation responded. According to Klein (Klein & Smith, 2008), these rescue workers responded only to AIG customers "who had paid about $20,000 a year" (p. 589). Klein accounts the experience of members of the community, adding "they were bragging that the houses next door were burning to the ground" (p. 589). These examples contextualize her theory that the tentacles of privatization have been spreading for many years and have made the movement of privatizing government services such an accepted and mainstream idea that the U.S. has gotten to the point that private companies are now working in the business of for-profit emergency response services. Klein argues that this radical privatization has occurred due to exploitation of moments of shock to further free-market ideology.

Over the past twenty years, several experts (Gutstein, 2009; Rampal, 2019; Stevenson, 2014; Washington et al., 2012) have expressed similar sentiments regarding the continuous crisis narrative in mathematics education. These scholars make claims that the crisis of mathematics education is in fact manufactured and that each iteration of crisis creates a period of disorientation during which responses that may not otherwise have been accepted are quickly put into effect with seemingly little resistance. One ongoing subordinate narrative of the mathematics crisis is one that claims alarm in a shortage of U.S. STEM-workers. This crisis has been met with calls of mitigation through improved STEM education in U.S. schools through increased focus, testing, teacher training and requirements, and large-scale curricular changes. Stevenson (2014) explains her view that the foundational claim of this narrative, that a STEM-worker shortage exists, is a myth. Through her deconstruction of the narrative, she reveals her evidence that there is, in fact, a STEM-qualified worker surplus. She proposes that the perpetuation of the narrative benefits STEM-related businesses who advocate for the government's issuance of an increased number of H-1B visas so they can hire employees from foreign countries at generally lower rates than their U.S. counterparts. This hypothesis aligns with Klein's (2007) proposal that during the shock of an event, political and / or corporate elites quickly implement policies as the general public looks for swift and effective solutions to mitigate the perceived issues. Also aligned is the idea that these players intentionally inflame the crisis through use of scare tactics in the wake of the shock. Stevenson (2014) goes as far as to point out a lack of attention to this topic in the related literature and calls on the research community to investigate the myths by intentionally making an effort to "research stem in the context of corporatization of education, so that the students' needs, not the financial bottom line, remain paramount in K-12 and teacher education" (p. 143). Stevenson's concern regarding the push toward corporate involvement and benefit within the educational sector aligns with criticisms from others regarding what are believed to be pervasive neoliberal influences to reform education through its privatization (Kleese, 2020).

Most recently, Rampal (2019) also discussed their belief that the crisis of mathematics education is manufactured, but on an international level. Rampal (2019) places a considerable amount of blame on the Programme for International Student Assessment (PISA) that is prominently used as a tool to measure success of mathematics teaching and learning. This assessment is designed and managed by the Organisation for Economic Cooperation and Development (IECD) who Rampal blames in conjunction with the national governments and media that perpetuate the narrative of crisis through their responses to the PISA assessments. She explains that while OECD promotes global competitiveness through free-market policies, they tangentially play a major role in creating reactions of global crisis and alarm upon the release of their exam results, a phenomenon that Rampal dubs, *PISA shocks.* Rampal (2019) points out that these reactions of crisis lead to countries being "coerced into impulsively changing curricula to suit its demands" to meet ongoing and cyclical reform efforts brought about by the results.

So, who is believed to benefit from generating such a long-standing discourse of crisis in mathematics education? All answers center on the belief that just as in Klein's (2008) theory of disaster capitalism, the crisis of mathematics education is manufactured to benefit a variety of stakeholders other than students it purports to advocate for (Rampal, 2019; Washington et al., 2012). Rampal posits that "several layers of interests from consultancies, business enterprises for teaching, tuitions and testing, use of information and communication technologies, increasingly for surveillance and monitoring or teaching and learning" (p. 57) stand to benefit from the narrative of crisis-attributed mathematics education. Additionally, Rampal claims that the crisis encourages corporations to expand their presence in the educational sector by establishing new *low-cost private schools* in poor communities. Similarly, Washington et al. (2012) are clear in their belief that the crisis is purposely manufactured and that it is done strictly to financially benefit members of the political and corporate elite. These authors all agree in their expression that not only is the crisis of mathematics education manufactured, but it is also done so to benefit others outside of school communities and further enables the for-profit outsourcing of public education.

Washington et al. (2012) state, "we assert that manufactured crises in mathematics education function more in the service of political, economic, and organizational agendas than the collective, best interest of children, teachers, school and communities" (p. 54). They posit that political elites are controlling the framing of issues such as the crisis of mathematics education, and that these frames shape the public discourse around the topic to influence and even control public opinion. They add, "we suggest that mathematics education is also a crisis-management project, wherein mathematics education research and policy are shaped by manufactured crises through frames generated by political elites, but generally unmoved by crises emanating from human experience" (p. 60). In relation to the concept of crises at large, Washington et al. (2012) proposes that the attachment of the notion of crisis to a problem, frames that problem in a way that demands an *immediate solution,* a sentiment that seems to closely echo Klein's (2007) proposal of the public's push for timely response during disorientation. In the case of mathematics education, this crisis framing positions students, teachers, and schools as entities that need to be fixed, which as Washington et al. (2012) explain, reinforces deficit views. Further, they advocate that the continued reiteration of the crisis frame functions in creating a public sense of urgency and perceived need for mitigation through educational reform which results in a normalization to "reduce children to objects in need of repair" (Washington, 2019, p. 62).

Gutstein (2009) is also in alignment with these views and express their belief that *crisis-management projects* are utilized to justify the "development of human capital for corporate interests" (Gutstein, 2009, p. 60) and concluding that "while the U.S. government and its corporate allies view global economic competition as a crisis and threat to their continued dominance, their proposed solution will benefit capital and not the majority of the U.S. people" (p. 138). As discussed by Klein (2008), in the wake of a crisis, policies are more easily implemented as they receive less resistance due to citizens being preoccupied with an immediate solution. In the case of mathematics education, the ongoing crisis of mathematics has presented the public with multiple iterations, each of which are followed by the implementation of a new type of *reform.* Ball and Youdell (2008) believe that the language of educational reform is most often used to camouflage the true intent of privatization through tendencies toward market-based, consumerist, and competitive policies in American society. As we see here, those who propose that the mathematics education crisis is of a manufactured nature also commonly believe that it is done for government and corporate gain.

Underpinnings and Influence. Privatization is a concept that Star (1988) advocates exists as "an idea, a theory and rhetoric, and a political practice" (p. 6). He explores the idea of privatization in an intensely intricate and thorough manner and seems to boil the description of the concept down to the "withdrawal from the state, not of individual involvement, but of assets, functions, indeed entire institutions" (p. 9). Star (1988) explains his belief that the idealization of a society that embodies laissez-faire individualism and free-market economics are utilized to justify the act of privatization. Ball & Youdell (2008) concur and explain "privatization in and of public education has its roots in the forms of *small state-free market* approaches to public services- what is sometimes called 'neo-liberalism'" (p. 11).

Neoliberal ideology of the late 20th century is often considered a resurgence of free-market capitalism dating back to the 1800s (Star, 1988). The ideology champions free-market capitalism, deregulation of government in the market, and reduction in government spending. In ideological terms, a free-market economy is "devoid of distortionary government interventions such as regulatory requirements and government subsidies, which often add unnecessary costs, constrain competition, redirect market allocations, or impede economic agents' decision-making ability" (Bennett, 2014). The idea is central around the concept of laissez-faire, a philosophy in that suggests that when markets are left alone by the government to act naturally on their own, producers of products in high demand will naturally continue to provide a supply to consumers and attract additional investment while products in low demand will inevitably be pushed out by market forces. Bennett (2014) argues that this process distinguishes the free-market system as a process of experimentation with success and failures serving as the premise for innovation and continued production of successful goods and services. As the government steps aside to allow for free-market forces to increasingly take effect, an increase in privatization results.

Ball and Youdell (2008) believe that, across the globe, there is an expanding propensity of governments to both institute privatization into public education and to privatize sectors of public education to outside sources. They explain that much of this privatization is hidden from obvious identification as it is "the result of deliberate policy often under the banner of 'educational reform'" (p. 3) and that many of the changes that are implemented "reflect an increasingly market-based, competitive and consumerist orientation in our societies" (p. 3). Ball and Youdell (2008) conceptualize the idea of privatization through two types: *endogenous privatization* and *exogenous privatization*. *Endogenous privatization* is described in simple terms as *privatization in education* and is defined by Ball and Youdell (2008) as involving "the importing of ideas, techniques and practices from the private sector in order to make the public sector more like business and more business-like" (p. 8). *Exogenous privatization*, *privatization of public education,* on the other hand, involves "the opening up of public education services to private sector participation on a for-profit basis and using the private sector to design, manage or deliver aspects of public education" (p. 9).

A strong emphasis on testing and measurement are typical characteristics of the neoliberal ideology in education (Adamson & Darling-Hammond, 2016; Klees, 2020). The implementation of these practices took centerstage in the 1980s to increase accountability, a major focus on the reform movement of that decade and those moving forward (Klees, 2020). This focus on accountability is intertwined with sentiments that schools are failing, and teachers are to blame. This perpetuated the idea that increased accountability efforts of testing and measurement were necessary to evaluate the effectiveness of schools and teachers. The passage of the No Child Left Behind Act in 2002 exacerbated the perception of public-school failure as it set the precedent that schools needed to achieve a level of 100 percent proficiency in the number of students passing the exam by 2014. Additionally, interwoven into the mindset of accountability, is the idea that competition breeds a rise in both the quality and effectiveness of performance, a characteristic of a free-market economy that sits as a major tenet of neoliberal ideology (Adamson & Darling-Hammond, 2016; Freidman, 1980; Steger & Roy, 2021). Adamson and Darling-Hammon (2016) highlight the characteristic in terms of its trait as a form of endogenous privatization (Bell & Youdell, 2008) explaining, "for some, the main idea is that competition will force schools to provide a good product or they will go out of business" (p. 131).

Competition and the intensive testing of students are actions encouraged and reinforced by what researcher Ravitch (2010) calls the *Billionaire Boys Club*. This group is comprised of the Broad, Gates, and Walton foundations, all founded and run by three enormously successful enterprises of Eli Broad of homebuilding and insurance, Bill Gates of Microsoft, and Sam Walton of Walmart. The three foundations have been known to have been highly influential to the Obama administration while also gaining high-level positions within the administration – enabling their ability to craft and implement educational policy. Each of these organizations have also been able to exert their influence directly onto school buildings by offering monetary incentives to districts in need of funds (Klees, 2020). The focus on competition and the concept of measuring the effectiveness of schools through rigorous testing can both easily be considered tactics of a corporate mindset, an idea to which Broad has not been shy. He expressed his belief in structuring and managing schools in a similar fashion to corporations quite clearly in stating, "schools should be redesigned to function like corporate enterprises" (Ravitch, 2010, p. 217). Ravitch (2013) believes that influence such as Broad's is vast and powerful and has "hijacked" US education policy. These direct influences of corporate mindsets reinforce the neoliberal discourses that reverberate within the educational sector.

Friedman (1980), supports the application of a corporate mindset into the education sector and spreads this doctrine amongst his followers. He was outspoken on issues involving the government's involvement in education, advocating for the privatization of schools and entities related to their success. Through empirical examples, Friedman (1980) highlights the benefits that he observed to students, parents, and teachers of parochial schools serving economically underprivileged families in New York City's Bronx and Harlem areas. He cites teacher dedication, calm school atmospheres, students' positive behaviors and their eagerness to learn as well as their advanced academic performance as results of the ability for parents to choose the school that their children attend. He asserts that this ability to choose is made possible in part by the sources that fund each child's education expenses: namely funds from charitable organizations, fees paid by parents, and funds from the churches themselves. He explains that this level of independent funding in lieu of public funds from the Board of Education provides schools the ability to be non-conforming with the Board's regulations, and therefore privatizes the schools. Friedman (1980) explains that the favorable conditions that he has witnessed result "because teachers and parents are free to choose how children shall be taught. Private money has replaced tax money. Control has been taken away from bureaucrats and put back where it belongs." (p. 159). Additionally, Friedman believes that, should parents be given the chance to choose which schools their children attend, families of all races and economic statuses would "desert schools that could not maintain order" (p. 165). This would therefore promote competition among the schools as staff and leaders attempt to make their school more desirable to the parents who are in the role of customer. The prevalence of privatization and free-market competition within public schools are evidence of the presence of hallmark attributes of the neoliberal ideology within the crisis narrative of today.

Great Minds' Positioning Within the Crisis of Mathematics Education

Positioned within the context of a crisis of mathematics education, the producers of the web pages designed for Great Minds of the Great Minds websites can build upon the ubiquitous belief that mathematics education needs a substantial and immediate change. Within the latest iteration of the crisis narrative, the need for change is being remedied through the Common Core State Standards Initiative. Great Minds' advertising is centered on the idea that mathematics classrooms need large-scale change, and their products of curriculum and services of professional development and coaching fulfill this need for change through its embodiment of the best practices called for in the CCSS-M such as *focus*, *coherence*, and *rigor*. This is made possible by the large cultural context that their actions of production and the viewer's action of interpretation are situated within (Fairclough, 1989).

They convey these messages in a variety of ways, as discussed in the sections of the micro- and meso- analysis above. To reinforce this belief that mathematics instruction is in need of change, they utilize approaches including persuasion through metaphor and moral obligation without including any justification of the foundational idea that change is even needed in the viewer's school or classroom. They do not include any text or images intended to convince the viewer that they need change in the first place. Their communication is not one of convincing, rather it is one of reproducing the same narrative that is already present in the discourse of U.S. mathematics education. The producers can advertise to the assumption that viewers of the website are already of the mindset that their classroom or school's math instruction are in need of change because as discussed in detail above, this belief is so ingrained in the discourse of U.S. education that it is a commonly held belief. Fairclough (1989) refers to these common beliefs as *common-sense assumptions,* which he explains are ideologies that people utilize and are largely unconscious of. As Chiapello and Fairclough (2002) advocate, underpinning these prevalent discourses such as those discussed in the section prior to this, are ideologies that they define as "a system of ideas, values, and beliefs oriented to explaining a given political order, legitimizing existing hierarchies and power relationships and preserving group identities" (p. 187). The common-sense beliefs expressed throughout this section find themselves most easily positioned within a neoliberal ideology. This ideology is attributed by numerous professionals (Klees, 2020; Monbiot, 2016; Washington et al., 2012) as the dominant discourse underpinning many of the policies we find influencing and directing U.S. education. Among these experts, Klees (2020) concurs and explains, "neoliberal discourses have become the new common sense; they pervade the policy air we breathe and so dominate the policy agenda worldwide" (p. 12). Monbiot (2016) describes neoliberalism as being so pervasive in our American society that it is rarely even perceived as an ideology, adding "we internalize and reproduce its creeds" (p. 2).

In building off their reinforcement of the discourse that mathematics education needs change, the producers utilize advertising techniques to attempt to convince the viewers that Great Minds holds a solution to their needs: their Eureka Math curriculum and their professional development and coaching services. Additionally, the producers employ strategies of advertisers, all unveiling the participation of Great Minds in the privatization of educational entities promoted by those subscribing to the neoliberal ideology (Friedman, 1980; Klees, 2020). Through their website, the producers attempt to establish legitimacy through functionalization, build a brand image, and establish a sense of trust in Great Minds.

In addition to the strategies detailed above, the producers attempt to portray a level of epistemic authority over the viewer. The ability to do this lies again within the broader context of discourse surrounding U.S. mathematics education. As Fairclough (1989, p. 87) succinctly states, "power behind discourse is also hidden power: the whole social order of discourse is put together and held together as a hidden effect of power". Within the "crisis" of mathematics education, the power is held by those who establish themselves in a position of epistemic authority. The belief that children lack proficiency in math and that it is the fault of school and teachers (Klees, 2020; Washington, 2012) allows Great Minds to take a position of superiority, dictating to viewers that they both know better than them and have the correct solutions to offer them in mitigating their problems. These are individuals, organizations, and sectors of the government that perpetuate the idea that change is necessary to address both the causes and symptoms of the educational crisis of mathematics education. As we have discussed, this ongoing discourse has convinced the masses in American culture that a crisis in mathematics education exists and needs to be resolved for the sake of the national security of the citizens of our country (Gutstein, 2009; Rampal, 2019; Stevenson, 2014; Washington et al., 2012). The average citizen, believing they do not possess the specialized knowledge or access necessary, looks to those who they perceive as the experts to solve this problem (Croce, 2020). Through this reliance, those perceived to possess the necessary knowledge are viewed as being in positions of epistemic authority and subsequently hold the power (Croce, 2020).

Standing upon this platform of common-sense assumptions and epistemic authority, made possible by the larger neoliberal discourse of U.S. mathematics education, the producers employ additional strategies to make their case to convince the viewer to utilize their products and / or purchase their services. Among these approaches as discussed in the micro- and meso-sections above is the suppression of data, legitimacy through functionalization, and differentiation through structural position. The discursive practices centered on this crisis are embedded in the culture in a way that allows the producers to utilize strategies such as suppression of data to make claims like Eureka Math being the *most utilized curriculum* without evidence or supporting information.

Positioning

My CDA analysis makes clear two points in response to the supporting questions I posed at the start of this Macro-Analysis section: Great Minds is not only aware of the current discourse of "crisis" of mathematics education that is positioned within, but willfully embraces the narrative as a platform to advertise their curriculum, professional development, and coaching services. The narrative, composed of various tenets of neoliberal ideology, is pervasive in discourse surrounding U.S. education. The neoliberal-aligned beliefs influence the discourse that U.S. students need to be outperforming their international counterparts and that their underperformance should be blamed on schools and teachers, and the beliefs set the stage for the notion that large scale reforms are necessary to preserve the security of the United States. The literature shows that this cyclical narrative of *crisis* has been deemed by some to be manufactured to be used as a tool to create a favorable environment that ultimately benefits various stakeholders by affording them power, control, and financial benefits. Regardless of the criticism as to the accuracy and truth of the crisis, the producers of the Great Minds website can use a variety of strategies to exploit the pervasive cultural discourse that is underpinned by neoliberal ideology. Their utilization of epistemic authority, suppression of data, differentiation through structural oppositions, and persuasion through metaphor and oral obligation, all absent of explanation or evidence support the characterization of the *crisis* narrative as a *common-sense assumption* (Fairclough, 1989). Additionally, the producers employ strategies of advertisers such as establishing legitimacy through functionalization, building a brand image and a sense of trust through overlexicalization, all unveiling the participation of Great Minds in the privatization of educational entities promoted by those subscribing to the neoliberal ideology. This CDA has revealed a great deal about Great Minds' position within the current state of education in the U.S. as positioned within the "crisis" of mathematics education and the subsequent claims made by the publisher. This analysis has led to my first finding that I will call, *Positioning within the Context of Mathematics Education Finding*: it appears that the Great Minds organization has chosen to utilize their placement within the macro- context of an educational *crisis* of mathematics education as the foundation upon which they have built their brand and attempt to build trust with consumers. The organization seems to use this platform to convince consumers that their products have a superior ability to provide their classrooms with the transformation necessary to implement the Common Core State Standards Initiative, which serves as the solution to the most recent iteration of crisis reform.

The CDA unveiled the discursive practices that Great Minds embraces within the context of the *crisis* of mathematics education to position itself as the curriculum best suited to mitigate the effects of the *crisis*. The publisher seems to utilize their website as an advertising platform to convince consumers that their products and services provide a solution to the issue of low student performance associated with the *crisis*. In this way, Great Minds discursively couples itself with the CCSS-M as whole and through specific characteristics that are considered central to best practices in mathematics education as a way of solidifying its claims. The following section presents a content analysis aimed at examining the ways the actual curricular tasks and content of Eureka Math furthers the standards to which it has positioned itself as furthering.

CHAPTER V

FINDINGS: CONTENT ANALYSIS

In the previous chapter, critical discourse analysis helped me to uncover how Great Minds discursively positioned Eureka Math in relation to the Common Core State Standards of Mathematical Practice through its use of pictures and language in its promotional materials. In this chapter, I look at the tasks within the curricular materials themselves to explore how Great Minds positions Eureka Math in relation to the CCSS-MP. In this chapter, I report my findings from my use of content analysis as a means for this exploration. The framework, as discussed in detail in Chapter III, was adapted from the work of Wijaya et al. (2015) and Remillard and Kim (2020) to explore three characteristics that they deemed essential to high-quality context-based tasks: the *nature of the context, the type of information given in the tasks,* and *the level of cognitive demand that the task requires.*

The data is organized into larger groupings and then subgroupings. The first main groupings delineate between the type of Eureka Math lesson component and then further into subgroupings of each of the three criteria of high-quality context-based tasks, and a fourth to report the data of the tasks when looked at in their final coding with the codes for all three criteria. Organization into the largest level of groupings was done to support Wijaya et al.'s (2015) fourth criteria, *purpose of the context-based task* which is intended to *distinguish whether a context-based task is used for mathematical modeling or applying mathematics* based on its position in the lesson. This position is described by Wijaya et al. (2015) as being either *before or* *after the explanation* of the mathematical skill or concept. In all the lessons in this specific Grade 5 Module 3, the Application Problem component came before the explanation section, all of part of the Concept Development served as the explanation, and the Problem Set was either after the *explanation* or was not included in the lesson due to a complete focus of dedicating time to the Concept Development (as was the case in two lessons). The fourth subgrouping is a summary that breaks down the findings to explain the *Overall Alignment to Criteria for High-Quality Tasks and the Overall Trends of Tasks Within Each Lesson Component*. I felt this section was important to gain a comprehensive summary of the data that I had previously reported but through a more precise lens.

Throughout this section of the chapter, I will utilize italicized font to identify direct language from the criteria and their descriptions as detailed by Wijaya et al. (2015) and Remillard & Kim (2020). I will also present tasks as shown directly from the Eureka Math curricular materials so that the reader can explore the visual models and teacher "exemplar vignettes" (Great Minds, 2023c, para. 68) that accompany some of them. While my study was not designed as a quantitative study, I provide tables that I feel are helpful in seeing the overall quantities in the data as I believe seeing the quantities can be helpful to gaining insight into the trends that emerged.

When reviewing the materials, I began at the start of the module with Lesson 1 and worked through the Teacher Edition in chronological order. I went about the same process within each of the 16 lessons, first beginning with the Application Problem section which is presented first. I began with the first task of the application problem and coded that task using all three characteristics: type of context, type of information, and type of cognitive demand. I wrote the three codes in the margin of the paper directly next to the start of the task. If the Application Problem section contained more than one task, I proceeded to the next task and coded it in the same manner. After coding all the tasks within the section of the lesson, I recorded the total number of each type of task within my designated spreadsheet on the tab dedication for that component of the lesson. There were three tabs utilized for recording data: Application Problem, Concept Development, and Problem Set – and used for data of the corresponding lesson component. Each tab on the spreadsheet featured the task characteristics and the sub-categories written out as well as the codes. This information was presented in the first three columns in a manner identical to what can be seen in Appendix 1. The next 16 columns were labeled Lesson 5.1 through 5.16 denoting the appropriate column for the data for each lesson. The last two columns were used to total the number of tasks across all 16 lessons for each of the code combinations and a formula to calculate and display the percentage of the total that those totals represented. After the data for the Application Problem section of the lesson was recorded into the spreadsheet, I proceeded to the Concept Development section and applied the same procedure, and then the Problem Set section. This process was conducted on all 16 lessons of the module.

Application Problem Component of the Lessons

The Application Problem component of the Eureka Math lessons in grades K-5 are intended to "provide students with an opportunity to apply their skills and understandings in new ways" (Anderson, 2017, para. 5) and in situations where this section precedes the Concept Development component is meant to "function as a springboard into the new learning of the day" (Anderson, 2017, para. 5).

If interpreting the criteria from Wijaya et al. (2015) regarding the task as being relative to its positioning within the lesson, it would meet the description of a task being *used for modeling* of a skill or concept for the fourth criteria of purpose of the context-based task. In the case of these Eureka Math lessons, I considered the specific skills there were required of students in each of the tasks featured in the Application Problem sections helping me to decipher that students were always utilizing skills or concepts that had previously been learned rather than the ones presented in the lesson that it was located within. This factor adjusted my interpretation of the criteria from meaning *before or after the explanation within the lesson* to meaning before or after the explanation of that specific skill concept. Applying this shift in interpretation led to the indication that the tasks in the Application Problem component of the lessons are in fact being presented *after* the explanation of the specific skill or concept necessary to solve the task. The explanation of the featured skill or concept was often in the lesson just prior or sometimes was detailed in the descriptive text to be from previous modules or previous grade levels. Therefore, all the tasks located within the Application Problem component served the purpose of *applying mathematics.*

Type of Context

There are 16 lessons that comprise Grade 5's module three. In those 16 lessons, there are 15 tasks provided in the Application Problem portions cumulatively. Three lessons did not feature any Application Problem section, and two lessons featured two tasks in the section.

In focusing on the first of three characteristics, we look at the type of context which sub-categories include no context, camouflage context, and relevant and essential context. Only the sub-category of relevant and essential context was considered by Wijaya et al. (2015) as meeting the standard of a high-quality task within the characteristic of Type of Task. As shown in Table

2, all 15 of the tasks were determined to be written within a camouflage context (T-CC) which can be described as lacking the need for *experiences from everyday life nor common sense reasoning* to solve because the *mathematical operations needed to solve the problems are already obvious*. This is the case in camouflage context tasks (T-CC) because the numbers within the tasks *are given in such a way that students can immediately interpret the problem as a mathematical problem and follow the common procedure for calculating the answer*. While some tasks could be considered relevant to students' daily lives, they fell short of meeting the criteria for the next highest level of context type of relevant and essential context (T-RE) because commonsense reasoning was not needed to identify the *relevant information and the solution strategy to solve the task*. Instead, the necessary information was given in the task within a context that was possibly relevant to their daily lives, but the pathway forward was obvious. In the case of these 15 Application Problem tasks, the operations were evident for a few reasons including the ways that the problem was presented in an obvious manner, the placement of the problem immediately following a lesson in which similar tasks focused on the same mathematical skills were mirrored. Figure 14 exhibits an example of a task that meets the criteria of *T-CC* from the Application Problem section of Lesson 13. While the context of jogging could be considered *relevant to students' daily lives,* as is a criterion for T-RE tasks, this task, like many others, does not meet the T-RE task criteria that *students need to identify the relevant information and solution strategy for solving the task*. Instead, the task meets the criteria for T-CC which is that *numbers are given in such a way that students can immediately interpret the problem as a mathematical problem and follow the common procedures for calculating the answer.* As is common fashion with the Application Problems, this task is at the start of Lesson 13 which immediately follows Lesson 12. Lesson 12 featured a stated objective of "subtract fractions greater than or equal to 1" (Great Minds, 2015, p. 188) and featured subtraction tasks in the Problem Set that were similar in structure to the Application Problem task provided in Lesson 13. An example of a task with a similar structure from the Problem Set of Lesson 12 is shown in Figure 15. None of the tasks met the type of context criteria for no context (T-NC) or relevant and essential (T-RE).

Table 2: Frequency of Types of Context of Application Problem Tasks

Type of Context	Total	% of Total
No Context T-NC	0	0%
Camouflage Context T-CC	15	100%
Relevant & Essential T-RE	0	0%

Application Problem (7 minutes)

Mark jogged $3\frac{5}{7}$ km. His sister jogged $2\frac{4}{5}$ km. How much farther did Mark jog than his sister?

Remind students to approach the problem with the RDW strategy. This is a very brief Application Problem. While circulating as students work, quickly assess which work to select for a short two- or three-minute Debrief.

Note: Students solve this Application Problem involving addition and subtraction of fractions greater than 2 and having unlike denominators, using visual models.

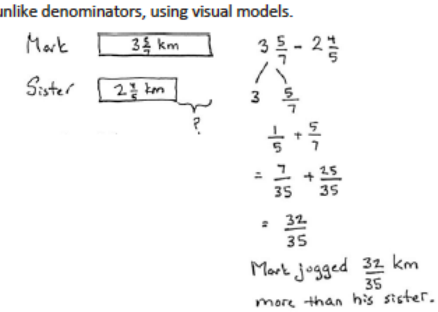


Figure 14: Application Problem T-CC Task
Note. Task featured in Lesson 13. *Grade 5 Module 3, p. 208*

3. Mr. Neville Iceguy mixed up $12\frac{3}{5}$ gallons of chili for a party. If $7\frac{3}{4}$ gallons of chili was mild, and the rest was extra spicy, how much extra spicy chili did Mr. Iceguy make?

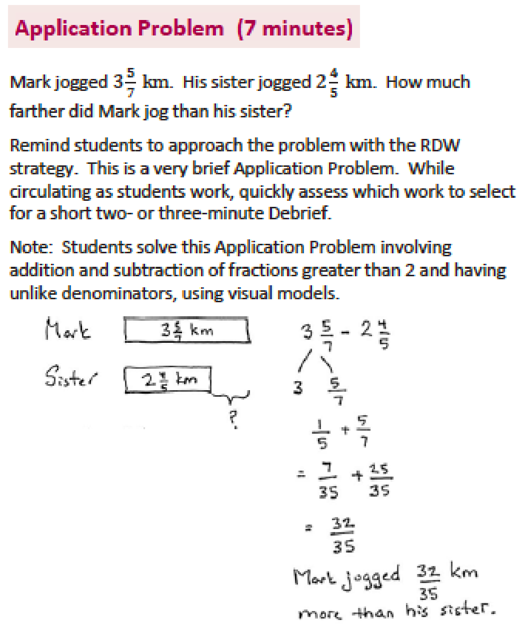
Figure 15: Problem Set T-CC Task
Note. Task featured in Problem Set of Lesson 12. Grade 5 Module 3, p. 199.

Type of Information

Fourteen of the 15 or 93% of the Application Problem tasks were written to provide a matching type of information (I-MATCH) and met the criteria of tasks that *contain exactly the information needed to find the solution*. See Table 3 for the breakdown. The task shown in Figure 16 (same as above), provides an example of a task that contains exactly the information that a student needs to solve without the need to derive more information, as with a missing type of information or select relevant information from what is given, as with a superfluous type of information. Here students are given the distance Mark jogged and the distance that his sister jogged as mixed numbers. They are then asked to find how much farther Mark jogged than his sister. It is easy to see that there are no additional values given that would require a student to determine which were relevant and necessary to solve the problem. Similarly, there is no information missing such as providing one distance in meters instead of kilometers to require students to use their knowledge of metric measurements of length to convert to a common unit and derive the values they need to solve.

Table 3: Frequency of Types of Information of Application Problem Tasks

Type of Information	Total	% of Total
Matching I-MATCH	14	93%
Missing I-MISS	1	7%
Superfluous I-S	0	0%



Application Problem (7 minutes)

Mark jogged $3\frac{5}{7}$ km. His sister jogged $2\frac{4}{5}$ km. How much farther did Mark jog than his sister?

Remind students to approach the problem with the RDW strategy. This is a very brief Application Problem. While circulating as students work, quickly assess which work to select for a short two- or three-minute Debrief.

Note: Students solve this Application Problem involving addition and subtraction of fractions greater than 2 and having unlike denominators, using visual models.

Figure 16: Application Problem I-MATCH Task
Note. Task featured in Lesson 13. Grade 5 Module 3, p. 208

One of the 15 (7%) tasks in the Application Problem portion of the lesson featured a missing type of information (I-MISS) which can be described as a task that *contains less information than needed, so students need to derive additional data.* The task featured in Figure 17 depicts a task that requires students to derive data. In this task, students need to generate the fractional amount of students who "call blue their favorite color" and who "prefer pink" (Great Minds, 2015, p. 43).

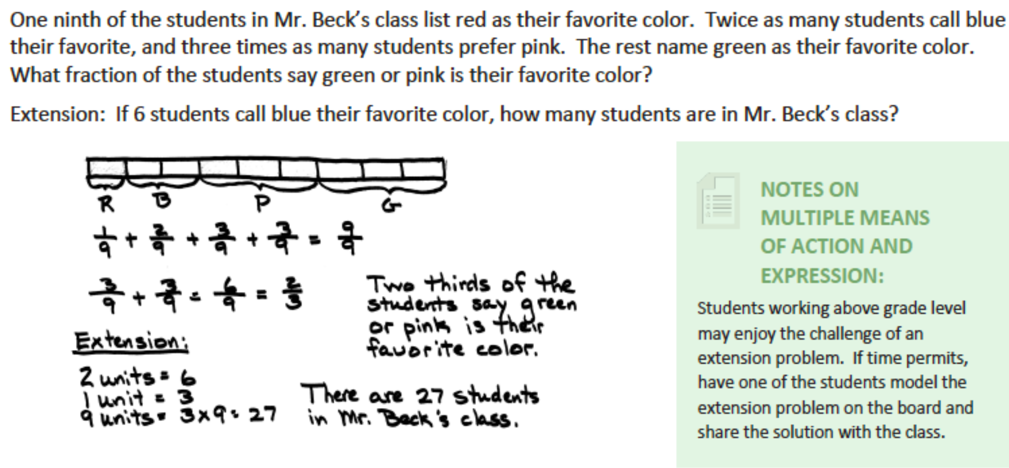


Figure 17: Application Problem I-MISS Task
Note. Task featured in Lesson 3. Grade 5 Module 3, p. 43

None of the tasks met the criteria for superfluous type of information (I-S) which would be a task that *contains more information than needed, so students need to select information.*

Type of Cognitive Demand

The tasks that comprised the Application Problem components of the lessons in this module reflected three of the five levels of cognitive demand. These levels were procedures without connections (CD-P), procedures with superficial connections (CD-PSC), and procedures with extended connections (CD-PEC). CD-P and CD-PSC tasks are considered as requiring low levels of cognitive demand by Remillard and Kim (2020) where CD-PEC tasks are considered to require a high level of cognitive demand. See Table 4.

8 of the 15 (53%) tasks in the Application Problem portion reflect a cognitive demand level of procedures without connections (CD-P) which are described as serving the purpose to *master procedures without meaning*. 6 of the 15 (40%) tasks comprising the Application Problem sections were designed to require one level of cognitive demand higher, reflecting their status as procedures with superficial connections (CD-PS) tasks. According to Remillard and Kim (2020), these tasks serve the purpose of *mastering the procedures using initial or limited*

connections as possible scaffolding. While both of these types of cognitive demands are considered by Remillard and Kim (2020) to require only low levels of cognitive demand, the next level above these, Procedures with extended connections (CD-PEC), encompass tasks requiring high levels and can be described as having the purpose to *understand and do the procedure in relation to underlying mathematical meaning.* Of the 15 questions that comprise Module Three's Application Problem sections, one task (7%) met the criteria for this type of cognitive demand. Examples of tasks meeting each of these three cognitive demand Levels are presented and explained in the next section.

Table 4: Frequency of Types of Cognitive Demand of Application Problem Tasks

Type of Cognitive Demand	Total	% of Total
Memorization (CD-M)	0	0%
Procedures Without Connections (CD-P)	8	53%
Procedures with Superficial Connections (CD-PSC)	6	40%
Procedures with Extended Connections (CD-PEC)	1	7%
Doing Mathematics (CD-DM)	0	0%

Three Criteria Combined

When considering each task as one featured within a Type of Context, with a type of information, and a level of cognitive demand that is required to solve the task, four distinct combinations emerged: *T-CC, I-MATCH, CD-P (47%); T-CC, I-MATCH, CD-PSC (40%); T-CC, I-MATCH, CD-PEC (7%); T-CC, I-MISS, CD-P (7%).* Tasks determined to be *T-CC, I-MATCH,* and *CD-P* comprised most of the tasks in the Application Problem component of the lessons in this module (47%). See Table 5.

All of the Application Problem tasks are written within camouflage contexts – meaning they were designed as math problems written within a context that did not require common sense reasoning from everyday life experiences because they lent themselves to be easily interpreted as a mathematical problem, and the operations needed to solve the problems were obvious or that a common or procedure that had just recently been learned – usually in the lesson prior – was to be applied. As a whole, the tasks in the Application Problem component differ in the type of information they feature [matching (I-MATCH) or missing (I-MISS)] and the level of cognitive demand that they require of students to solve [procedures without connections (CD-P), procedures with superficial connections (CD-PSC), and procedures with extended connections (CD-PEC)].

Table 5: Frequency of Combined Types of Context, Information, and Cognitive Demand of Application Problem Tasks

Type of Context	Type of Information	Type of Cognitive Demand	Code	Subtotals	% of Total
No Context T-NC		Memorization (CD-M)	T-NC, CD-M	0	0%
		Procedures without connections (CD-P)	T-NC, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-NC, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-NC, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-NC, CD-DM	0	0%
Camouflage Context T-CC	Matching I-MATCH	Memorization (CD-M)	T-CC, I-MATCH, CD-M	0	0%
		Procedures without connections (CD-P)	T-CC, I-MATCH, CD-P	7	47%
		Procedures with superficial connections (CD-PSC)	T-CC, I-MATCH, CD-PSC	6	40%
		Procedures with extended connections (CD-PEC)	T-CC, I-MATCH, CD-PEC	1	7%
		Doing mathematics (CD-DM)	T-CC, I-MATCH, CD-DM	0	0%
	Missing I-MISS	Memorization (CD-M)	T-CC, I-MISS, CD-M	0	0%
		Procedures without connections (CD-P)	T-CC, I-MISS, CD-P	1	7%
		Procedures with superficial connections (CD-PSC)	T-CC, I-MISS, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-CC, I-MISS, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-CC, I-MISS, CD-DM	0	0%
	Superfulous I-S	Memorization (CD-M)	T-CC, I-S, CD-M	0	0%
		Procedures without connections (CD-P)	T-CC, I-S, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-CC, I-S, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-CC, I-S, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-CC, I-S, CD-DM	0	0%
Relevant and Essential T-RE	Matching I-MATCH	Memorization (CD-M)	T-RE, I-MATCH, CD-M	0	0%
		Procedures without connections (CD-P)	T-RE, I-MATCH, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-RE, I-MATCH, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-RE, I-MATCH, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-RE, I-MATCH, CD-DM	0	0%
	Missing I-MISS	Memorization (CD-M)	T-RE, I-MISS, CD-M	0	0%
		Procedures without connections (CD-P)	T-RE, I-MISS, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-RE, I-MISS, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-RE, I-MISS, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-RE, I-MISS, CD-DM	0	0%
	Superfulous I-S	Memorization (CD-M)	T-RE, I-S, CD-M	0	0%
		Procedures without connections (CD-P)	T-RE, I-S, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-RE, I-S, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-RE, I-S, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-RE, I-S, CD-DM	0	0%

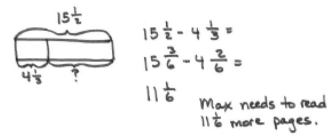
T-CC, I-MATCH, CD-P. Tasks coded as supporting a Cognitive Demand level of procedures without connections (*CD-P*) require students to *use procedures* that are *either specifically called for or their use is evident based on instruction.* They serve the purpose of supporting students to *master procedures without connections to the concepts or meanings underlying the procedures used. Figure* 18 is an example of a task coded *T-CC, I-MATCH, CD-P*. This task is presented to students in Lesson 12 of the module. As is written in the exemplar vignette that is provided, the teacher is encouraged to remind the fifth-grade students that they have been doing these types of problems for at least four years stating, "this is the same kind of subtraction problem we have been doing since first grade" (p. 188). This statement supports the idea that the use of the subtraction procedure should be evident based on previous instruction and that the task serves the primary purpose of mastering the procedure as intended by a CD-P task.

Application Problems (10 minutes)

Problem 1

Max's reading assignment was to read $15\frac{1}{2}$ pages. After reading $4\frac{1}{3}$ pages, he took a break. How many more pages does he need to read to finish his assignment?

T: Let's read the problem together.
S: (Read chorally.)



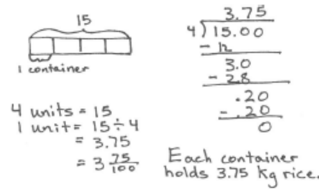
T: With your partner, share your thoughts about how to solve this problem. (Circulate and listen.)
T: Clara, can you please share your approach?
S: I said that you need to subtract $4\frac{1}{3}$ from $15\frac{1}{2}$ to find the part that is left.
T: Tell me the subtraction problem we need to solve.
S: $15\frac{1}{2} - 4\frac{1}{3}$.
T: Good. This is the same kind of subtraction problem we have been doing since first grade. A part is missing: the pages he has to read to finish.
T: Maggie, read your answer using a complete sentence.
S: Max needs to read 11 and 1 sixth more pages.

Figure 18: Application Problem T-CC, I-MATCH, CD-P Task
Note. Task featured in Lesson 12. Grade 5 Module 3, p. 188

T-CC, I-MATCH, CD-PSC. 6 of the 15 (40%) tasks in the Application Problem portions of the lessons were coded as T-CC, I-MATCH, CD-PSC. An example of this type of task can be found within the Application Problem portion of Lesson 1 and is featured in Figure 19 below. This task is designed with a context that does not require students to pull from experiences in their everyday lives, and the numbers are given in a way that makes it easy for students to interpret that problem as a mathematical problem and follow a common procedure for calculating the answer, indicating its coding as a camouflage context (T-CC). The amount of information given in the problem is exactly the amount needed to find the solution, deeming it as a matching type of information (I-MATCH). In following the exemplar vignette that is provided, there is a presence of an *initial or limited connection* that is being made. This limited connection is present when the teacher is encouraged to ask the students to draw a picture of what they see from the story and then give the total weight of the rice and the number of containers needed and the corresponding operation of division. However, in meeting the criteria of a *CD-PSC* task, these connections are *not maintained, and the primary focus becomes the procedure* of division which is seen in the writing of the expression and then moving into the computation of 15 divided by 4. There is a lack of discussion regarding the underlying mathematical meaning of division, such as how it was determined that the operation of division should be employed for this situation and what the operation of division does when utilized on quantities. The crux of that component of the discussion takes place when the teacher asks, “To find 1 container or 1 unit, we have to…?” and the students are expected to respond, “Divide” (Great Minds, 2015, p. 13). The suggested follow up by the teacher is, “Tell me the division expression” (Great Minds, 2015, p. 13). This exchange demonstrates the superficial level of connection present in this task as it is lacking further discussion about any underlying mathematical connections as the ones suggested above.

Application Problem (9 minutes)

15 kilograms of rice are separated equally into 4 containers. How many kilograms of rice are in each container? Express your answer as a decimal and as a fraction.



T: Let's read the problem together.
S: (Read chorally.)
T: Share with your partner: What do you see when you hear the story? What can you draw?
S: (Share with partners.)
T: I'll give you one minute to draw.
T: Explain to your partner what your drawing shows.
T: (After a brief exchange.) What's the total weight of the rice?
S: 15 kilograms.
T: 15 kilograms are being split equally into how many containers?
S: 4 containers.
T: So, the whole is being split into how many units?
S: 4 units.
T: To find 1 container or 1 unit, we have to...?
S: Divide.
T: Tell me the division expression.
S: 15 ÷ 4.
T: Solve the problem on your personal white board. Write your answer both in decimal form and as a whole number and a decimal fraction. (Pause.) Show your board.
T: Turn and explain to your partner how you got the answer. 15 ÷ 4 = 3.75.
T: (After students share.) Show the division equation with both answers.
S: $15 \div 4 = 3.75 = 3\frac{75}{100}$.
T: Express 75 hundredths in its simplest form.
S: 3 fourths.
T: Write your answer as a whole number and a fraction in its simplest form.
S: $15 \div 4 = 3.75 = 3\frac{75}{100} = 3\frac{3}{4}$.
T: So, 3 and 3 fourths equals 3 and 75 hundredths.
T: Tell me your statement containing the answer.
S: Each container holds 3.75 kg or $3\frac{3}{4}$ kg of rice.

Note: This Application Problem reviews division and partitioning as it relates to fractions. Also, it reviews replacing one fraction with another of the same value in anticipation of today's work with equivalent fractions.

Figure 19: Application Problem T-CC, I-MATCH, CD-PSC Task
Note. Task featured in Lesson 1. *Grade 5 Module 3, p. 74*

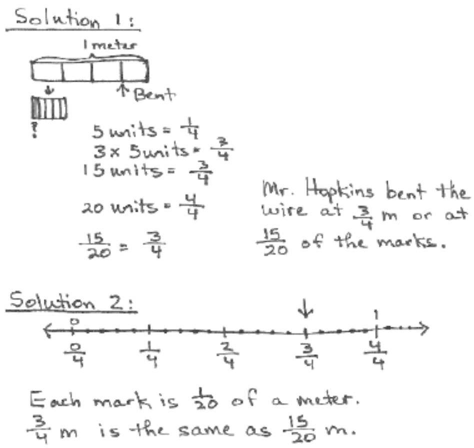
T-CC, I-MATCH, CD-PEC. Only 1 of the 15 (7%) tasks that make up the Application Problem portions of the lessons in this unit demonstrated the requirement of a high level of cognitive demand to solve. It was considered a high-quality context-based task in the characteristic of type of cognitive demand only. This task, featured in Figure 20, is unique among the rest as it meets the criteria for cognitive demand level of Procedures with extended connections (CD-PEC). Tasks of this nature serve the purpose of supporting students to *understand and do the procedure in relation to underlying mathematical meaning*. The task *makes the underlying meaning visible* as *students are expected to use the meanings to make connections to mathematical procedures or notation.* In this task, it is expected that students will utilize the Draw component of the Read-Draw-Write (RDW) process to create an area model or number line to represent the mathematical problem. While creating and utilizing one of these visual models could be considered procedural, as Remillard and Kim (2021) are clear that procedures do not need to be computation, the teacher-led discussion guides students to make connections between the meanings (equivalent fractions relative to whole numbers as represented on the area model or number line) and the mathematical procedure (creating the area model or number line).

Application Problem (8 minutes)

Mr. Hopkins has a 1-meter wire he is using to make clocks. Each fourth meter is marked off and divided into 5 smaller equal lengths. If Mr. Hopkins bends the wire at $\frac{3}{4}$ meter, what fraction of the smaller marks is that?

S: (Solve the problem, possibly using the RDW process independently or in partners.)

T: Let's look at two of your solutions and compare them.



T: When you look at these two solutions side by side, what do you see? (Consider using the following set of questions to help students compare the solutions as a whole class or to encourage inter-partner communication while circulating as they compare.)

- What did each of these students draw?
- What conclusions can you make from their drawings?
- How did they record their solutions numerically?
- How does the tape diagram relate to the number line?
- What does the tape diagram/number line clarify?
- What does the equation clarify?
- How could the statement with the number line be rephrased to answer the question?

NOTES ON SOLVING APPLICATION PROBLEMS:

Since Grade 1, students have used the Read, Draw, Write (RDW) approach to solve Application Problems. The method is as follows:

1. Read the problem.
2. Draw to represent the problem.
3. Write one or more equations that either help solve the problem or show how the problem was solved.
4. Write a statement that answers the question.

Embedded within Draw are important reflective questions:

- What do I see?
- Can I draw something?
- What conclusions can I reach from my drawing?

Note: This two-step Application Problem offers a problem-solving context for students to review making equivalent fractions with the number line or the area model as taught in Lesson 1.

Figure 20: Application Problem T-CC, I-MATCH, CD-PEC Task
Note. Task featured in Lesson 2. Grade 5 Module 3, p. 27

T-CC, I-MISS, CD-P. One of the 15 tasks (7%) in the Application Problem portions of the lessons met the criteria of T-CC, I-MISS, CD-P. This task was featured in Lesson 3 and is shown in Figure 21. This task was considered a high-quality context-based task in the characteristic of type of information only. The task is written regarding a classroom of students and their favorite colors. While one might argue that the context of favorite color among students in a classroom may serve as relevant to students' everyday lives, it does not require them to apply real-world common sense to solve it, deeming it camouflage context. The task contains a missing type of information as the fractions for the number of students whose favorite colors are blue and pink are given as *twice* and *three times* rather than a fractional quantity. This structure requires students to derive the fractions to represent these amounts. The task requires a procedure without connections (CD-P) level of cognitive demand from students as the purpose of the task is to *master a procedure*, which in this case is addition and subtraction of fractions as learned in previous lessons in this module.

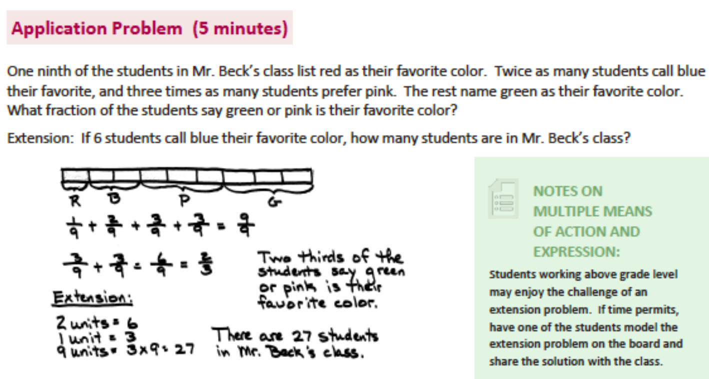


Figure 21: Application Problem T-CC, I-MISS, CD-P Task
Note. Task featured in Lesson 3. Grade 5 Module 3, p. 43

Summary of Application Problem Tasks

The Application Problem section of the lessons contained 15 tasks. All the tasks were considered context-based tasks, as all of them were written within a context. Taking a further look into the context that was given yielded coding all the tasks as featuring a camouflage context type of context (T-CC). For the most part (93%), the contexts were written to provide students with the exact amount of information that they needed to solve the task. This took away the ability for students to engage with tasks requiring them to either derive additional information to be used to solve the task or to select relevant information from a superfluous amount. The tasks demonstrated a variety of three different cognitive demand levels. The majority (93%) of the tasks required a low level of cognitive demand for students to complete them as they were coded as CD-P and CD-PSC tasks. The tasks most closely meeting the standards of high-quality tasks in the Application Problem component was one (7%) T-CC, I-MATCH, CD-PEC task, which met the criteria for a high level of cognitive demand only and one (7%) T-CC, I-MISS, CD-P task which met the criteria for a high level of type of information only. These results are summarized in my second finding which I will refer to as *Application Problems Finding*: in the Application Problem component of the lesson all the tasks were context-based tasks written within a camouflage context. Very few of the tasks were context-based and met the criteria for high-quality in the characteristics of type of information (7%) and type of cognitive demand (7%).

Concept Development Component of the Lessons

The Concept Development is the component of the lesson designed to "address new content being studied" (Anderson, 2017, para. 4). As discussed in Chapter III, I chose to examine the lessons by exploring each of the components (Application Problem, Concept Development, Problem Set) separately because of the unique purposes that they were designed to serve in the lesson and its lack of adherence to the high-quality task characteristic of purpose of the context-based task, emphasized by Wijaya et al. (2015). Within this characteristic, Wijaya et al. (2015) distinguished two sub-categories of *Application* and *Modeling*, which are described as a task that is given "after the explanation section" or "before the explanation section" (p. 52), respectively. The Concept Development section is the component of the Eureka Math lesson that, as a whole, does not fit into this structure from Wijaya et al. (2015) as it is neither before nor after the explanation, but rather serves as the portion of the lesson where the explanation takes place. Great Minds describes the Concept Development component as being "generally comprised of carefully sequenced problems centered within a specific topic to begin developing mastery via gradual increases in complexity" (Anderson, 2017, para. 4).

The tasks that comprise the Concept Development portion of the 16 lessons proved to be more diverse among the three criteria utilized to review them than the Application Problem tasks. Unlike the Application Problems, all 16 of the lessons have a Concept Development component. The number of tasks in the Concept Development component of each lesson ranges from two to six tasks. There are 78 total tasks that made up the Concept Development sections of the 16 lessons in Grade 5 Module 3.

Type of Context

In the Concept Development component of the 16 lessons in Grade 5 Module 3, 12 of the 78 total tasks were written within a context. These 12 tasks were contained in 5 lessons. The Concept Development component in 11 of the 16 lessons featured only tasks with no context (T-NC). These tasks featuring no context (T-NC) made up most of the tasks in this lesson component as 66 of the 78 (85%), were written referring *only to mathematics objects, symbols, or structures.* See Table 6 for a breakdown.

Figure 22 is a task in the Concept Development component of Lesson 2. It matches the criteria for no context type of context (T-NC), also called a bare task. While this task features a visual model, it is composed strictly of mathematical objects, symbols and structures rather than a context.

Table 6: Frequency of Types of Context of Concept Development Tasks

Type of Context	Total	% of Total
No Context T-NC	66	85%
Camouflage Context T-CC	12	15%
Relevant & Essential T-RE	0	0%

Problem 1: $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$.

1 third + 1 third = 2 thirds.

T: Draw a number line. Mark the endpoints as 0 and 1. Between zero and one, estimate to make three units of equal length and label them as thirds.

S: (Work.)

T: On your number line, show 1 third plus 1 third with arrows designating lengths. (Demonstrate, and then pause as students work.)

T: The answer is …?

S: 2 thirds.

T: Talk to your partner. Express this as an addition sentence and a multiplication equation.

S: $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$. → $2 \times \frac{1}{3}=\frac{2}{3}$.

T: Following the same pattern of adding unit fractions by joining lengths, show 3 fourths on a number line.

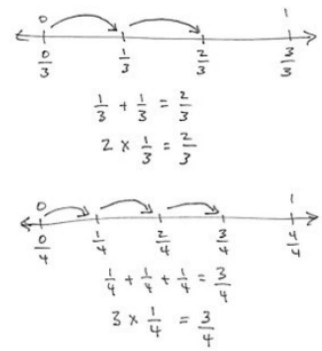


Figure 22: Concept Development T-NC Task
Note. Task featured in Lesson 2. Grade 5 Module 3, p. 28

The remaining 12 (15%) tasks met the criteria for camouflage context (T-CC) as they either did *not require experiences from students' everyday lives,* or *common-sense reasoning was not necessary to complete the task*. Like the explanation regarding the T-CC tasks featured in the Application Problem component of the lessons, the tasks in the Concept Development featured processes that allowed students to *immediately interpret the problem as a mathematical problem and follow the common procedures for calculating the answer.* The common procedures were often apparent from previous lessons and featured a minor change to accommodate the new skill or concept being learned. This can be seen in the task featured in Figure 23 that met the T-CC criteria from Lesson 7. This task follows two lessons (Lessons 5 and 6) that both focus on the subtraction of fractions and in which the same type of tape diagram and visual models were drawn. The additional complexity of tasks in this lesson compared to those in previous ones is that this task is named as a two-step word problem rather than a one-step. This is reflected in the lesson's objective which reads, "Solve two-step word problems" (Great Minds, 2015, p. 103). Also significant is that this task is the fourth task in the Concept Development of Lesson 7. These features of the task result in the use of the same visual model as previously utilized, not only in the lessons prior, but also in the three preceding tasks of this lesson's Concept Development. By the fourth task in the Concept Development, the repetitive nature of the tasks' structure allows students to *immediately interpret the problem as a mathematical problem and follow the common procedures for calculating the answer.*

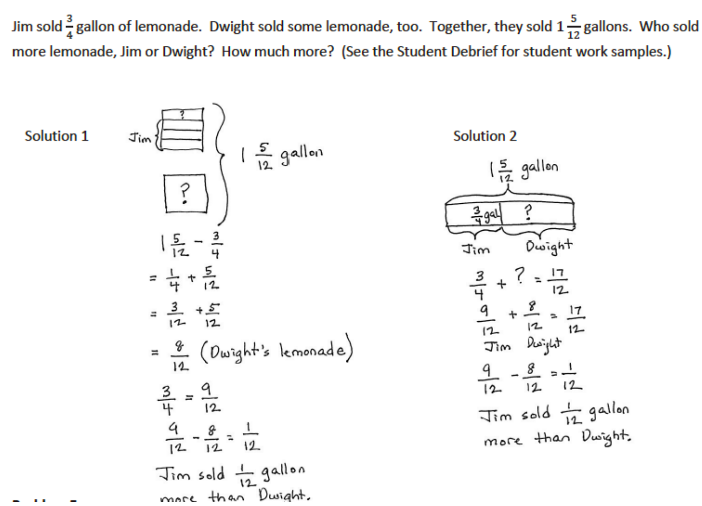


Figure 23: Concept Development T-CC Task
Note. Task featured in Lesson 7. Grade 5 Module 3, p. 106

None of the Concept Development tasks were determined to be a relevant and essential type of context (T-RE). The criteria for such a task would involve that the *context is relevant to students' daily lives*, *requires mathematical modeling,* and is developed in such a way that it allows students to *identify the relevant information and solution strategy for solving the task.*

The situations, like those of the tasks featured in this section, do not meet the criteria of T-RE because they are either not relevant to students' daily lives, feature a context that does not allow students to sift through the given information to decipher which information is relevant, or feature a combination of both.

The data regarding the type of context of the tasks in the Concept Development component of the lessons revealed a pattern regarding their appearance in the module. As discussed earlier, the Overview of Module Topics and Lesson Objectives (Great Minds, 2015, p. 7) lays out the structure of the module as a whole. Grade 5 Module 3 is split into four Topics labeled A, B, C, and D, and can also be seen to be divided into two halves with the use of a Mid-Module Assessment following Lesson 7. The Concept Development in lessons 1-6 and lessons 8-14 almost exclusively feature only T-NC tasks. The exceptions to this are lessons 3 and 5, which each feature a single T-CC task in each. In contrast, the Concept Development component of lessons 7 and 15 contain only T-CC tasks. Looking at this data in concert with the Overview of Module Topics and Lesson Objectives table (Appendix 1), it is revealed that tasks set within a context are almost exclusively utilized in the last lesson (Lesson 7) of Topic B when the first-half of the module is coming to a close and in the last two lessons (Lessons 15 and 16) of Topic D as the module as a whole is coming to a close. This indicates that, through the Concept Development portion of the lessons, which is meant to serve the purpose of "address new content being studied" (Anderson, 2017, para. 4), students engage in learning new skills and concepts through the beginning and middle of the first and second halves of the module through tasks that contain *only mathematics objects, symbols, or structures* and then at the end of both the first- and second- halves of the module through tasks set in a camouflage context.

The two exceptions to this pattern are two tasks that are both presented at the start of the Concept Development in Lesson 3 and 5 as students first learn to add and subtract fractions with unlike denominators, respectively. Both tasks feature exemplar vignettes that are teacher-led discussions through which students are encouraged to speak with a partner and then full class. Lesson 3 is the first time that students engage with the concept of adding fractions that have unlike denominators. The objective for the lesson is “add fractions with unlike units using the strategy of creating equivalent fractions” (Great Minds, 2015, p. 42). The teacher presents the students with a situation by writing “1 adult + 3 adults” (Great Minds, 2015, p. 44) on the board, followed by “1 fifth plus 3 fifths” (Great Minds, 2015, p. 44). The teacher then explains that “we can add 1 fifth plus 3 fifths because the units are the same” (Great Minds, 2015, p. 44) and rewrites the sentence using mathematical symbols. The teacher then asks the class as a whole, “what is 1 child plus 3 adults?” (Great Minds, 2015, p. 44). Through the exemplar vignette, it is expected that students will answer that children and adults cannot be added. The teacher then sends children to discuss with their partner why they cannot be added. Upon returning, the exemplar vignette suggests that the teacher should share that they heard some of the partnerships discuss that children and adults are unlike units and therefore should be replaced with equivalent units to allow for the addition of the two. The teacher then asks the class, “what do children and adults have in common?” (Great Minds, 2015, p. 44). In response, students are expected to say that they are people. The teacher then guides students to rename the units in the situation and add 1 person + 3 people to get a sum of 4 people. This is the first task that students encounter with adding fractions with unlike denominators (units). It seems to serve the purpose of establishing an analogy between the need to have like units to add within. It is followed by a bare task that is presented simply as ½ + ¼.

In Lesson 5, which is the first time that students encounter subtracting fractions with unlike denominators, they are led through a similar analogy to start the Concept Development component. In this task, instead of children and adults, students are presented with needing to subtract 1 girl from 3 boys. Through the exemplar vignette the teacher guides them to conclude that the units need to be changed to *students* in order to subtract. In both tasks, while written in a context, the *operations are obvious*, making both T-CC tasks.

Type of Information

The types of information utilized are matching, missing, and superfluous. Because these classifications are designed for context-based tasks, they are not as applicable to tasks that are void of a context, such as those meeting the criteria of no context (T-NC) because, by their very nature, they feature *only to mathematics objects, symbols, or structures* tasks without a context (Wijaya et al., 2015). Therefore, for this study, tasks coded as T-NC were only coded as matching in their type of information. As discussed in the above section, these types of tasks comprised 66 of the 78 tasks, indicating that all of them featured a matching type of information by default.

Of the 78 tasks featured in the Concept Development component, 75 (96%) featured a matching type of information (I-MATCH) and 3 (4%) provided a missing type of information (I-MISS). These 75 I-MATCH tasks provided *exactly the information needed to find the solution.* See Table 7 for a breakdown. As discussed in the Type of Context subsection above, 12 of the 78 tasks were written within a context, therefore deeming them context-based tasks. The remaining 66 I-MATCH tasks were not considered context-based tasks because they were void of a context and were instead bare. All 3 of the I-MISS tasks of the Concept Development component were of the 12 tasks written within a context, a characteristic which allows for them to feature missing information and also makes them context-based tasks.

Similar to those seen in the Application Problem component, the I-MATCH tasks in the Concept Development tend to be straightforward in the information provided to the students. An example of one of these tasks can be seen in Figure 24 from Lesson 7. While the objective of Lesson 15 is for students to “solve two-step word problems” (Great Minds 2015, p. 103), the *exact* numbers needed for solving the problem are given – nothing more and nothing less. There is no need for a student to derive additional information to solve the problem as would be required for an I-MISS task, nor is there additional information given that requires the students to sift through the information to select the information that is necessary to solve the task as would be required for a I-S task. Instead, the values of 1/5 and 2/3 will be utilized using one or more operations to solve this two-step word problem, as is specified in the lesson objective.

Table 7: Frequency of Types of Information of Concept Development Tasks

Type of Information	Total	% of Total
Matching I-MATCH	75	96%
Missing I-MISS	3	4%
Superfluous I-S	0	0%

George weeded $\frac{1}{5}$ of the garden, and Summer weeded some, too. When they were finished, $\frac{2}{3}$ of the garden still needed to be weeded. What fraction of the garden did Summer weed?

Figure 24: Concept Development I-MATCH Task
Note. Task featured in Lesson 7. *Grade 5 Module 3, p. 112*

In contrast to the I-MATCH tasks featured above, the three I-MISS tasks *contain less information than is needed* to solve the task, which requires that students *derive additional data.* Figure 53 depicts an I-MISS task that is featured in the Concept Development of Lesson 7. This task requires students to derive the fractional amount of money that Leonard spent on a gift for his brother, which the reader is told is “2 times as much as on some comic books” (Great Minds, 2015, p. 106). In looking at this task juxtaposed to the I-MATCH task featured in Figure 24, the difference between the two can be highlighted as the values given in the matching type of information task are of the exact values that will be used to solve the problem, and in the missing information task featured in Figure 25 there is a number that is not given in a straightforward way and instead needs to be derived by the students.

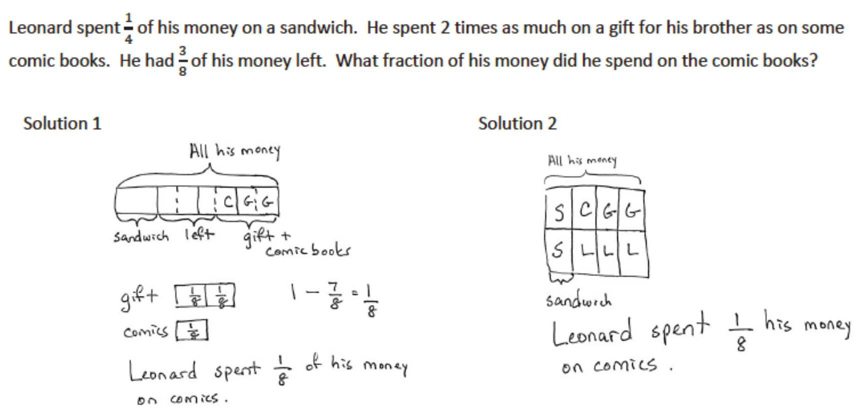


Figure 25: Concept Development I-MISS Task
Note. Task featured in Lesson 7. Grade 5 Module 3, p. 106

Type of Cognitive Demand

The three levels of cognitive demand that made up the tasks found within the Concept Development component of the lessons in Module 3 were the same that comprised the

Application Problem tasks. These levels were procedures without connections (CD-P), procedures with superficial connections (CD-PSC), and procedures with extended connections (CD-PEC). As has already been discussed, but is certainly worth repeating, Remillard and Kim (2020) consider CD-P and CD-PSC tasks as requiring low levels of cognitive demand and CD-PEC tasks as requiring high levels of cognitive demand.

Tasks matching the criteria for procedures without connections (CD-P) comprised 19 of the 78 tasks (24%), and procedures with superficial connections (CD-PSC) comprised 43 (55%). This meant that 62 of the 78 total tasks (79%) were considered to require low levels of cognitive demand. At a high level of cognitive demand, 16 of the 78 tasks (21%) met the criteria for procedures with extended connections (CD-PEC). The two levels that were found to be absent from the Concept Development portion of the lessons were the lowest cognitive demand level of memorization (CD-M) and the highest cognitive demand level of doing mathematics (CD-DM). See Table 8 for a breakdown.

Table 8: Frequency of Types of Cognitive Demand of Concept Development Tasks

Type of Cognitive Demand	Total	% of Total
Memorization (CD-M)	0	0%
Procedures Without Connections (CD-P)	19	24%
Procedures with Superficial Connections (CD-PSC)	43	55%
Procedures with Extended Connections (CD-PEC)	16	21%
Doing Mathematics (CD-DM)	0	0%

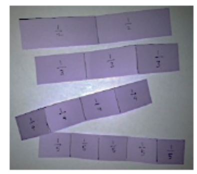
CD-PEC tasks were present in the Concept Development component in 10 of the 16 lessons of this module. Each of these ten lessons feature either one or two CD-PEC tasks and contain one to five tasks of lower cognitive demand levels (CD-P and / or CD-PSC). Therefore, the CD-PEC task does not stand on its own in the Concept Development portion of the lessons, but it shares the ten-to-thirty-eight-minute time allotment with other tasks. It is important to note again, that the Concept Development component of the lessons also contains the Problem Set which is usually recommended to be given at least 10 minutes of the time. As discussed earlier, these two components of the lesson were explored separately.

Figure 26 depicts a CD-PEC task that is given as one of four problems in a 30-minute Concept Development component of Lesson 1. The task features a teacher-led discussion that supports students through the steps of creating fraction manipulatives out of paper strips to represent halves, thirds, fourths, and fifths and connects the idea to fraction placement on the number line. Students are then guided to generate equivalent fractions by partitioning the whole into more, smaller pieces. The exemplar vignette that accompanies this task and serves as a teacher-led discussion demonstrates the deliberate guidance that is provided to support students to *understand and do the procedure* of representing fractions on a number line and generating equivalent fractions *in relation to underlying mathematical meaning* of referring to the smaller pieces as “groups” (Great Minds, 2015, p. 14) and utilizing multiplication to demonstrate the consistent proportional increase in the number of pieces. This deliberate connection between the two is what makes this task a CD-PEC task, providing the opportunity for students to engage in a task requiring a high level of cognitive demand.

This task did not meet the criteria for CD-DM tasks because, while essential to supporting students in making the connections between the procedure and the underlying mathematical meaning, the teacher-led discussion removed the opportunity for students to *develop their own strategies to learn and use mathematical concepts* by *imposing their own structure and procedure.* Rather, the teacher guided students through a predetermined process of folding the paper and creating number lines from it, led by step-by-step instruction.

Materials: (S) 4 paper strips $8\frac{1}{2}" \times 1"$

Problem 1: Make fractions equal to $\frac{1}{2}$.



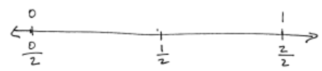
T: Take a paper strip. Hold it horizontally. Fold it vertically down the middle. How many equal parts do you have in the whole?

S: 2.

T: What fraction of the whole is 1 part?

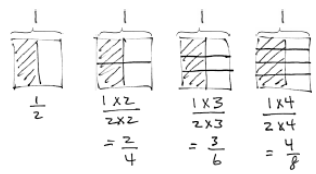
S: 1 half.

T: Draw a line to show where you folded your paper and label each half, 1 out of 2 units.



T: Make additional paper strips that show thirds, fourths, and fifths.

T: (After about 3 minutes.) Draw a number line that is a little longer than your paper strip. Use your strip as a ruler to mark zero and 1 above the line, as well as $\frac{0}{2}$, $\frac{1}{2}$, and $\frac{2}{2}$ below the line.



T: (Allow work time.) Sketch a square that is approximately 1 inch × 1 inch beneath your number line. This is representing the same 1 whole as the number line. For today, show half by vertically dividing the square. Shade 1 half on the left.

T: (Allow work time.) Draw another square to the right of that one. Shade it in the same way to represent $\frac{1}{2}$.

T: Partition 1 half horizontally across the middle.

T: What fraction is shaded now?

S: $\frac{1}{2}$ or $\frac{2}{4}$.

T: (Record numerically, referring to the picture.) 1 group of 2 out of two groups of 2.

$$\frac{1}{2} = \frac{1 \text{ group of two}}{2 \text{ groups of two}} \text{ or } \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

T: Explain how we have represented the equivalent fractions to your partners. (Students discuss.)

T: Show me $\frac{2}{4}$ on the number line. (Students show.) Yes, it is exactly the same value as 1 half. It is exactly the same point on the number line.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Sentence frames help students remember the linguistic and numerical patterns. As they gain confidence, gradually retract the frames. A suitable sentence frame for this lesson would be the equation to the left without any of the digits included.

Figure 26: Concept Development CD-PEC Task
Note. Task featured in Lesson 1. Grade 5 Module 3, p. 14

Looking at an example within the same Concept Development portion of Lesson 1 can help illuminate the difference found between CD-PEC and CD-PSC tasks and paint a picture of a pattern that was found to be common in this component of the lessons throughout the module. The tasks presented in Figures 26, 27, and 28 appear consecutively as problems 1, 2, and 3 in the Concept Development of Lesson 1. As previously discussed, problem 1, as displayed in Figure 26, meets the criteria for a CD-PEC task. Problems 2 and 3 featured in Figures 27 and 28 were coded as CD-PSC tasks. The *primary purpose* of the CD-PSC tasks featured here demonstrates the difference in the cognitive demand level between the two types of tasks as it shifts from *understanding the procedure in connection to the underlying mathematical meaning* (CD-PEC) to *mastering* the procedure by repeating the process (CD-PSC). While these takes tend to *make the underlying meaning visible,* the use of the connections to underlying meanings that have been previously learned are *superficial*. Once the students have engaged with the first task of the Concept Development component of this lesson (Figure 26), they are now able to utilize the procedure that was established in the first to complete the next two tasks (Figures 27 and 28). The positioning of these three tasks is, in part, what requires a lower cognitive demand level of the second and third tasks, as the *presentation of the task allows students to follow a pattern* as learned in problem 1 *to solve the problem,* a criterion of CD-PSC tasks. Once the procedure has been established in the first task, the tasks following are no longer able to provide the opportunity for students to *develop strategies* or *impose their own structures or procedures* as are hallmarks of high-level cognitive demand tasks. Rather, students are then simply following the previously learned procedure to apply it to subsequent problems. Bolstering this concept is a statement made by the curriculum writers themselves in a note to teachers in completing the fourth and final problem of the Concept Development with the students. They write "Note: The final complexity prior to working independently is to model a fraction greater than 1. The same exact process is used. Rectangles are used in the example just to break rigidity. This is not unique to squares!" (Great Minds, 2015, p. 16). With the focus on the process, we can see that the teacher-led discussion that provided guidance for students to *understand the underlying mathematical meaning* in Figure 26 dissolves here in favor of repeated practice by utilizing "the same exact process" (Great Minds, 2015, p. 16). It was determined through coding that CD-PSC tasks that require students to focus on repeating a procedure with teacher-led guidance make up the majority of the tasks in the Concept Development portions of the lessons in Module 3.

The sequencing of one CD-PEC task, followed by tasks requiring a lower-level of cognitive demand (CD-PSC and CD-P), was a common approach utilized in the Concept Development Components of module 3. Like the progression seen in problems 1, 2, and 3 in Figures 26, 27, and 28, the first task tends to feature an exemplar vignette through which the teacher leads the students through *structured steps.* In these steps, there is usually a combination of teacher explanation and direct questioning to the class that is utilized to explicitly explain the mathematical meaning underlying the procedures being conducted. As previously mentioned, Remillard and Kim (2020) make clear that procedures do not need to be computational, rather they can also be the creation or use of a visual model, which is the case in Lesson 1. After the completion of the first task, the next in the sequence tended to instruct the teacher to have students repeat the same process with different numbers, with less teacher guidance, and less or no connection to the underlying mathematical meaning. The number of tasks that required students to repeat the same process as learned in the preceding problem averaged approximately 4 in each lesson. Many of these sequences within the Concept Development ended with a completely procedural CD-P task serving the purpose of *mastering a procedure without meaning* as is discussed in the paragraph below.

Problem 3: Make fractions equal to $\frac{2}{3}$.

The next complexity is working with a non-unit fraction.

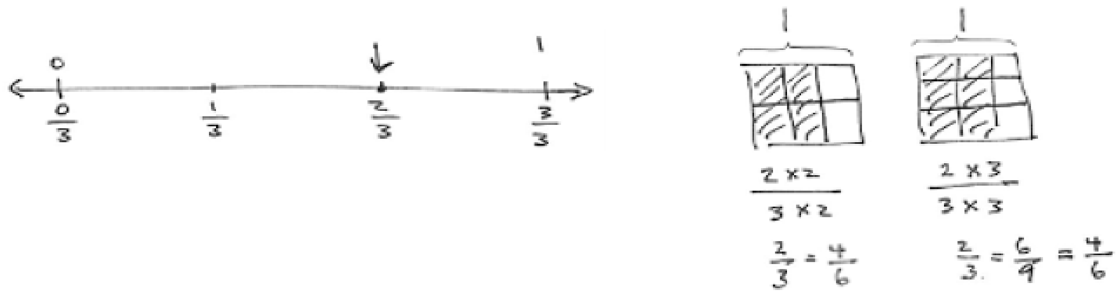


Figure 27: Concept Development CD-PSC Task
Note. Task featured in Lesson 1. Grade 5 Module 3, p. 15

Problem 4: Make fractions equal to $\frac{5}{4}$.

Note: The final complexity prior to working independently is to model a fraction greater than 1. The same exact process is used. Rectangles are used in the example just to break rigidity. This is not unique to squares!

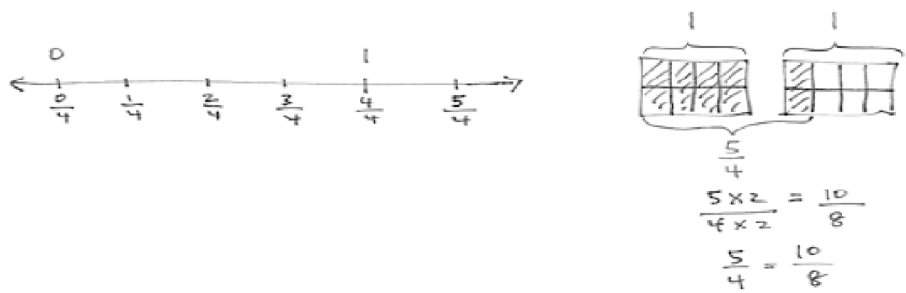
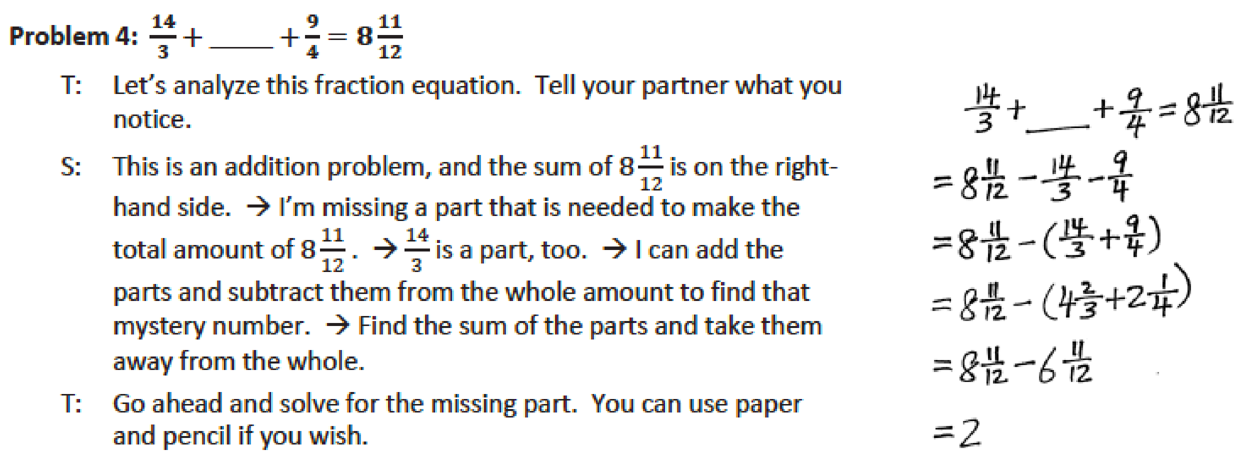


Figure 28: Concept Development CD-PSC Task
Note. Task featured in Lesson 1. Grade 5 Module 3, p. 16

The lowest-level of cognitive demand tasks present in the Concept Development component of the lessons were CD-P tasks which are described as serving the purpose to *master procedures without meaning*. In these types of tasks, *the use of procedures are either specifically called for, or their use is evident based on instruction* and *there is little ambiguity* *about what needs to be done*. CD-P tasks comprised 19 of the 77 (25%) Concept Development tasks. A CD-P task featured in Lesson 14 is shown in Figure 29. This task provides students with a clear idea of what is being asked for as it is presented with only mathematical numbers and a blank indicating the spot where a value needs to be determined. This task also gives students a strong indication of what needs to be done to solve it due to its positioning in both the lesson and the unit as students encountered tasks of similar or identical structure in the tasks preceding this both in the same lesson and throughout those prior. Depending on their positioning in the lesson, many of these tasks were also coded as CD-P tasks.



Problem 4: $\frac{14}{3} + \_\_\_\_ + \frac{9}{4} = 8\frac{11}{12}$

T: Let's analyze this fraction equation. Tell your partner what you notice.

S: This is an addition problem, and the sum of $8\frac{11}{12}$ is on the right-hand side. → I'm missing a part that is needed to make the total amount of $8\frac{11}{12}$. → $\frac{14}{3}$ is a part, too. → I can add the parts and subtract them from the whole amount to find that mystery number. → Find the sum of the parts and take them away from the whole.

T: Go ahead and solve for the missing part. You can use paper and pencil if you wish.

$$\frac{14}{3} + \_\_\_ + \frac{9}{4} = 8\frac{11}{12}$$
$$= 8\frac{11}{12} - \frac{14}{3} - \frac{9}{4}$$
$$= 8\frac{11}{12} - \left(\frac{14}{3} + \frac{9}{4}\right)$$
$$= 8\frac{11}{12} - \left(4\frac{2}{3} + 2\frac{1}{4}\right)$$
$$= 8\frac{11}{12} - 6\frac{11}{12}$$
$$= 2$$

Figure 29: Concept Development CD-P Task
Note. Task featured in Lesson 14. Grade 5 Module 3, p. 222

A significant pattern emerged in my exploration of the cognitive demand levels required by the tasks in the Concept Development component of these lessons. Of the 16 lessons in this module, 7 lessons featured one CD-PEC task, 3 lessons featured 2 CD-PEC tasks, and one lesson featured 2 CD-PEC tasks. In all 7 lessons that featured one CD-PEC task, that task was featured as the first task of the Concept Development section and was then followed up by one to five tasks of a low cognitive demand level (CD-P or CD-PSC). In the 3 lessons that each featured two CD-PEC tasks, one of those tasks was utilized as the first task of the Concept Development section. The single lesson that featured three CD-PEC tasks is the last lesson of the module. The way that the tasks unfolded within these lessons was fairly consistent and became predictable: the first task was of a higher level followed by the tapering off of cognitive demand required to a low level.

I focus on CD-PEC tasks because these tasks are the highest level of cognitive demand found in the Concept Development component of the lessons and are also considered by Remillard and Kim (2020) to be high-level tasks. Therefore, their positioning in the Concept Development portion of the lesson is worth noting as this component is intended to "address new content being studied" (Anderson, 2017, para. 4).

Three Criteria Combined

When considering each task as one featured within a type of context, with a type of information, and a type of Cognitive Demand that is required to solve the task, seven distinct combinations emerged: T-NC, I-MATCH, CD-P (25%); T-NC, I-MATCH, CD-PSC (47%); T-NC, I-MATCH, CD-PEC (14%); T-CC, I-MATCH, CD-PSC (6%) ; T-CC, I-MATCH, CD-PEC (4%); T-CC- I-MISS, CD-PSC (1%) and T-CC- I-MISS, CD-PEC (3%). See Table 9.

Table 9: Frequency of Combined Types of Context, Information, and Cognitive Demand of Concept Development Tasks

Type of Context	Type of Information	Type of Cognitive Demand	Code	Subtotals	% of Total
No Context T-NC		Memorization (CD-M)	T-NC, CD-M	0	0%
		Procedures without connections (CD-P)	T-NC, CD-P	19	25%
		Procedures with superficial connections (CD-PSC)	T-NC, CD-PSC	36	47%
		Procedures with extended connections (CD-PEC)	T-NC, CD-PEC	11	14%
		Doing mathematics (CD-DM)	T-NC, CD-DM	0	0%
Camouflage Context T-CC	Matching I-MATCH	Memorization (CD-M)	T-CC, I-MATCH, CD-M	0	0%
		Procedures without connections (CD-P)	T-CC, I-MATCH, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-CC, I-MATCH, CD-PSC	6	8%
		Procedures with extended connections (CD-PEC)	T-CC, I-MATCH, CD-PEC	3	4%
		Doing mathematics (CD-DM)	T-CC, I-MATCH, CD-DM	0	0%
	Missing I-MISS	Memorization (CD-M)	T-CC, I-MISS, CD-M	0	0%
		Procedures without connections (CD-P)	T-CC, I-MISS, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-CC, I-MISS, CD-PSC	1	1%
		Procedures with extended connections (CD-PEC)	T-CC, I-MISS, CD-PEC	2	3%
		Doing mathematics (CD-DM)	T-CC, I-MISS, CD-DM	0	0%
	Superfulous I-S	Memorization (CD-M)	T-CC, I-S, CD-M	0	0%
		Procedures without connections (CD-P)	T-CC, I-S, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-CC, I-S, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-CC, I-S, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-CC, I-S, CD-DM	0	0%
Relevant and Essential T-RE	Matching I-MATCH	Memorization (CD-M)	T-RE, I-MATCH, CD-M	0	0%
		Procedures without connections (CD-P)	T-RE, I-MATCH, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-RE, I-MATCH, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-RE, I-MATCH, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-RE, I-MATCH, CD-DM	0	0%
	Missing I-MISS	Memorization (CD-M)	T-RE, I-MISS, CD-M	0	0%
		Procedures without connections (CD-P)	T-RE, I-MISS, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-RE, I-MISS, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-RE, I-MISS, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-RE, I-MISS, CD-DM	0	0%
	Superfulous I-S	Memorization (CD-M)	T-RE, I-S, CD-M	0	0%
		Procedures without connections (CD-P)	T-RE, I-S, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-RE, I-S, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-RE, I-S, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-RE, I-S, CD-DM	0	0%

T-NC, I-MATCH, CD-PSC. The most prevalent type of task was T-NC, I-MATCH, CD-PSC which made up 36 of the 78 (46%) Concept Development tasks. These tasks feature mathematical symbols and numbers, are presented to students with the exact information needed to solve the problem, and require a cognitive demand level that is considered to be of low level (Remillard & Kim, 2020) because its purpose is to master a procedure while using or making only initial or limited connections to the underlying mathematical meaning. These tasks are not context-based tasks as they are not written within a context. An example of such a task can be seen in Figure 30. This T-NC, I-MATCH, CD-PSC task is the first task in the Concept Development of Lesson 5. This lesson serves as the first time that students encounter subtraction of fractions in this module, and this task is the second in the Concept Development, only after the teacher-led discussion of the metaphor of the addition of girls and boys needing to be renamed to the addition of students in comparison to the need to rename fractions with unlike denominators. Meaning this T-NC, I-MATCH, CD-PSC task is students' first ever opportunity to learn the new skill of subtracting fractions with unlike denominators in the K-12 Eureka Math curriculum as a whole.

The task was coded as T-NC because it features *only mathematical objects, symbols, or structures* and subsequently is coded as I-MATCH as discussed prior regarding the default coding of the type of information associated with a T-NC type of context. The task shows a CD-PSC level of cognitive demand for three significant reasons. First, the primary focus of this task is the procedure of creating two area models to represent ½ and ⅓, followed by the processes of representing both as equivalent fractions with like denominators and crossing out the 2/6 to represent subtraction to determine the solution. Secondly, the task adheres to the CD-PSC criteria: *tasks make underlying meaning visible* and *use of these connections is superficial, involving filling in blanks or using a predetermined outline or structure.* The teacher-led process, as laid out in the exemplar vignette, provides students with a predetermined structure that they follow to engage in the given process to subtract two fractions with unlike denominators for their first time. Guidance provided by the teacher such as "I'll draw one fraction model and partition it into 2 equal units" (Great Minds, 2015, p. 75) and "Now let's make equivalent units. (Model.)" (Great Minds, 2015, p. 75) provide the predetermined structure that students move through. Lastly, the task meets the CD-PSC criteria that *students follow a pattern to solve the problem often without understanding why*. The exemplar vignette prompts the teacher to "Write ½ - 1/3 = 3/6 – 2/6. Cross out 2 of 3 shaded sixths" (Great Minds, 2015, p. 75) without fully examining or even explaining to students why this is being done. Students may also wonder why they are drawing two area models at the start of the problem, then working with those two area models to create equivalent fractions and lastly moving to one area model when applying the operation of subtraction. This is not explained. So, while the use of the area model does serve as a connection to the underlying mathematical meanings of fractions, equivalent fractions, and the subtracting of such fractions, these connections are limited and are not utilized to make sense of the underlying mathematical meanings.

Problem 1: $\frac{1}{2} - \frac{1}{3}$

T: (Write $\frac{1}{2} - \frac{1}{3}$.) We'll need to change both units.

T: I'll draw one fraction model and partition it into 2 equal units. Then I'll write 1 half below one part and shade it to make it easier to see what 1 half is after I change the units. (Model.)

T: On the second fraction model, I'll make thirds with horizontal lines and write 1 third next to it after shading it. (Model.)

T: Now, let's make equivalent units. (Model.) How many new units do we have?

S: 6 units.

T: 1 half is how many sixths?

S: 1 half is 3 sixths.

T: 1 third is how many sixths?

S: 1 third is 2 sixths.

T: (Write $\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6}$. Cross out 2 of 3 shaded sixths.) Say the subtraction sentence with like units.

S: 3 sixths – 2 sixths = 1 sixth.

T: With unlike units?

S: 1 half – 1 third = 1 sixth.

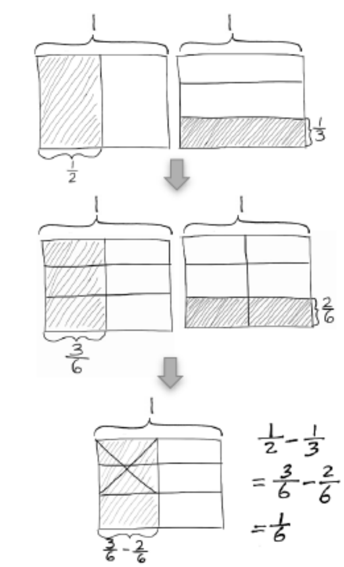


Figure 30: Concept Development T-NC, I-MATCH, CD-PSC Task
Note. Task featured in Lesson 5. Grade 5 Module 3, p. 75

While some lessons feature as many as 5 or 6 of the T-NC, I-MATCH, CD-PSC tasks, others feature only one. A pattern emerged as to when T-NC, I-MATCH, CD-PSC tasks were most utilized in the module. By comparing the number of these tasks featured in each lesson to the information displayed in the table of Objective of Module Topics and Lesson Objectives shown in Appendix 2, lessons containing the most T-NC, I-MATCH, CD-PSC tasks tended to be at the start of a new topic within the module. For example, Lesson 5 is the first time that students engage with subtraction of fractions in this grade level, and it is the first time that they learn to subtract fractions with unlike units. The Concept Development portion of this lesson is composed of six tasks, five of which have been coded T-NC, I-MATCH-PSC. The same is true in lessons 8 and 13 as students begin the new topics titled, Making Like Units Numerically (Great Minds, 2015, p. 126) and Further Applications (Great Minds, 2015, p. 204), respectively. The first lessons within these topics contain 5 to 6 T-NC, I-MATCH-PSC tasks. Using these observations, it appears that T-NC, I-MATCH-PSC tasks are most utilized as students begin their work with a new concept or skill within each Topic.

T-CC, I-MISS, CD-PEC. There were no tasks within the Concept Development component of the lessons that met the highest levels of type of context (T-RE) and type of cognitive demand (CD-DM). T-CC, I-MISS, CD-PEC tasks were revealed to be the highest-quality tasks according to the three combined characteristics, being of high-quality for type of information and high-level of type of cognitive demand. These tasks engage students in a mathematical situation set in a context while providing a missing type of information that requires them to derive information on their own and employ a level of cognitive demand considered to require their understanding of the procedure they are performing in relation to underlying mathematical meaning.

There were two T-CC, I-MISS, CD-PEC tasks not only within all the Concept Development components of the module, but within the entire module. Figure 31 displays an example of one of the tasks. These two tasks were the only tasks in the module that met the criteria for high-quality in two of the characteristics. Both tasks were featured in Lesson 16, the last lesson of the module. This lesson featured a unique structure in that the three total tasks featured in this lesson's Concept Development served also as the Problem Set. These tasks were described in the Teacher's Guide as "ribbon and wire problems" (Great Minds, 2015, p. 252) and were scored with a rubric, which is unlike any of the other tasks in the module.

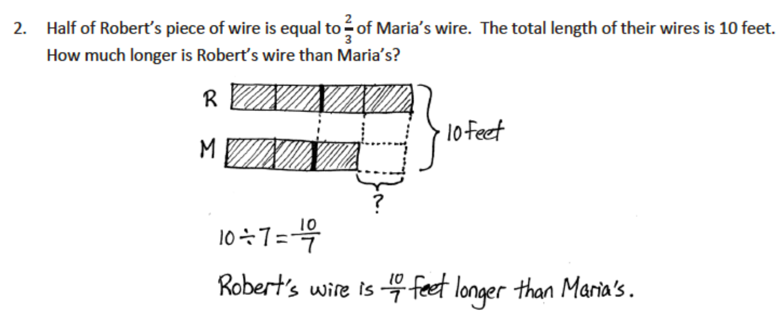


Figure 31: Concept Development T-CC, I-MISS, CD-PEC Task
Note. Task featured in Lesson 16. Grade 5 Module 3, p. 253

Summary of Concept Development Tasks

There were 78 tasks featured in the Concept Development component of the lessons in Module 3. 66 (85%) of the tasks were bare tasks, presented using *only objects, symbols, or structures* and 12 (15%) were context-based, featuring a camouflage context. Of the 12 context-based task, 10 were featured in lessons focused on the culmination of learning new skills or concepts. Three of the 12 context-based tasks met the criteria for high-quality in the characteristics of type of information and three met the criteria for high-quality in the characteristics of type of cognitive demand. Two of those tasks overlapped with a high-quality of both characteristics, making them T-CC, I-MISS, CD-PEC tasks and the highest-quality within the entire module. Both tasks were featured in Lesson 16, the last lesson of the module.

T-NC, I-MATCH, CD-PSC is the most utilized type of task. These bare tasks feature a matching type of information and low level of cognitive demand. They are present in high volumes at the start of new topics as students are introduced with new skills or concepts throughout the module.

This exploration of the Concept Development components of the lessons led to my third finding, which I will call the *Concept Development Finding* which concludes that the Concept Development section consisted of very few (15%) context-based tasks. Bare tasks, requiring a low level of cognitive demand, were heavily utilized at the start of new topics, and the highest-quality tasks were utilized at the culmination. The tasks that were found to be context-based tasks and to meet the criteria for a characteristic of high-quality were also very few: type of information (4%), type of cognitive demand (4%), and in both type of information and type of cognitive demand (3%).

Problem Set Component of the Lessons

The Problem Set component of the Eureka Math lessons is considered a part of the Concept Development section but is separated into its own distinct section in each lesson. This component is described by Great Minds as "an additional set of carefully crafted problems" and is intended to be "generally about 10 minutes of additional practice" (Anderson, 2017, para. 4). The tasks featured in the Problem Set are intended to be completed independently by students and therefore include instructions directed to the students. They, therefore, do not include an exemplar vignette for the teachers.

14 of the 16 lessons in this module featured a Problem Set section. Two lessons (Lesson 7 and Lesson 16) did not have an additional Problem Set section, but rather instructed the teacher to use the time in the Concept Development portion for students to work on the given problems. The Problem Set component of the lessons yielded the greatest quantity of tasks – featuring 135 total tasks in 14 lessons. Overall, the number of tasks in the Problem Set of each lesson ranged

from 3 to 15. The average number of tasks in each of the Problem Sets was approximately 9.6 tasks.

Type of Context

The tasks featured in the Problem Set component of this module were of N-TC or T-CC type of context. None of the tasks met the criteria for T-RE. Of the 14 lessons that had a Problem Set component, 13 of them contained both T-NC and T-CC tasks. Lesson 1 was the only lesson that featured only one Type of Context task, which were all T-NC tasks. The 13 lessons that featured a mixture of T-NC and T-CC tasks all contained more T-NC tasks than T-CC tasks. While the number of T-NC tasks in each lesson ranged from 6 to 13, the number of T-CC tasks ranged from 1 to 3 at most. Overall, 103 of the 135 (76%) tasks in the Problem Set component of this module were T-NC tasks and 32 of the 135 (24%) tasks were T-CC tasks. See Table 10 for a breakdown. This indicates that 32 of the 135 (24%) tasks were context-based tasks. Being that all of the tasks were coded as T-CC tasks, none of the context-based tasks were considered high-quality in the characteristics of type of context.

Table 10: Frequency of Types of Context of Problem Set Tasks

Type of Context	Total	% of Total
No Context T-NC	103	76%
Camouflage Context T-CC	32	24%
Relevant & Essential T-RE	0	0%

Figure 32 features two typical examples of the T-NC tasks featured in the Problem Set of this module. These tasks are a part of Lesson 11. Tasks from this lesson are featured frequently in this section because it contains tasks of a wide variety in the Problem Set component. The continuity of providing multiple examples from this lesson can support in demonstrating the different structures of tasks within the same mathematical concept of focus. While this task is labeled as task #1, it features 8 different tasks labeled a – h. Each of these 8 tasks was coded as its own. This task can be seen to be T-NC because it refers *only to mathematics objects, symbols, or structures.*

1. Generate equivalent fractions to get like units. Then, subtract.

    a. $\frac{1}{2} - \frac{1}{3} =$ b. $\frac{7}{10} - \frac{1}{3} =$

Figure 32: Problem Set T-NC, I-MATCH Task
Note. Task featured in Lesson 11. Grade 5 Module 3, p. 183

The very next numbered task in the lesson is featured in Figure 33 as an example of a T-CC task. While this task is written within a context, it meets the criteria for a T-CC task because it matches the criteria that *experiences from everyday life or common sense reasoning are not needed* and *the mathematical operations needed to solve the problem are already obvious.* This type of task structure that requires students to analyze the work or thinking of another individual to either determine its correctness and correct or offer to give suggestions to improve the work if necessary was a type of problem found every so often throughout the module. While this type of thinking may provide students with a higher level of cognitive demand than other tasks, the context meets the level of camouflage rather than relevant and essential because it falls short in several ways. Tasks of this type of structure do not authentically require students to engage in *identifying the relevant information and solution strategy for solving* nor do they *require mathematical modeling and common-sense reasoning with the context to understand and solve the problem.* Instead, these types of tasks are written within a context that is almost irrelevant to the purpose of the task, which, in this case, is the simple act of determining a denominator common to 6 and 8 that is smaller than 48.

2. George says that, to subtract fractions with different denominators, you always have to multiply the denominators to find the common unit; for example:

$$\frac{3}{8} - \frac{1}{6} = \frac{18}{48} - \frac{8}{48}.$$

Show George how he could have chosen a denominator smaller than 48, and solve the problem.

Figure 33: Problem Set T-CC, I-MATCH Task
Note. Task featured in Lesson 11. Grade 5 Module 3, p. 184

A second type of T-CC is featured in Figure 34. This task, which also comes from the Problem Set of Lesson 11 demonstrates an even more common type of task. This task looks like a typical word problem that might be featured in an average mathematics textbook. While this task is written within a context, the context of using sand to fill hourglasses, does not require *experiences from everyday life or common-sense reasoning* to solve. Additionally, one might also argue that such a context may not be relevant to a child's everyday life experiences. Similar contexts of tasks in the Problem Set that deemed them to be of the camouflage type include drinking different amounts of orange juice and determining the amount remaining (Lessons 3 and 11), spending fractional parts of a whole amount of money on different items and determining how much money was remaining (Lessons 3 and 7), fractional parts of time spent doing different activities while serving a 30-minute block in time-out (lesson 8), making different amounts of spicy and extra spicy chili (Lesson 12), deliberately wearing boots and inserting insoles in an attempt to meet a height requirement to ride a roller coaster (Lesson 13).

4. Harlan used $3\frac{1}{2}$ kg of sand to make a large hourglass. To make a smaller hourglass, he only used $1\frac{3}{7}$ kg of sand. How much more sand did it take to make the large hourglass than the smaller one?

Figure 34: Problem Set T-CC, I-MATCH Task
Note. Task featured in Lesson 11. Grade 5 Module 3, p. 184

Type of Information

132 of the 135 (98%) tasks feature a matching type of information (I-MATCH) *as they contain exactly the amount of information needed to find the solution*. See Table 11 for details. 3 of the 135 (2%) tasks provided missing type of information (I-MISS) as they *contained less information than needed so students need to derive additional data* to solve. As discussed twice before in this chapter, T-NC tasks were coded as I-MATCH tasks by default because by their very nature, tasks composed of only mathematical objects, symbols, and structures almost always contain the exact information needed to solve the problem. These T-NC tasks make up 103 of the 132 I-MATCH tasks. An example of such a task can be seen in Figure 32 above. The remaining 29 I-MATCH tasks are context-based tasks written within a camouflage context (T-CC) and contain exactly the amount of information needed to solve the problem. Examples of these types of tasks as featured in the Problem Set can be seen in Figures 33 and 34 above. Overall, in relation to the type of information, the Problem Sets contain 3 tasks that are high-quality context-based tasks within the characteristic of type of information. These tasks are T-CC, I-MISS tasks.

Table 11: Frequency of Types of Information of Problem Set Tasks

Type of Information	Total	% of Total
Matching I-MATCH	132	98%
Missing I-MISS	3	2%
Superfluous I-S	0	0%

The 3 I-MISS tasks are each featured in a different lesson. Figure 35 demonstrates an example of one of the I-MISS tasks. I believe that considering these two tasks as I-MISS may be due to a generous interpretation of the criteria on my part. This could be true because one might argue that in the Concept Development task given in Figure 25, "2 times as much" (Great Minds, 2015, p. 106) could translate to expecting a student to add the fraction value of ¼ twice. This might be considered by some as still being given the *exact* amount of information needed to solve the problem, therefore deeming it an I-MATCH rather than I-MISS task. However, after some contemplation, I felt that I-MISS was the appropriate coding as these two similar tasks stood out from the rest in the way that the information was presented. The third of the Problem Set I-MISS tasks is featured in Lesson 15 and is shown in Figure 36. In this task, students are expected to bring, or derive from another source, the information of how many seconds are in a minute to compute their final answer. A similar task was completed together with teacher guidance during the Concept Development of the same lesson, so students could derive the information that there are 60 seconds in a minute and unitize that information as they did in the earlier task.

Meiling has $1\frac{1}{4}$ liter of orange juice. She drinks $\frac{1}{3}$ liter. How much orange juice does she have left? (Extension: If her brother then drinks twice as much as Meiling, how much is left?)

Figure 35: Problem Set I-MISS Task
Note. Task featured in Lesson 11. Grade 5 Module 3, p. 184

Problem 4

Gavin had 20 minutes to do a three-problem quiz. He spent $9\frac{3}{4}$ minutes on Problem 1 and $3\frac{4}{5}$ minutes on Problem 2. How much time did he have left for Problem 3? Write the answer in minutes and seconds.

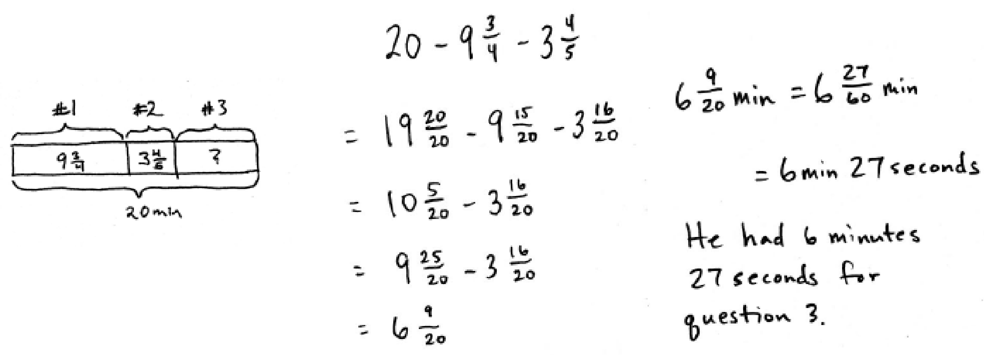


Figure 36: Problem Set I-MISS Task
Note. Task featured in Lesson 15. Grade 5 Module 3, p. 237

Type of Cognitive Demand

The tasks that comprise the Problem Set components in this module demonstrated three of the five levels of cognitive demand. These levels were the same as for the Application Problem and Concept Development components: procedures without connections (CD-P), procedures with superficial connections (CD-PSC), and procedures with extended connections (CD-PEC). As discussed earlier in this chapter, Remillard and Kim (2020) consider CD-P and CD-PSC tasks as requiring low levels of cognitive demand and CD-PEC tasks as requiring a high level of cognitive demand.

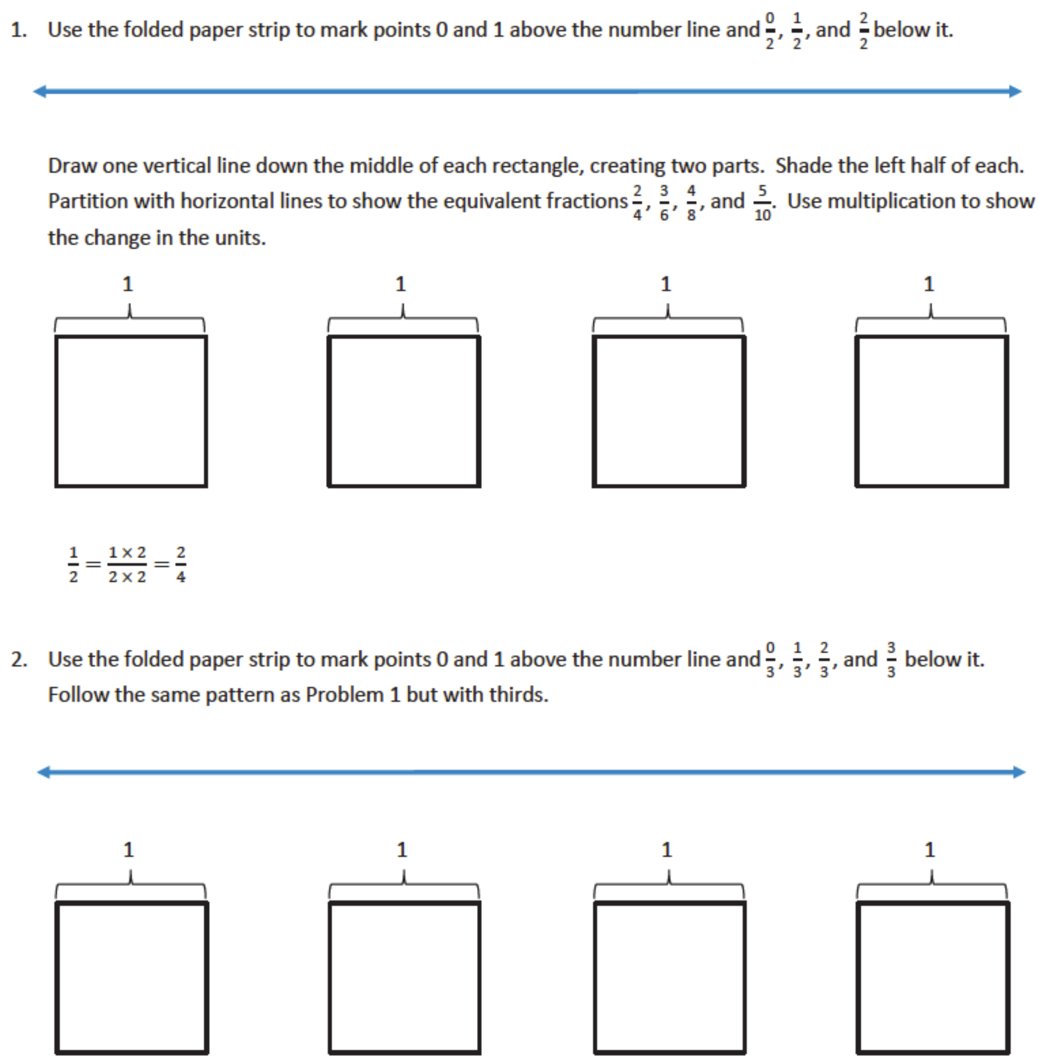
96 of the 135 tasks (71%) in the Problem Sets demonstrated the criteria of CD-P tasks. See Table 12 for a breakdown. The primary purpose of these tasks was determined to be for

students *to master procedures without meaning.* It is important to reiterate that, according to Remillard and Kim (2020), *procedures do not need to be computational.* I found that many of the tasks throughout this module required students to engage in enacting a procedure that was not computational in nature, but rather required them to produce a visual model that was previously demonstrated by a teacher or utilized in tasks earlier in the module or that same lesson. Essentially students were expected to reproduce the same type of visual model using the same process that was utilized earlier with a change in the fractions being represented. Although these tasks required the use of a visual model, they require a lower-level of cognitive demand and were coded as CD-P tasks because they met the matching criteria that *there is little ambiguity about what needs to be done and how to do it* and they *have no (or limited) connection to the concepts or meaning underlying the procedure used.*

Table 12: Frequency of Types of Cognitive Demand of Problem Set Tasks

Type of Cognitive Demand	Total	% of Total
Memorization (CD-M)	0	0%
Procedures Without Connections (CD-P)	96	71%
Procedures with Superficial Connections (CD-PSC)	36	27%
Procedures with Extended Connections (CD-PEC)	3	2%
Doing Mathematics (CD-DM)	0	0%

I have chosen to demonstrate an example of such a CD-P tasks paired with a CD-PSC task to articulate the factors that led to the difference in the two and how they work together to result in differing levels of cognitive demand required of students. The Problem Set in Lesson 1 features four tasks. Two of the tasks are shown in Figure 37 as the first and second tasks. In the first of these, students are required to produce a visual model to represent the equivalent fractions of 2/4, 3/6, 4/8, and 5/10. This requirement is paired with the written instructions to students instructing them how to go about this process. A generous interpretation of this guidance might deem it as a structure of *scaffolding,* however I believe it more likely meets the criteria of serving as *structured steps that direct attention away from the underlying meaning* – both criteria of CD-PSC tasks. While this task utilizes a visual model that supports students in making an *initial connection to underlying meaning,* the *primary focus becomes the procedure,* again meeting the criteria of CD-PSC and quite literally demonstrating a procedure with superficial connections. The juxtaposition of the second task following the first is what helps to clarify the determination of its CD-P coding. The first task supports students in making connections to an underlying mathematical concept and is then followed up by three more tasks that look the same but are presented without the same written instructions or other type of scaffolding. It is clear from this ordering that the exact same process is expected to be reproduced in the following tasks. This structure is consistent throughout the Problem Set tasks in this module. In this case, the second task even states for students, "Follow the same pattern as Problem 1 but with thirds" (Great Minds, 2015, p. 21), directly communicating to students that they should engage in the same process as before. This type of task structure provides a strong demonstration of the CD-P criteria that *use of procedures is either specifically called for, or their use is evident based on instruction* as the instructions instruct students to utilize the procedure and instructs them to look to the first task to imitate it. This structure leaves *little ambiguity about what needs to be done and how to do it,* matching another CD-P criterion. The purpose of these, and the following two tasks in Lesson 1's Problem Set, are for students to practice applying the procedure of creating a visual model that was previously demonstrated in the Concept Development component and then reiterated for students using written structured steps in the first task.


1. Use the folded paper strip to mark points 0 and 1 above the number line and $\frac{0}{2}$, $\frac{1}{2}$, and $\frac{2}{2}$ below it.

Draw one vertical line down the middle of each rectangle, creating two parts. Shade the left half of each. Partition with horizontal lines to show the equivalent fractions $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, and $\frac{5}{10}$. Use multiplication to show the change in the units.

1 1 1 1

$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$

2. Use the folded paper strip to mark points 0 and 1 above the number line and $\frac{0}{3}$, $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$ below it.
Follow the same pattern as Problem 1 but with thirds.

1 1 1 1

Figure 37: Problem Set CD-PS and CD-P Tasks
Note. Task featured in Lesson 1. Grade 5 Module 3, p. 21

As discussed regarding the criteria for CD-P tasks in the preceding paragraphs, these tasks serve the primary purpose for students to master a procedure without connection to the underlying mathematical meaning, and they made up a large portion of the tasks within the Problem Set. The large quantity of CD-P tasks lent themselves to come in a variety of forms. Figure 38 demonstrates an alternative structure that CD-P tasks were presented in. This figure shows an expanded view of the task in Figure 32 and is meant to show the cluster of eight tasks that all together are labeled number one. In this cluster, students would complete seven tasks of the same structure, each featuring different mixed numbers to be subtracted. This type of repetitive structure is a common occurrence found in this Grade 5 Module 3 Problem Sets.

Name ______________________________ Date ______________

1. Generate equivalent fractions to get like units. Then, subtract.

a. $\frac{1}{2} - \frac{1}{3} =$

b. $\frac{7}{10} - \frac{1}{3} =$

c. $\frac{7}{8} - \frac{3}{4} =$

d. $1\frac{2}{5} - \frac{3}{8} =$

e. $1\frac{3}{10} - \frac{1}{6} =$

f. $2\frac{1}{3} - 1\frac{1}{5} =$

g. $5\frac{6}{7} - 2\frac{2}{3} =$

h. Draw a number line to show that your answer to (g) is reasonable.

Figure 38: Problem Set CD-P Task
Note. Task featured in Lesson 11. Grade 5 Module 3, p. 182

Also common is the type of CD-P task featured in Figure 39 which differs from those in Figures 29 and 38 in that this CD-P task is written within a context. Although it is situated within a context, the task requires a low level of cognitive demand as its primary purpose is still to support students in learning a procedure, which in this case is subtracting two mixed numbers within the stated lesson objective, "Subtract fractions making like units numerically" (Great Minds, 2015, p. 175).

4. Harlan used $3\frac{1}{2}$ kg of sand to make a large hourglass. To make a smaller hourglass, he only used $1\frac{3}{7}$ kg of sand. How much more sand did it take to make the large hourglass than the smaller one?

Figure 39: Problem Set CD-P Task
Note. Task featured in Lesson 11. Grade 5 Module 3, p. 184

36 of the 135 tasks (27%) in the Problem Set of this module met the criteria for CD-PSC. CD-PSC tasks are considered by Remillard and Kim (2020) to be of a low level of cognitive demand and are described as making *initial or limited connections to underlying mathematical meanings, but not maintaining the connection.* Therefore, these tasks serve the primary focus of *practicing a procedure.* The task labeled *h* in the cluster of eight tasks featured in Figure 38 is an example of a CD-PSC task. In this task, students are instructed to utilize a specific visual model to demonstrate the reasonableness of the preceding task. While this task *makes underlying meaning visible,* the connection to the underlying meaning is superficial as it is a predetermined structure of a number line that was practiced with different mixed numbers earlier in the lesson.

In looking at tasks considered by Remillard & Kim (2020) as requiring a high-level of cognitive demand, only 3 of the 135 tasks (2%) in the Problem Set of this module met the criteria for CD-PEC and none met the CD-DM criteria. These three tasks are high-quality context-based tasks within the characteristic of cognitive demand. We can look at an example from Lesson 5 to see a CD-PEC task as shown in Figure 40. CD-PEC tasks make *underlying meaning visible* as *students are expected to use the meanings to solve problems and, in particular, make connections to mathematical procedures or notations.* In the task below, students are required to analyze what they are told is an incorrect solution from another person (Sandra) and are instructed to prove to her that her answer is wrong. It is then suggested that the students may want to draw a rectangular fraction model to support their thinking. Through Sandra's work within this task students are encountering a common misconception regarding subtracting fractions. They are then required to work through the problem on their own with the suggestion of drawing a rectangular fraction model. The requirement to "convince Sandra that she is wrong" (Great Minds, 2015, p. 84) would ideally require students to use the underlying meanings of the procedure of creating a rectangular fraction model to make connections to subtracting fractions. Through this task, students would again ideally explain the differences in the procedures utilized by themselves and Sandra, enough to engage in truly *understanding and doing the procedure in relation to the underlying meaning*.

3. Sandra says that $\frac{4}{7} - \frac{1}{3} = \frac{3}{4}$ because all you have to do is subtract the numerators and subtract the denominators. Convince Sandra that she is wrong. You may draw a rectangular fraction model to support your thinking.

Figure 40: Problem Set CD-PEC Task
Note. Task featured in Lesson 5. Grade 5 Module 3, p. 84

Three Criteria Combined

When considering all three codes there were six distinct types of tasks present in the Problem Sets of this unit. These included: T-NC, I-MATCH, CD-P (57%); T-NC, I-MATCH, CD-PSC (19%); T-CC, I-MATCH, CD-P (12%); T-CC, I-MATCH, CD-PSC (7%); T-CC, I-MATCH, CD-PEC (2%); and T-CC, I-MISS, CD-P (2%). See Table 13 for details. The majority of tasks in the Problem Set were T-NC, I-MATCH, CD-P tasks, which composed 77 of the 135 total tasks. These T-NC, I-MATCH, CD-P tasks served the purpose for students to master a procedure when being given *only mathematical objects, symbols, or structures*. There were two other types of CD-P tasks present in this module's Problem Set: T-CC, I-MATCH, CD-P and T-CC, I-MISS, CD-P tasks. Both types of tasks were presented within a context, one with matching type of information and the other in missing type of information. However, neither of these came close to being as prevalent in number as those presented without a context as they only comprised 16 and 3 of the 135 total tasks, respectively.

Table 13: Frequency of Combined Types of Context, Information, and Cognitive Demand of Problem Set Tasks

Type of Context	Type of Information	Type of Cognitive Demand	Code	Subtotals	% of Total
No Context T-NC		Memorization (CD-M)	T-NC, CD-M	0	0%
		Procedures without connections (CD-P)	T-NC, CD-P	77	57%
		Procedures with superficial connections (CD-PSC)	T-NC, CD-PSC	26	19%
		Procedures with extended connections (CD-PEC)	T-NC, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-NC, CD-DM	0	0%
Camouflage Context T-CC	Matching I-MATCH	Memorization (CD-M)	T-CC, I-MATCH, CD-M	0	0%
		Procedures without connections (CD-P)	T-CC, I-MATCH, CD-P	16	12%
		Procedures with superficial connections (CD-PSC)	T-CC, I-MATCH, CD-PSC	10	7%
		Procedures with extended connections (CD-PEC)	T-CC, I-MATCH, CD-PEC	3	2%
		Doing mathematics (CD-DM)	T-CC, I-MATCH, CD-DM	0	0%
	Missing I-MISS	Memorization (CD-M)	T-CC, I-MISS, CD-M	0	0%
		Procedures without connections (CD-P)	T-CC, I-MISS, CD-P	3	2%
		Procedures with superficial connections (CD-PSC)	T-CC, I-MISS, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-CC, I-MISS, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-CC, I-MISS, CD-DM	0	0%
	Superfulous I-S	Memorization (CD-M)	T-CC, I-S, CD-M	0	0%
		Procedures without connections (CD-P)	T-CC, I-S, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-CC, I-S, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-CC, I-S, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-CC, I-S, CD-DM	0	0%
Relevant and Essential T-RE	Matching I-MATCH	Memorization (CD-M)	T-RE, I-MATCH, CD-M	0	0%
		Procedures without connections (CD-P)	T-RE, I-MATCH, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-RE, I-MATCH, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-RE, I-MATCH, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-RE, I-MATCH, CD-DM	0	0%
	Missing I-MISS	Memorization (CD-M)	T-RE, I-MISS, CD-M	0	0%
		Procedures without connections (CD-P)	T-RE, I-MISS, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-RE, I-MISS, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-RE, I-MISS, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-RE, I-MISS, CD-DM	0	0%
	Superfulous I-S	Memorization (CD-M)	T-RE, I-S, CD-M	0	0%
		Procedures without connections (CD-P)	T-RE, I-S, CD-P	0	0%
		Procedures with superficial connections (CD-PSC)	T-RE, I-S, CD-PSC	0	0%
		Procedures with extended connections (CD-PEC)	T-RE, I-S, CD-PEC	0	0%
		Doing mathematics (CD-DM)	T-RE, I-S, CD-DM	0	0%

T-NC, I-MATCH, CD-P. T-NC, I-MATCH, CD-P tasks varied in their appearance and structure. While some focused on students' mastery of an algorithmic procedure, others honed in on their repetitive reproduction of a visual model. For example, Figure 41 demonstrates eight consecutive T-NC, I-MATCH, CD-P tasks with the main goal of supporting students in mastering the algorithmic procedure of adding or subtracting whole and mixed numbers. After completing these eight tasks, students would then encounter two T-CC, I-MATCH, CD-P tasks directly after. Figure 42 shows an example of one of those tasks.

1. Add or subtract.

a. $2 + 1\frac{1}{5} =$

b. $2 - 1\frac{3}{8} =$

c. $5\frac{2}{5} + 2\frac{3}{5} =$

d. $4 - 2\frac{2}{7} =$

e. $9\frac{3}{4} + 8 =$

f. $17 - 15\frac{2}{3} =$

g. $15 + 17\frac{2}{3} =$

h. $100 - 20\frac{7}{8} =$

Figure 41: Problem Set T-NC, I-MATCH, CD-P Tasks
Note. Task featured in Lesson 8. Grade 5 Module 3, p. 135

2. Calvin had 30 minutes in time-out. For the first $23\frac{1}{3}$ minutes, Calvin counted spots on the ceiling. For the rest of the time, he made faces at his stuffed tiger. How long did Calvin spend making faces at his tiger?

Figure 42: Problem Set T-CC, I-MATCH, CD-P Task
Note. Task featured in Lesson 8. Grade 5 Module 3, p. 136

Summary of Problem Set Tasks

The majority (76%) of tasks that make up the Problem Set component of the lessons are bare tasks with 103 of the 135 tasks being T-NC tasks. Only 32 of the 135 total tasks in the Problem Sets are context-based tasks, all within a camouflage context (T-CC). Of the 135 total tasks, 132 contained a matching type of information. 132 of the 135 (98%) tasks require a low level of cognitive demand as CD-P and CD-PSC tasks. With only 32 context-based tasks in the Problem Set component, 6 met the standard for high-quality criteria in one characteristic. Three of the context-based tasks were deemed high-quality within the characteristic of type of information and three other tasks were deemed high-quality for type of cognitive demand. These results led to my fourth finding, which I will refer to as the *Problem Set Finding,* which is that the Problem Set component of the lesson features few (24%) context-based problems. The tasks that were found to be context-based tasks and meeting the criteria for a characteristic of high-quality were also very few: type of information (2%) and type of cognitive demand (2%). This component featured many (71%) tasks requiring a low level of cognitive demand level of procedures without connections – the most of any of the three lesson components.

Summary

Cumulative

Cumulatively, there were 228 tasks comprising the Application Problem, Concept Development, and Problem Set components of this module. Of these tasks, 59 (25%) were situated within a context, indicating that these tasks met the definition of context-based tasks. 169 (74%) of the tasks were *bare tasks,* void of any context, featuring *only objects, symbols, or structures.* 100% of the 59 context-based tasks were written within a camouflage context (T-CC). The specifics of the few tasks (25%) considered context-based tasks are discussed below.

It is worth noting here that since none of the 59 *context-based tasks* in this module met the criteria for high-quality in the type of context as relevant and essential (T-RE), none of the tasks in the module were considered *high-quality* as a whole, by meeting all three criteria of type of context, type of information, and type of cognitive demand. There were, however, a few tasks that met the criteria in two of the three characteristics, and some that met in one of the three criteria. However, most met none of the three criteria.

Overall Alignment to Criteria for High-Quality Tasks

According to researchers (De Lange, 1995; Remillard & Kim, 2020; Wijaya et al., 2015), the ideal high-quality context-based task would be situated within a relevant and essential context (T-RE), would provide a missing (I-MISS) or superfluous (I-S) amount of information, and would require a high level of cognitive demand by meeting the standards of a math task that would be considered doing mathematics (CD-DM). None of the 228 tasks within the three components of this module met this level for all three criteria. Of the 228 total tasks, 3 (1%) met the criteria for two of the three characters of high-quality context-based tasks, and 26 (11%) met the criteria for only one of the three characteristics. It is important to reiterate here that one of the criteria for a context-based task is that the task is written within a context. As discussed above, of the 228 total tasks in the module, 59 met this foremost criterion, rendering the remaining 169 as bare tasks that featured no context. The figures discussed here are discussed with the whole being all the 228 tasks in the module and then number of context-based tasks being 59. While these 59 tasks are written within a context, they did not meet the criteria for high-quality in the type of context as all of them were T-CC tasks. Therefore, it is easy to see that none of the tasks in this module, context-based nor bare, met the criteria of high-quality in all three of the characteristics. In the section below, I will discuss the findings by specifically articulating whether I am referring to context-based tasks which are the 59 tasks written within a context or tasks as a whole body, which refers to the 228 tasks total.

The highest-quality context-based tasks appearing in this module were T-CC, I-MISS, CD-PEC tasks, meeting the criteria of high-quality in two of the three characteristics: type of information (as an I-MISS) and type of cognitive demand (CD-PEC). Again, while these were written within a context, the type of context (T-CC) was not of high-quality. These tasks met the highest level in the type of information characteristic (I-MISS) as both I-MISS and I-S are considered high-quality characteristics for context-based tasks (Wijaya et al., 2015). While these tasks were below the highest CD-DM level for cognitive demand, the criteria for CD-PEC does meet the standard for a high-quality level of cognitive demand. The T-CC, I-MISS, CD-PEC code describes two tasks, both of which are featured in the Concept Development component of the last lesson of the module, Lesson 16. This information led to the fifth finding, which I will refer to as the *Full Module Findings* which showed that, overall, few tasks (25%) throughout the module were context-based tasks, all of which were situated within a camouflage context. Even fewer were context-based tasks that met the criteria for high-quality in one (7%) or two (1%) of the three characteristics. None met the criteria for high-quality in all three of the characteristics examined. Overall, very few (4%) of the tasks within the module were context-based tasks with a high-quality type of information and even fewer (3%) were context-based with a high-quality type of cognitive demand. Even more rare (1%) were context-based tasks with a high-quality of both type of information and type of cognitive demand.

Overall Trends of Tasks Within Each Lesson Component

The results of the study revealed differences in the prevalence of the types of tasks presented in the Application Problem, Concept Development, and Problem Set components of the 16 lessons of Grade 5 Module 3.

The Application Problem section of the lessons included only tasks that were set within a camouflage context. These contexts were designed in a way that, for the most part (93%), provided students with exactly the amount of information needed to solve the task, without a need to derive nor select information. While these tasks spanned an array of three different cognitive demand levels, the vast majority (93%) of them required only a low level of cognitive demand for students to complete them. The tasks most closely meeting the standards of high-quality tasks in the Application Problem component was one (7%) T-CC, I-MATCH, CD-PEC task, which met the criteria for a high level of cognitive demand only and one (7%) T-CC, I-MISS, CD-PD task which met the criteria for a high level of type of information only. These results are summarized in the *Application Problem Finding:* in the Application Problem component of the lesson, all the tasks were context-based tasks written within a camouflage context. Very few of the tasks were context-based and met the criteria for high-quality in the characteristics of type of information (7%) and type of cognitive demand (7%).

Conversely, of the 78 total tasks in the Concept Development component of the lessons, 66 (85%) were bare tasks that were presented using *only objects, symbols, or structures.* This meant that only 112 of the 78 (15%) tasks were context-based, all of which were written within a camouflage context. 10 of these 12 context-based tasks were featured in lessons focused on the culmination of learning new skills or concepts, in the lesson right before the mid-module assessment and the final two lessons of the module. The two exceptions to this are at the start of the Concept Development in Lessons 3 and 5, when students are presented with analogies meant to demonstrate the need for replacing unlike units with like units when adding and subtracting fractions.

Overall, 62 of the 77 tasks in the Concept Development component of the task required a low level of cognitive demand as they appeared to serve the purpose of helping students to master a procedure either without connections (CD-P) or with only superficial connections (CD-PSC). Of the 12 context-based tasks in this component, 3 required a high level of cognitive demand (CD-PEC), which is 4% of the total in this component. In many of the lessons' Concept Developments, a task (both bare and context-based) requiring a higher-level of cognitive demand was presented at the start of this section. The higher-level cognitive demand task often featured an exemplar vignette that promoted dialogue focused on establishing connections between the procedures being employed and the underlying mathematical meaning. This task was followed by multiple tasks that required lower levels of cognitive demand, as the connections between the procedures and underlying mathematical meaning quickly dissolved with the progression to each succeeding task, often concluding with a bare or procedural task to close.

As discussed above, the two highest-quality context-based tasks (T-CC, I-MISS, CD-PEC) within the entire module are in the Concept Development component of Lesson 16. The remainder of context-based tasks in the Concept Development components of the lessons meet the standard of high-quality in either one or none of the criteria. There are a small number of the remaining context-based tasks within the Concept Development component that met the standards for high-quality context-based tasks in only one of the criteria. This proficiency was seen in 1 (1%) T-CC, I-MISS, CD-PSC task and 3 (4%) T-CC, I-MATCH, CD-PEC tasks –

through which the first task met the criteria for high-quality in the type of information (I-MISS) and the other three tasks met the criteria in the cognitive demand level required (CD-PEC).

This led to the *Concept Development Finding* which summarizes that the Concept Development section consisted of very few (15%) context-based tasks. Bare tasks requiring a low level of cognitive demand were heavily utilized at the start of new topics, and the highest-quality tasks were utilized at the culmination. The tasks that were found to be context-based tasks and meet the criteria for a characteristic of high-quality were also very few: type of information (4%), type of cognitive demand (4%), and in both type of information and type of cognitive demand (3%).

Like the tasks of the Concept Development component of the Eureka Math lessons, a majority (76%) of those that made up the Problem Set section were tasks void of context. Only 32 of the 135 (24%) total tasks in the Problem Sets were context-based tasks; again, all of them were written within a camouflage context (T-CC). Almost all the total tasks were found to be of a low level of cognitive demand, and the vast majority of these tasks appear to have been designed to *master procedures without meaning (CD-P)*, as 132 of the 135 (98%) were of this level. Of the 32 context-based tasks in the Problem Set component, 6 (2% of the total Problem Set tasks) met the standard for high-quality criteria in one characteristic. These tasks are T-CC, I-MISS, CD-P tasks which are high-quality for the type of information and T-CC, I-MATCH, CD-PEC tasks which demonstrate a high level of cognitive demand.

These results help paint a detailed picture of the makeup of each component of the lesson and, therefore, Eureka Math lessons as a whole. This data has led me to the *Problem Set Finding* through which I conclude that the Problem Set component of the lesson features few (24%) context-based problems. The tasks that were found to be context-based tasks and meet the criteria for a characteristic of high-quality were also very few: type of information (2%) and type of cognitive demand (2%). This component featured many (71%) tasks with a low level of cognitive demand level of procedures without connections, the most of any of the three lesson components.

Conclusion

In this chapter I have detailed the findings of the content analysis that I employed to explore the Grade 5 Module 3 of the Eureka Math materials. The analysis gave me a look into how the content of Eureka Math positions the curriculum in relation to the Common Core State Standards of Mathematical Practice. Among other things, it was revealed that Great Minds utilizes only a very small number of context-based tasks within the module and that their use is, for the most part, as tasks for the procedural application of skills or concepts learned. In the next chapter, I will discuss the data collected in relation to the contemporary research that frames this study. The five findings from this chapter together with Chapter IV shape the discussion as I move from a comprehensive view of the data to more nuanced trends within each lesson component.

CHAPTER VI

INTERPRETATIONS, IMPLICATIONS, AND SIGNIFICANCE

Math textbooks and other curricular resources are highly influential to what and how educators teach mathematics (Grouws et al., 2004; Peterson, et al., 2021; Stacey, et al., 2009; Stein & Kim 2009; Schmidt et al., 2001; Stylianides, 2014; Thompson, et al., 2012). It has been found that teachers in the early stages of their careers often utilize mathematics textbooks and curricular materials in a verbatim manner (Brown et al., 2016; Drake, 2002; Kastberg, 2012), and teachers lacking strong foundational mathematical knowledge report feeling inadequately prepared and thus rely heavily on the textbooks they choose or are mandated to utilize (Cohen et al., 2003).

Very recently, Open Educational Resources (OERs), often celebrated for their claim as influential tools in "reducing inequalities of educational opportunity and promoting innovative strategies to improve educational problems" (Bliss & Smith, 2015, p. 9) have begun to take a strong footing in the math curriculum market (Center for Education Market Dynamics, 2023). In fact, Eureka Math, from the publisher Great Minds, originally designed as an OER by the New York State Department of Education (NYSED) touts itself as the most widely used mathematics curricular resource in the U.S (Great Minds, 2015). In an interesting connection, literature also shows that it is not uncommon that teachers are left out of the decision-making process and mandated to utilize particular materials (Phillips, 2014). Specific to Eureka Math, in a study of teachers implementing the curricular resource, 80 and 90 percent of participants reported that their district either required or recommended their use of the curriculum (Kaufman et al., 2017).

Eureka Math's status as the most utilized mathematics curricular resource in the U.S. (EngageNY, 2017; Kaufman et al., 2018; Opher et al., 2017) compelled me to seek to understand how their claims about the curriculum align with the reality of their products and, more broadly, to education in the U.S. This study examined how the Eureka Math curricular resources are positioned within the context of the CCSS-MP and mathematics education more broadly and subsequently how the content of the resource then positions it in relation to the constructivist-aligned practices of the CCSS-MP. A constructivist theoretical framework comprised the lens through which the findings of this study were interpreted. The well-known theory posits that children learn by linking new information to their previous knowledge (Bada & Olusegun, 2015; Philips, 1995), which, in mathematics, can be accomplished through engagement with high-quality, context-based tasks (Bada & Olusegun, 2015; DeLange, 1995; Wijaya et al., 2015). In the eyes of a constructivist, the opportunity to engage in mathematical experiences that are situated in real-world context is an essential aspect to a learner's development in mathematical proficiency. As discussed in Chapter I, researchers (De Lange, 1995; Remillard & Kim, 2020; Wijaya et al., 2015) advocate that three characteristics are essential in making context-based tasks high-quality. These characteristics include the type of context that the task is situated within, the type of information that is given in the task, and the level of cognitive demand required of students to complete the task.

To answer the research questions that frame this descriptive case study, I employed two data analysis approaches: discourse analysis and content analysis. The discourse analysis served the purpose of examining the discourse Great Minds utilizes to position itself within the greater context of mathematics education and more specifically, the CCSS-MP on two pages of their website. I utilized a critical discourse analysis approach (CDA) developed by Fairclough (1989) to evaluate the micro- and meso-levels of the text itself as well as the macro-level of the broader contexts of the CCSS-M and *crisis* of mathematics that Great Minds and Eureka Math are situated within. While critical discourse analysis allowed me to explore how Great Minds positions itself discursively, content analysis allowed me to critically examine how the actual curricular tasks that make up the curriculum position Eureka Math in relation to the standards to which it discursively couples itself as well as best practices in math education. The latter entailed a detailed analysis of tasks provided in a module from the Eureka Math curriculum using a framework adapted from the work of Remillard and Kim (2020) and Wijaya et al. (2015). I focused on the tasks provided in the Application Problem, Concept Development, and Problem Set components of the lessons. I utilized the framework to hone in on the three characteristics deemed essential for high-quality by Wijaya et al. (2015) and De Lange (1995), including the type of context (De Lange, 1995; Wijaya et al, 2015), type of information (Wijaya et al., 2015), and type of cognitive demand (De Lange, 1995; Remillard & Kim, 2020).

The findings from both my discourse analysis and content analysis were presented in the previous chapters. In this chapter, I offer an interpretation of these findings, explain how findings from both modes of analysis address my research questions, and discuss the implications of this study on both educational practice and further research.

Interpretation of Findings

The findings directly addressed the two research questions:

1. Research Question 1: How does Great Minds discursively position the Eureka Math curricular resources within the context of the Common Core State Standards of Mathematical Practice and more broadly, mathematics education?
2. Research Question 2: How does the content of Eureka Math curricular resources produced by Great Minds position the curriculum in relation to constructivist-aligned approaches to the Common Core State Standards of Mathematical Practice?

As Coburn et al. (2016) has noted, curriculum cannot be fruitfully explored apart from the context within which it was developed and the aims to which it is aligned. Therefore, in order to address the above research questions, I first set out to explore the discursive context of Eureka Math and its parent publisher Great Minds through critical discourse analysis. Addressing the potential of critical discourse analysis to illuminate the ways in which reality is discursively produced and marshaled, Luke (2002) explains that the approach "sets out to capture the dynamic relations between discourse and society, between the micropolitics of everyday texts and the macropolitical landscape of ideological forces and power relations, capital exchange, and material historical conditions" (p. 100). Findings from my critical discourse analysis illustrate the ways in which the Great Minds organization has discursively positioned themselves within the macro-context of an educational *crisis* of mathematics education. Their position in this crisis serves as the foundation upon which they have built their brand and attempt to build trust with consumers. The organization can be seen to use this platform to convince consumers that their products have a superior ability to provide their classrooms with the transformation necessary to implement the Common Core State Standards Initiative – which serves as the solution to the most recent iteration of crisis reform.

The data from my critical discourse analysis of Great Minds' website shows that the publisher utilizes the concept that classrooms need to be transformed in order to meet the requirements of the CCSS-M. This notion sits comfortably within the *common-sense assumption* that U.S. mathematics education is in a state of crisis and that the CCSS-M are the solution to the latest iteration of it. This belief is reiterated on the Great Minds' website and is utilized to build its position that the Eureka Math materials will not only provide the solution through its alignment to the CCSS-M, but will do so in a way that is superior to other unnamed curricula. Among these strategies is their approach of making claims by vaguely comparing the Eureka Math curriculum to other unnamed curricular resources. A claim that is consistently made is that the Eureka Math curriculum is superior to others in providing a high level of *rigor*, proclaiming that the curriculum *exhibits unparalleled rigor*, *develops conceptual understanding,* and *connects learning with real-life application problems.* The CCSS names *rigor,* among *coherence* and *focus,* as one of the three instructional shifts embodied in the Common Core State Standards Initiative. *Rigor* is defined by NCTM as "how to support students in developing deep understanding of each standard" through a "balance of conceptual understanding, procedural skills, and applying mathematical ideas to a variety of contexts" (Gojak & Miles, 2015, p. xxiv). The NCTM further advocates that understanding is developed by highly rigorous tasks and "does not develop by assigning more worksheets or more difficult examples and problems" (p. xxiv). Additionally, Great Minds compares Eureka Math's ability to provide students with *a deeper understanding of the why behind the numbers* in a manner that is superior to other mathematics curriculum. This is paired with claims that students *develop conceptual understanding and* *practice procedural skills and fluency and have opportunities to connect their learning with real-life application problems.* These claims run parallel to the work on NCTM in the realm of rigor.

The essence of these claims surrounding the development of conceptual understanding and deeper understanding, as well as making connections to real-life problems, is directly aligned to the intended outcomes of implementing high-quality, context-based tasks (DeLange, 1995; Bada & Olusegun, 2015, Wijaya et al., 2015). High-quality, context-based tasks written in an essential and relevant context require students to select relevant information from an overabundance or derive necessary information from a lack thereof and utilize the process of mathematical modeling to solve the task requiring them to engage in a high-level of cognitive demand. It has been well-documented that students engaging in this process improve their ability to problem-solve real-world situations (Pollack, 2011; Niss & Blum, 2020; Remillard & Kim, 2020; Riyanto et al., 2017), connect their learning to real-world situations (Stein & Smith, 1998), and develop deeper levels of understanding of mathematical concepts (Fennema et al., 1993; Hiebert & Carpenter, 1992; Remillard & Kim, 2020; Stein & Smith, 1998; Wijaya et al., 2015). Engaging in the content analysis supported me in exploring the extent to which these claims were upheld within a selection of the Eureka Math curriculum. Findings two through five provided insight into the extent to which these claims made by Great Minds played out in the Eureka Math curriculum and how the materials went about furthering these claims.

Content analysis showed that, overall, few tasks (25%) throughout the module were context-based tasks, all of which were situated within a camouflage context. Even fewer were context-based tasks that met the criteria for high-quality in one (7%) or two (1%) of the three characteristics. None met the criteria for high-quality in all three of the characteristics examined. Overall, very few (4%) of the tasks within the module were context-based tasks with a high-quality type of information and even fewer (3%) were context-based with a high-quality type of cognitive demand. Even more rare (1%) were context-based tasks with a high-quality of both type of information and type of cognitive demand.

As discussed in detail in Chapter I, the level of cognitive demand required of a context-based task is essential to yielding its full benefits. It is a factor that directly impacts the type of problem solving a learner is engaged in and the level of understanding they take away from their time engaging in the task (Amaral & Hollebrands, 2017; Doyle, 1983; Fennema et al., 1993; Henningsen & Stein, 1997; Hiebert & Carpenter, 1992; Remillard & Kim, 2020; Stein & Smith, 1998). Some actions requiring high levels of cognition include constructing, synthesizing, applying, and explaining or justifying one's reasoning. In contrast, according to Remillard and Kim (2020), context-based tasks requiring low levels of cognition are very commonly the most utilized in mathematics textbooks and tend to require skills of reproducing information, memorizing, recalling, and applying algorithms, routines, or procedures. Engagement with tasks requiring a high level of cognitive demand has been proven to promote connections between mathematical concepts and mathematical procedures (Amaral & Hollerbrands, 2017; Remillard & Kim, 2020) and to directly impact the type of problem solving a learner is engaged in and the level of understanding that the learner takes away from that engagement (Amaral & Hollerbrands, 2017; Doyle, 1983; Fennema et al., 1993; Henningsen & Stein, 1997; Hiebert & Carpenter, 1992; Remillard & Kim, 2020; Stein & Smith, 1998). These effects of context-based tasks requiring a high level of cognitive demand would meet Great Minds' claim that the Eureka Math curriculum supports students in developing both a *conceptual understanding* and *deeper understanding of the why behind the numbers*. With only 3% of the tasks in this module being context-based tasks that feature the level of cognitive demand necessary to develop the ability for students to connect concepts to procedures, the extent to which the curriculum meets these claims seems minimal. In my exploration of the module, it was most common to encounter context-based tasks that featured a low level of cognitive demand which instead simply require students' reenactment of a procedure that they have memorized (Stein and Smith, 1998).

In addition to the type of cognitive demand required by a context-based task, the type of information given in the task is essential to its quality and, therefore, the benefits it can yield for students (DeLange, 1995; Pollack, 2011; Wijaya et al., 2015). As Pollack (2011) points out, real-world situations require individuals to decipher which information is relevant and which is not in solving a problem. When it comes to tasks in the mathematics classroom, structuring context-based tasks which feature missing information requires students to use their knowledge of the situation's context to derive and add the necessary missing information (Wijaya et al., 2015). Additionally, on the other end, providing erroneous information within the problem requires students to decide which information is relevant and position it to be utilized to solve the given problem (Niss & Blum, 2020). A task structured with exactly the amount of information needed to solve the problem makes not only the mathematical nature of the problem obvious, but often also the mathematical operations that will be utilized to solve it. So, while a context-based task with a matching type of information may be written in a context, students are not able to engage in a manner that is authentic to the way real-world problems arise, are modeled, or are ultimately solved. With only 4% of the total tasks provided in this module being context-based tasks featuring a missing or superfluous type of information, it seems challenging to make the claim that such a curriculum provides students with *opportunities to connect their learning with real-life application problems* as Great Minds has said. Instead, there are only a small number of tasks of this nature that students have the opportunity to encounter.

Mathematics teaching that promotes rigor is described by NCTM as one that "balances conceptual understanding, procedural skills, and applying mathematical ideas to a variety of contexts" (Gojak & Miles, 2015, p. xxiv). Great Minds advertises Eureka Math's unique superiority in *exhibiting unparalleled rigor throughout the grades*. The data summarized in the *Full Module Finding* creates dissonance between these claims and what the curriculum showed to promote. With very few (4%) of the tasks within the module being context-based tasks with a high-quality type of information and even fewer (3%) being context-based with a high-quality type of cognitive demand, the opportunities to practice procedural skills seem to greatly outweigh those of developing conceptual understanding. If featured in equal capacity, high-quality, context-based tasks might be thought to make up approximately 1/3 of the total tasks in the module. Further, to ensure the process of modeling through the design of tasks with relevant and essential (T-RE) type of context and doing mathematics (CD-DM) type of cognitive demand, many of the tasks would ideally be presented when students were encountering new mathematical concepts for the first time. Within the Eureka Math lesson structure, this first-time encounter that would allow for process of mathematical modeling by reasoning with the givens of a task to develop their own strategies and procedures to solve it, and it would take place at the start of the Concept Development component as its purpose is to "address new content being studied" (Anderson, 2017, para. 4).

A look into each lesson component revealed important trends of their own. As I detailed below, the presence of high-quality context-based tasks within the characteristics of both type of information and type of cognitive demand were very low within all three of the lesson components explored. Above, I have discussed the benefits that students miss out on from the lack of these types of tasks. In the sections below I discuss the interesting features of these components beyond the missing opportunities for tasks featuring high-quality characteristics.

The *Concept Development Finding* provided a more detailed look into the Concept Development component of the lessons. As this finding articulates, I found that the Concept Development component consisted of very few (15%) context-based tasks. Bare tasks requiring a low level of cognitive demand were heavily utilized at the start of new topics and the highest-quality tasks were utilized at the culmination. The tasks that were found to be context-based tasks and meet the criteria for a characteristic of high-quality were also very few: type of information (4%), type of cognitive demand (4%), and in both type of information and type of cognitive demand (3%).

From this finding, it seems as though Great Minds has chosen to reserve context-based tasks featured in the Concept Development for use after the learning of a new skill or concept has been completed. Therefore, rather than being utilized to engender the new learning through exploration with the real-world situation which would allow for mathematical modeling, the context-based tasks were utilized as a culmination of the learning for the purpose of applying previously learned skills and concepts. This was the case for 10 of the 12 context-based tasks in the Concept Development.

The two context-based tasks that were situated at the start of new learning in the Concept Development component were featured in lessons 3 and 5 of the module. As detailed in Chapter V, these two tasks were strongly teacher-led, guided explanations regarding the inability to add or subtract unlike units of *children* and *adults* in Lesson 3 and *boys* and *girls* in Lesson 5 and to then replace the unlike units with like units of *people* and *students* to complete the computations. An analogy was then created as these concepts were connected to the necessity to do the same when adding and subtracting fractions with unlike units. The connection made was certainly within a context, however, was superficial rather than grounded in the deeper underlying mathematical meaning. Tasks like this are utilized to provide students with a connection that supports their application of the idea of *replacing unlike units with like units*; however, it misses the true reason of why in favor of providing a quick trick to instead be applied to future experiences with adding and subtracting fractions. For example, when students encounter a task that requires them to add ½ and ⅓, they might change the fractions to 3/6 + 2/6 and justify their approach by explaining their noticing of the difference of the sizes of the fractional pieces of ½ and ⅓ and how, while possible to add them as is using a difference approach, they found it helpful to represent them as fractional piece of the same size. In contrast, a student justifying the replacement of ½ and ⅓ in connection to the need to change the units of girls and boys to children is to me superficial and meant as a *quick trick* to hotwire their pathway to the end goal of finding an answer of 5/6.

A possible different approach for introducing the concept of changing fractions with unlike denominators to equivalent fractions with like denominators could be to explore the sizes of the fractions with fraction tiles or the fraction manipulatives that were created and utilized in Lesson 1 of the module. Providing students with a task set in a relevant and essential context and paired with the availability to utilize fraction manipulatives to develop an approach to adding fractions with unlike denominators would be a strong alternative. This would allow students to make the deeper connections grounded in mathematical understandings which, in turn, would develop a deeper level of understanding regarding the concept while also improving their ability to problem-solve in real-world situations and retain the information better (Clements & Battista, 1990; Fosnot, 1996; Simon, 1995). Alternatively, the only two context-based problems utilized for new learning in the Concept Development component of this module lack the support of students in developing a deeper mathematical understanding and instead utilize an analogy to support a superficial understanding.

Still, most of the context-based tasks that came at the conclusion of new skills were not considered high-quality. Very few (4%) tasks within the Concept Development were context-based and provided students with the opportunity to derive additional information to solve the problem by providing a missing type of information in the task. This made the skills and concepts to be utilized in the tasks obvious, causing their purpose to be for application rather than for modeling (Wijaya et al., 2015). Tasks that promote application of previous learning support students in practicing procedural skills, satisfying one aspect of rigor as defined by the NCTM (Gojak & Miles, 2015). Additionally, very few (4%) tasks were context-based that required a high level of cognitive demand to solve which removes the opportunity for students to develop deeper understanding of concepts through the development of their own strategies in favor of procedural application. The component of rigor that appears to be off-balance within the Concept Development component is the development of conceptual understanding, which is effectively promoted through engagement in high-quality, context-based tasks.

The Concept Development component, which serves the purpose of addressing the new content being studied, consisted of a majority of *bare tasks*. This means that, through the use of this module, the majority of engagement that students have with skills and concepts regarding the addition and subtraction of fractions of unlike units is to learn and apply procedures to tasks that involve only mathematical symbols and structures. This revelation from the data does not seem to match the message being portrayed on Great Minds' website.

In the *Problem Set Finding*, I articulated that the Problem Set component of the lesson features few (24%) context-based problems. The tasks that were found to be context-based tasks and meeting the criteria for a characteristic of high-quality were also very few: type of information (2%) and type of cognitive demand (2%). This component featured many (71%) tasks with a low level of cognitive demand level of procedures without connections – the most of any of the three lesson components.

With this component of the lesson intended for students' independent practice and many of the tasks featured being bare tasks (76%), students do not have much exposure to completing context-based tasks on their own. When exploring the Problem Set of a lesson, it seemed as though the intention of the curriculum developers was to expose students to a high volume of math problems that were similar to one another in structure. I hypothesized that the purpose of this could be to develop their procedural fluency with the skill or concept that had been learned in the featured lesson. This predicted intention is corroborated with the result that 71% of the tasks within the Problem Set component required a Procedures without connections type of cognitive demand. This level of cognitive demand is concerned with the procedural performance of a skill void of connection to the underlying mathematical understandings. This focus on procedure, paired with the repetitive nature of the problems, made the approaches utilized to solve the tasks extremely predictable. As shown in the tasks featured in Figure 38, it was common to see six to eight bare tasks of the same structure, with only their numbers changed, listed one after another for completion by students. As the NCTM advocates, true mathematical understanding is developed by highly rigorous tasks and "not by assigning more worksheets or more difficult examples and problems" (p. xxiv). Keeping this in mind, it may be wise to suggest that Eureka Math misses the mark on providing students opportunities to independently engage in meaningful tasks through the Problem Set component of their lessons.

The *Application Problem Finding* articulated that in the Application Problem component of the lessons, all the tasks were context-based tasks written within a camouflage context. Very few of the tasks were high-quality, context-based, meeting the criteria in the characteristics of type of information (7%) and type of cognitive demand (7%). As stated in this finding, all the tasks featured in the Application Problem component of the lessons were context-based tasks with a camouflage type of context. Tasks featuring camouflage contexts may look as though they are positioned within a real-world context using words but are described by Wijaya et al. (2015) as "merely dressed-up bare problems" (p. 45). These types of tasks do not require modeling because the process required to solve the task is obvious (de Lange, 1995; Wijaya et al., 2015). As discussed in Chapter I, the process of mathematical modeling ultimately requires students to employ the process of mathematization which is explained by Wijaya et al. (2015) as the requirement to "transform the context situation into a mathematical form" (p. 45). The process of mathematization is one that is not necessary when the procedures to be used are obvious. Instead, the tasks within the Application Problems seem to live up to their title of serving the purpose of application rather than modeling. Additionally, while these tasks are presented in the context of words, they are not in fact positioned within real-world contexts. This finding seems to challenge Great Minds' claim that the Eureka Math curriculum provides students with *opportunities to connect their learning with real-life application problems.* It seems that while, at first glance, the Application Problem component exposes students to tasks that are written in a context, upon further exploration they are merely engaging in tasks that do not require common sense reasoning or real-life situations and through which the mathematical process needed for solving is obvious and allows them to follow a procedure that was previously learned and practiced to determine the answer.

As a result of their study, Wijaya et al. (2015) concluded that students' difficulty in solving high-quality, context-based tasks was a direct result of the lack of opportunities they were given to engage in such tasks. In the same sense, it may be possible that the Application Problem, Concept Development, and Problem Set components featured in the lessons of this module may be setting students up for the same challenges. Great Minds portrays Eureka Math as the solution to the mathematical crisis that currently plagues our schools. They claim that the use of Eureka Math will transform a customer's math classroom through *unparalleled rigor,* provide students with a *deeper understanding of the why behind the numbers*, and support students to *develop conceptual understanding and practice procedural skills and fluency, have opportunities to connect their learning with real-life application problems,* and *develop conceptual understanding and practice procedural skills and fluency*. The findings of this study showed that, when explored through the utilization of high-quality context-based tasks, it is difficult to say that the Eureka Math curricular resource delivers on these notions.

Implications for Future Research

My experience exploring Eureka Math through the specific focus of my study caused me to view the materials through a different lens than I had viewed it prior. In the past, I had seen it through the lens of a teacher and an instructional coach, but through this study I had the opportunity to view it with a purpose to understand specific characteristics. As I progressed through my study, the in-depth understanding that I was developing engendered new wonderings and, consequently, ideas for further research. Similar to my feelings, there exists a call from researchers for increased focus on the analysis of mathematical textbooks (Ball & Feiman-

Nesmer, 1988; Baker et al., 2010; National Research Council, 2004; Reys et al., 2004; Senk & Thompson, 2003; Stylianides, 2014; Weiss et al., 2001; Woodward & Elliot, 1990).

Arguably, the most obvious place this study could lead the next researcher or myself is to replicate the study with a module from a different grade level or different module within Grade 5. As discussed in Chapters I and III, I utilized the work of Stylianides (2014) who believes in careful consideration for choosing the parts of a textbook to be of focus when conducting detailed analysis and advocated for strategically choosing topics that "are important from a disciplinary perspective and with which students and teachers tend to face significant difficulties" (p. 68). The prevalence of contemporary research detailing the struggle of U.S. with understanding concepts involving proportional reasoning (Gabriel et al., 2013; Siegler et al., 2011), and most specifically fractions (Carpenter et al., 1981; Namkung & Fuchs, 2019), led me to choose Grade 5 Module 3 for my study. Exploring the extent to which constructivist-aligned approaches to the Common Core State Standards of Mathematical Practice are advanced in other grade levels or other Grade 5 Eureka Math modules would be a helpful extension to this study. Then, investigating how those approaches are advanced would support educators, coaches, and administrators to capitalize on the strategies provided in Eureka Math. Possible additional modules of study could be the next module in Grade 5, which focuses on multiplication and division of fractions, or moving back to look at modules that focus on more fundamental skills and concepts related to fractions in Module 5 of Grade 3 or Modules 5 and 6 of Grade 4.

Additional wonderings arose for me as I worked through my own exploration of the tasks in the Application Problem, Concept Development, and Problem Set components of the lessons in Grade 5 Module 3. Wijaya et al.'s (2015) criteria of purpose of the context-based task was used to determine whether the tasks being assessed were intended for application or for modeling.

This determination is distinguished using two criteria: *after the explanation section,* indicating that the task is intended for application or *before the explanation section,* indicating that the task is for modeling. As detailed in Chapter III, I anticipated that the ability to distinguish between the two criteria did not seem as if it would be so clear cut in the Eureka Math curricular resource because the Concept Development component was intended *for* the explanation of the concept, subsequently deeming it neither *before* nor *after* the explanation, but rather as *being* the explanation. This detail led to my adaptation of the instrument by removing the *purpose of the context-based task* characteristic to instead explore and report the data via the three separate lesson components of Application Problem, Concept Development, and Problem Set. I did this because I knew that these fell *before the explanation, during the explanation,* and *after the explanation,* respectively. Once I began exploring the tasks with my instrument in hand, this classification of the parts of the Eureka Math lessons became much more nuanced than I had originally thought. As discussed earlier in this chapter, the Application Problem component was indeed positioned *before* the explanation in the lesson that it was featured in. However, in a more nuanced classification, this component came *after* the explanation of the skill or concept that was of focus in each Application Problem component. This skill or concept was usually learned in a lesson or more prior. The Problem Set was a bit more straightforward, as the intention as stated by Great Minds was for additional practice and therefore could be considered to be more easily aligned to serve the purpose of application of skills and concepts. However, the Concept Development was arguably the most nuanced of the three. The tasks within this section often served as the explanation of a skill or concept, sometimes requiring more than one task to fully explain. Many Concept Development components contained the entire explanation within the very first problem of the section, and so then featured multiple tasks that followed that explanation problem, in what would deem them as serving the purpose of application.

The nuances discussed above led me to desire additional criteria to evaluate the tasks or the text of the discussion that served the purpose of explanation within the Concept Development component of the lessons. Specifically, while reading through the exemplar vignettes provided in many of the tasks within the Teacher Guide, I noticed the differences in the teachers' and students' roles in the lesson. I wondered how the suggested words and actions of the teacher within the exemplar vignettes of the problems supported students' learning opportunities within the tasks. In their own study, Remillard and Kim (2020) evaluated the teacher's role as displayed in segments of the lessons of five mathematics textbooks. Their framework is based on the notion that teaching is an *interactive system* where the roles of the teacher and student are related to how students conceptualize mathematical ideas and where they believe the source of their knowledge to lie. Additionally, this type of exploration can shed light on where the mathematical authority is held when the teacher implements the curriculum. The indicators that comprised their framework and which they explored within the teacher's guides were *telling and showing, telling and showing through the student text, guiding, facilitating, orchestrating,* and *stepping back.* These indicators support researchers in exploring the places that a textbook communicates the mathematical authority to be through the words and actions recommended. The placement of authority tends to be identified on a continuum ranging from the teacher's possession by telling and showing to teacher and textbook possession by guiding and finally to student possession by a teacher orchestrating in a less active role as students use their authority to develop strategies and engage in mathematical modeling. Utilizing this framework from

Remillard & Kim (2020), or one that explores the essence of this, may be a helpful next step in continuing the research related to the pedagogical approaches of Eureka Math.

An aspect of the exemplar vignettes that were featured in the tasks of the Concept Development was the line of questioning that teachers are encouraged to engage their students in. While I did not in fact evaluate this characteristic specifically, through my in-depth reading, I did walk away with a sense of a pattern that was arising. Some may say questioning and student-talk was a central feature of the tasks in this component, but I noticed that many of the questions were closed in nature, and the response expected from students as written in the exemplar vignettes was one that could be classified as either correct or incorrect. Examples of these types of exemplar responses are *24* or a mathematical term like, denominator. Simpson et al. (2015) advocate that in a mathematics classroom, the right types of questioning can serve as a critical role in furthering the CCSS-MP by creating student-centered, inquiry-based classrooms that encourage high levels of student thinking and reasoning, and by engaging students in mathematical discourse that "elicit students' thinking and reasoning beyond that of procedural tasks" (p. 295). The researchers designed a two-dimensional matrix, intended for use to classify questions and tasks to identify higher-level tasks to utilize and lower-level tasks to adjust or revamp. The instrument features a revised version of Bloom's Taxonomy as one dimension and Webb's Depth of Knowledge (DOK) as the other and is titled, Cognitive Rigor Matrix.

Implications for Practice

This study provided insight into how the Eureka Math curricular resources are positioned within the context of the CCSS-MP and mathematics education more broadly and subsequently how the content of the resource then positions it in relation to the constructivist-aligned practices of the CCSS-MP. While this study focused specifically on the Eureka Math curriculum, the results could also serve a significant role in informing the broader discussions of mathematics curriculum and more broadly still, conversations about curriculum decision making more generally.

Research has shown that curriculum designed for the teaching and learning of mathematics often does not in fact do what it says it will do nor adhere to the standards that it says it does (EdReports, 2015; Polikoff, 2015; Remillard & Kim, 2020). This study demonstrated that while Eureka Math advertises a curriculum capable of *transforming* classrooms through its superior level of *unparalleled rigor,* ability to *build conceptual understanding* and *facilitate opportunities for students to make real world connections,* there was a lack of evidence to support their claims. The implications of the findings presented in this study may serve as a useful starting point for the teachers and school leadership members who are responsible for the implementation of the Eureka Math curriculum in their own schools and classrooms. While previous analyses have been conducted to explore the alignment of Eureka Math with the Common Core State Standards of Mathematical Content (NGA & CCSSO, 2010), only one has been conducted to explore the extent to which the curricular resource meets the requirements of the Common Core State Standards of Mathematical Practice (EdReports, 2018). Exposure to this study may spark a line of questioning with a new lens focusing on the pedagogical approaches promoted through the Eureka Math curriculum. While some may desire diving into the specifics revealed about Eureka Math's Grade 5 Module 3, others may want to use the structure to engage their own review of the Eureka Math curricular materials themselves. Even without their own exploration, teachers and school leaders can use the findings of this study to build a more comprehensive understanding of the Eureka Math curriculum. From this, these educators and leaders can hold the awareness of where Eureka Math is not in alignment with the type of pedagogical approaches that they want implemented in their classrooms to then build professional development plans to support teachers in personalizing the materials to meet their desired approaches.

The focus of this study on one specific Mathematics curriculum can inform more broad discussions about mathematics curriculum and decision-making as a whole. In regards to mathematics curriculum materials, it has been recognized that textbooks play a significant role is guiding mathematics teachers in what they teach and how they teach it (Arican, 2018; Brown et al., 2016; Cai & Cirillo, 2014; Cohen et al., 2003; Fan, 2013; Fan et al., 2013; Grouws et al., 2004; Johnson, 2017; Jones & Tarr, 2006; Kastberg, 2012; Livy & Herbert, 2013; Peterson et al. 2021; Stacey, et al., 2009; Stein & Kim, 2009; Schmidt et al., 2001; Stylianides, 2014; Thompson et al., 2012). For students, the textbook that their teacher utilizes often determines how they experience mathematics through both the content that they learn and the pedagogical approaches through which they engage with that content. With mathematical textbooks having such a strong influence, it makes sense that both educators and decision makers should possess a deep understanding of not only the mathematical content contained within the pages of the materials, but of the pedagogical strategies that are utilized to further specific goals for students' development of conceptual understanding, critical thinking skills, and problem-solving abilities. As discussed in Chapter I, a 2015 study from EdReports (2023a) analyzed the K-8 Eureka Math curriculum in its alignment to both the Common Core State Standards of Mathematical Content CCSS-MC and the Common Core State Standards of Mathematical Practice CCSS-MP. The report concluded that Eureka Math was rated as *Partially Meets Expectations* for meaningfully connecting the CCSS-MP and CCSS-MC. When the report was provided to the publisher with an invitation to provide a response to accompany it for public consumption, Great Minds participated. Their written response to EdReports addressed the lower-level rating of the CCSS-MP through which Great Minds responded, that the "curriculum was built to embody the math practices by embedding them in what the curriculum asks students to do every day" (EdReports, 2023d, para. 7) and specified that they made the decision to provide one indicator of the CCSS-MP for each lesson. They continued, "your review has encouraged us to more carefully spell out the full meaning of each mathematical practice, helping teachers to do the same in their instruction" (para. 8). Through this response, Great Minds seems to partially acknowledge that Eureka Math's K-8 materials fall short in the standard that EdReports (2015) had set for the CCSS-MP, however the suggested possible solution proposed is that they would explain the practices differently. An important thing I noticed about this response is that the shortcomings revealed through the findings do not seem to be in the labeling of the CCSS-MP, but in the actual embodiment of the pedagogical approaches within the curricular materials. In my own review of the contemporary literature available, I have found few studies regarding the pedagogical approaches furthered by mathematics curricular resources. This seems to be an issue worth addressing. The findings of such studies would provide essential information to mathematics curricular decision makers. In the absence of such information, this study may help to propel mathematics leaders to conduct their own independent explorations and to share the findings regarding specific mathematics curricular resources with the mathematics education community at large.

The implications of this study have the possibility of expanding ever farther beyond mathematics and to curriculum more generally. This study suggests that it may serve as time well spent for individuals making mathematics curriculum adoption decisions to engage in seeking out reliable resources to inform a deep knowledge of the pedagogical approaches employed within any curriculum they are considering. As we see from the results of this study, Great Minds positioned themselves in the role of advertisers in an attempt to convince consumers to utilize or purchase their products and services. A similar sentiment was found by Pini (2011) in their exploration of the websites of prominent Education Management Organizations. Pini (2011) explains, "corporate discourses are persuasive because they influence people's perceptions of reality. Private companies present themselves as being able to offer that which education has steamily lost: 1) in managerial terms, high quality and efficiency; and 2) in political terms credibility and legitimacy" (p. 269). I advocate that claims made on such websites should be investigated by curriculum decision makers, themselves, or they should seek out comprehensive reports from reliable, outside resources.

As this study has revealed, if anything like Great Minds, other textbook publishers' websites may in fact be considered more of an advertising platform than a conveyor of evidence-based information. A quick search of these websites also shows that some of the publishers have shared studies of their own on their sites. While each of these studies should be judged in their own right, I would imagine it would take a considerable amount of time for a curriculum decision maker to thoroughly look into the reliability of the source of the studies and through financial connections, the possible bias that may have affected the findings. When it comes to the manner in which curricular adoption decision makers in all content areas inform their decisions, I believe that the *Positioning within the Context of Mathematics Education Finding* from this study may serve as a helpful tool to build a lens of caution for them.

While this study can inform practitioners of mathematics education in a variety of different roles, it has the ability to serve a significant role in stimulating broader discussions about curriculum and curricular decision making – both in mathematics and of all content areas.

Beyond the educational sector, researchers alike may find value in expanding on this study or using it to inform their own.

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APPENDICES

APPENDIX A

APPENDIX A

TASK CHARACTERISTICS, SUB-CATEGORIES, & CODES

Task Characteristic	Sub-Category	Codes	Description
Types of Context	No Context	T-NC	Refers only to mathematics objects, symbols, or structures
	Camouflage Context	T-CC	Experiences from everyday life or common sense reasoning are not needed The mathematical operations needed to solve the probems are already obvious Numbers are given in a such a way that students can immediately interpret the problem as a mathematical problem and follow the common procedure for calculating the answer.
	Relevant and Essential Context	T-RE	Context is relevant to students' daily lives Common sense reasoning within the context is needed to understand and solve the problem The mathematical operation is not explicitly given Mathematical modeling is needed Students need to identify the relevant information and solution strategy for solving the task.
Types of Information	Matching	I-MATCH	The tasks contain exactlly the information needed to find the solution
	Missing	I-MISS	The tasks contain less information than needed, so studnets need to derive additional data.
	Superfluous	I-S	The tasks contain more information than needed so students need to select information
Types of Cognitive Demand	Memorization (Low-Level)	CD-M	To recall facts, rules, and definitions without meaning • Involves either producing previously learned facts, rules, formulae, or definitions OR committing them to memory
	Procedures without connections (Low-Level)	CD-P	To master procedures without meaning • Tasks are algorithmic. Use of procedures is either specifically called for or their use is evident based on instruction. • The primary purpose of the task is to learn a procedure. • Procedures do not need to be computational. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no [or limited] connection to the concepts or meaning underlying the procedure used
	Procedures with superficial connections (Low-Level)	CD-PSC	To master the procedures using initial or limited connections as possible scaffolding • Tasks make initial connections to underlying meaning, but this connection is not maintained or limited and the primary focus becomes the procedure • Seemingly *doing math* or *procedure with connections* tasks in which structured steps direct attention away from meaning entirely. • Tasks make underlying meaning visible. Students' use of these connections is superficial, involving filling in blanks or using a predetermined outline or structure: Presentation of task allows students to follow a pattern or use a shortcut to solve the problem. often without understanding why • Students can complete the tasks without fully engaging the mathematical meanings.
	Procedures with extended connections (High-Level)	CD-PEC	To understand and do the procedure in relation to underlying mathematical meaning • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Doing math tasks made more procedural through structured steps. • Tasks make underlying meaning visible. Students are expected to use the meanings to solve problems and, in particular, make connections to mathematical procedures or notations. • Students must engage the mathematical meanings to complete the task. • Can include memorization of facts, as long as it is with meaning.
	Doing mathematics (High-Level)	CD-DM	To develop and use strategies (including previously learned procedures) for solving unstructured problems in order to learn and use mathematical concepts as they apply, explain and justify their reasoning, determine whether different solution strategies make sense. • Students need to impose their own structure and procedure. • Tasks go beyond doing or using operations.

APPENDIX B

APPENDIX B

OVERVIEW OF MODULE TOPICS AND LESSON OBJECTIVES

Overview of Module Topics and Lesson Objectives

Standards		Topics and Objectives		Days
4.NF.1 4.NF.3c 4.NF.3d	A	**Equivalent Fractions**		2
		Lesson 1:	Make equivalent fractions with the number line, the area model, and numbers.	
		Lesson 2:	Make equivalent fractions with sums of fractions with like denominators.	
5.NF.1 5.NF.2	B	**Making Like Units Pictorially**		5
		Lesson 3:	Add fractions with unlike units using the strategy of creating equivalent fractions.	
		Lesson 4:	Add fractions with sums between 1 and 2.	
		Lesson 5:	Subtract fractions with unlike units using the strategy of creating equivalent fractions.	
		Lesson 6:	Subtract fractions from numbers between 1 and 2.	
		Lesson 7:	Solve two-step word problems.	
		Mid-Module Assessment: Topics A–B (assessment ½ day, return ½ day, remediation or further applications 2 days)		3
5.NF.1 5.NF.2	C	**Making Like Units Numerically**		5
		Lesson 8:	Add fractions to and subtract fractions from whole numbers using equivalence and the number line as strategies.	
		Lesson 9:	Add fractions making like units numerically.	
		Lesson 10:	Add fractions with sums greater than 2.	
		Lesson 11:	Subtract fractions making like units numerically.	
		Lesson 12:	Subtract fractions greater than or equal to 1.	
5.NF.1 5.NF.2	D	**Further Applications**		4
		Lesson 13:	Use fraction benchmark numbers to assess reasonableness of addition and subtraction equations.	
		Lesson 14:	Strategize to solve multi-term problems.	
		Lesson 15:	Solve multi-step word problems; assess reasonableness of solutions using benchmark numbers.	
		Lesson 16:	Explore part-to-whole relationships.	

VITA

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